

CREDIT SUPPLY AND THE HOUSING BOOM: ONLINE APPENDIX

This document contains some additional results not included in the main body of the paper. In particular, we present (i) the derivation of a different version of the borrowing constraint, for the case in which changes in maximum allowed LTVs only apply to newly issued mortgages, including refinanced ones; and (ii) a simulation of the effects of looser lending and collateral constraints in a model that includes this type of borrowing constraint, and in which financial intermediaries face a positive but finite cost of equity adjustment. This supplement is not entirely self-contained, so readers might need to refer to the main paper.

1. THE BORROWING CONSTRAINT IN A MODEL WITH REFINANCING

In the model with amortization described in Appendix B of the paper, households can borrow up to a share θ of the value of the collateralizable portion of their house. This collateralizable portion declines over time at rate ρ , which captures the empirical fact that borrowers usually pay down their mortgage over time. However, if θ increases, all borrowers are assumed to take immediate advantage of higher LTVs by borrowing all the way up to the new share θ of the collateralizable fraction of the house.

This assumption, which is standard in the literature, helps to make the model more transparent by allowing the derivation of a closed form solution, but it is extreme. In reality, some borrowers take advantage of looser collateral constraints when they take on new mortgages or refinance their debt, but many existing homeowners amortize their mortgages without re-leveraging their collateral, regardless of intervening changes in collateral requirements. In this section, we modify the collateral constraint to take into account this important aspect of reality.

To do so, assume that agents who buy a new house can borrow up to $\hat{h}_{t+1}\theta_t p_t$, where $\hat{h}_{t+1} \equiv [h_{t+1} - (1 - \delta)h_t]$ denotes the amount of newly purchased houses, p_t is their price, and θ_t is the maximum LTV for mortgages issued at time t . Borrowers must repay a constant fraction ρ of their loans in every period, as in the model of Appendix B, but they can also refinance their mortgage. For simplicity, we assume that refinancing happens with an exogenous probability π . These assumptions imply that the representative borrower is

subject to the following debt limit at time t :

$$\bar{D}_t = \sum_{s=0}^{\infty} \hat{h}_{t+1-s} \left[(1-\pi)^s (1-\rho)^s \theta_{t-s} p_{t-s} + \sum_{j=0}^{s-1} \pi (1-\pi)^j (1-\delta)^{s-j} (1-\rho)^j \theta_{t-j} p_{t-j} \right],$$

where the first sum is over the age of the collateral (s), while the second sum is over the time when the house was last refinanced (j). The first term in the square bracket is today's maximum debt against houses purchased s periods ago, and never refinanced. The other terms represent instead the maximum leverage on houses also purchased s periods ago, but refinanced in $t-j$. The weighting on these terms captures the fact that the share of houses purchased in $t-s$ and refinanced exactly in $t-j$ is $\pi (1-\pi)^j$. The last expression can also be written as

$$\bar{D}_t = \sum_{s=0}^{\infty} \hat{h}_{t+1-s} \left[(1-\pi)^{s+1} (1-\rho)^s \theta_{t-s} p_{t-s} + \sum_{j=0}^s \pi (1-\pi)^j (1-\delta)^{s-j} (1-\rho)^j \theta_{t-j} p_{t-j} \right]$$

by adding and subtracting $\pi (1-\pi)^s (1-\rho)^s \theta_{t-s} p_{t-s}$ inside the square bracket. After some manipulation, this expression can be written as

$$\bar{D}_t = \sum_{s=0}^{\infty} (1-\pi)^s (1-\rho)^s \theta_{t-s} p_{t-s} \left[(1-\pi) \hat{h}_{t+1-s} + \pi h_{t+1-s} \right],$$

from which we obtain the recursive expression

$$\bar{D}_t = (1-\pi)(1-\rho) \bar{D}_{t-1} + \theta_t p_t [h_{t+1} - (1-\pi)(1-\delta) h_t].$$

2. THE NEW BORROWING CONSTRAINT IN A MODEL WITH FINANCIAL INTERMEDIARIES

In this section, we analyze the problem of a representative borrower subject to the debt limit derived in the previous section. We then combine the equilibrium conditions of this problem with those of the lenders and financial intermediaries, in the case in which the latter face a positive but finite cost of equity adjustment, as in appendix A of the paper. Finally, we simulate the effects of relaxing both the lending and collateral constraints in this extended version of the model.

