Online Appendix for "A New Empirical Method for Valuing Product Variety"

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Online Appendix: Theory

1 Symmetric Model of Demand

Symmetric Discrete Choice Model

We consider a population of statistically identical and independent consumers indexed by i of mass unity who choose to purchase a single product $j \in \{1, ..., J\}$ or the outside option $j = 0$. The number of products in the market, J, is our measure of product variety.

Preferences. The indirect utility of individual i who purchases product j is given by:

$$u_{ij}(y_i, p_j) = \alpha(y_i - p_j) + \delta_j + (1 - \sigma)\nu_i + \sigma\varepsilon_{ij} \tag{21}$$

where y_i is the consumer's income, p_j is the price of good j, δ_j is a firm-level characteristic and $(1-\sigma)\nu_i + \sigma\varepsilon_{ij}$ is an idiosyncratic match value between consumer i and product j, which captures heterogeneity in tastes across consumers and products. The utility of individual i who chooses the outside option is given by $u_{i0} = \alpha y_i + \varepsilon_{i0}$. Although linearity in price in equation (21) seems like a special case, Nevo (2011) shows that it may rationalizable by a quasi-linear utility function (no income effects). Specifying a distribution for the random utility shocks $(\nu_i, \varepsilon_{ij})$ gives rise to different models of discrete choice including probit, Logit, Nested Logit or any from the Generalized Extreme Value (GEV) family.[1]

[1] See Train (2003).

In general, we assume that for $j \neq 0$, the random utility shocks (ε_{ij}) are identical and independently (continuously) distributed (i.i.d.) and independent of (ν_i), but we allow ε_{i0} to be correlated with ν_i. In section 3, we relax the i.i.d. assumption and allow for general distributions of the random utility shocks.

Demand. Given the indirect utility function in equation (21), assuming there is 0 probability of ties, we may define the demand for product j as

$$q_j(p_1, \ldots, p_J, J) = \mathbb{P}\left(u_{ij}(y_i, p_j) = \max_{j' \in \{0,\ldots,J\}} u_{ij'}(y_i, p_{j'})\right). \tag{22}$$

Imposing symmetry, $\delta_j = \delta$ and $p_j = p$ for all $j = 1 \ldots J$, we may express aggregate demand (for all products excluding the outside good) when J varieties are available as:

$$Q(p, J) = \sum_{j=1}^{J} q_j(p, J). \tag{23}$$

Similarly, define $P(Q, J)$ to be the inverse aggregate demand corresponding to $Q(p, J)$.[2]

Symmetric Continuous Choice Model

We now introduce a class of continuous choice models.

Preferences. Let the representative consumer's utility function given by

$$u_J(q_1, \ldots, q_J, m) = h_J(q_1, \ldots, q_J) + m$$

for any $h_J : \{1, \ldots, J\} \to \mathbb{R}$ which is symmetric in all its arguments, continuously differentiable, strictly quasi-concave and $h(0, \ldots, 0) = 0$ and where the linear good m is interpreted as money.

Demand. The consumer's problem is

$$\max u_J(q_1, \ldots, q_J, m) = h_J(q_1, \ldots, q_J) + m \tag{24}$$
$$\text{subject to } m + \sum_{j=1}^{J} p_j q_j = y.$$

When the consumer is facing symmetric prices $p_j = p$ for all j, we can transform the problem as follows. Define $H_J(Q) = h_J\left(\frac{Q}{J}, \ldots, \frac{Q}{J}\right)$ where we interpret Q as aggregate demand.

[2]Given symmetry in preferences (the quality parameter δ does not depend on i or j) the symmetry in prices $p_j = p$ is motivated by assuming symmetric firms and a symmetric price equilibrium.

The new problem then is given by

$$u^*(p, J, y) = \max_Q H_J(Q) + y - pQ.$$

From the first-order condition, we obtain the family of inverse demands $P(Q, J) = H'_J(Q)$. Furthermore, it is easy to see that given the optimal aggregate quantity $Q(p, J)$ for price p, the strict quasi-concavity of h_J implies the consumer chooses symmetric quantities $q_j = \frac{Q}{J}$ for all j in the original problem.

Furthermore, none of the assumptions on utility are too restrictive. We show that for any family of downward sloping aggregate demands there exists a utility function $u_J : \mathbb{R}^{J+1} \to \mathbb{R}$ satisfying the conditions above that rationalize the aggregate demands. Let $P(Q, J)$ be continuously differentiable and strictly decreasing in Q. Let H be any antiderivative $\int P(Q, J)dQ$, which exists because $P(Q, J)$ is differentiable. Then, for some $\rho \in (0, 1)$, the following is a strictly quasi-concave direct utility function that rationalizes $P(Q, J)$ for integer J when all prices p_j in the market are equal:

$$u(q_1, \ldots, q_J, m) = H\left(\left(J^{\rho-1}\sum_{j=1}^{J} q_j^{\rho}\right)^{\frac{1}{\rho}}\right) + m.$$

Furthermore, we can make sense of J as a continuous variable if we permit a continuum of varieties $q : [0, J] \to \mathbb{R}$ and let

$$u_J(q, m) = H\left(\left(\int_0^J J^{\rho-1} q^{\rho}(j)dj\right)^{\frac{1}{\rho}}\right) + m.$$

Consumer Surplus

In the discrete choice model we define consumer surplus as the expected maximum utility normalized by the marginal utility of income:

$$CS(p_1, \ldots, p_J, J) = \frac{1}{\alpha}\mathbb{E}\left[\max_{j\in\{0,\ldots,J\}} u_{ij}(y_i, p_j)\right]$$

while for the continuous choice model consumer surplus is given by the indirect utility function $u^*(p, J, y) = H_J(Q(p, J)) + y - p * Q(p, J)$.

2 Proofs of Claims, Propositions and Theorems

Lemma 1. *The variety effect can be equivalently represented as*

$$\Lambda(J) = \int_p^\infty \frac{\partial Q(s,J)}{\partial J} ds = \int_0^Q \frac{\partial P(t,J)}{\partial J} dt + O(dJ) \tag{25}$$

where $O(dJ)$ is a term which goes to zero at the same rate as dJ.

Proof. We now prove first equality in Equation (25) holds. We focus on the expression for variety effect. With Taylor expansion,

$$Q(J_1, s) - Q(J_0, s) = \frac{\partial Q(J,s)}{\partial J}|_{J=J_0} * (J_1 - J_0) + o((J_1 - J_0)^2).$$

Let $\Delta J = J_1 - J_0$, plug into the expression of variety effect,

$$\begin{aligned}\Lambda(J) &= \frac{1}{\Delta J}\int_{p_0}^\infty [Q^{J_1}(s) - Q^{J_0}(s)]ds \\ &= \frac{1}{\Delta J}\int_{p_0}^\infty [\frac{\partial Q(J,s)}{\partial J}|_{J=J_0} * \Delta J + (\Delta J)^2 O(1)]ds \\ &= \int_{p_0}^\infty [\frac{\partial Q(J,s)}{\partial J}|_{J=J_0} + (\Delta J) O(1)]ds.\end{aligned}$$

where the first equality holds by definition, the second equality holds by plugging in Taylor expansion, and third equality holds by moving $1/\Delta J$ into the integration.
When $J_1 \to J_0$, $\Delta J \to 0$, then $\Lambda(Q,J) \to \int_{p_0}^\infty [\frac{\partial Q(J,s)}{\partial J}|_{J=J_0}]ds$. This gives the first equality in equation (5).
We now prove second equality in equation (25) holds when ΔJ is sufficiently small. Let $Q^* = Q(p_0, J_1)$, with integration by parts,

$$\begin{aligned}\int_{p_0}^\infty Q(s,J_1)ds &= \left[sQ^{J_1}(s)\right]_{p_0}^\infty - \int_{Q(p_0,J_1)}^{Q(\infty,J_1)} P(t,J_1)dt \\ &= -p_0 Q(p_0,J_1) + \int_0^{Q^*} P(t,J_1)dt \\ &= \int_0^{Q^*} [P(t,J_1) - p_0]dt.\end{aligned}$$

