Quick Look at Slow and Fast Light
Concept of Phase Velocity of a Monochromatic Wave

Monochromatic plane wave

$$E(z, t) = E_o \ e^{i(kz - \omega t)} + \text{c.c.}$$

Phase \( \phi = kz - \omega t \)

Dispersion relation \( k = \frac{n \omega}{c} \)

Constant phase front moves a distance \( z \) in time \( t \)

$$k \Delta z = \omega \Delta t$$

Phase velocity \( v_p = \frac{\Delta z}{\Delta t} = \frac{\omega}{k} = \frac{c}{n} \)

\( v_p > c \) does not contradict special theory of relativity
Superposition of two single frequency plane waves

\[ E = E_0 \left[ \cos(k_1 z - \omega_1 t) + \cos(k_2 z - \omega_2 t) \right] \]

\[ = \{2E_0 \cos(\Delta k z - \Delta \omega t)\} \cos(k z - \omega t) \]

Group velocity

\[ v_g = \frac{\Delta \omega}{\Delta k} \]
\[ \Delta \omega = (\omega_1 - \omega_2)/2, \quad \Delta k = (k_1 - k_2)/2 \]

For non-dispersive medium

\[ v_g = \frac{c}{n} \]

Phase velocity

\[ v_p = \frac{\omega}{k} = \frac{c}{n} \]
\[ \omega = (\omega_1 + \omega_2)/2, \]
\[ k = (k_1 + k_2)/2, \quad k_i = \frac{n \omega_i}{c}, \quad i = 1, 2 \]
Pulse in a Dispersive Medium

In a dispersive medium, \( n(\omega) \), for no pulse distortion, frequency components add in phase at pulse peak.

\[
\phi(\omega) = k(\omega)z - \omega t, \quad k(\omega) = \frac{n(\omega)\omega}{c}
\]

\[
\frac{d\phi}{d\omega} = 0 \quad \Rightarrow \quad \frac{dn}{d\omega} \frac{\omega z}{c} + \frac{nz}{c} - t = 0, \quad z = v_g t
\]

Group Velocity

\[
v_g = \frac{c}{n + \omega \frac{dn}{d\omega}} = \frac{d\omega}{dk}
\]

Group Index

\[
n_g = n + \omega \frac{dn}{d\omega}
\]

Slow & fast light effects make use of large \( \frac{dn}{d\omega} \) in the vicinity of material resonance.

\[
\frac{dn}{d\omega} > 0 \quad \text{(normal dispersion)} \quad \Rightarrow \quad \text{Slow Light}
\]

\[
\frac{dn}{d\omega} < 0 \quad \text{(anomalous dispersion)} \quad \Rightarrow \quad \text{Fast Light}
\]
Dispersion and Slow Light using EIT in a System

Susceptibility to first order in probe field amplitude

\[ \chi = \frac{-i N |\mu_{21}|^2}{\hbar} \left[ \frac{i \delta_2 - \gamma_{31}}{\hbar} \right] \frac{[i \Delta + \gamma_{31}]}{[i \Delta + \gamma_{31} - g_s^2/4]}. \]

-- \_31 is decoherence rate for ground states

For large amplitude of strong field and \_1 = 0

\[ \chi = \frac{i 4N |\mu_{21}|^2}{\hbar} \left[ i(\omega - \omega_0) - \gamma_{31}, \right] n_g \approx \frac{2 \omega_0 N |\mu_{21}|^2}{\hbar g_s^2}. \]

\( n_g \) can be as large as \( O(10^7) \)

\( v_g (< c) \quad O(10^2) \) m/s
Slow Light in Pr:YSO

Energy Diagram

- Repump refills the spectral holes burned by pump and probe fields or prevents persistent SHB due to long population life time of ground state sublevels (100s @ 5K)
- Appropriate pulse sequences for the beams are generated using AOM switching
Observation of Slow Light in Pr:YSO

Measured group delay $\sim$ 100 s = 33 m/sec

Fast Light Using Anomalous Dispersion

Fast Light Using Anomalous Dispersion

Inside pulse delayed by:
\[ T = \frac{L}{V_g} - \frac{L}{C} = (n_g - 1)\frac{L}{C} \]

Inside pulse advanced by:
\[ -T = (1 - n_g)\frac{L}{C} \]

Role of Fresnel Drag in Sagnac Effect

\[ V_P : \text{Phase Velocity in Absence of Rotation} \]
\[ V_R^{\pm} : \text{Relativistic Phase Velocities Seen in an Inertial Frame} \]
\[ T^{\pm} : \text{time for the Phase Fronts to travel from BS1 to BS2} \]
\[ A : \text{Area normal to} \]

\[ V_R^{\pm} = \frac{V_P \pm \nu}{1 \pm V_P \nu / C_o^2} \]

\[ L^{\pm} = \pi R \pm \nu T^{\pm} \]

\[ T^{\pm} = L^{\pm} / V_R^{\pm} \]

\[ V_R^{\pm} = \frac{C_o}{n} \mp \nu \alpha_F; \quad \alpha_F = (1 - \frac{1}{n^2}) \]

