The evolution of binary fractions in globular clusters

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ABSTRACT
We study the evolution of binary stars in globular clusters using a new Monte Carlo approach combining a population synthesis code (STARTTRACK) and a simple treatment of dynamical interactions in the dense cluster core using a new tool for computing three- and four-body interactions (FEWBODY). We find that the combination of stellar evolution and dynamical interactions (binary—single and binary—binary) leads to a rapid depletion of the binary population in the cluster core. The maximum binary fraction today in the core of a typical dense cluster such as 47 Tuc, assuming an initial binary fraction of 100 per cent, is only \(\sim 5\)–10 per cent. We show that this is in good agreement with recent Hubble Space Telescope observations of close binaries in the core of 47 Tuc, provided that a realistic distribution of binary periods is used to interpret the results. Our findings also have important consequences for the dynamical modelling of globular clusters, suggesting that ‘realistic models’ should incorporate much larger initial binary fractions than has usually been the case in the past.

Key words: stellar dynamics – methods: N-body simulations – binaries: close – binaries: general – globular clusters: general – globular clusters: individual: NGC 104 (47 Tucanae).

1 INTRODUCTION
Binary stars play a fundamental role in the evolution of globular clusters for at least two important reasons. First, the evolution of stars in binaries, whether in a cluster or in the galactic field, can be very different from the evolution of the same stars in isolation. In a dense environment such as a globular cluster, this difference is exacerbated by dynamical encounters, which affect binaries much more than single stars. Secondly, binary stars crucially affect the dynamical evolution of globular clusters, providing (through inelastic collisions) the source of energy that supports them against gravitational collapse (Goodman & Hut 1989; Gao et al. 1991; Fregeau et al. 2003). In the ‘binary burning’ phase, a cluster can remain in quasi-thermal equilibrium with nearly constant core density and velocity dispersion for many relaxation times, in a similar way to that in which a star can maintain itself in thermal equilibrium for many Kelvin–Helmholtz times by burning hydrogen in its core. The binary fraction (and the initial, primordial binary fraction in particular), is therefore one of the most important parameters that determine the evolution of globular clusters. However, most previous dynamical studies of globular clusters – even those including binaries – have neglected stellar evolution, which can significantly impact the properties and survival of binaries and hence the reservoir of energy they provide.

At present, there are very few direct measurements of binary fractions in clusters. However, even early observations showed that binary fractions in globular cluster cores are smaller than in the solar neighbourhood (e.g. Cote et al. 1996). Recent Hubble Space Telescope observations have provided further constraints on the binary fractions in many globular clusters (Rubenstein & Bailyn 1997; Bellazzini et al. 2002b). The measured binary fractions in dense cluster cores are found to be very small. As an example, the upper limit on the core binary fraction of NGC 6397 is only 5–7 per cent (Cool & Bolton 2002). On the other hand, in very sparse clusters, such as NGC 288 (Bellazzini et al. 2002a), but also in some other ‘core-collapsed’ clusters, such as NGC 6752 (Rubenstein & Bailyn 1997), the upper limit for the binary fraction can be as high as \(\sim 30\) per cent.

For the initial binary fraction in globular clusters, there are of course no direct measurements. However, there are no observational or theoretical arguments suggesting that the formation of binaries and hierarchical multiples in dense stellar systems should be significantly different from other environments such as open clusters, the Galactic field, or star-forming regions. Binary frequencies \(\geq 50\) per cent are found in the solar neighbourhood and in open clusters (Hallwachs et al. 2003). T Tauri stars also have a very high binary fraction (Köhler, Leinert & Zinnecker 2001). For the range of separations between 120 and 1800 au, their binary fraction is
comparable to that of main-sequence stars in the solar neighbourhood (Brandner et al. 1996), while at shorter periods it is higher (Melo 2003). Furthermore, many stars are formed in systems of multiplicity 3 or higher: in the field their abundance is no less than 40 per cent for inner periods \( \leq 10 \) d (Tokovinin 1997). All of this suggests that, in dense stellar systems as well, most stars could be formed in binary and multiple configurations.

Most dynamical interactions in dense cluster cores tend to destroy binaries (the possible exception is tidal capture, which may be considered as a gravitational hardening event with unrealistically small numbers of binaries). For example, the time required to perform just one direct N-body simulation of a cluster containing \( 2 \times 10^5 \) stars with all stars formed initially in binaries would be at least a year on GRAPE-6, with some dependence on the initial binary parameters.\(^2\)

