# INVESTMENT SHOCKS AND BUSINESS CYCLES: TECHNICAL APPENDIX AND ADDITIONAL RESULTS 

ALEJANDRO JUSTINIANO, GIORGIO E. PRIMICERI, AND ANDREA TAMBALOTTI

This document contains technical details on the approximation, solution and estimation of our baseline model, as well as some additional results not included in the main body of the paper. In particular, we present: i) the derivation of the log-linearized baseline model; ii) the details of the dataset; iii) the prior densities and posterior estimates of the unknown coefficients; iv) an assessment of the fit of the model; v) additional tables and figures related to its estimation with the dataset of Smets and Wouters (2007) (SW hereafter); vi) additional tables related to the estimation of the model with durable goods in home production; vii) some robustness checks.

This supplement is not self-contained, so readers are advised to read the main paper first.

## 1. Derivation of the Log-Linearized Model

In this appendix, we report the first order conditions for the optimization problems described in the paper and the other relationships that define the equilibrium of the baseline model. We then compute the model's steady state and its log-linear approximation. A MATLAB code for the solution of the resulting system of linear rational expectations equations based on Chris Sims' gensys is available at:
http://faculty.wcas.northwestern.edu/~gep575/modelJPT.m.

### 1.1. Nonlinear equilibrium conditions.

1.1.1. Firms: production function and cost minimization. Production function for intermediate good producer $i$

$$
Y_{t}(i)=A_{t}^{1-\alpha} K_{t}^{\alpha}(i) L_{t}^{1-\alpha}(i)-A_{t} F .
$$

[^0]Cost minimization

$$
\begin{aligned}
M C_{t}(i) A_{t}^{1-\alpha}\left(\frac{L_{t}(i)}{K_{t}(i)}\right)^{-\alpha} & =W_{t} \\
M C_{t}(i) A_{t}^{1-\alpha}\left(\frac{L_{t}(i)}{K_{t}(i)}\right)^{1-\alpha} & =r_{t}^{k}
\end{aligned}
$$

where $M C_{t}(i)$ is marginal cost, the multiplier on the production function in the cost minimization problem. From this we obtain a common capital labor ratio across producers

$$
\frac{K_{t}(i)}{L_{t}(i)}=\frac{W_{t}}{r_{t}^{k}} \frac{\alpha}{1-\alpha}
$$

and thus a common marginal cost

$$
M C_{t}=\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} r_{t}^{k \alpha}\left(\frac{W_{t}}{A_{t}}\right)^{1-\alpha} .
$$

1.1.2. Firms: prices. Using the fact that, with our production function, average variable costs and marginal costs coincide, we can rewrite the objective function for firms setting their price optimally as

$$
\begin{aligned}
& \max _{\left\{P_{t}(i)\right\}_{t}} E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p}^{s} \frac{\beta^{s} \Lambda_{t+s}}{\Lambda_{t}}\left[\left(P_{t}(i) \Pi_{t, t+s}-M C_{t}\right) Y_{t+s}(i)\right]\right\}, \\
& \text { s.t. } Y_{t+s}(i)=\left(\frac{P_{t}(i) \Pi_{t, t+s}}{P_{t+s}}\right)^{-\frac{1+\lambda_{p, t+s}}{\lambda_{p, t+s}}} Y_{t+s}
\end{aligned}
$$

with

$$
\Pi_{t, t+s} \equiv \prod_{k=1}^{s}\left(\pi_{t+k-1}^{\iota_{p}} \pi^{1-\iota_{p}}\right)
$$

The first order condition is then

$$
0=E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p}^{s} \beta^{s} \Lambda_{t+s} \tilde{Y}_{t+s}\left[\tilde{P}_{t} \Pi_{t, t+s}-\left(1+\lambda_{p, t+s}\right) M C_{t+s}\right]\right\}
$$

where $\tilde{P}_{t}$ is the optimally chosen price, which is the same for all producers, and $\tilde{Y}_{t+s}$ is the demand they face in $t+s$.

Aggregate price index

$$
P_{t}=\left[\left(1-\xi_{p}\right)\left(\tilde{P}_{t}\right)^{\frac{1}{\lambda_{p, t}}}+\xi_{p}\left(\pi_{t-1}^{\iota_{p}} \pi^{1-\iota_{p}} P_{t-1}\right)^{\frac{1}{\lambda_{p, t}}}\right]^{\lambda_{p, t}} .
$$

1.1.3. Households: consumption. Marginal utility of nominal income

$$
P_{t} \Lambda_{t}=\frac{b_{t}}{C_{t}-h C_{t-1}}-h \beta E_{t} \frac{b_{t+1}}{C_{t+1}-h C_{t}}
$$

Euler equation

$$
\Lambda_{t}=\beta R_{t} E_{t} \Lambda_{t+1}
$$

or

$$
1=E_{t}\left[M_{t+1} R_{t} \frac{P_{t}}{P_{t+1}}\right],
$$

where $M_{t+1} \equiv \beta \frac{\Lambda_{t+1} P_{t+1}}{\Lambda_{t} P_{t}}$ is the "real" stochastic discount factor.
1.1.4. Households: investment and capital. Optimal choice of physical capital stock

$$
\Phi_{t}=\beta E_{t}\left[\Lambda_{t+1}\left(r_{t+1}^{k} u_{t+1}-P_{t+1} a\left(u_{t+1}\right)\right)\right]+(1-\delta) \beta E_{t} \Phi_{t+1}
$$

Optimal choice of investment

$$
P_{t} \Lambda_{t}=\Phi_{t} \mu_{t}\left[1-S\left(\frac{I_{t}}{I_{t-1}}\right)-\frac{I_{t}}{I_{t-1}} S^{\prime}\left(\frac{I_{t}}{I_{t-1}}\right)\right]+\beta\left[\Phi_{t+1} \mu_{t+1}\left(\frac{I_{t+1}}{I_{t}}\right)^{2} S^{\prime}\left(\frac{I_{t+1}}{I_{t}}\right)\right]
$$

where $\Phi_{t}$ is the multiplier on the capital accumulation equation - the shadow value of installed physical capital.

Defining Tobin's $q$ as $q_{t} \equiv \Phi_{t} / P_{t} \Lambda_{t}$, the relative marginal value of installed capital with respect to consumption, we can also write

$$
1=q_{t} \mu_{t}\left[1-S\left(\frac{I_{t}}{I_{t-1}}\right)-\frac{I_{t}}{I_{t-1}} S^{\prime}\left(\frac{I_{t}}{I_{t-1}}\right)\right]+\left[\beta M_{t+1} q_{t+1} \mu_{t+1}\left(\frac{I_{t+1}}{I_{t}}\right)^{2} S^{\prime}\left(\frac{I_{t+1}}{I_{t}}\right)\right]
$$

and

$$
1=E_{t}\left\{M_{t+1} \frac{1}{q_{t}}\left[\frac{r_{t+1}^{k}}{P_{t+1}} u_{t+1}-a\left(u_{t+1}\right)+(1-\delta) q_{t+1}\right]\right\}
$$

the Euler equation that prices the physical capital stock. Note that, without adjustment costs (i.e. $S=1$ and $S^{\prime}=0$ ), the first equation reduces to $q_{t}=\mu_{t}^{-1}$ : the relative price of capital/investment is equal to the inverse of the investment shock.

Optimal capital utilization

$$
r_{t}^{k}=P_{t} a^{\prime}\left(u_{t}\right)
$$

Definition of capital input

$$
K_{t}=u_{t} \bar{K}_{t-1}
$$

Physical capital accumulation

$$
\bar{K}_{t}=(1-\delta) \bar{K}_{t-1}+\mu_{t}\left[1-S\left(\frac{I_{t}}{I_{t-1}}\right)\right] I_{t}
$$

1.1.5. Households: wages. Wage setting equation for workers renegotiating their salary

$$
\begin{aligned}
0 & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s} \Lambda_{t+s} \tilde{L}_{t+s}\left[\tilde{W}_{t} \Pi_{t, t+s}^{w}-\left(1+\lambda_{w t+s}\right) b_{t+s} \varphi \frac{\tilde{L}_{t+s}^{\nu}}{\Lambda_{t+s}}\right]\right\} \\
\Pi_{t, t+s}^{w} & =\prod_{k=1}^{s}\left[\left(\pi e^{\gamma}\right)^{1-\iota_{w}}\left(\pi_{t+k-1} e^{z_{t+k-1}}\right)^{\iota w}\right]
\end{aligned}
$$

Wages evolve as

$$
W_{t}=\left\{\left(1-\xi_{w}\right)\left(\tilde{W}_{t}\right)^{\frac{1}{\lambda_{w}}}+\xi_{w}\left[\left(\pi e^{\gamma}\right)^{1-\iota_{w}}\left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota_{w}} W_{t-1}\right]^{\frac{1}{\lambda_{w}}}\right\}^{\lambda_{w}}
$$

1.1.6. Monetary policy. Monetary policy rule

$$
\frac{R_{t}}{R}=\left(\frac{R_{t-1}}{R}\right)^{\rho_{R}}\left[\left(\frac{\pi_{t}}{\pi}\right)^{\phi_{\pi}}\left(\frac{X_{t}}{X_{t}^{*}}\right)^{\phi_{X}}\right]^{1-\rho_{R}}\left[\frac{X_{t} / X_{t-1}}{X_{t}^{*} / X_{t-1}^{*}}\right]^{\phi_{d X}} \eta_{m p, t}
$$

### 1.1.7. Market clearing. Definition of GDP

$$
X_{t}=C_{t}+I_{t}+G_{t}
$$

Aggregate resource constraint, obtained by aggregating the households' budget constraint across households and combining it with the government budget constraint and the zero profit condition of the final good producers and of the employment agencies

$$
C_{t}+I_{t}+a\left(u_{t}\right) \bar{K}_{t-1}=\frac{1}{g_{t}} Y_{t}
$$

1.2. Stationary equilibrium. Total factor productivity $A_{t}$ is non stationary. Therefore, we define normalized stationary variables as follows

$$
\begin{aligned}
y & =Y / A \\
x & =X / A \\
k & =K / A \\
c & =C / A \\
i & =I / A \\
w & =W /(A P) \\
\rho & =r^{k} / P \\
s & =M C / P \\
\tilde{p} & =\tilde{P} / P \\
\pi & =P / P P_{-1} \\
\lambda & =\Lambda A P \\
\phi & =\Phi A
\end{aligned}
$$

1.2.1. Firms: production function and cost minimization. Production function for intermediate good producer $i$

$$
\begin{equation*}
y_{t}(i)=k_{t}^{\alpha}(i) L_{t}^{1-\alpha}(i)-F \tag{1.1}
\end{equation*}
$$

Capital labor ratio

$$
\begin{equation*}
\frac{k_{t}(i)}{L_{t}(i)}=\frac{w_{t}}{\rho_{t}} \frac{\alpha}{1-\alpha} \tag{1.2}
\end{equation*}
$$

Marginal cost

$$
\begin{equation*}
s_{t}=\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \rho_{t}^{\alpha} w_{t}^{1-\alpha} \tag{1.3}
\end{equation*}
$$

