Gradient Symbol Processing in Speech Production

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Outline

• The challenge: Reconciling symbols with pervasive gradience

• Gradient Symbol Processing: Integrating gradience and symbolic structure

• GSP: Enriching psycholinguistic theory
  – Gradient patterns and gradient representations

• GSP: Enriching grammatical theory
  – Opacity and gradient representations

• Conclusions
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Preliminaries: Why Symbolic Representations?

- **Symbolic representations**: Systematic, structured combinations of discrete constituents.

```
    o
   / \                        
  /   \                       
 /     \                      
|       |                      
|       |                      
| [lab]  | [cor]                 
         |                      
```

“bat”
Why Symbolic Representations?

• Systematic structure of sound patterns
  – Words are not made up of random sound strings; they form systematic, coherent sets.
  – Such systematic patterns are easily characterized by formalisms like generative phonological grammars—which are built upon symbolic representations.

• Psycholinguistic patterns
  – Speakers implicitly acquire structural patterns
    • English speakers: Onsets vs. rimes
    • Influences speech errors (MacKay, 1972), acquisition of novel patterns (Treiman, 1983; Kapatsinski, 2009).
  – *Structural*: Speakers are aware of patterns that are not specific to particular consonants (Lee & Goldrick, 2008)
Critiques of Pure Symbolic Accounts

• Challenge: Pervasive gradience
  – Gradient patterns are easily acquired and productively applied by learners.

  – Representational content is gradient
    • Systematic, within-category variation in realization of identical symbolic representations.
Gradient Patterns

• Phonological patterns are not simply free recombination of discrete elements; graded restrictions on combinations.

• Variable processes with acquired probabilities
  – left-\( \rightarrow \) [ft] 80% [f] 20% vs. [ft] 60% [f] 40%
  – Extensively studied in sociolinguistic literature (Weinreich et al 1968 et seq)

• Gradient phonotactics
  – Influence speed and accuracy of processing (Vitevitch et al. 2004)
  – Implicitly acquired (Goldrick & Larson 2008)

• n.b. Consistent with stochastic grammars over discrete symbolic elements (see Coetzee & Pater, in press for a review).
Gradient Content:
Incomplete neutralization/near merger

- (see Warner et al., 2004; Yu, 2007, for recent reviews).
  - Language exhibits contrast between two phone categories X and Y in environment A.
  - Contrast is ‘eliminated’ in some other environment B (listeners do not distinguish X and Y in B); however, instrumental studies reveal reliable differences.

- Dutch (Warner et al. 2004): voiced/voiceless (lenis/fortis) contrast ‘eliminated’ word finally.

<table>
<thead>
<tr>
<th>Vowel duration (ms)</th>
<th>Short vowel</th>
<th>Long vowel</th>
</tr>
</thead>
<tbody>
<tr>
<td>/−voi/</td>
<td>120</td>
<td>175</td>
</tr>
<tr>
<td>/+voi/</td>
<td>124</td>
<td>178</td>
</tr>
</tbody>
</table>

- Not simply “phonetic”—systematic, within-category variation
Gradient Content in Speech Production

• The acoustic/articulatory properties of speech errors are influenced by the intended target (Goldstein et al., 2007; McMillan et al., 2009; Pouplier, 2007).

  • E.g., /keff/ -> [geff]

• VOT of /k/->[g] errors is significantly longer than correct [g].
  • Reflects a trace of the intended voiceless target.

• Not simply phonetic variation across tokens; *systematic, within-category* differences across processing contexts.
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• The challenge: Reconciling symbols with pervasive gradience

• **Gradient Symbol Processing: Integrating gradience and symbolic structure**

  • GSP: Enriching psycholinguistic theory
    – Gradient patterns and gradient representations
  • GSP: Enriching grammatical theory
    – Opacity and gradient representations

• Conclusions
Current Perspectives on Gradience

• Eliminate symbolic representations
  – Replace with continuous dynamical systems (e.g., Browman & Goldstein, 1986, et seq.).
  – Replace with highly detailed, real-valued exemplars (e.g., Goldinger, 1998).

• Hybrid systems
  – Symbolic representations are associated/linked to detailed exemplars (Johnson, 1997; Pierrehumbert, 2001; see Pierrehumbert, 2006 for a recent review).
  – Note: Maintains *parallel*, independent systems of representation.
Gradient Symbol Processing

• Realize symbolic structures *precisely* as vectors in a high-dimensional representational space.
  – *Integrate* the continuous and the discrete.