Alternatively, a binary population synthesis code (e.g. Hurley, Tout & Pols 2002), normally used to evolve large numbers of stars and binaries without dynamical interactions, can be extended by introducing a simple treatment of dynamics. In this type of approach it is often assumed that all the relevant parameters of the cluster (e.g. central density and velocity dispersion) remain constant throughout each dynamical simulation, i.e. the dynamics is assumed to take place in a fixed background cluster. Many previous studies of dense stellar systems have been based on this type of approximation (see, e.g., Hut, McMillan & Romani 1992; Di Stefano & Rappaport 1994; Sigurdsson & Phinney 1995; Portegies Zwart et al. 1997a,b; Davies 1995; Davies & Benz 1995; Davies 1997; Rasio et al. 2000; Smith & Bonnell 2001). This approach, sometimes called the ‘encounter rate technique’ (Benacquista 2002), is computationally much less expensive than direct N-body simulations and hence allows the systematic exploration of the vast parameter space of initial conditions for clusters and their primordial binary populations. In addition, the use of sufficiently large numbers of stars and binaries makes the simulations more realistic. Although obviously much less accurate in its description of the overall cluster dynamics, this method opens the possibility of studying ‘star cluster ecology’ in considerably greater detail than has been possible with N-body simulations. In particular, it makes it possible to study in detail the rare but important evolutionary channels that may play a crucial role in the formation of some of the most interesting tracers of dynamical interactions in dense clusters, such as ultracompact X-ray binaries, millisecond pulsars and cataclysmic variables (Ivanova & Rasio 2004, 2005).

Unfortunately, it is difficult to compare these two approaches, as each is based on a very different set of simplifying assumptions. There are no comprehensive studies of dense stellar systems including a self-consistent treatment of both dynamics and binary star evolution. In many recent N-body simulations for large clusters (using either Aarseth-type codes or Hénon’s Monte Carlo method; Aarseth 2001; Fregeau et al. 2003), binary stars are treated in the point-mass limit and soft binaries are eliminated from the start. Binary destruction can then occur only through resonant four-body interactions. However, N-body studies of open clusters that incorporate realistic treatments of binary stellar evolution have shown that stellar evolution affects the binaries significantly, and that, even in these low-density environments, the complex interplay between binary evolution and dynamics, even for soft binaries, can play an important role in the overall cluster evolution and in determining the properties of surviving binaries (Hurley & Shara 2003).

The second approach, ‘binary population synthesis with dynamics’, which we have adopted in this work, suffers from the lack of self-consistent dynamical evolution of the cluster, which is assumed

\(^2\) J. Hurley, personal communication. The estimate is based on 5 d required to simulate an open cluster of 20000 stars with 2000 binaries on GRAPE-6 in Shara & Hurley (2002).
to remain in a constant state of thermal equilibrium for its entire
evolution. This state, where the energy production through ‘binary
burning’ in the core is balanced by the outer energy flux into the clus-
ter halo, does indeed provide nearly constant conditions throughout
the lifetime of a typical globular cluster. The exception might be
‘core-collapsed’ clusters, which may have run out of binaries and
evolved to a much more centrally concentrated state. Typically, the
density and ‘temperature’ profiles of a cluster do not change much
as long as there are enough binaries remaining to provide support
against gravothermal contraction. Stellar interiors provide a useful
analogy: as long as a star keeps enough hydrogen to burn in its core,
it can remain in thermal equilibrium on the main sequence and avoid
core contraction and envelope expansion. Just like main-sequence
stars, globular clusters can maintain a nearly constant interior struc-
ture for many thermal time-scales (i.e. many relaxation times) as
long as they do not run out of ‘fuel’ (binaries). This behaviour is ex-
pected qualitatively (see, e.g., Goodman & Hut 1989), and has now
been demonstrated quantitatively in many different studies using
various numerical techniques for cluster simulations. For example,
the recent study by Fregeau et al. (2003) considered the evolution of
idealized clusters of equal-mass stars without stellar evolution for
a range of initial binary fractions. For the case of an isolated Plun-
mer model with 10 per cent initial hard binaries they found (see their
fig. 4) that the core radius of this cluster can remain nearly constant
(to within a factor of ~2) for many tens of half-mass relaxation
times (i.e. more than a Hubble time for most Galactic globular clus-
ters, where the half-mass relaxation time is ~10^7 yr). For a more
realistic cluster model with 20 per cent hard binaries initially and
tidal truncation, they found that, after ~401_t_{t_{ddw}}, when the cluster is
about to disrupt in the Galactic tidal field, the core radius still has
not varied by more than a factor ~2 over the entire evolution (see
their fig. 11); and over the first 10_t_{t_{ddw}}, the core radius changed by
less than ~20 per cent. The central velocity dispersion also does
not vary much with time (see, e.g., Giersz & Spurzem 2003, fig. 1).
Similar results have been obtained from direct N-body simulations
of open clusters, where the central density and velocity dispersion
also remain nearly constant in models with significant numbers of
binaries (Hurley & Shara 2003).