1.2.2. Firms: prices. Price setting equation for firms changing prices

$$
\begin{align*}
0 & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p}^{s} \beta^{s} \lambda_{t+s} \tilde{y}_{t+s}\left[\tilde{p}_{t} \tilde{\Pi}_{t, t+s}-\left(1+\lambda_{p t+s}\right) s_{t+s}\right]\right\}  \tag{1.4}\\
\tilde{\Pi}_{t, t+s} & =\prod_{k=1}^{s}\left[\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\iota_{p}}\left(\frac{\pi_{t+k}}{\pi}\right)^{-1}\right] \\
\tilde{y}_{t+s} & =\left(\tilde{p}_{t} \tilde{\Pi}_{t, t+s}\right)^{-\frac{1+\lambda_{p, t+s}}{\lambda_{p, t+s}}} Y_{t+s}
\end{align*}
$$

Aggregate price index

$$
\begin{equation*}
1=\left[\left(1-\xi_{p}\right)\left(\tilde{p}_{t}\right)^{\frac{1}{\lambda_{p t}}}+\xi_{p}\left[\left(\frac{\pi_{t-1}}{\pi}\right)^{\iota_{p}}\left(\frac{\pi_{t}}{\pi}\right)^{-1}\right]^{\frac{1}{\lambda_{p t}}}\right]^{\lambda_{p t}} \tag{1.5}
\end{equation*}
$$

1.2.3. Households: consumption. Marginal utility of income

$$
\begin{equation*}
\lambda_{t}=\frac{e^{z_{t}} b_{t}}{e^{z_{t}} c_{t}-h c_{t-1}}-h \beta E_{t} \frac{b_{t+1}}{e^{z_{t+1}} c_{t+1}-h c_{t}} \tag{1.6}
\end{equation*}
$$

Euler equation

$$
\begin{equation*}
\lambda_{t}=\beta R_{t} E_{t}\left(\lambda_{t+1} \frac{e^{-z_{t+1}}}{\pi_{t+1}}\right) \tag{1.7}
\end{equation*}
$$

1.2.4. Households: investment and capital. Optimal capital utilization

$$
\begin{equation*}
\rho_{t}=a^{\prime}\left(u_{t}\right) \tag{1.8}
\end{equation*}
$$

Optimal choice of physical capital

$$
\begin{equation*}
\phi_{t}=\beta E_{t}\left\{e^{-z_{t+1}} \lambda_{t+1}\left[\rho_{t+1} u_{t+1}-a\left(u_{t+1}\right)\right]\right\}+(1-\delta) \beta E_{t}\left(\phi_{t+1} e^{-z_{t+1}}\right) \tag{1.9}
\end{equation*}
$$

Optimal choice of investment

$$
\begin{equation*}
\lambda_{t}=\phi_{t} \mu_{t}\left[1-S\left(\frac{i_{t}}{i_{t-1}} e^{z_{t}}\right)-\frac{i_{t}}{i_{t-1}} e^{z_{t}} S^{\prime}\left(\frac{i_{t}}{i_{t-1}} e^{z_{t}}\right)\right]+\beta\left[\phi_{t+1} e^{-z_{t+1}} \mu_{t+1}\left(\frac{i_{t+1}}{i_{t}} e^{z_{t+1}}\right)^{2} S^{\prime}\left(\frac{i_{t+1}}{i_{t}} e^{z_{t+1}}\right)\right] \tag{1.10}
\end{equation*}
$$

Definition of capital input

$$
\begin{equation*}
k_{t}=u_{t} \bar{k}_{t-1} e^{-z_{t}} \tag{1.11}
\end{equation*}
$$

Physical capital accumulation

$$
\begin{equation*}
\bar{k}_{t}=(1-\delta) e^{-z_{t}} \bar{k}_{t-1}+\mu_{t}\left[1-S\left(\frac{i_{t}}{i_{t-1}} e^{z_{t}}\right)\right] i_{t} \tag{1.12}
\end{equation*}
$$

1.2.5. Households: wages. Wage setting equation for workers renegotiating their salary

$$
\begin{align*}
0 & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s} \lambda_{t+s} \tilde{L}_{t+s}\left[\tilde{w}_{t} \tilde{\Pi}_{t, t+s}^{w}-\left(1+\lambda_{w, t+s}\right) b_{t+s} \varphi \frac{\tilde{L}_{t+s}^{\nu}}{\lambda_{t+s}}\right]\right\}  \tag{1.13}\\
\tilde{\Pi}_{t, t+s}^{w} & =\prod_{k=1}^{s}\left[\left(\frac{\pi_{t+k-1} e^{z_{t+k-1}}}{\pi e^{\gamma}}\right)^{s_{w}}\left(\frac{\pi_{t+k} e^{z_{t+k}}}{\pi e^{\gamma}}\right)^{-1}\right] \\
\tilde{L}_{t+s} & =\left(\tilde{w}_{t} \tilde{\Pi}_{t, t+s}^{w}\right)^{-\frac{1+\lambda_{w, t+s}}{\lambda_{w, t+s}}} L_{t+s}
\end{align*}
$$

Wages evolve as

$$
\begin{equation*}
w_{t}=\left\{\left(1-\xi_{w}\right)\left(\tilde{w}_{t}\right)^{\frac{1}{\lambda_{w, t}}}+\xi_{w}\left[\left(\frac{\pi_{t-1} e^{z_{t-1}}}{\pi e^{\gamma}}\right)^{\iota_{w}}\left(\frac{\pi_{t} e^{z_{t}}}{\pi e^{\gamma}}\right)^{-1} w_{t-1}\right]^{\frac{1}{\lambda_{w, t}}}\right\}^{\lambda_{w, t}} \tag{1.14}
\end{equation*}
$$

1.2.6. Monetary policy. Monetary policy rule

$$
\frac{R_{t}}{R}=\left(\frac{R_{t-1}}{R}\right)^{\rho_{R}}\left[\left(\frac{\pi_{t}}{\pi}\right)^{\phi_{\pi}}\left(\frac{x_{t}}{x_{t}^{*}}\right)^{\phi_{X}}\right]^{1-\rho_{R}}\left[\frac{x_{t} / x_{t-1}}{x_{t}^{*} / x_{t-1}^{*}}\right]^{\phi_{d X}} \eta_{m p, t}
$$

1.2.7. Market clearing. Definition of GDP

$$
x_{t}=c_{t}+i_{t}+\left(1-\frac{1}{g_{t}}\right) y_{t}
$$

Resource constraint

$$
\begin{equation*}
c_{t}+i_{t}+a\left(u_{t}\right) e^{-z_{t}} \bar{k}_{t-1}=\frac{1}{g_{t}} y_{t} \tag{1.15}
\end{equation*}
$$

1.3. Steady state. From (1.9) and (1.10) we get

$$
\rho=\frac{e^{\gamma}}{\beta}-(1-\delta) .
$$

With $\rho$, (1.3) and (1.4) imply

$$
w=\left[\frac{1}{1+\lambda_{p}} \alpha^{\alpha}(1-\alpha)^{1-\alpha} \frac{1}{\rho^{\alpha}}\right]^{\frac{1}{1-\alpha}} .
$$

With $\rho$ and $\omega$, we can use (1.2) to compute

$$
\begin{equation*}
\frac{k}{L}=\frac{w}{\rho} \frac{\alpha}{1-\alpha} \tag{1.16}
\end{equation*}
$$

The zero profit condition for intermediate goods producers

$$
y-\rho k-w L=\left(\frac{k}{L}\right)^{\alpha} L-F-\rho k-w L=0
$$

implies

$$
\frac{F}{L}=\left(\frac{k}{L}\right)^{\alpha}-\rho \frac{k}{L}-w .
$$

Therefore, we can compute

$$
\frac{y}{L}=\left(\frac{k}{L}\right)^{\alpha}-\frac{F}{L}
$$

Now, from (1.15) and (1.12)

$$
\begin{aligned}
\frac{i}{L} & =\left[1-(1-\delta) e^{-\gamma}\right] \frac{e^{\gamma} k}{L} \\
\frac{c}{L} & =\frac{y}{L} \frac{1}{g}-\frac{i}{L}
\end{aligned}
$$

from (1.6)

$$
\lambda L=\left(\frac{c}{L}\right)^{-1} \frac{e^{\gamma}-h \beta}{e^{\gamma}-h}
$$

so that from (1.14) and (1.13) we obtain an expression for $L$

$$
L=\left(\frac{w}{\left(1+\lambda_{w}\right) \varphi} \lambda L\right)^{\frac{1}{1+\nu}}
$$

This relationship provides a mapping between the parameter $\varphi$ and the steady state value of hours, $L$. It is convenient to parametrize the steady state in terms of the latter, since this choice immediately implies

$$
\begin{aligned}
k & =\frac{k}{L} L \\
y & =\frac{y}{L} L \\
i & =\frac{i}{L} L \\
c & =\frac{c}{L} L
\end{aligned}
$$

1.4. Log-linearized equilibrium. Log-linear deviations from steady state are defined as follows

$$
\hat{\zeta}_{t} \equiv \log \zeta_{t}-\log \zeta
$$

except for $\hat{z}_{t} \equiv z_{t}-\gamma, \hat{\lambda}_{p, t} \equiv \log \left(1+\lambda_{p, t}\right)-\log \left(1+\lambda_{p}\right)$ and $\hat{\lambda}_{w, t} \equiv \log \left(1+\lambda_{w, t}\right)-$ $\log \left(1+\lambda_{w}\right)$.
1.4.1. Firms: production function and cost minimization. Production function for intermediate good producer $i$

$$
\hat{y}_{t}=\frac{y+F}{y}\left[\alpha \hat{k}_{t}+(1-\alpha) \hat{L}_{t}\right]
$$

Capital labor ratio

$$
\hat{\rho}_{t}-\hat{w}_{t}=\hat{L}_{t}-\hat{k}_{t}
$$

Marginal cost

$$
\hat{s}_{t}=\alpha \hat{\rho}_{t}+(1-\alpha) \hat{w}_{t}
$$

1.4.2. Firms: prices. Price setting equation for firms changing prices

$$
\begin{aligned}
0 & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p}^{s} \beta^{s}\left[\hat{\tilde{p}}_{t}+\hat{\Pi}_{t, t+s}-\hat{\lambda}_{p t+s}-\hat{s}_{t+s}\right]\right\} \\
\hat{\Pi}_{t, t+s} & =\sum_{k=1}^{s}\left[\iota_{p} \hat{\pi}_{t+k-1}-\hat{\pi}_{t+k}\right]
\end{aligned}
$$