Similarly,

$$\int_{p_0}^\infty Q(t,J_0)ds = -p_0 Q(p_0,J_0) + \int_0^{Q_0} P(t,J_0)dt = \int_0^{Q_0} [P(t,J_0) - p_0]dt.$$

Substitute into the expression of variety effect,

$$\begin{aligned}
\Lambda(J) &= \frac{1}{\Delta J}\int_{p_0}^{\infty}[Q(J_1,s)-Q(J_0,s)]ds \\
&= \frac{1}{\Delta J}\left\{\int_0^{Q^*}[P(t,J_1)-p_0]dt-\int_0^{Q_0}[P(t,J_0)-p_0]dt\right\} \\
&= \frac{1}{\Delta J}\left\{\int_0^{Q_0}[P(t,J_1)-p_0]dt-\int_0^{Q_0}[P(t,J_0)-p_0]dt-\int_{Q_0}^{Q^*}[P(t,J_1)-p_0]dt\right\} \\
&= \frac{1}{\Delta J}\int_0^{Q_0}[P(t,J_1)-P(t,J_0)]dt-\frac{1}{\Delta J}\int_{Q_0}^{Q^*}[P(t,J_1)-p_0]dt \\
&= \frac{1}{\Delta J}\int_0^{Q_0}[\frac{\partial P(J,t)}{\partial J}|_{J=J_0}*\Delta J+(\Delta J)^2 O(1)]dt-\frac{1}{\Delta J}\int_{Q_0}^{Q^*}[P(t,J_1)-p_0]dt \\
&= \underbrace{\int_0^{Q_0}[\frac{\partial P(J,t)}{\partial J}|_{J=J_0}+(\Delta J)O(1)]dt}_{\text{first term in equation (5)}}-\underbrace{\frac{1}{\Delta J}\int_{Q^*}^{Q_0}[p_0-P(t,J_1)]dt}_{\text{second term in equation (5)}}.
\end{aligned}$$

The second to last equality holds by Taylor expansion,

$$P(J_1,s)-P(J_0,s)=\frac{\partial P(J,s)}{\partial J}|_{J=J_0}*(J_1-J_0)+(J_1-J_0)^2 O(1).$$

The last expression holds by moving $1/\Delta J$ into the integration.

When $J_1 \to J_0$, $\Delta J \to 0$, then the first term

$$\int_0^{Q_0}[\frac{\partial P(J,t)}{\partial J}|_{J=J_0}+(\Delta J)O(1)]dt \to \int_0^{Q_0}[\frac{\partial P(J,t)}{\partial J}|_{J=J_0}]dt.$$

This gives the expression in equation (25).

We now prove the second term in equation (5) is $O(dJ)$ as $J_1 \to J_0$.

$$
\begin{aligned}
\frac{1}{\Delta J}\int_{Q^*}^{Q_0}[p_0 - P(t,J_1)]dt &= \frac{1}{\Delta J}\int_{Q^*}^{Q_0}[p_0 - P(t,J_1)]dt \\
&= \frac{-1}{\Delta J}\int_{Q^*}^{Q_0}[P(t,J_1) - P(Q^*,J_1)]dt \\
&= \frac{1}{\Delta J}\int_{Q^*}^{Q_0}[\frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*} * (t-Q^*) + (t-Q^*)^2 O(1)]dt \\
&= \frac{1}{\Delta J}\left\{\frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*}\int_{Q^*}^{Q_0}(t-Q^*)dt - O(1)\int_{Q^*}^{Q_0}(Q^*-t)^2 dt\right\} \\
&= \frac{1}{\Delta J}\left\{\frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*}\frac{(Q^*-Q_0)^2}{2} + O(1)\frac{(Q^*-Q_0)^3}{3}\right\} \\
&= \frac{1}{\Delta J}\left\{\frac{1}{2}\frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*} + \frac{1}{3}O(1)(\frac{\partial Q(J,p_0)}{\partial J}|_{J=J_0} * (J_1-J_0) + (J_1-J_0)^2 O(1))\right\} \\
&\times (\frac{\partial Q(J,p_0)}{\partial J}|_{J=J_0} \times (J_1-J_0) + (J_1-J_0)^2 O(1))^2 \\
&= [\frac{1}{2}\frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*} + \frac{1}{3}O(1)(\frac{\partial Q(J,p_0)}{\partial J}|_{J=J_0}(\Delta J) + (\Delta J)^2 O(1))] \\
&\times [\frac{\partial Q(J,p_0)}{\partial J}|_{J=J_0} + (\Delta J)O(1)]^2 \Delta J \\
&\to 0.
\end{aligned}
$$

The first equality holds by definition.

The second equality holds by definition of Q^*.

The third equality doing Taylor expansion for $P(t,J_1) - P(Q^*,J_1)$ around Q^*.

$$
\begin{aligned}
P(t,J_1) - P(Q^*,J_1) &= \frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*} \times (t-Q^*) + O((t-Q^*)^2) \\
&= \frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*} \times (t-Q^*) + (t-Q^*)^2 O(1)
\end{aligned}
$$

The fourth equality holds by moving constant out of the integral. $\frac{\partial P(Q,J_1)}{\partial Q}|_{Q=Q^*}$ is a constant because when doing the expansion the derivative $\frac{\partial P(Q,J_1)}{\partial Q}$ takes value at $Q = Q^*$.

The fifth equality holds by calculating the integral.

The sixth equality holds by doing Taylor expansion for $Q^* - Q_0$ around J_0. This is because $Q^* = Q(p_0, J_1)$, $Q_0 = Q(p_0, J_0)$.

$$
\begin{aligned}
Q^* - Q_0 &= Q(p_0,J_1) - Q(p_0,J_0) \\
&= \frac{\partial Q(J,p_0)}{\partial J}|_{J=J_0} * (J_1-J_0) + (J_1-J_0)^2 O(1)
\end{aligned}
$$

The seventh equality holds by simplifying terms with ΔJ. The limit is zero because both $\frac{\partial P(Q,J_1)}{\partial Q}$ and $\frac{\partial Q(J,p_0)}{\partial J}$ are finite numbers. $\frac{\partial P(t,J_1)}{\partial t}$ is the slope of demand curve, $\frac{\partial Q(J,p_0)}{\partial J}$ is quantity change when variety changes holding price unchanged. We have thus proved $\lim_{\Delta J \to 0} \frac{1}{\Delta J} \int_{Q^*}^{Q_0} [p_0 - P(t, J_1)]dt = O(dJ)$ □

Proof of Corollary 1

Proof. Average change in willingness to pay for infra-marginal units is:

$$\overline{\frac{\partial P}{\partial J}}(Q, J) = \frac{1}{Q} \int_0^Q \frac{\partial P}{\partial J}(s, J)ds.$$

Integrating by parts we get:

$$\Lambda = \int_p^\infty \frac{\partial Q}{\partial J}(s, J)ds = \int_0^Q \frac{\partial P}{\partial J}(s, J)ds = Q\overline{\frac{\partial P}{\partial J}}.$$

□

Proof of Theorem 1

Proof. By Assumption 1, for some J, J' inverse aggregate demands are parallel, $\frac{\partial P}{\partial Q}(Q, J) = \frac{\partial P}{\partial Q}(Q, J')$, then there exists d such that $P(Q, J) = P(Q, J') + d$, the variety effect is given by

$$\Lambda(J, J') = \int_0^{Q(p,J)} P(s, J)ds - \int_0^{Q(p,J')} P(s, J')ds = Q(p, J) * d - \int_{Q(p,J)}^{Q(p,J')} P(s, J')ds.$$