There are several binary population synthesis codes in use today.
Only a few of them include, in addition to a detailed treatment of
mass transfer phases, stellar evolution along with tidal interaction of
binary components. The code of Hurley et al. (2002) was designed
to study low- and intermediate-mass stars leading to the formation
of white dwarf systems. The code of Belczynski, Kalogera & Bulik
(2002, Belczynski et al., in preparation) was originally developed
to investigate more massive stars – progenitors of neutron star and
black hole systems (calibrated against full binary stellar evolution
codes for mass-transfer phases) – and was expanded recently to treat
carefully binaries with white dwarfs as well. We use the latter code
(called STARTRACK) to follow the evolution of single and binary stars
in our simulations. As mentioned earlier, we also treat all dynamical
encounters explicitly by direct integration, using a recently devel-
oped numerical toolkit for small-N gravitational dynamics that is
particularly well suited to performing three- and four-body integra-
tions (Fregeau et al. 2004).

3 METHODS AND ASSUMPTIONS

3.1 Cluster initial conditions

Our initial conditions are described by the following parameters:
the total number of stars N (single or in a binary), initial mass func-
tion (IMF), binary fraction f_b, distribution of binary parameters
(period, P, eccentricity, e, and mass ratio, q = m_2/m_1 < 1). We typ-
ically adopt standard choices used in previous population synthesis
studies, which are based on available observations for stars in the
field and in young star clusters (Sills et al. 2003). For most of the
calculations reported here, we use the following ‘standard’ initial
conditions.

(i) We adopt the IMF of Kroupa (2002), which can be written as a
broken power law dN ∝ m^{-α} dm, where α = 0.3 for m/MM_⊙ < 0.08,
α = 1.3 for 0.08 ≤ m/MM_⊙ < 0.5, α = 2.3 for m/MM_⊙ ≥ 0.5. We
assume that all stars are formed in a single burst of star formation
at t = 0 in our simulations.

(ii) We consider a wide range of stellar masses from 0.05 to
100 M_⊙.3

(iii) The binary mass ratio, q, is assumed to be distributed uni-
formly in the range 0 < q < 1. This is in agreement with observations
for q ≥ 0.2 (Woitas, Leinert & Köhler 2001).

(iv) The binary period, P, is taken from a uniform distribution in
log_{10} P over the range P = 0.1–10^7 d.

(v) The binary eccentricity, e, follows a thermal distribution with
probability density p(e) = 2e.

(vi) We reject systems where binary components would overflow
their Roche lobe at pericentre.

The initial average stellar mass is then ⟨m⟩ ≈ 0.5 M_⊙ and, with
the flat mass ratio distribution, the average binary mass is ⟨m_b⟩ ≈
0.75 M_⊙.

3.2 Stellar evolution

We evolve all stars (single and binary) using the population synthesis
code STARTRACK (Belczynski et al. 2002), which has recently been
updated significantly (Belczynski et al., in preparation). This is the
only current population synthesis code that incorporates detailed
treatments of all possible types of mass transfer (MT) episodes:
stable MT (conservative or non-conservative), unstable MT (ther-
mal or dynamically) and thermal time-scale MT. Also included
are the effects of Eddington-limited mass accretion and transient
behaviour of accretion discs (based on the disc instability model).
STARTRACK allows us to follow the evolution of binaries with a large
range of stellar masses, metallicities and star formation histories
(constant rate, sudden or exponential bursts, etc.). STARTRACK also
models in detail the loss of mass and angular momentum through
stellar winds (dependent on metallicity) and gravitational radiation,
asymmetric core collapse events with a realistic spectrum of comp-
act object masses, and the effects of magnetic braking and tidal
circularization on close binaries. In our simulations, we adopted the
new prescription for magnetic braking given by Ivanova & Tauru
(2003). Compared with the older prescription (Verbunt & Zwaan
1981) closer binaries lose their angular momentum at a slower rate
and hence survive as binaries longer. The evolution of single stars in
STARTRACK is based on the analytic fits provided by Hurley, Pols &
Tout (2000), but includes a more realistic determination of compact
object masses (Fryer & Kalogera 2001).

We treat the evolution of stellar collision and binary merger prod-
ucts following the general ‘rejuvenation’ prescription of Hurley et al.
(2002). It ensures that the merger product has the same total amount
of hydrogen, helium and carbon as the two parent stars together. For
some stars, the assumptions made in the treatment of the merger

3 The lower limit is chosen in order to provide a self-consistent mass-ratio
distribution for binaries with primaries down to 0.15 M_⊙.
depend on the types of stars and the type of merger (collision versus binary coalescence). For example, we assume that there is no accretion on to a neutron star during a physical collision, and that the other star, if it is unevolved, is destroyed completely (e.g. we do not consider the possible formation of a Thorne–Zytkow object). We treat as a ‘dynamical common envelope (CE)’ event the outcome of a physical collision between a compact object and a red giant, applying a standard ‘alpha prescription’ (where we adopt $\alpha_{\text{CE},a} = 1$; see Iben & Livio 1993), but taking into account the initial positive energy. In particular, if this compact object is a neutron star, a compact binary containing a neutron star and a white dwarf is formed. We assume that, during the CE phase, the neutron star will accrete a significant amount of the envelope material and will become a millisecond pulsar (Bethe & Brown 1998). If the resulting mass of the neutron star exceeds the limit for a neutron star (taken in our simulations to be $2 M_\odot$), we assume that a black hole is formed.