Solving for the summation

$$
\begin{aligned}
\frac{1}{1-\xi_{p} \beta} \widehat{\tilde{p}}_{t} & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{p}^{s} \beta^{s}\left[-\hat{\Pi}_{t, t+s}+\hat{\lambda}_{p t+s}+\hat{s}_{t+s}\right]\right\} \\
& =-\hat{\Pi}_{t, t}+\hat{\lambda}_{p t}+\hat{s}_{t}-\frac{\xi_{p} \beta}{1-\xi_{p} \beta} \hat{\Pi}_{t, t+1}+\xi_{p} \beta E_{t}\left\{\sum_{s=1}^{\infty} \xi_{p}^{s-1} \beta^{s-1}\left[-\hat{\Pi}_{t+1, t+s}+\hat{\lambda}_{p t+s}+\hat{s}_{t+s}\right]\right\} \\
& =\hat{\lambda}_{p t}+\hat{s}_{t}+\frac{\xi_{p} \beta}{1-\xi_{p} \beta} E_{t}\left[\widehat{\tilde{p}}_{t+1}-\hat{\Pi}_{t, t+1}\right]
\end{aligned}
$$

where we used $\hat{\Pi}_{t, t}=0$.
Prices evolve as

$$
0=\left(1-\xi_{p}\right) \widehat{\tilde{p}}_{t}+\xi_{p}\left(\iota_{p} \hat{\pi}_{t-1}-\hat{\pi}_{t}\right)
$$

from which we obtain

$$
\hat{\pi}_{t}=\frac{\beta}{1+\iota_{p} \beta} E_{t} \hat{\pi}_{t+1}+\frac{\iota_{p}}{1+\iota_{p} \beta} \hat{\pi}_{t-1}+\kappa \hat{s}_{t}+\kappa \hat{\lambda}_{p t}
$$

with $\kappa \equiv \frac{\left(1-\xi_{p} \beta\right)\left(1-\xi_{p}\right)}{\xi_{p}\left(1+\iota_{p} \beta\right)}$.
1.4.3. Households: consumption. Marginal utility

$$
\begin{aligned}
\hat{\lambda}_{t}= & \frac{e^{\gamma}}{e^{\gamma}-h \beta}\left[\hat{b}_{t}+\hat{z}_{t}-\left(\frac{e^{\gamma}}{e^{\gamma}-h}\left(\hat{c}_{t}+\hat{z}_{t}\right)-\frac{h}{e^{\gamma}-h} \hat{c}_{t-1}\right)\right] \\
& -\frac{h \beta}{e^{\gamma}-h \beta} E_{t}\left[\hat{b}_{t+1}-\left(\frac{e^{\gamma}}{e^{\gamma}-h}\left(\hat{c}_{t+1}+\hat{z}_{t+1}\right)-\frac{h}{e^{\gamma}-h} \hat{c}_{t}\right)\right]
\end{aligned}
$$

or

$$
\begin{aligned}
\hat{\lambda}_{t}= & \frac{h \beta e^{\gamma}}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} E_{t} \hat{c}_{t+1}-\frac{e^{2 \gamma}+h^{2} \beta}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} \hat{c}_{t}+\frac{h e^{\gamma}}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} \hat{c}_{t-1} \\
& +\frac{h \beta e^{\gamma} \rho_{z}-h e^{\gamma}}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} \hat{z}_{t}+\frac{e^{\gamma}-h \beta \rho_{b}}{e^{\gamma}-h \beta} \hat{b}_{t}
\end{aligned}
$$

Euler equation

$$
\hat{\lambda}_{t}=\hat{R}_{t}+E_{t}\left(\hat{\lambda}_{t+1}-\hat{z}_{t+1}-\hat{\pi}_{t+1}\right)
$$

1.4.4. Households: investment and Capital. Capital utilization

$$
\hat{\rho}_{t}=\chi \hat{u}_{t}
$$

Capital

$$
\hat{\phi}_{t}=(1-\delta) \beta e^{-\gamma} E_{t}\left(\hat{\phi}_{t+1}-\hat{z}_{t+1}\right)+\left(1-(1-\delta) \beta e^{-\gamma}\right) E_{t}\left[\hat{\lambda}_{t+1}-\hat{z}_{t+1}+\hat{\rho}_{t+1}\right]
$$

Investment

$$
\lambda_{t}=\hat{\phi}_{t}+\hat{\mu}_{t}-e^{2 \gamma} S^{\prime \prime}\left(\hat{\imath}_{t}-\hat{\imath}_{t-1}+\hat{z}_{t}\right)+\beta e^{2 \gamma} S^{\prime \prime} E_{t}\left[\hat{\imath}_{t+1}-\hat{\imath}_{t}+\hat{z}_{t+1}\right]
$$

Capital input

$$
\hat{k}_{t}=\hat{u}_{t}+\hat{\bar{k}}_{t-1}-\hat{z}_{t}
$$

Capital accumulation

$$
\widehat{\bar{k}}_{t}=(1-\delta) e^{-\gamma}\left(\widehat{\bar{k}}_{t-1}-\hat{z}_{t}\right)+\left(1-(1-\delta) e^{-\gamma}\right)\left(\hat{\mu}_{t}+\hat{\imath}_{t}\right)
$$

1.4.5. Households: wages. Wage setting equation for workers renegotiating their salary

$$
\begin{aligned}
0 & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[\widehat{\tilde{w}}_{t}+\hat{\Pi}_{t, t+s}^{w}-\hat{\lambda}_{w, t+s}-\hat{b}_{t+s}-\nu \hat{\tilde{L}}_{t+s}+\hat{\lambda}_{t+s}\right]\right\} \\
\hat{\Pi}_{t, t+s}^{w} & =\sum_{k=1}^{s}\left[\iota_{w}\left(\hat{\pi}_{t+k-1}+\hat{z}_{t+k-1}\right)-\left(\hat{\pi}_{t+k}+\hat{z}_{t+k}\right)\right]
\end{aligned}
$$

and using the labor demand function

$$
0=E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[\widehat{\tilde{w}}_{t}+\hat{\Pi}_{t, t+s}^{w}-\hat{\lambda}_{w, t+s}-\hat{b}_{t+s}-\nu\left(\hat{L}_{t+s}-\left(1+\frac{1}{\lambda_{w}}\right)\left(\widehat{\tilde{w}}_{t}+\hat{\Pi}_{t, t+s}^{w}-\hat{w}_{t+s}\right)\right)+\hat{\lambda}_{t+s}\right]\right\}
$$

Solving for the summation

$$
\begin{aligned}
\frac{\nu_{w}}{1-\xi_{w} \beta} \widehat{\tilde{w}}_{t} & =E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[-\left(1+\nu\left(1+\frac{1}{\lambda_{w}}\right)\right) \hat{\Pi}_{t, t+s}^{w}+\hat{\psi}_{t+s}\right]\right\} \\
& =-\nu_{w} \hat{\Pi}_{t, t}^{w}+\hat{\psi}_{t}+E_{t}\left\{\sum_{s=1}^{\infty} \xi_{w}^{s} \beta^{s}\left[-\nu_{w} \hat{\Pi}_{t, t+s}^{w}+\hat{\psi}_{t+s}\right]\right\} \\
& =\hat{\psi}_{t}-\frac{\xi_{w} \beta}{1-\xi_{w} \beta} \nu_{w} \hat{\Pi}_{t, t+1}^{w}+\xi_{w} \beta E_{t}\left\{\sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{s}\left[-\nu_{w} \hat{\Pi}_{t+1, t+1+s}^{w}+\hat{\psi}_{t+1+s}\right]\right\} \\
& =\hat{\psi}_{t}+\frac{\xi_{w} \beta}{1-\xi_{w} \beta} \nu_{w} E_{t}\left[\widehat{\tilde{w}}_{t+1}-\hat{\Pi}_{t, t+1}^{w}\right]
\end{aligned}
$$

where

$$
\begin{align*}
\hat{\psi}_{t} & \equiv \hat{\lambda}_{w, t}+\hat{b}_{t}+\nu \hat{L}_{t}-\hat{\lambda}_{t}+\nu\left(1+\frac{1}{\lambda_{w}}\right) \hat{w}_{t}  \tag{1.18}\\
\nu_{w} & \equiv 1+\nu\left(1+\frac{1}{\lambda_{w}}\right)
\end{align*}
$$

and recall that $\hat{\Pi}_{t, t}^{w}=0$.
Wages evolve as

$$
\hat{w}_{t}=\left(1-\xi_{w}\right) \widehat{\tilde{w}}_{t}+\xi_{w}\left(\hat{w}_{t-1}+\iota_{w} \hat{\pi}_{t-1}+\iota_{w} \hat{z}_{t-1}-\hat{\pi}_{t}-\hat{z}_{t}\right)
$$

or

$$
\begin{equation*}
\hat{w}_{t}=\left(1-\xi_{w}\right) \widehat{\tilde{w}}_{t}+\xi_{w}\left(\hat{w}_{t-1}+\hat{\Pi}_{t-1, t}^{w}\right) . \tag{1.19}
\end{equation*}
$$

The combination of equations (1.17), (1.18) and (1.19) yields the wage Phillips curve

$$
\begin{aligned}
\hat{w}_{t}= & \frac{1}{1+\beta} \hat{w}_{t-1}+\frac{\beta}{1+\beta} E_{t} \hat{w}_{t+1}-\kappa_{w} \hat{g}_{w, t}+ \\
& +\frac{\iota_{w}}{1+\beta} \hat{\pi}_{t-1}-\frac{1+\beta \iota_{w}}{1+\beta} \pi_{t}+\frac{\beta}{1+\beta} E_{t} \hat{\pi}_{t+1}+ \\
& +\frac{\iota_{w}}{1+\beta} z_{t-1}-\frac{1+\beta \iota_{w}-\rho_{z} \beta}{1+\beta} z_{t}+\kappa_{w} \hat{\lambda}_{w, t}
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{g}_{w, t} & =\hat{w}_{t}-\left(\nu \hat{L}_{t}+\hat{b}_{t}-\hat{\lambda}_{t}\right) \\
\kappa_{w} & \equiv \frac{\left(1-\xi_{w} \beta\right)\left(1-\xi_{w}\right)}{\xi_{w}(1+\beta)\left(1+\nu\left(1+\frac{1}{\lambda_{w}}\right)\right)}
\end{aligned}
$$

1.4.6. Monetary policy. Monetary policy rule

$$
\hat{R}_{t}=\rho_{R} \hat{R}_{t-1}+\left(1-\rho_{R}\right)\left[\phi_{\pi} \hat{\pi}_{t}+\phi_{X}\left(\hat{x}_{t}-\hat{x}_{t}^{*}\right)\right]+\phi_{d X}\left[\left(\hat{x}_{t}-\hat{x}_{t-1}\right)-\left(\hat{x}_{t}^{*}-\hat{x}_{t-1}^{*}\right)\right]+\hat{\eta}_{m p, t}
$$

### 1.4.7. Market clearing. Definition of GDP

$$
\hat{x}_{t}=\hat{y}_{t}-\frac{\rho k}{y} \hat{u}_{t}
$$

Resource constraint

$$
\frac{c}{y} \hat{c}_{t}+\frac{i}{y} \hat{\imath}_{t}+\frac{\rho k}{y} \hat{u}_{t}=\frac{1}{g} \hat{y}_{t}-\frac{1}{g} \hat{g}_{t}
$$