Taking the limit as $J' \to J$ then $d \to \frac{\partial P}{\partial J}(Q, J)$, furthermore $\frac{\partial P}{\partial J}(Q, J)$ is constant so

$$\overline{\frac{\partial P}{\partial J}}(Q, J) = \frac{\partial P}{\partial J}(Q, J) = \frac{dP(Q(J), J)}{dJ} - \frac{\partial P}{\partial Q}(Q, J)\frac{dQ(J)}{dJ} = \left(\frac{dP}{dQ} - \left.\frac{dP}{dQ}\right|_J\right)\frac{dQ}{dJ}$$

where $\frac{dP}{dQ} = \frac{dP(Q(J),J)}{dJ} / \frac{dQ}{dJ}$. □

3 Revealed Preference Approach with Asymmetric Products

This section derives an expression for the variety effect in the case where products are non-symmetric. The following proposition contains an expression that generalizes the integral form of the variety effect in Definition 2 to the case of asymmetric demands and prices.[3]

Lemma. *Consider the case where the number of products goes from J to M (with $M > J$). The variety effect is equal to :*

$$\Lambda = \int_0^\infty \sum_{j=J+1}^{M} q_j(\mathbf{p_J}, p_{J+1} + s, p_{J+2} + s, \ldots, p_M + s)ds$$

where $\mathbf{p_J} = (p_1, p_2, ..., p_J)$ are the prices for the existing J products, and $(p_{J+1}, \ldots, p_M)$ are the prices at which the new $M - J$ products are introduced.

Proof. We have $\frac{\partial CS(p_1,\ldots,p_J,J)}{\partial p_j} = -q_j(p_1, \ldots, p_J, J)$. Therefore, consider the case where the number of products goes from J to M (with $M > J$), where$\mathbf{p_J} = (p_1, p_2, ..., p_J)$ are the prices for the existing J products, and $(p_{J+1}, \ldots, p_M)$ are the prices at which the new $M - J$ products are introduced. Let $\mathbf{p_M} = (p_{1,} \ldots, p_M)$ and $\mathbf{p'_M} = (\mathbf{p_J}, \infty, \ldots, \infty)$ then evaluating the line integral:

$$\begin{aligned} \Lambda &= CS(\mathbf{p_M}, M) - CS(\mathbf{p_J}, J) = CS(\mathbf{p_M}, M) - CS(\mathbf{p'_M}, M) \\ &= \int_0^\infty \sum_{j=J+1}^{M} q_j(\mathbf{p_J}, p_{J+1} + s, p_{J+2} + s, \ldots, p_M + s)ds \end{aligned}$$

To complete the proof, we only need to show $CS(p_J, J) = CS(p'_M, M)$.
We claim that under price $p'_M = (p_J, \infty, \ldots, \infty)$, the choice bundle $\{q_j(p_J, J)\}_{j=1}^J$ is the same as $\{q_j(p'_M, M)\}_{j=1}^M$. The problem can be solved with two-stage budgeting: Step 1, allocate income y to products 1 to J, denoted by y_1, and J to M, denoted by y_2. Step 2, allocate y_1 within 1 to J products, and allocate y_2 within J to M products. Notice that the price of J to M products are infinity, optimization gives $y_1 = y$, $y_2 = 0$. The allocation between 1 to J with income $y_1 = y$ in step 2 when there are M products is thus the same as the allocation between 1 to J with income $y_1 = y$ in when there are J products. The arguments above prove that

$$q_j(p'_M, M) = \begin{cases} q_j(p_J, J) & \forall j = 1, \ldots, J \\ 0 & \forall j = J + 1, \ldots, M \end{cases}.$$

[3]This holds for both continuous and discrete choice as long as preferences are quasi-linear in money. In the case where there are income effects the analogous expression holds by substituting the Marshallian demands with Hicksian compensated demands. For a derivation in the discrete choice case when only one variety is introduced and income effects are allowed see Bhattacharya (2015).

Thus $Q(p'_M, M) = \sum_{j=1}^{M} q_j(s, M)ds = \sum_{j=1}^{J} q_j(s, J)ds = Q(p_J, J)$.
Then we show $CS(p_J, J) = CS(p'_M, M)$.

$$CS(p'_M, M) = \int_{p'_M}^{\infty} Q(s, M)\mathrm{ds} = \int_{p_J}^{\infty} Q(p_J, J)ds = CS(p_J, J).$$

□

The intuition is that the welfare gain from the new varieties can be calculated by integrating the demand for new varieties.[4] This result implies that if we non-parametrically identify the demand curves before and after the change in variety, we can exactly compute the variety effect. More often, in practice the econometrician does not have enough information to identify the demands non-parametrically (e.g. this would require to observe prices for which the demand of some variety is arbitrarily close to 0 but these prices might never show up in the data), however the econometrician always has information that is local to the market equilibrium. The main objective in what follows is to give plausible and parsimonious sufficient conditions to identify the variety effect using reduced-form methods based on local information.

Similar to the symmetric case, the variety effect can also be stated in terms of aggregate demands as:

$$\Lambda = \int_0^{\infty} Q_M(\mathbf{p_M} + s\mathbf{1_M})ds - \int_0^{\infty} Q_J(\mathbf{p_J} + s\mathbf{1_J})ds$$

where $\mathbf{1_K}$ is a $K-$dimensional vector of ones, and $\mathbf{p_M} = (\mathbf{p_J}, p_{J+1}, \ldots, p_M)$.

Next, we assume that there exists some price index d such that $Q_M(\mathbf{p_M} + (s + d)\mathbf{1_M}) = Q_J(\mathbf{p_J} + s\mathbf{1_J})$ for all $s \in \mathbb{R}$.[5] In other words, increase prices starting from $\mathbf{p_M}$ by some constant amount d until total quantity demanded equals quantity demanded when there are J products in the market. Under this assumption, it follows that:

$$\Lambda = \int_0^{d} Q_J(\mathbf{p_J} - s\mathbf{1_J})ds.$$

By the mean value theorem for integrals, there exists $d' \in [0, d]$ such that

$$\Lambda = \int_0^{d} Q_J(\mathbf{p_J} - s\mathbf{1_J})ds = d * Q_J(\mathbf{p_J} - d'\mathbf{1_J}).$$

[4] Notice that the prices of the old varieties are kept fixed, while the integral is taken over uniform increases in the price of the new varieties.

[5] This is related to the price index in Feenstra (1994). However, in Feenstra (1994), the price index is defined as the (common) price reduction that would have to occur when there are J goods in the market in order to give the same *utility* as when there are M goods.

Intuitively, as $M \to J$ we have $d' \to d$ and we obtain a first order approximation for the variety effect:

$$\Lambda \approx d * Q_J(\mathbf{p_J}).$$

We summarize these observations in the following theorem.

Theorem. *Assume that for all M and J there exists some d such that $Q_M(\mathbf{p_M}+(s+d)\mathbf{1}_{\boldsymbol{M}}) = Q_J(\mathbf{p_J}+s\mathbf{1}_{\boldsymbol{J}})$ for all $s \in \mathbb{R}$. Then there exists $d' \in [0,d]$ such that*

$$\Lambda = d * Q_J(\mathbf{p_J} - d'\mathbf{1}_{\boldsymbol{J}}). \tag{26}$$

Furthermore, let $\mathbf{p}_M^1 = \mathbf{p}_M + s\mathbf{1_M}$, $\Delta P = s$, and $\Delta Q = Q_M(\mathbf{p_M^1}) - Q_J(\mathbf{p_J})$ then

$$d = \left(\frac{\Delta P}{\Delta Q} - \left.\frac{dP}{dQ_J}\right|_J\right)\Delta Q + o\left((s-d)^2\right) \tag{27}$$

where $\left.\frac{dP}{dQ_J}\right|_J = \left.\left(\frac{dQ_J(\mathbf{p_J}+t\mathbf{1_J})}{dt}\right)^{-1}\right|_{t=0}$.