To evolve the cluster population of single stars and binaries in our code, we consider two basic time-scales. One is associated with the evolutionary changes in the stellar population, $\Delta t_{\text{ev}}$, and the other with the rate of encounters, $\Delta t_{\text{enc}}$ (see Section 3.4). The evolution time-step $\Delta t_{\text{ev}}$, is computed so that no more than 2 per cent of all stars change their properties (mass and radius) by more than 5 per cent. The global time-step for the cluster evolution is taken to be $\Delta t = \min\{t_{\text{ev}}, t_{\text{enc}}\}$.

### 3.3 Dynamical cluster model

As we described in Section 2, our model for the cluster dynamics is highly simplified. We adopt a simple two-zone, core–halo model for the cluster. We assume that the core number density, $n_c$, and one-dimensional (1D) velocity dispersion, $\sigma_{\text{1D}}$, and the half-mass relaxation time, $t_{\text{rh}}$, remain strictly constant throughout the evolution. While dense globular clusters of interest have $\sigma_{\text{1D}} \sim 10 \text{ km s}^{-1}$, the core density can vary by several orders of magnitude. Here we set $n_c = 10^5 \text{ pc}^{-3}$ for most calculations, representative of a fairly dense cluster such as 47 Tuc. In general, $n_c$ is the main ‘knob’ that we can turn to increase or decrease the importance of dynamics. Setting $n_c = 0$ corresponds to a traditional population synthesis simulation, where all binaries and single stars evolve in isolation after a single initial burst of star formation. To model a specific cluster, we match its core mass density today, $\rho_c^{\text{bg}}$, central velocity dispersion and half-mass relaxation time.

The escape speed from the cluster core can be estimated from observations as $v_c \approx 2.5 \sigma_{\text{3D}}$ (Webbink 1985), where $\sigma_{\text{3D}}$ is the three-dimensional (3D) central velocity dispersion. Following an interaction or a supernova explosion, any object that has acquired a recoil speed exceeding $v_c$ is removed from the simulation. Acquiring a large recoil velocity in a dynamical encounter is a very efficient mechanism for ejecting low-mass objects from the cluster. We find generally that recoil to the halo does not play a significant role: the recoil velocity into the halo differs by $\sim 10$ per cent from the escape velocity from the cluster, and affects only a small number of objects.

For computing interactions in the core, the velocities of all objects are assumed to be distributed according to a lowered Maxwellian (King 1966), with

$$f(v) = \frac{v^2}{\sigma^2(m)} \left\{ \exp[-1.5v^2/\sigma^2(m)] - \exp\left[-1.5v_e^2/\sigma^2(m)\right] \right\},$$

where $\sigma(m) = \langle m \rangle / \langle m \rangle_{1/2,\text{3D}}$ (assuming energy equipartition in the core) and $v_e$ is the escape speed. Here $\langle m \rangle_{1/2}$ is the average mass of an object in the core. In addition, we use $\sigma$ to impose a cut-off for soft binaries entering the core: any binary with maximum orbital speed $< 0.1 \sigma_{\text{3D}}$ is immediately broken into two single stars (Hills 1990).

In the presence of a broad mass spectrum, the cluster core is always dominated by the most massive objects in the cluster, which tend to concentrate there via mass segregation. As stars evolve, the composition of the core will therefore change significantly over time. Mass segregation in globular clusters was investigated recently in Fregoeau et al. (2002) by considering both light and heavy tracers in two-component models. It was found that the characteristic mass-segregation time-scale is given by

$$t_c \approx 10 C (\langle m \rangle_h/m) t_{\text{rh}},$$

where $C$ is a constant of the order of unity and $\langle m \rangle_h$ is the average mass of an object in the halo, and $m$ is the current mass of the object. This equation represents a diffusion process and can be applied to all stars, not just to those more massive than average. Indeed, even low-mass objects may (rarely) diffuse into the cluster core on a long time-scale, although on average they will tend to drift outwards. However, for very light objects with masses $\lesssim 0.4 (\langle m \rangle_h$ which is typically $\sim 0.3 M_\odot$ at the beginning and $\sim 0.15 M_\odot$ after $\sim 10 \text{ Gyr}$), this expression becomes less accurate, although it remains valid qualitatively. For a more recent discussion of mass segregation in the presence of a broad mass spectrum, see Gürkan, Freitag & Rasio (2004).

To model mass segregation in our simulations, we adopted the time-scale given by equation (2), but treated the process as stochastic: the probability for an object of mass $m$ to enter the core after a time $t_i$ is sampled from a Poisson distribution,

$$p(t_i) = \frac{1}{t_c} \exp(-t_i/t_c).$$

This treatment ensures that all stars heavier than $\sim 0.4 \langle m \rangle_h$ diffusing into the core will have the appropriate mass spectrum and that interactions will occur between objects drawn from the correct distribution.