### 1.5. Linear rational expectations model. The following 16 equations

$$
\begin{aligned}
& \hat{y}_{t}= \frac{y+F}{y}\left[\alpha \hat{k}_{t}+(1-\alpha) \hat{L}_{t}\right] \\
& \hat{\rho}_{t}= \hat{w}_{t}+\hat{L}_{t}-\hat{k}_{t} \\
& \hat{s}_{t}= \alpha \hat{\rho}_{t}+(1-\alpha) \hat{w}_{t} \\
& \hat{\pi}_{t}= \frac{\beta}{1+\iota_{p} \beta} E_{t} \hat{\pi}_{t+1}+\frac{\iota_{p}}{1+\iota_{p} \beta} \hat{\pi}_{t-1}+\kappa \hat{s}_{t}+\kappa \hat{\lambda}_{p, t} \\
& \hat{\lambda}_{t}= \frac{h \beta e^{\gamma}}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} E_{t} \hat{c}_{t+1}-\frac{e^{2 \gamma}+h^{2} \beta}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} \hat{c}_{t}+\frac{h e^{\gamma}}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} \hat{c}_{t-1} \\
&+\frac{h \beta e^{\gamma} \rho_{z}-h e^{\gamma}}{\left(e^{\gamma}-h \beta\right)\left(e^{\gamma}-h\right)} \hat{z}_{t}+\frac{e^{\gamma}-h \beta \rho_{b}}{e^{\gamma}-h \beta} \hat{b}_{t} \\
& \hat{\lambda}_{t}= \hat{R}_{t}+E_{t}\left(\hat{\lambda}_{t+1}-\hat{z}_{t+1}-\hat{\pi}_{t+1}\right) \\
& \hat{\rho}_{t}= \chi \hat{u}_{t} \\
& \hat{\phi}_{t}=(1-\delta) \beta e^{-\gamma} E_{t}\left(\hat{\phi}_{t+1}-\hat{z}_{t+1}\right)+\left(1-(1-\delta) \beta e^{-\gamma}\right) E_{t}\left[\hat{\lambda}_{t+1}-\hat{z}_{t+1}+\hat{\rho}_{t+1}\right] \\
& \hat{\lambda}_{t}= \hat{\phi}_{t}+\hat{\mu}_{t}-e^{2 \gamma} S^{\prime \prime}\left(\hat{\imath}_{t}-\hat{\imath}_{t-1}+\hat{z}_{t}\right)+\beta e^{2 \gamma} S^{\prime \prime} E_{t}\left[\hat{\imath}_{t+1}-\hat{\imath}_{t}+\hat{z}_{t+1}\right] \\
& \hat{k}_{t}= \hat{u}_{t}+\hat{\bar{k}}_{t-1}-\hat{z}_{t} \\
& \hat{\bar{k}}_{t}=(1-\delta) e^{-\gamma}\left(\hat{\bar{k}}_{t-1}-\hat{z}_{t}\right)+\left(1-(1-\delta) e^{-\gamma}\right)\left(\hat{\mu}_{t}+\hat{\imath}_{t}\right) \\
& \hat{w}_{t}= \frac{1}{1+\beta} \hat{w}_{t-1}+\frac{\beta}{1+\beta} E_{t} \hat{w}_{t+1}-\kappa_{w} \hat{g}_{w, t}+ \\
&+\frac{\iota_{w}}{1+\beta} \hat{\pi}_{t-1}-\frac{1+\beta \iota_{w}}{1+\beta} \pi_{t}+\frac{\beta}{1+\beta} E_{t} \hat{\pi}_{t+1}+ \\
&+\frac{\iota_{w}}{1+\beta} z_{t-1}-\frac{1+\beta \iota_{w}-\rho_{z} \beta}{1+\beta} z_{t}+\kappa_{w} \hat{\lambda}_{w, t} \\
& \hat{x}_{t}= \hat{x}_{t}-\left(\nu \hat{L}_{t}+\hat{b}_{t}-\hat{\lambda}_{t}\right) \\
& \hat{g}_{w, t}=\frac{\rho k}{y} \hat{u}_{t} \\
& \hat{R}_{t}= \rho_{R} \hat{R}_{t-1}+\left(1-\rho_{R}\right)\left[\phi_{\pi} \hat{\pi}_{t}+\phi_{X}\left(\hat{x}_{t}-\hat{x}_{t}^{*}\right)\right]+\phi_{d X}\left[\left(\hat{x}_{t}-\hat{x}_{t-1}\right)-\left(\hat{x}_{t}^{*}-\hat{x}_{t-1}^{*}\right)\right]+\hat{\eta}_{m p, t} \\
& \hat{y}_{t}= \frac{1}{g} \hat{g}_{t}+\frac{c}{y} \hat{c}_{t}+\frac{i}{y} \hat{\imath}_{t}+\frac{\rho k}{y} \hat{u}_{t}, \\
& \hat{y}_{t}
\end{aligned}
$$

form a linear system of rational expectations equations, together with the 15 equations describing the evolution of the economy with flexible prices, flexible wages and no markup shocks, whose allocation we denote with a "*" superscript. We solve this system of equations
for the 31 endogenous variables

$$
\left[\begin{array}{l}
\hat{y}_{t}, \hat{k}_{t}, \hat{L}_{t}, \hat{\rho}_{t}, \hat{w}_{t}, \hat{s}_{t}, \hat{\pi}_{t}, \hat{c}_{t}, \hat{\lambda}_{t}, \hat{R}_{t}, \hat{u}_{t}, \hat{\phi}_{t}, \hat{\imath}_{t}, \widehat{\bar{k}}_{t}, \hat{x}_{t}, \hat{g}_{w, t}, \\
\hat{y}_{t}^{*}, \hat{k}_{t}^{*}, \hat{L}_{t}^{*}, \hat{\rho}_{t}^{*}, \hat{w}_{t}^{*}, \hat{s}_{t}^{*}, \hat{c}_{t}^{*}, \hat{\lambda}_{t}^{*}, \hat{R}_{t}^{*}, \hat{u}_{t}^{*}, \hat{\phi}_{t}^{*}, \hat{\imath}_{t}^{*}, \widehat{\bar{k}}_{t}^{*}, \hat{x}_{t}^{*}, \hat{g}_{w, t}^{*}
\end{array}\right],
$$

conditional on the evolution of the exogenous shocks (and the normalization of three of them), as reported in the main body of the paper.
1.6. Normalization of the shocks. As in Smets and Wouters (2007), some of the exogenous shocks are re-normalized by a constant term. In particular, we normalize the price and wage markups shocks and the intertemporal preference shock, but not the investment shock.

More specifically, the log-linearized Phillips curve is

$$
\hat{\pi}_{t}=\frac{\beta}{1+\beta \iota_{p}} E_{t} \hat{\pi}_{t+1}+\frac{1}{1+\beta \iota_{p}} \hat{\pi}_{t-1}+\kappa \hat{s}_{t}+\kappa \hat{\lambda}_{p, t} .
$$

The normalization consists of defining a new exogenous variable, $\hat{\lambda}_{p, t}^{*} \equiv \kappa \hat{\lambda}_{p, t}$, and estimating the standard deviation of the innovation to $\hat{\lambda}_{p, t}^{*}$ instead of $\hat{\lambda}_{p, t}$. We do the same for the wage markup and the intertemporal preference shock, for which the normalizations are

$$
\begin{aligned}
\hat{\lambda}_{w, t}^{*} & =\left(\frac{\left(1-\beta \xi_{w}\right)\left(1-\xi_{w}\right)}{\left(1+\nu\left(1+\frac{1}{\lambda_{w}}\right)\right)(1+\beta) \xi_{w}}\right) \hat{\lambda}_{w, t} \\
\hat{b}_{t}^{*} & =\left(\frac{\left(1-\rho_{b}\right)\left(e^{\gamma}-h \beta \rho_{b}\right)\left(e^{\gamma}-h\right)}{e^{\gamma} h+e^{2 \gamma}+\beta h^{2}}\right) \hat{b}_{t}
\end{aligned}
$$

These normalizations are chosen so that these shocks enter their equations with a coefficient of one. In this way, it is easier to choose a reasonable prior for their standard deviation. Moreover, the normalization is a practical way to impose correlated priors across coefficients, which is desirable in some cases. For instance, imposing a prior on the standard deviation of the innovation to $\hat{\lambda}_{p, t}^{*}$ corresponds to imposing priors that allow for correlation between $\kappa$ and the standard deviation of the innovations to $\hat{\lambda}_{p, t}$. Often, these normalizations improve the convergence properties of the MCMC algorithm.

## 2. The Data

Our dataset spans the period from 1954QIII to 2004QIV. All data are extracted from the Haver Analytics database (series mnemonics in parenthesis). Following Del Negro, Schorfheide, Smets, and Wouters (2007), real GDP is constructed by diving the nominal series (GDP) by population (LF and LH) and the GDP Deflator (JGDP). Real series for
consumption and investment are obtained in the same manner, although consumption corresponds only to personal consumption expenditures on non-durables (CN) and services (CS), while investment is the sum of personal consumption expenditures on durables (CD) and gross private domestic investment (I). Real wages correspond to nominal compensation per hour in the non-farm business sector (LXNFC), divided by the GDP deflator. The labor input is measured by the log of hours of all persons in the non-farm business sector (HNFBN), divided by population. The quarterly log difference in the GDP deflator and the effective Federal Funds rate are our measures of inflation and the nominal interest rate. No series is demeaned or detrended.

## 3. Prior Densities and Posterior Estimates

Table 1 presents the details of the prior densities on the model's structural coefficients, as well as posterior medians, standard deviations and 90 percent probability intervals.

## 4. Variance Decomposition Implied by Prior and Posterior

Here we report two tables that are briefly discussed, although not presented, in the main text of the paper. Table 2 summarizes the implications of our priors for the variance decomposition of the observable variables in the baseline model. We report means, medians and 90 percent credible intervals. Table 3 reports instead medians and 90 percent credible intervals of the posterior variance decomposition, i.e. the contribution of each shock to the unconditional variance of the observable variables.

## 5. Model Fit

How well does our model fit the data? We address this question by comparing a set of statistics implied by the model to those measured in the data. In particular, we study the standard deviation and the complete correlation structure of the observable variables included in the estimation.

Table 4 reports the standard deviation of our seven observable variables, in absolute terms as well as relative to that of output growth. For the model, we report the median and the 90 percent probability intervals that account for both parameter uncertainty and small sample uncertainty. The model overpredicts the volatility of output growth, consumption and investment, but it matches their relative standard deviations fairly well. The match with
hours is close in both cases. There is also a tendency to underpredict the volatility of nominal interest rates and inflation, which might be due to the fact that the model does not replicate the very high correlation between these two variables.

With as many shocks as observable variables, why does the model not capture their standard deviation perfectly? The reason is that a likelihood-based estimator tries to match the entire autocovariance function of the data, and thus must strike a balance between matching standard deviations and all the other second moments, namely autocorrelations and crosscorrelations. These other moments are displayed in figure 1, for the data (grey line) and the model (back line), along with the 90 percent posterior intervals for the model implied by parameter uncertainty and small sample uncertainty.

Focus first on the upper-left 4-by-4 block of graphs, which includes all the quantities in the model. On the diagonal, we see that the model captures the decaying autocorrelation structure of these four variables very well. The success is particularly impressive for hours, for which the model-implied and data autocorrelations lay virtually on top of each other. In terms of cross-correlations, the model does extremely well for output (the first row and column) and for hours (the fourth row and column), but fails to capture the contemporaneous correlation between consumption and investment growth. This correlation is slightly positive in the data, but essentially zero in the model.

In sum, relative to smaller scale RBC models (Cooley and Prescott (1995), King and Rebelo (1999)), we do slightly worse in matching the properties of consumption, especially its correlation with investment. However, our model performs considerably better in terms of hours worked. This is an important result, because one of our main objectives is to investigate the sources of fluctuations in hours.