2.1. The model's equilibrium conditions. The representative borrower maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_{b,t}) + v_b(h_{b,t})],$$

subject to

$$c_{b,t} + p_t [h_{b,t+1} - (1 - \delta) h_{b,t}] + R_{t-1} D_{b,t-1} \leq y_{b,t} + D_{b,t}$$

$$D_{b,t} \leq \bar{D}_{b,t}$$

$$\bar{D}_{b,t} = (1 - \pi) (1 - \rho) \bar{D}_{b,t-1} + \theta_t p_t [h_{b,t+1} - (1 - \pi) (1 - \delta) h_{b,t}].$$

The first order conditions of this optimization problem are

$$u' (c_{b,t}) (1 - \mu_t) = \beta_b R_t E_t u' (c_{b,t+1})$$

$$u' (c_{b,t}) p_t (1 - \zeta_t \theta_t) = \beta_b v' (h_{b,t+1}) + \beta_b (1 - \delta) E_t [u' (c_{b,t+1}) p_{t+1} (1 - (1 - \pi) \zeta_{t+1} \theta_{t+1})]$$

$$u' (c_{b,t}) (\zeta_t - \mu_t) = \beta_b (1 - \pi) (1 - \rho) E_t u' (c_{b,t+1}) \zeta_{t+1},$$

where $u' (c_{b,t}) \mu_t$ and $u' (c_{b,t}) \zeta_t$ are the Lagrange multipliers of the second and third constraint, respectively.

The usual assumptions of linear utility and fixed supply of housing for the borrowers yield the following set of equilibrium conditions

$$(2.1) \quad p_t (1 - \zeta_t \theta_t) = \beta_b \cdot mrs + \beta_b (1 - \delta) E_t [p_{t+1} (1 - (1 - \pi) \zeta_{t+1} \theta_{t+1})]$$

$$(2.2) \quad 1 - \mu_t = \beta_b R_t$$

$$(2.3) \quad \zeta_t - \mu_t = \beta_b (1 - \pi) (1 - \rho) E_t \zeta_{t+1}$$

$$c_{b,t} + p_t \delta \bar{h}_b + R_{t-1} D_{b,t-1} = y_{b,t} + D_{b,t}$$

$$\mu_t (D_{b,t} - \bar{D}_{b,t}) = 0, \quad \mu_t \geq 0, \quad D_{b,t} \leq \bar{D}_{b,t}$$

$$(2.4) \quad \bar{D}_{b,t} = (1 - \pi) (1 - \rho) \bar{D}_{b,t-1} + \theta_t p_t [1 - (1 - \pi) (1 - \delta)] \bar{h}_b.$$

Appendix A derives the equilibrium conditions of the problem of the lenders and financial intermediaries. When combined with the market clearing condition for debt, they can be summarized by the following upward sloping supply of funds

$$(2.5) \quad R_t = \frac{1}{\beta_l} \left[1 + \frac{D_{b,t}^\gamma}{\tilde{L}} \right],$$

where $\tilde{L} = \frac{\chi}{\tau(1+\gamma)} (\chi \bar{E})^\gamma$, and γ captures the elasticity of the equity issuance cost function.

2.2. Steady state. To compute the steady state of the model, suppose first that the borrowing constraint is not binding. In this case, $\mu = \zeta = 0$, $R = \frac{1}{\beta_b}$, $p = \frac{\beta_b mrs}{1 - \beta_b(1 - \delta)} \equiv \underline{p}$, $D_b = \left[\left(\frac{\beta_l}{\beta_b} - 1 \right) \tilde{L} \right]^{\frac{1}{\gamma}}$ and $\bar{D}_b = \theta p^{\frac{1 - (1 - \pi)(1 - \delta)}{1 - (1 - \pi)(1 - \rho)}} \bar{h}_b$. For this to be an equilibrium, the collateral constraint must actually not be binding, as assumed above. This requires $D_b < \bar{D}_b$.