Equation (2) was derived for the restricted case of a two-component cluster – without a realistic IMF – therefore $C$ is unknown by a factor of a few. We find in simulations that the final core mass is nearly proportional to $1/C(\langle m \rangle_h$. In order to obtain a better fit to observations for the core mass versus total cluster mass relation, we have fine-tuned equation (2) using data for 47 Tuc, in particular the ratio of the core mass to the total mass of the cluster. For this cluster we adopted a core mass of $10^5 M_\odot$ and a total cluster mass of $10^6 M_\odot$ at present (Freire et al. 2001); we also take $t_{\text{rh}} = 3 \times 10^6 \text{ yr}$ (Harris 1996) and an age of 11 Gyr (Gratton et al. 2003). While the core mass can be found directly from our simulations, the total cluster mass has an uncertainty due to the IMF cut-off at the low-mass end in our standard cluster model. First, we found the total mass of a cluster model evolved to 11 Gyr and the initial number of very massive stars (defined as those producing a black hole at the end of their stellar evolution). We find that at 11 Gyr, the cluster has $145 M_\odot$ per black hole (or per heavy primordial star), when the IMF extends down to 0.01 $M_\odot$. This allows us to normalize our model to the real cluster mass: with this ratio we have an estimate for the total cluster mass when we use a higher cut-off for the IMF (0.05 $M_\odot$). This now gives us the ratio of the core mass to the total cluster mass corresponding to our simulations. We find that the best fit for 47 Tuc gives $C(\langle m \rangle_h = 3 M_\odot$.
3.4 Treatment of dynamical interactions

All objects in our simulations are allowed to have dynamical inter-
actions only after they have entered the cluster core. We use a simple
Monte Carlo prescription to decide which pair of objects actually
have an interaction during each time-step.

The cross-section for an encounter between two objects, of masses
$m_i$ and $m_j$, with relative velocity at infinity $v_{ij}$, is computed as
\[ S_{ij} = \pi d_{\text{max}}^2 \left(1 + v_{\text{esc}}^2/v_{ij}^2\right), \]  
(4)
where $d_{\text{max}}$ is the maximum distance of closest approach that defines a
significant encounter and $v_{\text{esc}} = 2G(m_i + m_j)/d_{\text{max}}$ is the velocity at
pericentre. Here the index $i$ (lowercase) reflects an individual object
in the core, while $J$ (uppercase) denotes a random representative
object from the subclass of objects $J$. In order to more accurately
determine encounter rates, at each time-step binaries and single
stars in the core are divided into 100 subclasses: 10 by size (radius for
single stars or semimajor axis for binaries) and 10 by mass. Boundaries
between mass subclasses are fixed approximately as 0.1 × 2$^m$. Subclasses by size depend on the current sizes of single and
binary populations (separately), with the step between subclasses
taken as $\delta \log_{10} R_{\text{bin}} = 0.1 \left(\log_{10} R_{\text{max}} - \log_{10} R_{\text{min}}\right)$. The encounter
rate for a given object $i$ and an object from subclass $J$ is $\Gamma_{ij} = n_J S_{ij} v_{ij}$. Where $n_J$ is the number density of objects in subclass $J$, and
the cross-section and relative velocity are defined for an average
object in subclass $J$.

The total interaction rate for a given object $i$ is the sum of the in-
teraction rates with all relevant subclasses, $\Gamma_i = \sum_J n_J S_{ij} v_{ij}$. The corresponding interaction time is $t_i = 1/\Gamma_i$. The actual time for an
encounter $t_i$ follows a Poisson distribution with mean $\Gamma_i$. In practice, we
generate a random number $0 < X < 1$, and assume that the en-
counter happened if $t_i = X t_i \leq \Delta t$. The time-step is limited so that
$\Delta t_{\text{dyn}} \leq 0.25$ min, $t_i$. We keep track separately of the time-scales
$t_{i,J}$ for interactions with each subclass $J$, and the corresponding
$t_{i,J} = X t_i t_{i,J}$ is generated from an independent random number. If
an encounter happened, it is assumed to be with an object from the
subclass with the smallest $t_{i,J}$. The actual interacting object $J$ from
that subclass $J$ is randomly selected from the list of non-interacted
objects in this subclass.

In this paper we consider separately binary–binary, binary–single
and single–single interactions. The cross-sections and rates are cal-
culated using $d_{\text{max}} = 5R_i + R_j$ for single–single, $d_{\text{max}} = 3(b_i + b_j)$
for binary–single and $d_{\text{max}} = 3(b_i + b_j)$ for binary–binary. Here $b_i = a_i(1 + e_i)$ is the apocentre separation of the binary ($a_i$ is
the semimajor axis and $e_i$ the eccentricity), $R_j$ is the aver-
age stellar radius in subclass $J$ and $b_j$ is the average apocentre
separation in subclass $J$. A single–single interaction with pericen-
tre distance $d_i < 2(R_i + R_j)$ is treated as a physical collision and
assumed to lead to a merger or a dynamical CE phase. If $2 < d_i/(R_i + R_j) < 5$, we check whether a binary could be produced via tidal
capture using the approach described in Portegies Zwart & Meinen
(1993). If a tidal-capture binary is formed, its eccentricity is set to
zero and its semimajor axis set to $2d_i$, assuming rapid circularization
($\sim 10$ yr) as predicted by the standard model described in McMillan,
McDermott & Taam (1987).