With respect to prices, the model is overall quite successful in reproducing the main stylized facts. We emphasize two issues: first, the model does not capture the full extent of the persistence of inflation and the nominal interest rate, even in the presence of inflation indexation and of a fairly high smoothing parameter in the interest rate rule. Second, we match very closely the correlation between output and inflation, which is highlighted for example by Smets and Wouters (2007) as an important measure of a model's empirical success.

## 6. Additional Results on the Comparison with SW

In our dataset, consumption is personal consumption expenditures on nondurable goods and services, while investment is the sum of personal consumption expenditures on durable goods and gross private domestic investment. SW instead define consumption as personal consumption expenditures on durables, non durables and services, and investment as private fixed investment, which excludes changes in inventories. ${ }^{1}$ Figure 2 displays the sample autocovariances for output, consumption and investment growth in the two datasets.

Tables 5 and 6 report the posterior parameter estimates and the complete business cycle variance decomposition of our baseline model, estimated with SW's definition of all the observables. Finally, figure 3 plots the impulse responses of GDP, consumption, investment and hours to an investment shock and an intertemporal preference shocks. The solid and dashed line correspond to our baseline model estimated with our and SW's definition of the observables respectively.
7. The Model with Durable Goods: Details and some Additional Results

Section 5 of the main body of the paper presents the results of a variance decomposition exercise based on a model with an explicit role for durable goods in home production. This appendix describes it in some detail. The optimization problems of the final and intermediate goods producers, the employment agencies and the behavior of the government are identical to those in the baseline model of section 2 of the paper. The household problem is instead somewhat more involved.

Each household maximizes the utility function

$$
E_{t} \sum_{s=0}^{\infty} \beta^{s} b_{t+s}\left[\log \left(C_{t+s}-h C_{t+s-1}\right)-\varphi \frac{\left(L_{t+s}+L_{h, t+s}\right)^{1+\nu}}{1+\nu}\right],
$$

where $L_{t}$ now denotes market hours and $L_{h, t}$ is the amount of hours spent in home production. Unlike in the baseline specification, we follow SW and assume that households' labor is homogenous and gets differentiated by labor unions with market power. These unions purchase labor from the households at wage $W_{t}^{h}$ and sell it to the employment agencies as a differentiated product at wage $W_{t}(j)$ for labor of type $j$.

[^1]$C_{t}$ is now a constant elasticity of substitution (CES) aggregate of consumption of nondurable goods and services $\left(N_{t}\right)$ and of the service flow from durable goods $\left(S_{t}\right)$
$$
C_{t}=\left[\theta N_{t}^{\frac{\eta-1}{\eta}}+(1-\theta) S_{t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}
$$
where $\eta$ is the elasticity of substitution between the two components. The service flow from durables is itself a CES aggregate of the stock of durable goods available to the household and the time spent in the home-production process:
$$
S_{t}=\left[\psi\left(A_{t} L_{h, t}\right)^{\frac{\tau-1}{\tau}}+(1-\psi) D_{t}^{\frac{\tau-1}{\tau}}\right]^{\frac{\tau}{\tau-1}}
$$
where $D_{t}$ is the stock of durable goods and we are assuming that neutral technology also affects the efficiency of home production. This specification encompasses cases in which time and durables are complements or substitutes. If $\tau=1$, the home technology reduces to a Cobb-Douglas production function. If $\psi=0$, the service flow from durable goods is simply a constant share of the stock of durables.

With this generalization, the household's budget constraint becomes

$$
P_{t} N_{t}+P_{t} I_{d, t}+P_{t} I_{t}+T_{t}+B_{t} \leq R_{t-1} B_{t-1}+Q_{t}+\Pi_{t}+W_{t}^{h} L_{t}+r_{t}^{k} u_{t} \bar{K}_{t-1}-P_{t} a\left(u_{t}\right) \bar{K}_{t-1} .
$$

$I_{d, t}$ denotes purchases of durable goods, whose stock evolves according to

$$
D_{t+1}=(1-\delta) D_{t}+\mu_{t} \zeta_{t}\left(1-S\left(\frac{I_{d, t}}{I_{d, t-1}}\right)\right) I_{d, t}
$$

Note that the accumulation of durable goods is affected by two shocks: the same investment shock that impinges on the standard capital accumulation, $\mu_{t}$, and a shock specific to the accumulation of durables, which evolves as

$$
\log \zeta_{t}=\rho_{\zeta} \log \zeta_{t-1}+\varepsilon_{\zeta, t}
$$

where $\varepsilon_{\zeta, t}$ is i.i.d. $N\left(0, \sigma_{\zeta}^{2}\right)$.
This model involves six additional parameters with respect to the baseline, which correspond to the coefficients of the CES aggregators $(\theta, \eta, \psi$ and $\tau)$ and the autocorrelation and innovation variance parameters $\rho_{\zeta}$ and $\sigma_{\zeta}$. We derive the mapping between $[\theta, \psi]$ and $\left[\frac{L_{h}}{L}, \frac{I_{d}}{I}\right]$, and estimate the latter instead of the former. Our prior for the two elasticities of substitution ( $\eta$ and $\tau$ ) is centered on the Cobb-Douglas case, using a Gamma density with mean equal to 1 and standard deviation equal to 0.2 . In line with Chang and Schorfheide (2003), we adopt a Gamma prior for the two steady-state ratios ( $\frac{L_{h}}{L}$ and $\frac{I_{d}}{I}$ ) with mean 0.7
and standard deviation 0.1. The prior for $\rho_{\zeta}$ is a Beta with mean 0.4 and standard deviation 0.2 , while the prior for $\sigma_{\zeta}$ is an Inverse-Gamma centered with mean 0.25 and standard deviation 1. The priors on the remaining coefficients are identical to those of the baseline model.

We estimate this version of the model with the growth rate of consumer durables as an additional observable variable. As we show in the online appendix, the posterior modes of $\frac{L_{h}}{L}$ and $\frac{I_{d}}{I}$ are 0.47 and 0.72 respectively. The former is broadly consistent with the estimates of Chang and Schorfheide (2003), while the latter is in line with the average ratio of durable to investment goods in the data. The posterior modes of $\eta$ and $\tau$ are 0.58 and 0.61 respectively. These estimates imply that nondurable consumption and the service flow from durables, as well as durable goods and time, are complements. This result is at odds with the estimates of Chang and Schorfheide (2003), but is consistent with the findings of Greenwood and Hercowitz (1991), who stress the importance of technological complementarity in home production for the allocation of capital and time across sectors.

Table 7 reports the prior densities and parameter estimates of the extended model in which durable goods play an explicit role in home production. Table 8 presents the business cycle variance decomposition implied by this model.

## 8. Robustness Analysis

This section investigates the robustness of our main finding to a number of alternative specifications of the model. The results of these robustness checks are summarized in table 9 , which reports the share of the variance of output and hours explained by the investment shock at business cycle frequencies for the baseline and several alternative specifications.
8.1. Standard calibration of capital income share and labor supply elasticity ( $\alpha=$ 0.3 and $\nu=1$ ). The baseline estimates of the share of capital income $(\alpha)$ and of the Frisch elasticity of labor supply $(1 / \nu)$ are different from the typical values used in the RBC literature. However, the second column of table 9 shows that the contribution of investment shocks to the business cycle fluctuations of output and hours increases with respect to the baseline, if these two parameters are calibrated to the more typical values of $\alpha=0.3$ and $\nu=1$.
8.2. No ARMA shocks. Following Smets and Wouters (2007), the baseline model includes an ARMA $(1,1)$ specification for the wage and price markup shocks. Results are very similar
when markup shocks are assumed to follow an $\operatorname{AR}(1)$ process instead, as illustrated in column three of table 9 .
8.3. GDP growth in the policy rule. We also experimented with a model in which the interest rate responds to output growth, rather than to the output gap, since both specifications are common in the literature. In this case, the contribution of investments shocks falls slightly with respect to the baseline case, as shown in column four of table 9 .
8.4. Maximum likelihood. The last robustness check is with respect to the priors on the model parameters. The baseline exercise uses the prior information reported in table 1, following the recent literature on Bayesian estimation of DSGE models. One objection to this methodology is that the results might be unduly influenced by this information, although the role of investment shocks is negligible in the prior variance decomposition described in section ??. As a further check, we also estimated the model by maximum likelihood. Maximizing the likelihood is numerically much more challenging than maximizing the posterior, since the use of weakly informative priors ameliorates the problems caused by flat areas in the likelihood surface and by multiple local modes. These difficulties notwithstanding, we were able to compute maximum likelihood estimates for the model parameters. ${ }^{2}$ The implications of these estimates for the variance decomposition are illustrated in the last column of table 9 , which makes clear that investment shocks still account for around 60 percent of the business cycle fluctuations in output and hours.

## References

Chang, Y., and F. Schorfheide (2003): "Labor-Supply Shifts and Economic Fluctuations," Journal of Monetary Economics, 50(8), 1751-1768.
Cooley, T. F., And E. Prescott (1995): "Economic Growth and Business Cycles," in Frontiers of Business Cycle Research, ed. by T. F. Cooley, chap. 1, pp. 1-38. Princeton University Press, Princeton, NJ.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007): "On the Fit and Forecasting Performance of New Keynesian Models," Journal of Business and Economic Statistics, 25(2), 123-162.
Greenwood, J., And Z. Hercowitz (1991): "The Allocation of Capital and Time over the Business Cycle," Journal of Political Economy, 99(6), 1188-214.

[^2]King, R. G., and S. T. Rebelo (1999): "Resuscitating Real Business Cycles," in Handbook of Macroeconomics, ed. by J. B. Taylor, and M. Woodford, Amsterdam. North-Holland.
Smets, F., AND R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian Approach," American Economic Review, 97(3), 586-606.