• Computation over these vectors is stochastic and graded.
  – Gradience in representation and processing.

• Successful optimization over symbol structures requires discretization.
  – Distinct dynamic process that drives network towards pure symbolic states.
Gradient Symbol Processing

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Gradient symbolic representations

• Origins: Connectionist computation with symbolic structures
  – Issue: Connectionist networks compute over vectors. How can vectors realize symbols?
  – General solution: Organize activation space to reflect symbolic structure (see Tesar & Smolensky, 2006, for a review)
Structured space of activations

• Toy example: consonant cluster voicing (e.g., dz in “dudes”)
  – Axes reflect symbolic *roles* (e.g., positions in cluster)
  – Different *fillers* represented by regions along each axis
Structured space of activations

- If properly structured, can exhibit properties of symbols
  - Filler occupies same region of space, regardless of what else is present ([voi] for C1 is the same region across different C2s)
  - *Compositionality*—Supports generalization
Gradient symbolic representations

• Embedding symbols in continuous space increases expressive power

• Example: 0.8 [voi]C1
  0.8 [voi]C2
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Connectionist Processing of Vectors

**Real brains**
- Basic processing element: neuron
- Processing mechanism: series of action potentials propagated via axons and dendrites
- Whether or not a neuron sends out an action potential is determined by integrating all incoming electrical activity.

**Connectionist Network**
- Basic processing element: unit
- Processing mechanism: propagation of “activation” (abstract representation of activity levels) via connections between units
- Activation of a unit is a function of incoming activation from units connected to it
Processing Dynamics: Harmonic Networks

\[ \frac{da_1}{dt} = -a_1 + i_1 - \lambda a_2 \]

IAC network shown; Also Boltzmann, Hopfield, Harmonium, BSB...

Slides cribbed from Paul Smolensky
Harmonic Network Processing as Optimization

Processing is parallel satisfaction (measured by Harmony) of violable well-formedness constraints on linguistic representations.

\[ H = \sum_{\alpha\beta} a_\alpha W_{\alpha\beta} a_\beta \]
Symbolic Knowledge as Constraint Satisfaction

- Using violable, weighted constraints, can define **Harmonic Grammars** over linguistic representations (Legendre, Miyata & Smolensky, 1990; Smolensky & Legendre, 2006)
  - As powerful as Turing machines; can specify Type 0 (recursively enumerable) languages (Hale & Smolensky, 2006)
  - Successfully used to model phonological (Pater, 2009) and syntactic (Legendre et al., 1990) patterns as well as speech error data (Goldrick & Daland, 2009).

- Assuming that constraints strictly dominate one another leads to another formalism, **Optimality Theory** (Prince & Smolensky, 1993)
Stochastic Optimization

• Deterministic algorithms lead you to a *good* network state, but are not guaranteed to find *the best* state.
  – “Climbing harmony hill” only works if there’s a smooth path to the best solution

• Solution: Stochastic optimization techniques (Hopfield, 1982; Smolensky, 1986; Movellan & McClelland, 1993)
  – Randomness superimposed on algorithm that maximizes harmony.
  – Over time, amount of randomness decreases.
  – Converges to a probability distribution over network states that reflects harmony (more harmonic ≈ more probable).

• **Note:** inherent gradience in computation as well as representation
Gradient Symbol Processing

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  – Gradience in representation and processing.

• Successful optimization over symbol structures requires discretization.
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Connectionist Networks Love Gradience

• Although the network can represent symbols, there’s nothing in the structure of computation that limits the network to these states.
  – Spread of activation within the network is not determined by symbolic structure

• In fact, networks prefer to be in non-symbolic states.
Connectionist Networks Love Gradience

• Example: Computing the English plural
  – dudes-> [dz] not [ds]
  – cats-> [ts] not [tz]

• Input
  – Consonant 1 is voiced
  – Consonant 2 not specified (/s/ or /z/ depending on C1)

• Constraints
  – **Agree**: C1 and C2 should agree in voicing ([dz] not [ds])
  – **NoVoi**: C2 should not be voiced ([s] not [z])
  – Assume (following English) that Agree is stronger than NoVoi.
Connectionist Networks Love Gradience

• Simple IAC activation equation from above.