Each dynamical interaction involving a binary is calculated using
\textit{FEWBODY}, a new numerical toolkit for simulating small-$N$ gravita-
tional dynamics that provides automatic calculation termination and
classification of outcomes (for a detailed description see Fregeau
et al. 2004). \textit{FEWBODY} numerically integrates the orbits of small-$N$
objects, and performs collisions in the sticky-star approximation.
The ability of \textit{FEWBODY} to automatically classify and terminate cal-
culations as soon as the outcome is unambiguous makes it well
suited for carrying out large sets of binary interactions, for which
calculations must be as computationally efficient as possible.

4 TEST CALCULATIONS AND SIMPLE ESTIMATES

4.1 Comparison with $N$-body simulations

We have compared our simple dynamical model to recent re-
results from fully self-consistent Monte Carlo simulations based on
Hénon’s algorithm for solving the Fokker–Planck equation (Joshi,
Rasio & Portegies Zwart 2000; Joshi, Nave & Rasio 2001). For our
work we used the results obtained for an idealized model cluster con-
taining 20 per cent primordial hard binaries (binding energies in the
range 1–133 kT, where $kT$ is the average kinetic energy of an object
in the cluster) for a King model with dimensionless central potential
$W_0 = 7$ (Fregeau et al. 2003, hereafter F03). In this simulation all
stars had equal masses, were treated as point masses (no physical
collisions), no stellar evolution was taken into account and all binary
interactions were treated using simple recipes.

We used our code to perform a similar simulation: we considered a
cluster consisting of equal-mass stars, and we turned off all stellar
evolution and physical collisions. To fit the F03 model, we took the
core mass as 8.3 per cent of the total cluster mass (corresponding to a
King model with $W_0 = 7$), and we took an average star mass of 1 $M_\odot$ and $\sigma_c = 10$ km s$^{-1}$. For a total cluster mass of $3 \times 10^5$ $M_\odot$
these conditions imply $t_{\text{bin}} = 8 \times 10^9$ yr and $n_i = 2000$ pc$^{-3}$. We
evolved the cluster for $20 t_{\text{bin}}$.

In Fig. 1 we show our results for the core and halo binary frac-
tions as a function of time, compared with the model of F03. The
agreement with F03 is excellent: the core binary fraction rises
very quickly to ~35 per cent and then remains close to this value
for ~10 $t_{\text{bin}}$. In contrast, the halo binary fraction decreases more

![Figure 1](https://example.com/figure1.png)

Figure 1. Evolution of the core and halo binary fractions (top) and the core
radius (bottom) in our test model, compared with the F03 model (see the
text). In the top panel, the solid line shows the binary fraction in the core
and the dashed line shows the binary fraction in the halo, both for the test model.
The dotted line shows the binary fraction in the core in the F03 model and
the dash-dotted line shows the binary fraction in the halo in the F03 model.
In the bottom panel the solid line shows the core radius in the test model and
the dotted line shows the core radius in the F03 model.

gradually from 20 to 10 per cent. Considering the differences between the two treatments, especially for binary interactions, this agreement is quite remarkable.

In the bottom panel of Fig. 1 we show the evolution of the core radius $r_c$. The core radius evolves in the same way as in the dynamical simulation of F03. Overall, the core radius does not change much during the entire evolution and its value is consistent with the measured values for observed globular clusters with similar parameters (e.g. NGC 3201 or 6254, in which the total cluster mass, the central number density and the central velocity dispersion are similar to those in our model).

### 4.2 Semi-analytic estimates

One can estimate the final binary fraction $f_{bc}$ in a dense environment by considering several mechanisms of binary destruction.

- (i) All soft binaries are usually destroyed during a strong encounter.
- (ii) Some fraction of hard binaries is destroyed through stellar evolution (mergers or disruptions after supernova explosions).
- (iii) When a hard binary has a strong encounter with another hard binary or a single star, it can exchange its less massive component for a more massive star, shrink its orbit or be destroyed in a collisional merger.

It should be noted that in our simplified semi-analytical treatment we neglect the effect of mass segregation, which tends to increase the core binary fraction. (This issue is discussed in more detail in Section 5.2.)