Federal Reserve Bank of Chicago
E-mail address: ajustiniano@frbchi.org

Northwestern University, CEPR and NBER
E-mail address: g-primiceri@northwestern.edu

Federal Reserve Bank of New York
E-mail address: andrea.tambalotti@ny.frb.org

Table 1: Prior densities and posterior estimates for the baseline model

|  |  | Prior |  |  | Posterior ${ }^{2}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Coefficient | Description | $\begin{gathered} \hline \hline \text { Prior } \\ \text { Density }^{1} \\ \hline \end{gathered}$ | Mean | Std | Median | Std | [ | 5 |  | 95 | ] |
| $\alpha$ | Capital share | N | 0.30 | 0.05 | 0.17 | 0.01 | [ | 0.16 |  | 0.18 | ] |
| $l_{p}$ | Price indexation | B | 0.50 | 0.15 | 0.24 | 0.08 | [ | 0.12 |  | 0.38 | ] |
| $l_{w}$ | Wage indexation | B | 0.50 | 0.15 | 0.11 | 0.03 | [ | 0.06 |  | 0.16 | ] |
| $100 \gamma$ | SS technology growth rate | N | 0.50 | 0.03 | 0.48 | 0.02 | [ | 0.44 |  | 0.52 | ] |
| $h$ | Consumption habit | B | 0.50 | 0.10 | 0.78 | 0.04 | [ | 0.72 |  | 0.84 | ] |
| $\lambda_{p}$ | SS price markup | N | 0.15 | 0.05 | 0.23 | 0.04 | [ | 0.17 |  | 0.29 | ] |
| $\lambda_{w}$ | SS wage markup | N | 0.15 | 0.05 | 0.15 | 0.04 | [ | 0.08 |  | 0.22 | ] |
| $\log L^{s s}$ | SS log-hours | N | 0.00 | 0.50 | 0.38 | 0.47 | [ | -0.39 |  | 1.15 | ] |
| $100(\pi-1)$ | SS quarterly inflation | N | 0.50 | 0.10 | 0.71 | 0.07 | [ | 0.58 |  | 0.82 | ] |
| $100\left(\beta^{-1}-1\right)$ | Discount factor | G | 0.25 | 0.10 | 0.13 | 0.04 | [ | 0.07 |  | 0.21 | ] |
| $v$ | Inverse Frisch elasticity | G | 2.00 | 0.75 | 3.79 | 0.76 | [ | 2.70 |  | 5.19 | ] |
| $\xi_{p}$ | Calvo prices | B | 0.66 | 0.10 | 0.84 | 0.02 | [ | 0.80 |  | 0.87 | ] |
| $\xi_{w}$ | Calvo wages | B | 0.66 | 0.10 | 0.70 | 0.05 | [ | 0.60 |  | 0.78 | ] |
| $\chi$ | Elasticity capital utilization costs | G | 5.00 | 1.00 | 5.30 | 1.01 | [ | 3.84 |  | 7.13 | ] |
| $S^{\prime \prime}$ | Investment adjustment costs | G | 4.00 | 1.00 | 2.85 | 0.54 | [ | 2.09 | , | 3.88 | ] |
| $\phi_{\pi}$ | Taylor rule inflation | N | 1.70 | 0.30 | 2.09 | 0.17 | [ | 1.84 | , | 2.39 | ] |
| $\phi_{X}$ | Taylor rule output | N | 0.13 | 0.05 | 0.07 | 0.02 | [ | 0.04 | , | 0.10 | ] |
| $\phi_{d X}$ | Taylor rule output growth | N | 0.13 | 0.05 | 0.24 | 0.02 | [ | 0.20 | , | 0.28 | ] |
| $\rho_{R}$ | Taylor rule smoothing | B | 0.60 | 0.20 | 0.82 | 0.02 | [ | 0.79 | , | 0.86 | ] |

Table 1: Prior densities and posterior estimates for the baseline model

| Coefficient | Description | Prior |  |  | Posterior ${ }^{2}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Prior Density ${ }^{1}$ | Mean | Std | Median | Std | [ | 5 |  | 95 | ] |
| $\rho_{m p}$ | Monetary policy | B | 0.40 | 0.20 | 0.14 | 0.06 | [ | 0.05 |  | 0.25 | ] |
| $\rho_{z}$ | Neutral technology growth | B | 0.60 | 0.20 | 0.23 | 0.06 | [ | 0.14 |  | 0.32 | ] |
| $\rho_{g}$ | Government spending | B | 0.60 | 0.20 | 0.99 | 0.00 | [ | 0.99 |  | 0.99 | ] |
| $\rho_{\mu}$ | Investment | B | 0.60 | 0.20 | 0.72 | 0.04 | [ | 0.65 |  | 0.79 | ] |
| $\rho_{p}$ | Price markup | B | 0.60 | 0.20 | 0.94 | 0.02 | [ | 0.90 |  | 0.97 | ] |
| $\rho_{w}$ | Wage markup | B | 0.60 | 0.20 | 0.97 | 0.01 | [ | 0.95 |  | 0.99 | ] |
| $\rho_{b}$ | Intertemporal preference | B | 0.60 | 0.20 | 0.67 | 0.04 | [ | 0.60 |  | 0.73 | ] |
| $\theta_{p}$ | Price markup MA | B | 0.50 | 0.20 | 0.77 | 0.07 | [ | 0.61 |  | 0.85 | ] |
| $\theta_{w}$ | Wage markup MA | B | 0.50 | 0.20 | 0.91 | 0.02 | [ | 0.88 |  | 0.94 | ] |
| $100 \sigma_{m p}$ | Monetary policy | I | 0.10 | 1.00 | 0.22 | 0.01 | [ | 0.20 |  | 0.25 | ] |
| $100 \sigma_{z}$ | Neutral technology growth | I | 0.50 | 1.00 | 0.88 | 0.05 | [ | 0.81 | , | 0.96 | ] |
| $100 \sigma_{g}$ | Government spending | I | 0.50 | 1.00 | 0.35 | 0.02 | [ | 0.33 |  | 0.38 | ] |
| $100 \sigma_{\mu}$ | Investment | I | 0.50 | 1.00 | 6.03 | 0.96 | [ | 4.71 |  | 7.86 | ] |
| $100 \sigma_{p}$ | Price markup | I | 0.10 | 1.00 | 0.14 | 0.01 | [ | 0.12 |  | 0.17 | ] |
| $100 \sigma_{w}$ | Wage markup | I | 0.10 | 1.00 | 0.20 | 0.02 | [ | 0.18 | , | 0.24 | ] |
| $100 \sigma_{b}$ | Intertemporal preference | I | 0.10 | 1.00 | 0.04 | 0.00 | [ | 0.03 |  | 0.04 | ] |

[^3]Table 2: Prior variance decomposition for observable variables in the baseline model

Medians (first row), Means (second row) and [5th,95th] percentiles (third row)

| Series \Shock | Policy | Neutral | Government | Investment | Price mark-up | Wage mark-up | Preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output growth | 0.01 | 0.26 | 0.23 | 0.00 | 0.00 | 0.01 | 0.08 |
|  | 0.06 | 0.34 | 0.31 | 0.01 | 0.03 | 0.07 | 0.18 |
|  | [ 0.00, 0.32] | [ 0.02, 0.89] | [ 0.02, 0.85] | [ 0.00, 0.04] | [ 0.00, 0.12] | [ 0.00, 0.43] | [ 0.00, 0.75] |
| Consumption growth | 0.01 | 0.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.42 |
|  | 0.06 | 0.38 | 0.02 | 0.01 | 0.02 | 0.05 | 0.46 |
|  | [ $0.00,0.33$ ] | [ 0.01, 0.93] | [ 0.00, 0.10] | [ 0.00, 0.03] | [ 0.00, 0.09] | [ 0.00, 0.29] | [ 0.02, 0.98] |
| Investment growth | 0.01 | 0.38 | 0.00 | 0.03 | 0.00 | 0.01 | 0.05 |
|  | 0.08 | 0.43 | 0.03 | 0.09 | 0.05 | 0.12 | 0.22 |
|  | [ 0.00, 0.46] | [ 0.01, 0.95] | [ 0.00, 0.14] | [ 0.00, 0.41] | [ 0.00, 0.26] | [ 0.00, 0.70] | [ 0.00, 0.94] |
| Hours | 0.01 | 0.17 | 0.07 | 0.00 | 0.00 | 0.04 | 0.06 |
|  | 0.09 | 0.29 | 0.17 | 0.03 | 0.05 | 0.20 | 0.17 |
|  | [ 0.00, 0.52] | [ 0.00, 0.90] | [ 0.00, 0.66] | [ 0.00, 0.12] | [ 0.00, 0.31] | [ 0.00, 0.92] | [ 0.00, 0.81] |
| Wage growth | 0.00 | 0.73 | 0.00 | 0.00 | 0.04 | 0.08 | 0.00 |
|  | 0.01 | 0.66 | 0.01 | 0.00 | 0.11 | 0.18 | 0.03 |
|  | [ 0.00, 0.03] | [ 0.11, 0.98] | [ 0.00, 0.03] | [ 0.00, 0.01] | [ 0.00, 0.52] | [ 0.00, 0.71] | [ 0.00, 0.18] |
| Inflation | 0.01 | 0.11 | 0.01 | 0.00 | 0.07 | 0.07 | 0.03 |
|  | 0.11 | 0.24 | 0.04 | 0.02 | 0.19 | 0.25 | 0.16 |
|  | [ 0.00, 0.68] | [ 0.00, 0.87] | [ 0.00, 0.19] | [ 0.00, 0.07] | [ 0.00, 0.80] | [ 0.00, 0.95] | [ 0.00, 0.84] |
| Interest Rates | 0.02 | 0.15 | 0.02 | 0.00 | 0.02 | 0.03 | 0.11 |
|  | 0.09 | 0.28 | 0.07 | 0.03 | 0.09 | 0.18 | 0.27 |
|  | [ 0.00, 0.44] | [ 0.00, 0.92] | [ 0.00, 0.34] | [ 0.00, 0.13] | [ 0.00, 0.50] | [ 0.00, 0.89] | [ 0.00, 0.94] |

Mean shares add up to one, but median shares do not, due to the sweness induced by the dispersed prior distribution for the standard deviation of the shocks.

Table 3: Posterior variance decomposition for observable variables in the baseline model
Medians and [5th,95th] percentiles

| Series \Shock | Monetary Policy | Neutral | Government | Investment | Price mark-up | Wage mark-up | Preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output growth | $\begin{gathered} 0.05 \\ {[0.03,0.07]} \end{gathered}$ | $\begin{gathered} 0.21 \\ {[0.16,0.28]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.06,0.09]} \end{gathered}$ | $\begin{gathered} 0.49 \\ {[0.42,0.56]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.05]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.03,0.08]} \end{gathered}$ | $\begin{gathered} 0.09 \\ {[0.06,0.11]} \end{gathered}$ |
| Consumption growth | $\begin{gathered} 0.02 \\ {[0.01,0.04]} \end{gathered}$ | $\begin{gathered} 0.26 \\ {[0.20,0.32]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.02,0.03]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.04,0.12]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.04,0.13]} \end{gathered}$ | $\begin{gathered} 0.54 \\ {[0.46,0.61]} \end{gathered}$ |
| Investment growth | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.03,0.08]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.86 \\ {[0.80,0.90]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ |
| Hours | $\begin{gathered} 0.03 \\ {[0.01,0.06]} \end{gathered}$ | $\begin{gathered} 0.04 \\ {[0.02,0.06]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.24 \\ {[0.12,0.38]} \end{gathered}$ | $\begin{gathered} 0.06 \\ {[0.03,0.10]} \end{gathered}$ | $\begin{gathered} 0.58 \\ {[0.38,0.77]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.01,0.04]} \end{gathered}$ |
| Wage growth | $\begin{gathered} 0.00 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.32 \\ {[0.24,0.40]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.24 \\ {[0.19,0.31]} \end{gathered}$ | $\begin{gathered} 0.40 \\ {[0.33,0.48]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.01]} \end{gathered}$ |
| Inflation | $\begin{gathered} 0.03 \\ {[0.02,0.06]} \end{gathered}$ | $\begin{gathered} 0.08 \\ {[0.04,0.13]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.04 \\ {[0.01,0.11]} \end{gathered}$ | $\begin{gathered} 0.25 \\ {[0.16,0.35]} \end{gathered}$ | $\begin{gathered} 0.56 \\ {[0.43,0.71]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ |
| Interest Rates | $\begin{gathered} 0.10 \\ {[0.07,0.13]} \end{gathered}$ | 0.05 $[0.04,0.08]$ | 0.01 $[0.01,0.01]$ | 0.45 $[0.30,0.58]$ | 0.03 $[0.02,0.04]$ | 0.22 $[0.11,0.41]$ | 0.13 $[0.08,0.19]$ |

Posterior median and mean shares are almost identical, in contrast to the prior variance decomposition (Table 1).