Fresnel Drag Coefficient
Role of Fresnel Drag in Sagnac Effect

\[ V_R^\pm = \frac{V_P \pm \nu}{1 \pm V_P \nu / C_o} \]

\[ L^\pm = \pi R \pm \nu T^\pm \]

\[ T^\pm = L^\pm / V_R^\pm \]

\[ V_R^\pm = \frac{C_o}{n} \mp \nu \alpha_F; \quad \alpha_F = (1 - \frac{1}{n^2}) \]

\[ \Delta t = (n^2 (1 - \alpha_F))\Delta t_o = \Delta t_o \]

\[ \Delta \phi = (n^2 (1 - \alpha_F))\Delta \phi_o = \Delta \phi_o \]

\[ \text{Fresnel Drag Effect is Included in the Proper Description of the Sagnac Effect} \]
Doppler Shift and Laub Drag in Sagnac Effect

No Doppler Effect if the Laser is stationery, but the stage rotates, with the no relative motion between the mirrors and the medium.
Doppler Shift and Laub Drag in Sagnac Effect

Laser and MZI frame are stationery, and the medium moves with a relative Velocity of \(V_M\).

\[ \Delta \omega^\pm = \pm \omega V_M / C_o \]

The relativistic velocities are then given by:

\[ V_R^\pm \approx \frac{C_o}{n_o} \left(1 - \frac{\Delta \omega^\pm}{n_o} \frac{\partial n}{\partial \omega}\right) \mp \nu \alpha_F = \frac{C_o}{n_o} \mp V_M \frac{\omega}{n_o^2} \frac{\partial n}{\partial \omega} \mp \nu \alpha_F = \frac{C_o}{n_o} \mp V_M \left(\frac{n_g - n_o}{n_o^2}\right) \mp \nu \alpha_F; \]
Doppler Shift and Laub Drag in Sagnac Effect

Laser and MZI frame are stationery, and the medium remains stationery (or vice versa)

Here $V_M = -(-v) = -R$, so that the relativistic velocities are then given by:

$$V_R^\pm \approx \frac{C_o}{n_o} \mp v\alpha_L; \quad \alpha_L = \left[1 - \frac{1}{n_o^2} - \frac{(n_g - n_o)}{n_o^2}\right]$$

The Laub Drag Coefficient

Doppler Shift and Laub Drag in Sagnac Effect

Laser and MZI frame are stationary, and the medium remains stationary (or vice versa)

\[
\begin{align*}
V_R^\pm & \approx \frac{C_o}{n_o} \mp v\alpha_L; \\
\alpha_L & = \left[ 1 - \frac{1}{n_o^2} - \frac{\left(n_g - n_o\right)}{n_o^2} \right]
\end{align*}
\]

\[
\Delta t \approx \left(n^2(1 - \alpha_L)\right)\Delta t_o; \\
\Delta \phi = \left(n^2(1 - \alpha_L)\right)\Delta \phi_o
\]

(For \(n_g > n_o\))

\[
\Delta t \approx n_g \Delta t_o; \\
\Delta \phi = n_g \Delta \phi_o
\]
Optical Sagnac Effect in a Passive Ring Cavity

Optical Sagnac Effect in a Passive Ring Cavity

No Rotation:

\[ \omega_o = \frac{C_o}{n_o} \frac{2\pi N}{P} \]

With Rotation:

\[ \omega^\pm = V^\pm_E \cdot \frac{2\pi N}{P} \equiv \omega_o \pm \frac{\Delta \omega_o}{2} ; \quad V^\pm_E = V^\pm_R \pm v \]

\[ \Delta \omega_o = \frac{2 \Omega R \omega_o}{C_o n_o} = \frac{\omega_o}{C_o n_o} \cdot \frac{\Omega A}{P} \]
In general:

\[ \omega^\pm = \omega_o \pm \frac{\Delta \omega}{2} = V_E^\pm \cdot \frac{2\pi N}{P} \]

\[ V_E^\pm \equiv V_R^\pm \pm \nu = \frac{C_o}{n(\omega^\pm)} \cdot \left[ 1 \pm \frac{\nu}{C_o n(\omega^\pm)} \right] \]

(here \( \nu \) is considered a parameter whose amplitude is to be determined)
Enhancement of Sagnac Effect in a PRC using Fast Light

\[ V_E^\pm \equiv V_R^\pm \pm v = \frac{C_0}{n(\omega^\pm)} \cdot \left[ 1 \pm \frac{v}{C_0 n(\omega^\pm)} \right] \]

\[ V_E^\pm = \frac{C_0}{n_o} \cdot \left[ 1 \pm \frac{v}{C_0 n_o} \mp \tilde{n} \frac{\Delta \omega}{2} \right]; \quad \tilde{n} \equiv \frac{[\partial n / \partial \omega]}{n_o} \]

\[ \omega^\pm = \omega_o \pm \frac{\Delta \omega}{2} = V_E^\pm \cdot \frac{2\pi N}{P} \]