Let us consider a dense environment with number density $10^6$ pc$^{-3}$, $\sigma_{1D} = 10$ km s$^{-1}$ and with an average mass of 0.5 M$_\odot$. With our choices of initial parameters, and an average mass of 0.5 M$_\odot$, 40 per cent of all primordial binaries initially are soft (this fraction would be 50 per cent with respect to the average mass of 1 M$_\odot$). We introduce $\eta$ — the hardness of a binary system — as

$$\eta = \frac{G m_1 m_2}{a \sigma^2(m)},$$

where $a$ is the binary separation, $m_1$ and $m_2$ are the masses of the binary components and $m$ is the average mass of a single star. Binaries that have $\eta < 1$ are termed soft, and those with $\eta > 1$ are termed hard.

To find how many hard binaries will be destroyed by stellar evolution alone, we calculated the probability of binary destruction as a function of its initial total mass and orbital period, using the binary population synthesis code (see Fig. 2). This simulation was performed with $1.25 \times 10^5$ binaries distributed initially flat in log $P_d$ and log $M_{\text{tot}}$ (in order to have better resolution for destruction rates for high-mass binaries), where $P_d$ is the binary period in days and $M_{\text{tot}} = M_1 + M_2$ is the total binary mass in M$_\odot$, and was evolved for 14 Gyr.

The result is striking: most of the very hard binaries, with hardness ratios $\eta \gtrsim 100$, are destroyed by stellar evolution. The empty space near the bottom right-hand corner of Fig. 2 reflects the absence of systems below the minimum period for binaries on the main sequence (the period at which stars come into contact). For binaries with total mass $\gtrsim 10$ M$_\odot$, destructions mainly occur during the first $10^8$ yr of cluster evolution. Binaries with period $\gtrsim 10^4$ d are mainly destroyed through SN explosions. Binaries with period $\lesssim 10^4$ d are destroyed mainly via mergers at the MS stage. For periods in the range $10^4$–$10^5$ d the destructions are associated with common envelope evolution and occur at later times. Destructions in binaries of smaller masses are not much different from more massive binaries at small periods, but are not destroyed through SN explosions at large periods. Also, the CE event in less massive binaries occurs when the donor is a less evolved giant (at the first red giant branch).

One may expect that destruction rates should vary smoothly; however, binary evolution involves many qualitatively different events. In particular, an interesting fluctuation in destruction rates can be seen at log $P \sim 1.75$ and log $M_{\text{tot}} \sim 0.95$, where local destruction rates are lower than for nearly binaries. For these and nearby binaries the destruction rates are approximately the same for mass ratio $q \lesssim 0.5$ but different for larger $q$. For binaries of masses close to and smaller than these, most destructions for $q \gtrsim 0.5$ occur during the CE event between a WD and a giant. This CE phase is the second interaction in the binary and follows the stable MT event with the other donor. When the total binary mass is smaller, the WD mass is $\lesssim 0.9$ M$_\odot$, and a CE event leads to a merger. For binaries of higher total mass, the second interaction occurs between a He star or a WD and a star at the Hertzsprung gap. This MT is unstable and leads to a merger. This, therefore, provides for a small local decrease in destruction rates.

With our IMF and considering binaries of all masses, the fraction of hard binaries destroyed during their evolution is 20 per cent. Among binaries where at least one star is more massive than 0.5 M$_\odot$ the destruction fraction is 50 per cent, and for those with total initial mass above 1 M$_\odot$, this fraction is closer to 60 per cent. The fraction of hard binaries that is not destroyed but is instead softened by evolution is very small, $\leq$ 1 per cent. In Section 4.3 we will discuss in more detail how binary destruction rates change with time.

Therefore, even before any dynamical processes are considered for the hard binary population, we estimate that the final binary fraction cannot be higher than $\sim 50$ per cent, and for binaries with at least one star more massive than 0.5 M$_\odot$ this upper limit becomes 30 per cent. For relatively massive binaries, with total initial mass above 1 M$_\odot$, the upper limit for $f_{bc}$ is only 24 per cent. Overall,
this estimate already shows that (i) the expected final binary fraction in a dense star cluster will be low and (ii) stellar evolution cannot be neglected when estimating binary fractions from dynamical models of dense star clusters.

Let us now consider the effects of dynamical interactions. The time-scale for a binary to undergo a strong encounter with another single star, the collision time, can be estimated as \( \tau_{\text{coll}} = 1/\sigma v_\infty \). Assuming that the strong encounter occurs when the distance of closest approach \( d_{\text{max}} \leq k a \) with \( k \simeq 2 \), we obtain

\[
\tau_{\text{coll}} = 3.4 \times 10^{13} \text{ yr} \times k^{-2} P_a^{3/2} M_{\text{tot}}^{3/2} n_s^{-1} v_{10}^{-1} \times \left[ 1 + 913 \left( \frac{M_{\text{col}} + (M)}{M_1 M_2} \right) \right]^{-1}.
\]