Proof. Observe by assumption $Q_M(\mathbf{p_M^1}) = Q_J(\mathbf{p_J}+(s-d)\mathbf{1_J})$, then the second part of the theorem follows directly from the first-order Taylor approximation:

$$Q_M(\mathbf{p_M^1}) = Q_J(\mathbf{p_J}) + (s-d)\frac{dQ_J(\mathbf{p_J}+t\mathbf{1_J})}{dt} + o\left((s-d)^2\right)$$

where $\frac{dQ_J(\mathbf{p_J}+t\mathbf{1_J})}{dt}$ is the directional derivative in the direction $\mathbf{1_J}$. □

We now have an expression for the variety effect in (26) that is similar to equation (4), and an expression for the average change in the willingness-to-pay (27) similar to (7), therefore Theorem 2 is a general version of Theorem 1.

Several features of the Theorem are worth highlighting. First, observe we defined $\mathbf{p}_M^1 = \mathbf{p}_M + s\mathbf{1_M}$, this is equivalent to assuming that all prices adjust uniformly after the introduction of the new varieties. Although it seems restrictive, having prices adjust in the same direction $\mathbf{1_J}$ as the vertical shift d allows us to identify d by a simple application of the Taylor approximation.[6]Suppose that this is not the case and let $\mathbf{v} = \mathbf{p}_M^1 - \mathbf{p}_M$ and observe for some

[6]A class of models for which we obtain symmetric pass-through is that of linear-quadratic revenues with constant marginal costs and Bertrand pricing. However, it might be unreasonable to assume uniform price adjustments when, e.g., there is one large incumbent with several products and several small entrants with one product each.

$s \in [\min v_j, \max v_j]$ then $Q_M(\mathbf{p}_M + s\mathbf{1_M}) = Q_M(\mathbf{p}_M^1)$. Therefore

$$d = \left(\frac{\Delta P}{\Delta Q} - \left. \frac{dP}{dQ_J} \right|_J \right) \Delta Q + o\left((s-d)^2\right) \tag{28}$$

still holds for $\Delta P = s$. Furthermore, in practice it is easy to approximate s by the average price change $\frac{1}{M}\sum_{j=1}^{M} v_j$. In this sense the sufficient statistics formula is robust to the assumption of long-run uniform pass-through. However, the assumption of short-run uniform pass-through is crucial, since we use it to calculate the directional derivative $\left.\frac{dQ_J}{dP}\right|_J = \frac{dQ(\mathbf{p_J}+t\mathbf{1_J})}{dt} = \sum_{j=1}^{J} \frac{\partial Q_J}{\partial p_j}$.

Second, we interpret the directional derivative $\left.\frac{dQ_J}{dP}\right|_J = \frac{dQ(\mathbf{p_J}+t\mathbf{1_J})}{dt} = \sum_{j=1}^{J} \frac{\partial Q_J}{\partial p_j}$ as the short-run slope of aggregate demand in the direction of uniform price changes, that connects the interpretation of (27) with equation (7) in the symmetric model, namely that d can be captured using the difference in the slopes of inverse demands when variety changes and when variety is fixed:

$$d = \left(\frac{dP}{dQ} - \left. \frac{dP}{dQ_J} \right|_J \right) dQ. \tag{29}$$

Furthermore, if we observe the change in aggregate demand Q_J when all prices are increased simultaneously, we do not need to estimate each partial derivative separately and we can directly estimate $\left.\frac{dQ_J}{dP}\right|_J$. When we take the model to the data, we assume a tax change is passed on to the consumers symmetrically across products, therefore the tax inducing the uniform price change that allows to estimate the total derivative $\left.\frac{dQ_M}{d\mathbf{p}}\right|_M$ without having to estimate the partial derivatives.

Finally, the most important assumption in Theorem 2 was the existence of the price index d such that $Q_M(\mathbf{p_M} + (s+d)\mathbf{1}_{\boldsymbol{M}}) = Q_J(\mathbf{p_J} + s\mathbf{1}_{\boldsymbol{J}})$ for all s. To understand the result in terms of inverse demands, observe that for any $Q \in \mathbb{R}^+$ there is a unique s such that $Q_J(\mathbf{p^J}+s\mathbf{1_J}) = Q$, this implicitly defines the inverse aggregate demand $P(Q, J)$. Furthermore, $Q_M(\mathbf{p_M}+(s+d)\mathbf{1}_{\boldsymbol{M}}) = Q_J(\mathbf{p_J}+s\mathbf{1}_{\boldsymbol{J}})$ is equivalent to $P(Q, J)+d = P(Q, M)$ for all $Q \in \mathbb{R}^+$, which shows that the inverse aggregate demands are parallel. Moreover, by theorem 3, the price index d is equal to the average change in willingness-to-pay.

4 Extensions

4.1 Probabilistic Entry

In this section, we extend the symmetric firm model to allow for probabilistic entry.[7] We assume that nature draws a fixed cost. Let the equilibrium price and quantity functions be given respectively by $p(J)$ and $q(J)$. Assume that for every draw of the fixed cost, there is a uniquely determined number J of firms that enter the market. Then the distribution of fixed costs determines an equilibrium distribution F of variety J and consumer surplus from an ex ante perspective is given by:

$$CS = \int \int_p^\infty Q(s, J) ds dF(J) \tag{30}$$

Moreover, when there is an exogenous change in variety from the distribution F_1 to F_2 we may calculate, for the discrete case, that the variety effect is:

$$\Lambda = \int \int_p^\infty Q(s, J) ds dF_2(J) - \int \int_p^\infty Q(s, J) ds dF_1(J) \tag{31}$$

Suppose there exists $d(J_2, J_1)$ such that $Q_{J_2}(p + (s + d(J_2, J_1)) = Q_{J_1}(p)$ for all s. Let the conditional distribution of new variety be given by $F_{2|1}(J_2|J_1)$. Then,

$$\begin{aligned}
\Lambda &= \int \int \left[\int_p^\infty Q(s, J_2) ds - \int_p^\infty Q(s, J_1) ds \right] dF_{2|1}(J_2|J_1) dF_1(J_1) \\
&= \int \int \left[\int_0^{d(J_2, J_1)} Q(p + s, J_2) ds \right] dF_{2|1}(J_2|J_1) dF_1(J_1) \\
&= \int \int \left[\int_0^{d(J_2, J_1)} Q(p - d(J_2, J_1) + s, J_1) ds \right] dF_{2|1}(J_2|J_1) dF_1(J_1) \\
&\approx \int \int d(J_2, J_1) dF_{2|1}(J_2|J_1) Q(p, J_1) dF_1(J_1) \\
&= E[d(J_2, J_1) * Q(p, J_1)].
\end{aligned}$$

Thus, we obtain the familiar formula for the variety effect in terms of the product of the aggregate demand and the expected vertical shift of the inverse demand, the second of which

[7] In light of proposition 1 everything that follows goes through for both the continuous and discrete choice models.

is the average change in willingness to pay. Similarly, letting the bars denote expectations:

$$
\begin{aligned}
d\bar{Q} &= E_{F_2}(Q(p(J), J) - E_{F_1}(Q(p(J), J)) \\
&= E_{F_2}(Q(p(J) + d(J, J_0), J_0) - E_{F_1}(Q(p(J) + d(J, J_0), J_0)) \\
&\approx E_{F_2 - F_1}\left[\frac{\partial Q}{\partial p}(p_0, J_0) * (p(J) + d(J, J_0) - p_0)\right] \\
&= \frac{\partial Q}{\partial p}(p_0, J_0) E_{F_2 - F_1}(p(J) + d(J, J_0)) \\
&= \frac{\partial Q}{\partial p}(p_0, J_0)(d\bar{p} + E(d)).
\end{aligned}
$$

Therefore $E(d) = \left(\frac{dp}{dQ}|_J - \frac{d\bar{p}}{d\bar{Q}}\right) d\bar{Q}$ corresponds to the probabilistic version of $d \approx \left(\frac{dP}{dQ} - \frac{dP}{dQ}\Big|_J\right) dQ$.