Self-Consistent Solution:

\[ \Delta \omega = \frac{\Delta \omega_o}{1 + \omega_o \tilde{n}} = \Delta \omega_o \cdot \frac{n_o}{n_g} \]
Enhancement of Sagnac Effect in a PRC using Fast Light

\[ \Delta \omega = \frac{\Delta \omega_o}{1 + \omega_o \tilde{n}} = \Delta \omega_o \cdot \frac{n_o}{n_g} \equiv \Delta \omega_o \cdot \xi \]

**Constraint:**
\[ \Delta n = n_o \tilde{n} \Delta \omega << 1 \]

\[ \Delta \omega = \Delta \omega_o \cdot \xi; \quad 1 << \xi << C_o n_o / v; \quad v = \Omega R; \quad \partial n / \partial \omega = -\left(n_o / \omega_o \right) [1 - \xi^{-1}] \]

**Critically Anomalous Dispersion (CAD):**
\[ \partial n / \partial \omega = -\left(n_o / \omega_o \right) \]
Numerical Example for the Constraint:

\[ \Delta \omega = \Delta \omega_o \cdot \xi; \quad 1 \ll \xi \ll C_o n_o / v; \quad v = \Omega R; \quad \partial n / \partial \omega = -\left( n_o / \omega_o \right)[1 - \xi^{-1}] \]

Consider a ring cavity with \( R = 1 \) meter, a rotation rate of \( \sim 73 \) micro-radian per second (earth rate), and \( n_o = 1.5 \):

The enhancement factor can be as high as \( 10^{12} \) while still satisfying the constraints.
Enhancement of General Purpose Interferometric Sensing Using Fast Light
Enhancement of General Purpose Interferometric Sensing Using Fast Light

Model:

ref region: \( n(\omega) = n_o + \Delta \omega \cdot \frac{\partial n}{\partial \omega}; \quad \{\frac{\partial n}{\partial \omega} = -\left(\frac{n_o}{\omega_o}\right)[1 - \xi^{-1}]\}; \quad \xi = \frac{n_o}{n_g} >> 1\} \)

test region: \( n(\omega) = n_o + \Delta \omega \cdot \frac{\partial n}{\partial \omega} + \Delta S \cdot \frac{\partial n}{\partial S}; \quad \{\frac{\partial n}{\partial S} \equiv \sigma, \text{independent of } \omega\} \)

With no dispersion: \( \Delta \omega = \omega_o \cdot \frac{\sigma}{n_o} \cdot \Delta S \equiv \Delta \omega_o \)

With anomalous dispersion: \( \Delta \omega = \Delta \omega_o \cdot \xi; \quad \{\xi = \frac{n}{n_g} >> 1; \text{ the CAD condition}\} \)
Slow-Light Enhanced Rotation Sensing: Experiment

![Diagram of experimental setup](image_url)
**Slow-Light Enhanced Rotation Sensing: Experiment**

![Graph showing frequency and magnitude](image)

- **5P_{1/2}**
- **5S_{3/2}**
- **F=1**
- **F=2**
- Saturated pump absorption
- Probe absorption in EIT cell

**Diagram:**
- Pump
- Probe
- 1.772 GHz

**Figure:**
- Photodiode output
- Lock-in-detection

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Center for Photonic Communication and Computing  
Laboratory for Atomic and Photonic Technology
Anomalous Dispersion Enhanced Rotation Sensing: Experiment

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Anomalous Dispersion Enhanced Rotation Sensing: Experiment

Experimental Set-Up: vapor-cells
Experimental Set-Up: Trapped Atoms
Artificial Black-Hole Using Slow Light
Analogy Between
Charged Particles in a Magnetic Field
And
Photons in a Rotating Medium (Gravimagentism)
Artificial Blackhole with Slow-Light in a Rotating Medium

$B_{\text{eff}}$ (effective magnetic field)

$A_{\text{eff}}$ (effective vector potential)

Rotating Medium (Vortex)

Force

Slow-photons (1 cm/sec)

$\mathbf{v}$
Artificial Blackhole with Slow-Light in a Rotating Medium

Artificial Blackhole with Slow-Light in a Rotating Medium

GR-Relevant Terrestrial Experiments

- Sagnac Effect for Sensing of Lense-Thirring Rotation
  - Using Fast-Light Interferometry
  - Using Atomic Interferometry

- Artificial Blackhole Using Slow Light

- GPS and Quantum Clock-Synchronization

- Equivalence Principle and Slow-Light

- LIGO Project for Detecting Gravitational Waves

- Fast-Light and Atomic Inter. for Det. Grav. Waves

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