• Input strength = 1

• Vary strength of Agree (NoVoi = 1-Agree): .99 vs. .9

• Under non-zero weighting of conflicting constraints, gradient states are optimal.
The Goldilocks Zone of Gradience

- Some gradience is critical
  - But too much gradience is problematic.
- Empirically incorrect
  - Speech errors involving voicing: Gradient shifts are limited in strength.
- Prevents system from finding solutions to problems involving mutually dependent choices.
  - E.g. OCP (*C1=C2): To determine if C1 is well formed, need to select a particular C2 (and vice versa).
Constraining Gradience
Smolensky, Goldrick, & Mathis (submitted)

• Subsymbolic Optimization-Quantization
  – In addition to dynamic processes that maximize harmony, include discretization dynamics.

• Attractors exist for each point on the ‘grid’ defined by purely symbolic representations.
  – All attractors are equivalent; only harmony process is sensitive to relative well-formedness.

• Two dynamical processes operate in parallel
  – Harmony process dominates early; discretization dominates later
  – **Note**: Neither process completely dominates computation at any point.
Illustration: Constraining Gradience

• Example: 0.8 [voi]C1
  0.8 [voi]C2

• Suppose filled blue circle is the best symbolic state, but this gradient state is the most harmonic.

• Next graph: Harmony shown on z-axis.
Subsymbolic Optimization-Quantization

Smolensky, Goldrick, & Mathis (submitted)
Subsymbolic Optimization-Quantization
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Smolensky, Goldrick, & Mathis (submitted)
Subsymbolic Optimization-Quantization

• Symbolic structures are realized as vectors within a continuous activation space.

• Well-formedness of symbolic structures is specified by a set of violable constraints, realized by patterns of interconnections among simple processing units.

• Output of system is computed in parallel by:
  – Stochastic process optimizing well-formedness over the continuous activation space.
  – Discretization process that drives activations towards points corresponding to symbolic states.
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• Conclusions
Testing this Framework: Speech Error data

• No closed-form solution to optimization-quantization method.
  – Simulation code available online

• Simulate production of mini tongue twisters
  – Two syllables: sag knack. Simulate consonants only.
  – Input: Target sequence (/sg nk/).
  – Constraints: Reproduce input + phonotactics (simulation 1)

• Normal processing: Slow transition from optimization- to
discretization-dominant processing

• Tongue twisters: Start at later point in transition.
  – Speeds RTs
  – Results in errors
Testing this Framework: Speech Error data

• Observation 1: Speech errors reflect gradient phonotactic regularities (Goldrick & Larson, 2008)
  – sag knack -> sag nag vs. sag knack -> sag gack
  – Error where /g/ appears in coda is more likely when /g/ appears in coda in 80% vs. 20% of sequences
Gradient Phonotactics

• Across simulations, alter relative preference of /g/ for onset vs. coda.
Account of Gradient Phonotactics

• Stochastic optimization process
  – Converges to a probability distribution over network states that reflects harmony (more harmonic $\approx$ more probable).

• Motivated by the need to solve complex optimization problems.

• Predicts: When processing is sped up, network falls into non-target states with probability $\approx$ relative harmony.
Testing this Framework: Speech Error data

• Observation 2: Speech errors reflect gradient activation of target, error representations (Goldrick & Blumstein, 2006).
  – VOT of /g/-> [k] errors is significantly shorter than correct [k], reflecting a trace of the intended voiced target.
Phonetic Gradience

Q: How does “activation” correspond to articulatory/acoustic properties?
A: Degree to which filler influences articulatory plan.
See Karen Chu’s poster for a detailed discussion!
Account of Phonetic Traces in Errors

• Optimization and discretization operate in parallel; neither completely dominates computation.

• Optimization process prefers preservation of the input.
  – This is a necessary feature of any system where output is partially determined by input.

• When, in an error, the system has become trapped in the incorrect attractor basin, the optimization process will continue to exert a pull towards the target state.
  – Result in small deviations within the continuous space of activations towards the target.
  – N.b. similar principles apply for modeling incomplete neutralization (where Markedness drives system to a non-Faithful attractor basin)
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Counterbleeding

- “Canadian Raising”
  - Raise vowels before voiceless:
    - write [əɪt] ride [aɪd]
  - Raise vowels before underlying voiceless:
    - writer [əɪd] rider [aɪd]
- Transformational account: Voicing counterbleeds Raising:
  - Raising: [+low]→[−low]/_[−voi]
  - Voicing: [−voi]→[+voi]/V_V