4.2 Outside markets and Parallel Inverse Demands

A concern with using short-run and long-run price elasticities of demand to identify the variety effect is that differences between the two may conflate changes in the level of variety with changes that might occur for other reasons, such as adjustment costs. In this section, we extend the model by incorporating an outside market represented by the variable y and we assume the consumer can adjust y in response to price and variety changes. We show that, under certain conditions, the sufficient statistics formula is robust to the existence of this outside market by an application of the envelope theorem.

We start from a continuous choice model where all firms in the inside market are symmetric, and let p be the symmetric equilibrium price of the inside market, and Q the aggregate quantity. Let $u(Q, y, J) - pQ$ be the utility function of the consumer and assume u is supermodular and quasi-concave. Let

$$Q^*(y, p, J) = \underset{Q}{\operatorname{argmax}}\, u(Q, y, J) - pQ$$

be the aggregate demand of the inside good conditional on (p, y, J), and let

$$y(p, J) = \underset{y}{\operatorname{argmax}}\, u(Q^*(y, p, J), y, J) - pQ^*(y, p, J)$$

be the optimal choice of y given (p, J). Finally, define the aggregate demand where variety is allowed to vary to be $Q(J) = Q^*(y(p(J), J), p(J), J)$.

Observe the change in "variable-variety" aggregate demand for the inside market given an exogenous change in variety J has three components:

$$\frac{dQ(J)}{dJ} = \frac{\partial Q^*}{\partial p}\frac{dp(J)}{dJ} + \frac{\partial Q^*}{\partial y}\frac{dy(p(J),J)}{dJ} + \frac{\partial Q^*}{\partial J}, \tag{32}$$

the indirect effect of variety through equilibrium price p, the indirect effect of variety through the outside variable y, and the direct effect of variety J.

Assume the following parallel inverse demands condition:

Assumption. *(Parallel Inverse demands) For all* J *and all* y *there exists* d *such that* $Q(y(p,J),p,J) = Q(y(p+d,J_0),p+d,J_0)$ *for all* p. d *is the vertical distance between two fixed-variety demand curves with variety* J *and* J_0 *separately.*

Let J_1 be the variety after changes. Under this assumption,

$$\begin{aligned} dQ = Q(J_1) - Q(J_0) &= Q(y(p_1+d,J_0),p_1+d,J_0) - Q(y(p_0,J_0),p_0,J_0) \\ &\approx \frac{\partial Q^*}{\partial p}|_{p=p_0}\,(p_1+d-p_0) \\ &= \frac{\partial Q^*}{\partial p}|_{p=p_0}\,(dp+d)\,. \end{aligned}$$

So we can calculate the vertical shift

$$d \approx \left(\frac{dp}{dQ^*} - \frac{dp}{dQ}\right) * dQ. \tag{33}$$

Define the indirect utility function

$$w(y,p,J) = u(Q^*(y,p,J),y,J) - pQ^*(y,p,J),$$

and note that in a variable-variety equilibrium, welfare is $v(J) = w(y^*(p(J),J),p(J),J)$ from the consumer perspective. Taking the first-order derivatives yields the following:

$$\frac{dv(J)}{dJ} = \underbrace{\frac{\partial w}{\partial p}\frac{dp}{dJ}}_{\text{price effect}} + \underbrace{\frac{\partial w}{\partial J}}_{\text{variety effect}} + \frac{\partial w}{\partial y}\frac{dy}{dJ} = -Q\frac{dp}{dJ} + \Lambda.$$

where the second equality holds by the envelope theorem.[8]

Theorem. *Under the parallel inverse demand assumption,*

$$-Q * d \approx \Lambda dJ, \tag{34}$$

so

$$dv(J) \approx -Q * (dp + d). \tag{35}$$

The bias of approximation in equation (35) goes to zero when J is sufficiently small.

Proof. We know $Q(y(p_1, J_1), p_1, J_1) = Q_1$. The welfare when $J = J_1$ is

$$\begin{aligned} w(y(p_1, J_1), p_1, J_1) &= u(Q_1, p_1, J_1) - p_1 Q_1 \\ &= \int_0^{Q_1} [P(y(t, J_1), t, J_1) - p_1] dt \\ &= \int_{p_1}^{\infty} Q(y(s, J_1), s, J_1) ds. \end{aligned}$$

The first equality holds by definition.

The second equality holds by result of maximization.

$$Q_1 = argmax_Q \, u(Q, p, J) - pQ.$$

FOC gives,

$$\frac{\partial u}{\partial Q}(Q, y, J) = p(Q, y, J).$$

The third equality holds with integration by parts,

$$\begin{aligned} \int_0^{Q_1} [P(y(t, J_1), t, J_1) - p_1] dt &= -p_1 Q(y(p_1, J_1), p_1, J_1) + \int_0^{Q_1} P(y(t, J_1), t, J_1) dt \\ &= [sQ(y(p, J_1), p, J_1)]_{p_1}^{\infty} - \int_{Q(y(p_1, J_1), p_1, J_1)}^{Q(y(\infty, J_1), \infty, J_1)} P(y(t, J_1), t, J_1) dt \\ &= \int_{p_1}^{\infty} Q(y(s, J_1), s, J_1) ds. \end{aligned}$$

[8] By definition,

$$y(p(J), J) = \underset{y}{\operatorname{argmax}} \; u(Q(y(p(J), J), p, J), y, J) - pQ(y(p(J), J), p, J) = \underset{y}{\operatorname{argmax}} \; w(y(p(J), J), p, J).$$

Applying the envelop theorem we get

$$\frac{\partial w}{\partial y} = 0.$$

Similarly, we can derive expressions welfare when $J = J_0$. Since $Q(y(p_0, J_0), p_0, J_0) = Q_0$

$$\begin{aligned} w(y(p_0, J_0), p_0, J_0) &= u(Q_0, p_0, J_0) - p_0 Q_0 \\ &= \int_0^{Q_0} [P(y(t, J_0), t, J_0) - p_0] dt \\ &= \int_{p_0}^{\infty} Q(y(s, J_0), s, J_0) ds. \end{aligned}$$

The long term (when J changes from J_0 to J_1) welfare change $\Delta w = w(y(p_1, J_1), p_1, J_1) - w(y(p_0, J_0), p_0, J_0)$ is

$$\begin{aligned} \Delta w &= \int_{p_1}^{\infty} Q(y(s, J_1), s, J_1) - \int_{p_0}^{\infty} Q(y(s, J_0), s, J_0) ds \\ &= -\int_{p_0}^{p_1} Q(y(s, J_0), s, J_0) ds + \int_{p_1}^{\infty} Q(y(s, J_1), s, J_1) ds - \int_{p_1}^{\infty} Q(y(s, J_0), s, J_0) ds \\ &= \underbrace{-\int_{p_0}^{p_1} Q(y(s, J_0), s, J_0) ds}_{\text{welfare change due to price effect}} + \underbrace{\int_{p_1}^{\infty} [Q(y(s, J_1), s, J_1) - Q(y(s, J_0), s, J_0)] ds}_{\text{welfare change due to variety effect}}. \end{aligned}$$

Let $\Delta w_p = -\int_{p_0}^{p_1} Q(y(s, J_0), s, J_0) ds$, $\Delta w_v = \int_{p_1}^{\infty} [Q(y(s, J_1), s, J_1) - Q(y(s, J_0), s, J_0)] ds$.