## Table 4: Standard deviations and relative standard deviations for observable variables in the baseline model

|  | Standard deviation |  |  |  |  | Relative standard deviation ${ }^{2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Baseline Model |  |  |  |  | Baseline Model |  |  |  |
| Series | Data | Median | [ 5th | 95th | ] | Data | Median | [ 5th | 95th ] |
| Output growth | 0.94 | 1.12 | [ 1.04 | 1.20 | ] | 1.00 | 1.00 |  |  |
| Consumption growth | 0.51 | 0.74 | [ 0.68 | 0.82 | ] | 0.54 | 0.67 | [ 0.56 | 0.79 ] |
| Investment growth | 3.59 | 4.60 | [ 4.20 | 5.05 | ] | 3.83 | 4.13 | [ 3.70 | 4.63 ] |
| Hours | 4.10 | 5.03 | [ 4.12 | 6.49 | ] | 4.36 | 3.81 | [ 2.76 | $5.54]$ |
| Wage growth | 0.52 | 0.64 | [ 0.60 | 0.70 | ] | 0.55 | 0.57 | [ 0.49 | 0.67 ] |
| Inflation | 0.60 | 0.55 | [ 0.47 | 0.67 | ] | 0.64 | 0.45 | [ 0.35 | 0.59 ] |
| Interest Rates | 0.84 | 0.70 | [ 0.62 | 0.81 | ] | 0.90 | 0.59 | [ 0.46 | 0.76 ] |

${ }^{1}$ For each parameter draw, we generate 1000 samples of the observable variables from the model with same length as our dataset (202 observations) after discarding 50 initial observations.
${ }^{2}$ Standard deviation relative to the standard deviation of output growth

Table 5: Posterior estimates in the baseline model with SW definition of the observables ${ }^{1}$

| Coefficient | Description | Posterior ${ }^{2}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Median | Std | [ | 5 |  | 95 | ] |
| $\alpha$ | Capital share | 0.13 | 0.02 | [ | 0.10 | , | 0.16 | ] |
| $l_{p}$ | Price indexation | 0.28 | 0.09 | [ | 0.14 | , | 0.44 | ] |
| $l_{w}$ | Wage indexation | 0.09 | 0.03 | [ | 0.04 | , | 0.14 | ] |
| $100 \gamma$ | SS technology growth rate | 0.49 | 0.02 | [ | 0.45 | , | 0.53 | ] |
| $h$ | Consumption habit | 0.66 | 0.05 | [ | 0.57 | , | 0.75 | ] |
| $\lambda_{p}$ | SS price markup | 0.28 | 0.04 | [ | 0.22 | , | 0.35 | ] |
| $\lambda_{w}$ | SS wage markup | 0.12 | 0.05 | [ | 0.05 | , | 0.21 | ] |
| $\log L^{s s}$ | SS log-hours | 0.33 | 0.02 | [ | 0.29 | , | 0.37 | ] |
| $100(\pi-1)$ | SS quarterly inflation | 0.65 | 0.08 | [ | 0.51 | , | 0.77 | ] |
| $100\left(\beta^{-1}-1\right)$ | Discount factor | 0.14 | 0.05 | [ | 0.07 |  | 0.22 | ] |
| $v$ | Inverse Frisch elasticity | 3.88 | 0.79 | [ | 2.75 | , | 5.33 | ] |
| $\xi_{p}$ | Calvo prices | 0.82 | 0.03 | [ | 0.78 | , | 0.87 | ] |
| $\xi_{w}$ | Calvo wages | 0.80 | 0.05 | [ | 0.70 | , | 0.86 | ] |
| $\chi$ | Elasticity capital utilization costs | 5.01 | 0.98 | [ | 3.59 | , | 6.81 | ] |
| $S^{\prime \prime}$ | Investment adjustment costs | 6.47 | 1.22 | [ | 4.63 | , | 8.74 | ] |
| $\phi_{\pi}$ | Taylor rule inflation | 1.81 | 0.18 | [ | 1.53 | , | 2.13 | ] |
| $\phi_{X}$ | Taylor rule output | 0.09 | 0.03 | [ | 0.05 | , | 0.14 | ] |
| $\phi_{d X}$ | Taylor rule output growth | 0.26 | 0.03 | [ | 0.22 | , | 0.30 | ] |
| $\rho_{R}$ | Taylor rule smoothing | 0.82 | 0.02 | [ | 0.77 | , | 0.85 | ] |

(Continued on the next page )

Table 5: Posterior estimates in the baseline model with SW definition of the observables ${ }^{1}$

| Coefficient | Description | Posterior ${ }^{2}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Median | Std | [ | 5 |  | 95 | ] |
| $\rho_{m p}$ | Monetary policy | 0.12 | 0.05 | [ | 0.04 |  | 0.22 | ] |
| $\rho_{z}$ | Neutral technology growth | 0.09 | 0.04 | [ | 0.04 |  | 0.17 | ] |
| $\rho_{g}$ | Government spending | 0.99 | 0.00 | [ | 0.98 |  | 0.99 | ] |
| $\rho_{\mu}$ | Investment | 0.75 | 0.04 | [ | 0.68 |  | 0.81 | ] |
| $\rho_{p}$ | Price markup | 0.97 | 0.02 | [ | 0.94 |  | 0.99 | ] |
| $\rho_{w}$ | Wage markup | 0.96 | 0.02 | [ | 0.92 |  | 0.97 | ] |
| $\rho_{b}$ | Intertemporal preference | 0.59 | 0.09 | [ | 0.42 | , | 0.72 | ] |
| $\theta_{p}$ | Price markup MA | 0.86 | 0.06 | [ | 0.73 |  | 0.92 | ] |
| $\theta_{w}$ | Wage markup MA | 0.98 | 0.01 | [ | 0.95 |  | 0.99 | ] |
| $100 \sigma_{m p}$ | Monetary policy | 0.23 | 0.01 | [ | 0.21 |  | 0.26 | ] |
| $100 \sigma_{z}$ | Neutral technology growth | 0.89 | 0.05 | [ | 0.81 | , | 0.98 | ] |
| $100 \sigma_{g}$ | Government spending | 0.62 | 0.03 | [ | 0.57 | , | 0.67 | ] |
| $100 \sigma_{\mu}$ | Investment | 6.07 | 1.22 | [ | 4.38 | , | 8.49 | ] |
| $100 \sigma_{p}$ | Price markup | 0.18 | 0.01 | [ | 0.15 | , | 0.20 | ] |
| $100 \sigma_{w}$ | Wage markup | 0.30 | 0.02 | [ | 0.28 | , | 0.33 | ] |
| $100 \sigma_{b}$ | Intertemporal preference | 0.08 | 0.02 | [ | 0.06 |  | 0.11 | ] |

[^4]Table 6: Posterior variance decomposition at business cycle frequencies in the baseline model with SW definition of the observables

Medians and [5th,95th] percentiles

| Series \Shock | Policy | Neutral | Government | Investment | Price mark-up | Wage mark-up | Preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output | $\begin{gathered} 0.09 \\ {[0.06,0.12]} \end{gathered}$ | $\begin{gathered} 0.38 \\ {[0.31,0.46]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.05,0.10]} \end{gathered}$ | $\begin{gathered} 0.19 \\ {[0.12,0.28]} \end{gathered}$ | $\begin{gathered} 0.04 \\ {[0.02,0.07]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.20 \\ {[0.14,0.27]} \end{gathered}$ |
| Consumption | $\begin{gathered} 0.07 \\ {[0.04,0.10]} \end{gathered}$ | $\begin{gathered} 0.37 \\ {[0.30,0.45]} \end{gathered}$ | $\begin{gathered} 0.08 \\ {[0.06,0.11]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.03,0.08]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.39 \\ {[0.31,0.48]} \end{gathered}$ |
| Investment | $\begin{gathered} 0.02 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.04,0.10]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.86 \\ {[0.80,0.90]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.06]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.01]} \end{gathered}$ |
| Hours | $\begin{gathered} 0.13 \\ {[0.08,0.18]} \end{gathered}$ | $\begin{gathered} 0.17 \\ {[0.12,0.22]} \end{gathered}$ | $\begin{gathered} 0.10 \\ {[0.08,0.13]} \end{gathered}$ | $\begin{gathered} 0.22 \\ {[0.14,0.31]} \end{gathered}$ | $\begin{gathered} 0.06 \\ {[0.03,0.10]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.05]} \end{gathered}$ | $\begin{gathered} 0.29 \\ {[0.20,0.38]} \end{gathered}$ |
| Wages | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.40 \\ {[0.30,0.51]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.28 \\ {[0.21,0.38]} \end{gathered}$ | $\begin{gathered} 0.3 \\ {[0.22,0.39]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ |
| Inflation | $\begin{gathered} 0.01 \\ {[0.00,0.03]} \end{gathered}$ | $\begin{gathered} 0.3 \\ {[0.19,0.40]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.57 \\ {[0.46,0.68]} \end{gathered}$ | $\begin{gathered} 0.1 \\ {[0.06,0.17]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.00,0.02]} \end{gathered}$ |
| Interest Rates | $\begin{gathered} 0.30 \\ {[0.23,0.38]} \end{gathered}$ | $\begin{gathered} 0.13 \\ {[0.09,0.19]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.12 \\ {[0.07,0.20]} \end{gathered}$ | $\begin{gathered} 0.08 \\ {[0.06,0.12]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.04]} \end{gathered}$ | $\begin{gathered} 0.31 \\ {[0.18,0.44]} \end{gathered}$ |

Table 7: Prior densities and posterior estimates in the model with durables

| Coefficient | Description | Prior |  |  | Posterior |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Prior Density ${ }^{1}$ | Mean | Std | Mode | Std | [ 5 | 95 |
| $\alpha$ | Capital share | N | 0.30 | 0.05 | 0.105 | 0.004 | [ 0.097 | 0.111 |
| $l_{p}$ | Price indexation | B | 0.50 | 0.15 | 0.210 | 0.073 | [ 0.099 | 0.333 ] |
| $l_{w}$ | Wage indexation | B | 0.50 | 0.15 | 0.120 | 0.037 | [ 0.072 | 0.194 ] |
| $100 \gamma$ | SS technology growth rate | N | 0.50 | 0.03 | 0.485 | 0.027 | [ 0.430 | 0.523 ] |
| $h$ | Consumption habit | B | 0.50 | 0.10 | 0.755 | 0.013 | [ 0.733 | 0.777 |
| $\lambda_{p}$ | SS price markup | N | 0.15 | 0.05 | 0.231 | 0.030 | [ 0.181 | 0.283 ] |
| $\lambda_{w}$ | SS wage markup | N | 0.15 | 0.05 | 0.188 | 0.050 | [ 0.118 | 0.282 ] |
| $\log L^{s s}$ | SS log-hours | N | 0.00 | 0.50 | 1.102 | 0.291 | [ 0.615 | 1.557 ] |
| 100( $\pi-1$ ) | SS quarterly inflation | N | 0.50 | 0.10 | 0.667 | 0.071 | [ 0.510 | 0.734 ] |
| $100\left(\beta^{-1}-1\right)$ | Discount factor | G | 0.25 | 0.10 | 0.115 | 0.034 | [ 0.070 | 0.189 |
| $v$ | Inverse Frisch elasticity | G | 2.00 | 0.75 | 1.701 | 0.248 | [ 1.270 | 2.120 ] |
| $\xi_{p}$ | Calvo prices | B | 0.66 | 0.10 | 0.835 | 0.019 | [ 0.811 | 0.869 ] |
| $\xi_{w}$ | Calvo wages | B | 0.66 | 0.10 | 0.866 | 0.009 | [ 0.857 | 0.885 |
| $\chi$ | Elasticity capital utilization costs | G | 5.00 | 1.00 | 5.346 | 1.171 | [ 3.843 | 7.800 ] |
| $S^{\prime \prime}$ | Investment adjustment costs | G | 4.00 | 1.00 | 2.072 | 0.106 | [ 1.827 | 2.180 ] |
| $\phi_{\pi}$ | Taylor rule inflation | N | 1.70 | 0.30 | 1.854 | 0.101 | [ 1.715 | 2.059 |
| $\phi_{X}$ | Taylor rule output | N | 0.13 | 0.05 | 0.044 | 0.007 | [ 0.030 | 0.052 ] |
| $\phi_{d X}$ | Taylor rule output growth | N | 0.13 | 0.05 | 0.188 | 0.017 | [ 0.161 | 0.216 |
| $\rho_{R}$ | Taylor rule smoothing | B | 0.40 | 0.20 | 0.789 | 0.018 | [ 0.757 | 0.819 |