<table>
<thead>
<tr>
<th></th>
<th>aɪtV</th>
<th>aɪdV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raising</td>
<td>əɪtV</td>
<td>Ø</td>
</tr>
<tr>
<td>Voicing</td>
<td>əɪdV</td>
<td>aɪdV</td>
</tr>
<tr>
<td>c.f.:voicing bleeds raising</td>
<td>aɪdV</td>
<td></td>
</tr>
</tbody>
</table>

- N.b counterbleeding on environment (Bakovic, in press)
Counterbleeding: Impossible in OT/HG

<table>
<thead>
<tr>
<th>/aɪtV/</th>
<th>*[+low] [–voi]</th>
<th>*[–voi]/ V_V</th>
<th>ID(voi)</th>
<th>ID(low)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faithful: aɪtV</td>
<td>−1</td>
<td>−1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raising: əɪtV</td>
<td>−1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bleed: aɪdV</td>
<td></td>
<td>−1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.Bleed: əɪdV</td>
<td></td>
<td>−1</td>
<td>−1</td>
<td></td>
</tr>
</tbody>
</table>

- Faithful, raising candidates fail to satisfy Markedness.
- The counterbleeding candidate necessarily has greater violations than the bleeding candidate.
  - Unmotivated, gratuitous violation of Faithfulness; Markedness has eliminated [–voi]
- What if some ‘trace’ of [–voi] remained?
  - Could this provide sufficient motivation for raising?
Harmonic Grammars in GSP

• Gradient candidates
  – Rather than limiting to 1.0 [+voi] 0 [–voi] allow for partial activation of multiple fillers
  – Specific candidate considered: 0.9 [+voi] .01 [–voi]
    • Note: Filler activation not constrained to sum to 1.
    • Activation of filler ≈ how much evidence for actual presence of filler in articulatory plan [as opposed to ‘neutral’ value]
    • Here: Neither filler is completely present

• Constraint violations
  – *[+low][–voi]: ¬(Activation of [+low]*activation of [–voice])
  – *[–voi]/V_V: ¬(Activation of [–voi])
  – ID(x): ¬(Activation of non-underlying feature in given position)
### Counterbleeding in GSP

<table>
<thead>
<tr>
<th>/aɪtV/</th>
<th>*[+]low [−voi]</th>
<th>*[−voi]/V_V</th>
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<th>ID(low)</th>
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<td></td>
<td>−1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.Bleed: əɪdV</td>
<td></td>
<td>−1</td>
<td>−1</td>
<td></td>
</tr>
<tr>
<td>Bleed: Partially voiced</td>
<td>−.01</td>
<td>−.01</td>
<td>−0.9</td>
<td></td>
</tr>
<tr>
<td>C.Bleed: Partially voiced, partially raised</td>
<td>−.0001</td>
<td>−.01</td>
<td>−0.9</td>
<td>−0.9</td>
</tr>
</tbody>
</table>

- Gradient candidates have a smaller violation of ID(voi) than the bleeding candidate; intervocalic voicing constraint prefers elimination of all traces of voicing.
  - If ID(voi) is sufficiently strong relative to Markedness, voicing will not be complete.
### Counterbleeding in GSP

<table>
<thead>
<tr>
<th>/aɪtV/</th>
<th>*[+low] [–voi]</th>
<th>*[–voi]/ V_V</th>
<th>ID(voi)</th>
<th>ID(low)</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td>–1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Bleed: əɪdV</td>
<td></td>
<td>–1</td>
<td>–1</td>
<td>–1</td>
</tr>
<tr>
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<td>–.0001</td>
<td>–.01</td>
<td>–0.9</td>
<td>–0.9</td>
</tr>
</tbody>
</table>

- Among gradient candidates, raising constraint (*+low–voi) slightly prefers gradient candidate with raising; ID(low) prefers no raising.
  - If ID(low) is very weak relative to markedness, raising will occur.
Simulation Analysis

• Two roles: Vowel [low], Consonant [voice]
  – Fillers: +/-

• Parameters set to insure non-errorful processing

• Constraints as above. Range (0.5-2) & 0.1 for ID(low)
  – Limited to weightings that allow for application of Voicing, Raising.

• Examine input /+low–voi/: /aItV/
  – Bleeding: Output [+low+voi] [aIdV]
  – Counterbleeding: Output [–low+voi] [.UserInfo()]
• Counterbleeding mapping can be optimal. Conditions:
  – X axis: ID(voi) must be strong enough to allow some preservation of voicing.
  – Y axis: Raising constraint must be strong enough to overcome Faithfulness violation.
Account of Counterbleeding

• Optimization and discretization operate in parallel; neither completely dominates computation.
  – Gradient states can be optimal.