We now show price effect $\frac{\Delta w_p}{\Delta J} \to -Q \frac{dp}{dJ}$ as $\Delta J \to 0$, which is the expression above equation (20) in the paper.

$$\begin{aligned} \Delta w_p &= -\int_{p_0}^{p_1} Q(y(s, J_0), s, J_0) ds \\ &= -(p_1 - p_0) Q(y(\hat{p}, J_0), \hat{p}, J_0) \\ &= -[\frac{dp}{dJ}|_{J=J_0} * (J_1 - J_0) + (J_1 - J_0)^2 O(1)] Q(y(\hat{p}, J_0), \hat{p}, J_0). \end{aligned}$$

The first equality holds by definition.

The second equality holds by applying mean value theorem, where $\hat{p} \in (p_0, p_1)$.

The third equality holds by doing Taylor expansion for $p_1 - p_0$ around J_0,

$$p_1 - p_0 = p(J_1) - p(J_0) = \frac{dp}{dJ}|_{J=J_0} * (J_1 - J_0) + (J_1 - J_0)^2 O(1).$$

Let $\Delta J = J_1 - J_0$. As $\Delta J \to 0$, $\hat{p} \to p_0$.

$$\frac{\Delta w_p}{\Delta J} = -[\frac{dp}{dJ}|_{J=J_0} + (\Delta J) O(1)] Q(y(\hat{p}, J_0), \hat{p}, J_0) \to -Q(y(p_0, J_0), p_0, J_0) \frac{dp}{dJ} = -Q_0 \frac{dp}{dJ}.$$

We now derive the expression for variety effect Λ. Let $Q^* = Q(y(p_1, J_0), p_1, J_0)$.

$$\begin{aligned}
\Delta w_v &= \int_{p_1}^{\infty} [Q(y(s, J_1), s, J_1) - Q(y(s, J_0), s, J_0)]ds \\
&= \int_{p_1}^{\infty} [Q(y(s + d, J_0), s + d, J_0) - Q(y(s, J_0), s, J_0)]ds \\
&= \int_{0}^{Q_1} P(y(t, J_0), t, J_0)dt - Q_1(p_1 + d) - \int_{0}^{Q^*} P(y(t, J_0), t, J_0)dt + p_1 Q^* \\
&= -\int_{Q_1}^{Q^*} [P(y(t, J_0), t, J_0) - p_1]dt - Q_1 d.
\end{aligned}$$

The first equality holds by definition.

The second equality holds by parallel inverse demand assumption

$$Q(y(s, J_1), s, J_1) = Q(y(s + d, J_0), s + d, J_0).$$

The third equality holds with integration by parts,

$$\begin{aligned}
\int_{p_1}^{\infty} Q(y(s + d, J_0), s + d, J_0)ds &= \int_{p_1+d}^{\infty} Q(y(s, J_0), s, J_0)ds \\
&= [sQ(y(s, J_0), s, J_0)]_{p_1+d}^{\infty} - \int_{Q(y(p_1+d,J_0),p_1+d,J_0)}^{Q(y(\infty,J_0),\infty,J_0)} P(y(t, J_0), t, J_0)dt) \\
&= -Q_1(p_1 + d) + \int_{0}^{Q_1} P(y(t, J_0), t, J_0)dt.
\end{aligned}$$

$$\begin{aligned}
\int_{p_1}^{\infty} Q(y(s, J_0), s, J_0)]ds &= \int_{p_1}^{\infty} Q(y(s, J_0), s, J_0)ds \\
&= [sQ(y(s, J_0), s, J_0)]_{p_1}^{\infty} - \int_{Q(y(p_1+d,J_0),p_1,J_0)}^{Q(y(\infty,J_0),\infty,J_0)} P(y(t, J_0), t, J_0)dt) \\
&= -p_1 Q + \int_{0}^{Q} P(y(t, J_0), t, J_0)dt.
\end{aligned}$$

The fourth equality holds by rearranging terms.

In the paper, welfare change due to variety effect is approximated by $-Q_0 * d$. The bias of this approximation is

$$bias = -\int_{Q_1}^{Q^*} [P(y(t, J_0), t, J_0) - p_1]dt - Q_1 d + Q_0 d.$$

We can prove the bias goes to zero as $\Delta J \to 0$.

$$
\begin{aligned}
bias &= -\int_{Q_1}^{Q^*}[P(y(t,J_0),t,J_0)-p_1]dt - Q_1 d + Q_0 d \\
&= -\int_{Q_1}^{Q^*}[(\frac{\partial P}{\partial y}\frac{\partial y}{\partial Q}+\frac{\partial P}{\partial Q})|_{Q=Q^*}(t-Q^*)+(t-Q^*)^2O(1)]dt-(Q_1-Q_0)d \\
&= \frac{1}{2}(\frac{\partial P}{\partial y}\frac{\partial y}{\partial Q}+\frac{\partial P}{\partial Q})|_{Q=Q^*}(Q_1-Q^*)^2+\frac{1}{3}(Q_1-Q^*)^3O(1)-(Q_1-Q_0)d \\
&= \frac{1}{2}(\frac{\partial P}{\partial y}\frac{\partial y}{\partial Q}+\frac{\partial P}{\partial Q})|_{Q=Q^*}(Q_1-Q^*)^2+\frac{1}{3}(Q-Q^*)^3O(1)-(Q_1-Q_0)^2(\frac{dp}{dQ^*}|_{p=p_0}-\frac{dp}{dQ}|_{p=p_0}) \\
&= \frac{1}{2}(\frac{\partial P}{\partial y}\frac{\partial y}{\partial Q}+\frac{\partial P}{\partial Q})|_{Q=Q^*}[(\frac{\partial Q}{\partial y}\frac{\partial y}{\partial J}+\frac{\partial Q}{\partial J})|_{J=J_0}(\Delta J)+(\Delta J)^2O(1)]^2 \\
&+ \frac{1}{3}[(\frac{\partial Q}{\partial y}\frac{\partial y}{\partial J}+\frac{\partial Q}{\partial J})|_{J=J_0}(\Delta J)+(\Delta J)^2O(1)]^3O(1) \\
&+ [(\frac{dQ}{dJ})|_{J=J_0}(\Delta J)+(\Delta J)^2O(1)]^2(\frac{dp}{dQ^*}|_{p=p_0}-\frac{dp}{dQ}|_{p=p_0}) \\
&\to 0
\end{aligned}
$$

The first equation holds by definition.

The second equation holds by doing Taylor expansion for $P(y(t,J_0),t,J_0)-p_1$ around Q^*,

$$P(y(t,J_0),t,J_0)-p_1 = P(y(t,J_0),t,J_0)-P(y(Q^*,J_0),Q^*,J_0) = (\frac{\partial P}{\partial y}\frac{\partial y}{\partial Q}+\frac{\partial P}{\partial Q})|_{Q=Q^*}(t-Q^*)+(t-Q^*)^2O(1).$$

The third equation holds by calculating the integral.

The fourth equation holds by plug in expression for d as calculated above.