If the borrowing constraint is instead binding, $D_b = \bar{D}_b = \theta p^{\frac{1 - (1 - \pi)(1 - \delta)}{1 - (1 - \pi)(1 - \rho)}} \bar{h}_b$, but the computation of the steady state level of house prices is more involved. It requires the numerical solution of the following two equations

$$p = \frac{\beta_b \cdot mrs}{1 - \zeta\theta - \beta_b(1 - \delta)(1 - (1 - \pi)\zeta\theta)}$$

$$\zeta = \frac{1 - \frac{\beta_b}{\beta_l} \left[1 + \frac{1}{\tilde{L}} \left(\theta^{\frac{1 - (1 - \pi)(1 - \delta)}{1 - (1 - \pi)(1 - \rho)}} \bar{h}_b \right)^{\gamma} p^{\gamma} \right]}{1 - \beta_b(1 - \pi)(1 - \rho)},$$

which are obtained from combining (2.1), (2.2), (2.3), (2.4) and (2.5). Given p , it is easy to compute D_b , R , μ and ζ using (2.5), (2.2), and (2.3).

2.3. Dynamics. Unlike in the baseline model, characterizing the transition dynamics between two steady states requires numerical methods, because in the extended model the existing debt limit $\bar{D}_{b,t-1}$ is a state variable. The solution relies on the following shooting algorithm:

- (1) Solve for the initial and final steady state.
- (2) Guess a transition path of $\bar{D}_{b,t}$ between the two steady states, assuming that the new steady state is reached before time T (T can be arbitrarily large).
- (3) Guess that the borrowing constraint is binding at time $T - 1$. Given the guess of $\bar{D}_{b,t}$, solve for the equilibrium in $T - 1$, and verify that it is consistent with the assumption of a binding borrowing constraint. If it is not, re-compute the equilibrium under the assumption that the borrowing constraint is not binding. Repeat the same steps moving back in time, from time $T - 1$ to 1.
- (4) Use the path of house prices obtained in (3) to update the guess of the transition path of $\bar{D}_{b,t}$ based on equation (2.4). Stop if the new guess of $\bar{D}_{b,t}$ is the same as the previous one. Otherwise repeat (3) until convergence.

2.4. Parameter values. We calibrate the model to match the same targets as in the baseline. In addition, this model has implications for the ratio between new and total

mortgages. Using data from the Federal Housing Finance Agency, the average annual ratio between new originations and total mortgage debt during the 1990s is 0.25. To match this target, together with the ratio of debt to real estate of 0.43, as in the baseline, we choose $\rho = 0.0328$ and $\pi = 0.0307$. The other parameters can be set to the values reported in table 1 in the paper. Finally, we experiment with different values of γ , the parameter that controls the curvature of credit supply. As long as we can choose the magnitude of the shift in $\tilde{L}$ so as to deliver the observed fall in interest rates during the boom, as in the baseline calibration, the magnitude of this parameter only influences the profile of interest rates, with little effect on the other variables.

2.5. Results. Figure 2.1 reports the results of a contemporaneous slackening of collateral and lending limits in the extended model. This exercise combines a gradual rise in θ from 0.8 to 1.02, with an increase in $\tilde{L}$ that produces a decline in mortgage rates of 2.5 percentage points between 2000 and 2006, as in section 4.2 of the paper. The response of house prices and debt to the looser credit conditions is similar to that in the baseline model, although quantitatively somewhat smaller. The key difference is in the behavior of the debt-to-real estate ratio, which was rising significantly in the simulation reported in figure 4.2. In contrast, this measure of leverage is essentially flat in the model with refinancing, because the increase in leverage among the borrowers that refinance as θ rises is roughly balanced by the reduction among those who continue to accumulate equity as scheduled. These results corroborate the conclusions regarding the relative roles of credit supply and demand shifts in the boom that we drew from the baseline model. In particular, they demonstrate that extending that framework to allow for some heterogeneity in refinancing behavior across borrowers allows us to account for a flat debt-to-real estate ratio, as in fact 3, without altering substantially the ability of the model to account for the other fundamental facts of the boom.



FIGURE 2.1. Response of macro variables to a combined relaxation of lending and collateral constraints.