• Partially activated symbolic elements contribute to harmony.
  – Same constraints that determine harmony of categorical states apply to gradient structures.

• n.b. Other counterbleeding processes—relying on deeper derivational history—may require distinct mechanisms.
Predictions: Phonetic Traces of Opacity

- Trigger for counterbled process should be partially present
  - ≈ ‘traces’ in speech error data
- Gafos & Benus (2006): Transparent vowels in Hungarian
  - Hungarian [back] harmony
    - Root determines whether suffix is [+/-back]
  - Front unrounded vowels do not participate, but do not block harmony.
    - Ultrasound reveals phonetic trace of [+/-back]—a trace of the vowel harmony trigger.

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Tongue posture:
neutral vowel /i/

---

The same effect is observed with the ultrasound data. In Fig. 11, we have superimposed tongue postures of /í/, /é/ in a back (dotted lines) versus front harmony (solid lines) context from participant ZZ. The direction of the effect is the same as that observed with the EMMA data. Transparent vowels in the front context are more advanced than those in the back context.

We observed differences in effect size between the two methodologies. For example, in ultrasound, averaged differences in tongue position between the two harmonic contexts reach up to 2.5 mm. In EMMA, the maximal average difference was 1.4 mm. Such differences in magnitude are at least in part due to the fact that ultrasound allows access to almost the entire surface of the tongue as opposed to the position of a few flesh points with EMMA.

---

Table 1

<table>
<thead>
<tr>
<th>Subject</th>
<th>Rec.</th>
<th>F</th>
<th>B</th>
<th>MD</th>
<th>F</th>
<th>B</th>
<th>MD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZZ</td>
<td>Trisyllabic (3-syll)</td>
<td>TD</td>
<td>–48.02</td>
<td>–48.97</td>
<td>0.95**</td>
<td>–43.12</td>
<td>–43.51</td>
</tr>
<tr>
<td></td>
<td>TB</td>
<td>–38.65</td>
<td>–40.05</td>
<td>1.40**</td>
<td>–30.89</td>
<td>–31.48</td>
<td>0.59**</td>
</tr>
<tr>
<td></td>
<td>TT</td>
<td>–23.41</td>
<td>–24.73</td>
<td>1.32**</td>
<td>–21.68</td>
<td>–22.07</td>
<td>0.39**</td>
</tr>
<tr>
<td></td>
<td>Monosyllabic (1-syll)</td>
<td>TD</td>
<td>–46.67</td>
<td>–46.93</td>
<td>0.26</td>
<td>–42.08</td>
<td>–42.61</td>
</tr>
<tr>
<td></td>
<td>TB</td>
<td>–36.17</td>
<td>–36.81</td>
<td>0.64*</td>
<td>–29.54</td>
<td>–30.38</td>
<td>0.84**</td>
</tr>
<tr>
<td></td>
<td>TT</td>
<td>–20.35</td>
<td>–20.62</td>
<td>0.27</td>
<td>–20.09</td>
<td>–20.60</td>
<td>0.51**</td>
</tr>
</tbody>
</table>

Note. F = front; B = back; MD = mean difference; TT = tongue tip; TB = body; TD = dorsum. The values under F, B are negative because the origin of the coordinate system is approximately at the subject’s upper incisors with receivers positions farther inside the mouth represented with progressively decreasing values.
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Balancing Gradience and Discreteness

• Symbolic theories cannot account for gradient effects simply be appealing to gradient knowledge over discrete elements.
  – Gradience pervades cognition

• However, symbolic representations have proven to be powerful tools for explaining a wide variety of empirical data
  – Symbolic structures are the cornerstone of theories of cognition
Subsymbolic Optimization-Quantization

• Principles of phonological knowledge and its processing
  – Symbolic structures are realized as vectors within a continuous activation space.
  – Well-formedness of symbolic structures is specified by a set of violable constraints.
  – Computation is the interleaving of stochastic optimization and discretization processes.

• Allows for gradient symbolic computation
  – Gradient knowledge of how symbolic structures recombine
  – Gradient content of symbolic structures

• Strengthens both psycholinguistic and linguistic accounts.
Thanks!

• Paul Smolensky; Don Mathis

• Helpful comments
  – Northwestern: SoundLab, Phonatics groups

• Research support
  – National Science Foundation CAREER Grant BCS0846147

• Papers, simulation package for Gradient Symbol Processing
  http://faculty.wcas.northwestern.edu/matt-goldrick/gsp