The fifth equation holds by doing Taylor expansion for Q_1-Q^* around J_0, and Q_1-Q_0 around J_0.

$$Q_1-Q^* = Q(y(p_1,J_1),p_1,J_1)-Q(y(p_1,J_0),p_1,J_0) = (\frac{\partial Q}{\partial y}\frac{\partial y}{\partial J}+\frac{\partial Q}{\partial J})|_{J=J_0}(J_1-J_0)+(J_1-J_0)^2O(1),$$

$$Q_1-Q_0 = Q(y(p_1,J_1),p_1,J_1)-Q(y(p_0,J_0),p_0,J_0) = (\frac{dQ}{dJ})|_{J=J_0}(J_1-J_0)+(J_1-J_0)^2O(1).$$

We can map the derived expressions to the graph. In Figure 1, welfare changes

$$\begin{aligned}
\Delta w &= area(CEF) - area(AGH) \\
&= \int_{p_1}^{\infty} Q(y(s, J_1), s, J_1) - \int_{p_0}^{\infty} Q(y(s, J_0), s, J_0) ds \\
&= \underbrace{-area(EGHI)}_{\Delta \text{w due to price effect}} - \underbrace{area(ACFI)}_{\Delta \text{w due to variety effect}} \\
&= \underbrace{-\int_{p_0}^{p_1} Q(y(s, J_0), s, J_0) ds}_{\Delta \text{w due to price effect}} + \underbrace{\int_{p_1}^{\infty} [Q(y(s, J_1), s, J_1) - Q(y(s, J_0), s, J_0)] ds.}_{\Delta \text{w due to variety effect}}
\end{aligned}$$

Welfare change due to variety effects

$$\begin{aligned}
\Delta w_v &= -area(ACFI) \\
&= area(CEF) - area(AEI) = \int_{p_1}^{\infty} [Q(y(s, J_1), s, J_1) - Q(y(s, J_0), s, J_0)] ds \\
&= area(ADB) - area(AEI) = \int_{p_1}^{\infty} [Q(y(s + d, J_0), s + d, J_0) - Q(y(s, J_0), s, J_0)] ds \\
&= -area(BDEI) = - \int_{Q_1}^{Q^*} [P(y(t, J_0), t, J_0) - p_1] dt - Q_1 d.
\end{aligned}$$

The third equality holds because demand curves are parallel, so $area(CEF) = area(ADB)$. Approximation as in equation (34) is

$$-Q_0 d = -area(DEJK).$$

Bias for approximation of variety effect is

$$bias = \Delta w_v + Q_0 d = -area(BDEI) + area(DEJK) = area(BIJK)$$

□

In other words, we can estimate the welfare effect in equation (35) by estimating pass-through (dp) and the vertical shift parameter (d) through equation (33). To estimate the latter, we use the short-run slope of demand to estimate the fixed-variety price elasticity of demand and use the long-run slope of demand to estimate the variable-variety price elasticity of demand. However, note that if in practice adjustment costs also imply that $dy/dp = 0$, then our empirical estimates will not identify the correct elasticity. Therefore we have shown that our formula for the variety effect is robust to the existence of an outside market which adjusts in response to price changes. However, this is not true if there exists some externality from the outside market as we explain in the next section.

4.3 The Principle of Le Chatelier and Externalities

A concern with using short-run and long-run price elasticities of demand to identify the variety effect is that differences between the two may conflate changes in the level of variety with changes that might occur for other reasons. In this section, we extend the model by incorporating an outside market represented by the variable y and we assume the consumer can only adjust y in the long run and the firms can only adjust p_y in the long run. We start from a continuous choice model where all firms in the inside market are symmetric, we denote p the symmetric equilibrium price of the inside market, and Q the aggregate quantity.

Let $u(Q, y, J) - pQ - p_y y$ be the utility function of the consumer and assume u is supermodular and quasiconcave. Let

$$Q^*(y, p, p_y, J) = \underset{Q}{\operatorname{argmax}}\, u(Q, y, J) - pQ - p_y y$$

be the aggregate demand of the inside good conditional on (p, y, J), and let

$$y^*(p, p_y, J) = \underset{y}{\operatorname{argmax}}\, u(Q^*(y, p, J), y, J) - pQ^*(y, p, J) - p_y y$$

be the optimal choice of y given $(p, p_{y,} J)$. Finally, define the long-run aggregate demand $Q(J) = Q^*(y^*(p(J), p_y(J), J), p(J), p_y(J), J)$.

Observe the long-run change in aggregate demand for the inside market given an exogenous change in variety J has three components:

$$\frac{dQ(J)}{dJ} = \frac{\partial Q^*}{\partial p}\frac{dp(J)}{dJ} + \frac{\partial Q^*}{\partial p_y}\frac{dp_y(J)}{dJ} + \frac{\partial Q^*}{\partial y}\frac{dy(p(J), p_y(J), J)}{dJ} + \frac{\partial Q^*}{\partial J} \tag{36}$$

the indirect effect of variety through equilibrium price p, the indirect effect of variety through the outside variable y, and the direct effect of variety J.

Assume the following parallel inverse demands condition:

Assumption. *(Parallel Inverse demands) For all J and all y there exists d such that for all p then $Q(y, p, p_{y,} J) = Q(y_{0,} p + d, p_{y0}, J_0)$.*

As before we can calculate the vertical shift

$$d \approx \left(\frac{dp}{dQ^*} - \frac{dp}{dQ} \right) * dQ. \tag{37}$$

Define the indirect utility function

$$w(y, p, p_y, J) = u(Q^*(y, p, p_y, J), y, J) - pQ^*(y, p, p_y, J) - p_y y$$

and note from the consumer perspective in a long-run equilibrium welfare is

$$v(J) = w(y^*(p(J), p_y(J), J), p(J), p_y(J), J).$$

Taking the first-order conditions:

$$\begin{aligned} \frac{dv(J)}{dJ} &= \frac{\partial w}{\partial y}\frac{dy^*}{dJ} + \frac{\partial w}{\partial p}\frac{dp}{dJ} + \frac{\partial w}{\partial p_y}\frac{dp_y}{dJ} + \frac{\partial w}{\partial J} \\ &= \frac{\partial w}{\partial y}\frac{dy^*}{dJ} - Q\frac{dp}{dJ} + \frac{\partial w}{\partial p_y}\frac{dp_y}{dJ} + \Lambda \\ &= -Q\frac{dp}{dJ} + \frac{\partial w}{\partial p_y}\frac{dp_y}{dJ} + \Lambda \end{aligned}$$

where the last line follows from the envelope theorem. Furthermore, the parallel inverse demands condition implies $-Q * d \approx \left(\Lambda + \frac{\partial w}{\partial p_y}\frac{dp_y}{dJ} \right) dJ$ and so

$$dv(J) \approx -Q * (dp + d). \tag{38}$$

In other words, we can estimate the welfare effect in (38) by estimating pass-through (dp) and the vertical shift parameter (d) through equation (37). To estimate the latter, we need the short-run slope of demand (keeping both variety J and the outside market demand y fixed) and the long-run slope of demand when both y and J are adjusted. However, estimating the vertical shift parameter is not enough to estimate the variety effect, Λ, since the vertical shift includes indirect effects of variety through the outside market price p_y. An application of the Le Chatelier Principle (Samuelson 1947, Milgrom and Roberts 1996) shows the slope of demand in the very long run (when both J and p_y are adjusted) is steeper than when only variety J adjusts, therefore $-Q * d$ would be overestimating Λ. In summary, the love for variety assumption and the Le Chatelier Principle together imply the following bounds:

$$0 \leq \Lambda \leq -Q * d'(J).$$

We have shown how to apply the parallel demands assumption in a model with an outside market y to calculate the welfare effect $\frac{dv}{dJ}$ with the reduced-form estimates that are analogous to those used in the baseline model. If we are interested in calculating the variety effect Λ we need one more estimate: the long-run slope of demand where J is variable but p_y is kept constant $\left.\frac{dQ}{dJ}\right|_{p_y} = \frac{dQ^*(y,p(J),p_y,J)}{dJ}$. Then

$$\Lambda = Q * \left(\frac{\frac{dp}{dJ}}{\left.\frac{dQ}{dJ}\right|_{p_y}} - \frac{1}{\frac{\partial Q^*}{\partial p}} \right) * \left.\frac{dQ}{dJ}\right|_{p_y}.$$

To put diferently, if in the long run the price of the outside market p_y is correlated with J, the variety effect is not identified with the two instruments we described and in that case we need an instrument which is uncorrelated with p_y. However, it is important to notice, that y has no direct effect on welfare given the envelope theorem $\frac{\partial w}{\partial y} = 0$, so only changes in the price p_y (which are not controlled by the consumer) affect the estimation of Λ.