( Continued on the next page )

Table 7: Prior densities and posterior estimates in the model with durables

| Coefficient | Description | Prior |  |  | Posterior |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Prior Density ${ }^{1}$ | Mean | Std | Median | Std | [ 5 | 95 |
| $\eta$ | CES consumption aggreg. | G | 1.00 | 0.20 | 0.548 | 0.032 | 0.500 , | 0.605 ] |
| $\tau$ | CES service flow aggreg. | G | 1.00 | 0.20 | 0.374 | 0.036 | 0.326 | 0.448 ] |
| $I_{d} / I$ | SS durables-to-investment ratio | G | 0.70 | 0.10 | 0.692 | 0.061 | [ 0.625, | 0.821 ] |
| $L_{h} / L$ | SS home-to-mkt hours ratio | G | 0.70 | 0.10 | 0.468 | 0.063 | [ 0.354 , | 0.560 ] |
| $\rho_{m p}$ | Monetary Policy | B | 0.40 | 0.20 | 0.091 | 0.044 | [ 0.040 , | 0.186 ] |
| $\rho_{z}$ | Neutral technology growth | B | 0.60 | 0.20 | 0.241 | 0.038 | [ 0.166 , | 0.293 ] |
| $\rho_{g}$ | Government spending | B | 0.60 | 0.20 | 0.990 | 0.001 | [ 0.986 , | 0.990 ] |
| $\rho_{\mu}$ | Investment | B | 0.60 | 0.20 | 0.882 | 0.023 | [ 0.830 , | 0.907 ] |
| $\rho_{\mu d}$ | Durables specific | B | 0.40 | 0.20 | 0.928 | 0.011 | 0.926 | 0.963 |
| $\rho_{p}$ | Price markup | B | 0.60 | 0.20 | 0.990 | 0.004 | [ 0.976, | 0.989 ] |
| $\rho_{w}$ | Wage markup | B | 0.60 | 0.20 | 0.702 | 0.015 | [ 0.676, | 0.725 ] |
| $\rho_{b}$ | Intertemporal preference | B | 0.60 | 0.20 | 0.078 | 0.035 | [ 0.035 , | 0.141 ] |
| $\theta_{p}$ | Price markup MA | B | 0.50 | 0.20 | 0.708 | 0.015 | [ 0.717 , | 0.764 ] |
| $\theta_{w}$ | Wage markup MA | B | 0.50 | 0.20 | 0.889 | 0.010 | [ 0.871 , | 0.899 ] |
| $100 \sigma_{m p}$ | Monetary policy | I | 0.10 | 1.00 | 0.221 | 0.009 | [ 0.207 | 0.235 ] |
| $100 \sigma_{z}$ | Neutral technology growth | I | 0.50 | 1.00 | 0.801 | 0.037 | [ 0.760 | 0.887 ] |
| $100 \sigma_{g}$ | Government spending | I | 0.50 | 1.00 | 0.351 | 0.018 | [ 0.328 | 0.387 ] |
| $100 \sigma_{\mu}$ | Investment | I | 0.50 | 1.00 | 5.391 | 0.288 | [ 4.804, | 5.732 ] |
| $100 \sigma_{\mu}$ | Durables specific | I | 0.25 | 1.00 | 9.316 | 0.495 | [ 8.825, | 10.225 ] |
| $100 \sigma_{p}$ | Price markup | I | 0.10 | 1.00 | 0.131 | 0.003 | [ 0.124 , | 0.134 ] |
| $100 \sigma_{w}$ | Wage markup | I | 0.10 | 1.00 | 0.217 | 0.005 | [ 0.214 , | 0.231 ] |
| $100 \sigma_{b}$ | Intertemporal preference | I | 0.10 | 1.00 | 0.032 | 0.001 | [ 0.031, | 0.032 ] |

[^5]Table 8: Posterior variance decomposition at business cycle frequencies in the model with durables

|  |  |  |  | Medians and [ | 5th] percentiles |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Series \Shock | Monetary Policy | Neutral | Government | Investment | Price mark-up | Wage mark-up | Preference | Durables Specific |
| Output | $\begin{gathered} 0.04 \\ {[0.03,0.06]} \end{gathered}$ | $\begin{gathered} 0.17 \\ {[0.14,0.23]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.01]} \end{gathered}$ | $\begin{gathered} 0.63 \\ {[0.58,0.68]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.04,0.07]} \end{gathered}$ | $\begin{gathered} 0.04 \\ {[0.03,0.07]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.02,0.03]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ |
| Consumption | $\begin{gathered} 0.04 \\ {[0.03,0.05]} \end{gathered}$ | $\begin{gathered} 0.25 \\ {[0.21,0.33]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.01]} \end{gathered}$ | $\begin{gathered} 0.14 \\ {[0.09,0.21]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.12 \\ {[0.09,0.16]} \end{gathered}$ | $\begin{gathered} 0.40 \\ {[0.34,0.47]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ |
| Investment | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.90 \\ {[0.89,0.92]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.05]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.01]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ |
| Hours | $\begin{gathered} 0.05 \\ {[0.03,0.06]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.05,0.07]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.72 \\ {[0.69,0.76]} \end{gathered}$ | $\begin{gathered} 0.06 \\ {[0.05,0.08]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.03,0.08]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ |
| Wages | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.40 \\ {[0.31,0.51]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.06 \\ {[0.03,0.09]} \end{gathered}$ | $\begin{gathered} 0.29 \\ {[0.23,0.36]} \end{gathered}$ | $\begin{gathered} 0.19 \\ {[0.15,0.26]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.05]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ |
| Inflation | $\begin{gathered} 0.04 \\ {[0.03,0.06]} \end{gathered}$ | $\begin{gathered} 0.08 \\ {[0.05,0.13]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.19 \\ {[0.12,0.29]} \end{gathered}$ | $\begin{gathered} 0.36 \\ {[0.26,0.45]} \end{gathered}$ | $\begin{gathered} 0.24 \\ {[0.19,0.32]} \end{gathered}$ | $\begin{gathered} 0.07 \\ {[0.05,0.10]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ |
| Interest Rates | $\begin{gathered} 0.13 \\ {[0.10,0.15]} \end{gathered}$ | $\begin{gathered} 0.05 \\ {[0.04,0.08]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.45 \\ {[0.37,0.52]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[0.02,0.04]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.31 \\ {[0.25,0.39]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.00,0.01]} \end{gathered}$ |
| Durables | $\begin{gathered} 0.02 \\ {[0.01,0.03]} \end{gathered}$ | $\begin{gathered} 0.11 \\ {[0.08,0.16]} \end{gathered}$ | $\begin{gathered} 0.00 \\ {[0.00,0.00]} \end{gathered}$ | $\begin{gathered} 0.61 \\ {[0.55,0.68]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.01,0.02]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[0.00,0.01]} \end{gathered}$ | $\begin{gathered} 0.22 \\ {[0.17,0.28]} \end{gathered}$ |

# Table 9: Variance share of output and hours at business cycles frequencies ${ }^{1}$ due to investment shocks, robustness 

|  | Baseline | $\begin{gathered} v=1 \text { and } \\ \alpha=0.3 \end{gathered}$ | $\begin{gathered} \text { No MA } \\ \text { components }^{2} \end{gathered}$ | Taylor rule with output growth ${ }^{3}$ | MLE ${ }^{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Series |  |  |  |  |  |
| Output | 0.50 | 0.63 | 0.54 | 0.47 | 0.59 |
| Hours | 0.59 | 0.75 | 0.56 | 0.52 | 0.65 |
| ${ }^{1}$ Business cycle frequencies correspond to periodic components with cycles between 6 and 32 quarters. Variance decompositions are performed at the mode of each specification. |  |  |  |  |  |
| ${ }^{2}$ Moving average component for price and wage mark-up shocks calibrated to zero. |  |  |  |  |  |
| ${ }^{3}$ Taylor rule responds to observable output growth instead of the output gap. |  |  |  |  |  |
| ${ }^{4}$ Baseline specification estimated by maximum likelihood. |  |  |  |  |  |

Fig 1: Autocorrelation for baseline specification, dsge median (dark), dsge 5-95 (dotted) \& data (grey)


Autocovariances in the data: JPT (solid) and SW (dashed)


Figure 3: Impulse responses to a one standard deviation shock









[^0]:    Date: December 2009. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Chicago, the Federal Reserve Bank of New York or any other person associated with the Federal Reserve System.

[^1]:    1 SW also use different series for hours and wages, but this does not have any material impact on the results.

[^2]:    ${ }^{2}$ More precisely, to maximize the likelihood we need to calibrate $\varkappa$, since the likelihood is not very informative on this parameter and this creates convergence problems in the maximization routine. Therefore, we calibrated $\varkappa=5$, which is our prior mean. This value of $\varkappa$ implies a low elasticity of capital utilization, which makes the propagation of investment shocks if anything more problematic.

[^3]:    ${ }^{1} \mathrm{~N}$ stands for Normal, B Beta, G Gamma and I Inverted-Gammal distribution
    ${ }^{2}$ Median and posterior percentiles from 3 chains of 120,000 draws generated using a Random walk Metropolis algorithm. We discard the initial 20,000 and retain one every 10 subsequent draws.

[^4]:    ${ }^{1}$ Priors are identical to those reported in table 1 of the paper, except for the mean of SS log-hours. The prior for this parameter is a normal with mean 0.33 and standard deviation 0.05 .
    ${ }^{2}$ Median and posterior percentiles from 3 chains of 120,000 draws generated using a Random walk
    Metropolis algorithm. We discard the initial 20,000 and retain one every 10 subsequent draws.

[^5]:    ${ }^{1} \mathrm{~N}$ stands for Normal, B Beta, G Gamma and I Inverted-Gamma1 distribution