Online Appendix: Data Appendix

Nielsen Retail Scanner Data

We obtained the Nielsen scanner data from the Kilts Marketing Data Center at the University of Chicago Booth School of Business. The micro data records weekly prices and quantities by product at the barcode level (Universal Product Code, UPC) for over 35,000 stores from approximately 90 retail chains across the United States (except for Hawaii and Alaska), covering the years 2006-2014.[9] Each store, geolocated at the county level, is assigned one of five possible store types ("channels"), and can be matched with its parent chain.[10] Products are organized in a hierarchical structure: There are over 2.5 million different UPCs, which are categorized into approximately 1,200 *product-module*s. Each module is then assigned to one of roughly 120 *product-groups*, which in turn is part of one of 10 broader *product-departments*.

[9] Products without a barcode such as random weight meat, fruits, and vegetables are not included in the data set.

[10] The five channels are grocery, drug, mass merchandise, convenience and liquor stores. The dataset covers more than half of the total sales volume of U.S. grocery stores, but only 2 percent of sales in convenience stores. Each store and each parent chain has a unique identifier. Retail chain names are confidential and unknown to researchers.

Table A1 shows a few examples of UPCs included in the retail data.

We restrict our sample to grocery stores, and our main analyses focus on products from top selling modules that rank above the 80th percentile of total U.S. sales in the distributions of food and non-food modules. These modules account for almost 80% of the total value of sales in grocery stores in the scanner data. To implement our uniform pricing instrument, we further restrict the sample to stores that are assigned to the same chain throughout the 2006-2014 period, are present in the data for at least two years, and belong to retailers that were associated with the same parent company throughout the period.

From the scanner data, we construct two samples that we use for our empirical analysis: 1) repeated cross-sections where the unit of observation is at the store-module-year level, 2) panel data where the unit of observation is at the store-module-quarter level. For each sample, we generate measures of price (p), quantity (Q) and product variety (J) at the module level.

Prices To measure price for each module-store-period combination, we take several steps. First, we average prices across weeks to obtain either quarterly or yearly measures:

$$p_{jmrt} = \frac{\sum_{w \in t} (q_{jmrw} \times p_{jmrw})}{\sum_{w \in t} q_{jmrw}}.$$

where j = UPC, m = module, r = store, and w = week. Formally, q_{jmrw} denotes the number of units of product (UPC) j in module m sold in store r, located in county c in state s, in week w. Similarly, p_{jmrw} is the associated per-unit average weekly price. Second, we average prices across UPCs to obtain module-level price indices. Handbury and Weinstein (2015) show that comparing standard indices across locations can be problematic if consumer preferences are heterogeneous across locations, and if some varieties are unavailable in some places. For example, if consumers in a given location tend to buy larger packages of a given beverage than in other locations, the average *per-unit* price will be higher in that location even though the *per liter* average price might be lower. To correct for these sources of bias, we follow Handbury and Weinstein (2015) and adjust prices by estimating the following regression separately for each module:

$$\log p_{jmrt} = \alpha_j + \alpha_{mrt} + \varepsilon_{jmrt}$$

where α_j and α_{mrt} are UPC and module-store-time fixed effects, respectively.[11] We keep the estimated module-store-time fixed effects, $\hat{\alpha}_{mrt}$ as price indices (which are in logs).

Quantity To measure output, we first calculate "expected" sales using fixed prices of each UPC. That is, we compute $\bar{p}_{jt}$, the simple average price for product j at time t across all stores, and denote the expected revenue from sales of product j in store r by $Q_{jmrt} = q_{jmrt} \times \bar{p}_{jt}$. We then aggregate expected revenue across all UPCs to the module-store-year level for the cross-sectional analysis and to the module-store-quarter level for the time-series analysis. The unit of time, t, is either a year for equation (18) or a quarter for equation (19). Note that variation in Q_{mrt} over time and across stores reflects differences in quantity since prices are held constant. For instance, because our empirical specifications are in logs, if we were to use UPC-level data in our analyses the output elasticities obtained using Q_{jmrt} as our dependent variable would be numerically identical to elasticities estimated using q_{jmrt} directly.[12]

Variety Finally, our main measure of variety is obtained by counting the number of unique UPCs per module sold each period t in store r: $J_{mrt} = \{j \in J_m | q_{jmrt} > 0\}$. We also consider an alternative share-weighted measure of product variety G_{mrt} that puts more weight on more popular products. First, we obtain each UPC's market share s_{jt} by dividing total national sales of each product by the total national sales of the module it belongs to.[13] Then, for each store-time-module cell, variety shares G_{mrt} are obtained by adding up the UPC-level market shares s_{jt} for products with positive sales in store r at time t. For instance, for a given module, a store that has positive sales of nearly all products that appear in the data set would have a variety share close to 1. Differences in G_{mrt} across stores and modules therefore reflect both differences in the number of products (J) as well as in the market shares of products with positive sales. The empirical relationship between G and J is positive and concave, where stores with few different products tend to mainly sell high market-share products.

[11]Observations are weighted by expenditures, $R_{jmrw} = q_{jmrw} \times p_{jmrw}$. We run these regressions twice – once at the yearly and once at the quarterly level, for long-run and short-run specifications respectively.

[12]Because $\log Q_{jmrt} = \log q_{jmrt} + \log \bar{p}_{jt}$, the inclusion of product-time fixed effects would fully absorb variation in $\log \bar{p}_{jt}$.

[13]In practice, we calculate these shares separately for each UPC-store cell leaving out the store's sales from national sales, so that the market shares are store-specific. However, since every store only makes up a very small portion of total national shares, leaving out own-store sales only trivially affect the resulting measures.

References

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Table A9: Plug-In Estimates of the Variety Effect

	Leave-self-out reduced-form estimates					
Cross-sectional reduced-form estimates (Table 2)	Odd # cols.	Even # cols.	Odd # cols.	Even # cols.		
Short-run reduced-form estimates (Table 3)	Odd # cols.	Odd # cols.	Even # cols.	Even # cols.	Odd # cols.	Even # cols.
	(1)	(2)	(3)	(4)	(5)	(6)
Cross-sectional estimates (endogenous J)						
Price response, dlog(p)/dz	1.00	0.92	1.00	0.92	-	-
Output response, dlog(Q)/dz	-1.88	-1.53	-1.88	-1.53	-	-
Variety response, dlog(J)/dz	-0.69	-0.48	-0.69	-0.48	-	-
Short-run estimates (fixed J)						
Price response, dlog(p)/dz |J	0.99	0.99	0.96	0.96	0.99	0.96
Quantity response, dlog(Q)/dz |J	-1.03	-1.03	-1.02	-1.02	-1.03	-1.02
Variety effect "plug-in" estimate						
Variety Effect Parameter	1.193	1.178	1.131	1.105	-	-
Variety effect CES/Logit						
Variety Effect Parameter	-	-	-	-	0.966	0.943

Notes: Columns (1) through (4) report "plug-in" estimates of the variety effect using different combinations of estimates from Tables 2 and 3. In columns (5) and (6), the variety effect parameter is equal to -(dlog(p)/dz |J)/(dlog(Q)/dz |J).