Here \( (M) \) is the mass of an average single star in \( M_\odot \), \( v_{10} = v_\infty / (10 \text{ km s}^{-1}) \), where \( v_\infty \) is the relative velocity at infinity and \( n_s = n/(10^5 \text{ pc}^{-3}) \), where \( n \) is the number density. Using equation (5), equation (6) can be rewritten in a more convenient form:

\[
\tau_{\text{coll}} = 1.7 \times 10^6 \text{ yr} \times \eta^2 k^{-2} n_s^{-1} \left( \frac{M_{\text{col}} + (M)}{M_1 M_2} \right)^2 \times \left[ 1 + \frac{2 M_{\text{tot}} + (M)}{k M_1 M_2} \right]^{-1}.
\]

It can be seen from equation (7) that the collision time for a binary with \( \eta = 1 \) and \( M_{\text{col}} = 1 M_\odot \) is only \( \sim 10^9 \) yr. Overall, with our IMF, \( \sim 50 \) per cent of all hard binaries have collision times shorter than \( \sim 10 \) Gyr (see also Fig. 2), and 15 per cent have collision times as short as \( \sim 1 \) Gyr. In addition, most of the hard binaries that could have experienced an encounter are binaries with \( \eta = 1 \), and therefore they are from the population that is destroyed by stellar evolution to a lesser degree than binaries harder than \( \eta = 100 \). While this estimate considered only binary–single encounters, binary–binary encounters will further enhance the destruction rate. Moreover, when the binary fraction is higher than \( \sim 25 \) per cent, binary–binary encounters dominate over binary–single encounters.

Each binary–single encounter with a hard binary can result in a hardening of this binary, an exchange of a companion with a more massive single star, or binary destruction in a physical collision. The probability of a physical collision during an encounter increases strongly as the binary becomes harder (Fregeau et al. 2004). In addition, a very hard binary can be ejected from the core if its recoil velocity exceeds the escape velocity from the cluster. Each of these three processes, directly or indirectly, leads to the depletion of binaries (immediate or delayed): acquiring a more massive companion, and orbital shrinkage or the eccentricity increase, makes systems more likely be destroyed through stellar evolution. As a conservative lower limit, we assume that half of the interacting hard binaries will be destroyed as a result of an encounter (immediately or later). This is clearly a lower limit, as scattering experiments show that, for hard binaries containing main-sequence stars most encounters will lead to a physical collision (Fregeau et al. 2004).

Taking everything into account, we conclude that 64 per cent of all binaries will be destroyed – only \( k = 36 \) per cent of primordial binaries can survive to the present. The expected final binary fraction is therefore \( f_b = \frac{N_b}{N_0 + N_b} = k_b/(2 - k_b) = 22 \) per cent (for stars over the entire mass range) and it is 13 per cent (\( k_b = 23 \) per cent) for binaries with at least one star more massive than \( 0.5 M_\odot \), while for \( M_{\text{tot}} \geq 1 M_\odot \) it is only 10 per cent (\( k_b = 18 \) per cent). We note again that this upper limit for the final binary fraction does not take into account several other mechanisms leading to binary destruction, such as binary–binary encounters, which are more likely to cause binary destruction than binary–single encounters.

### 4.3 Encounters in dense environments

In order to verify our encounter rates, we considered the evolution of a binary population that is completely immersed into a dense environment with \( n_c = 10^3 \text{ pc}^{-3} \) for its entire evolution. We adopted a velocity dispersion \( \sigma_{1D} = 10 \text{ km s}^{-1} \) and evolved the system for 14 Gyr. This is an even simpler model of a star cluster, where all objects are effectively inside the ‘core’ at all times and the effects of mass segregation and diffusion between a core and a halo are completely ignored. It allows us to study more clearly the various types of dynamical interactions as separate processes. We ran a sequence of simulations, with: (i) only binary–single encounters, with no physical collisions allowed during an encounter; (ii) binary–single encounters with physical collisions; (iii) both binary–single and binary–binary encounters but still without physical collisions; (iv) binary–single and binary–binary encounters with physical collisions; and (v) all types of encounters allowed, including single–single collisions. In this last case we also eliminated from the system any object that acquired a recoil speed exceeding the escape velocity \( v_c = 2.5 \sigma_{1D} \).

In Section 4.2 we estimated that, with only binary–single encounters taken into account, the final binary fraction should not exceed 22 per cent. This assumed that all objects in the core had a mass equal to the average mass \( 0.5 M_\odot \). With the complete IMF, a single star participating in the encounter can have mass higher than \( 0.5 M_\odot \). Also, a fraction of initially hard binaries can become soft during the evolution, e.g. because of mass loss. As a result, we obtain even lower remaining binary fractions: \( f_{b,c} = 16.5 \) per cent for case (i) and \( f_{b,c} = 16.4 \) per cent for case (ii). These numbers are in agreement with the upper limits we estimated in Section 4.2.

When binary–binary encounters are taken into account, the result is even more dramatic: we obtain \( f_{b,c} = 8.0 \) per cent and \( f_{b,c} = 7.9 \) per cent for cases (iii) and (iv), respectively. As expected, we see that binary–binary encounters further enhance the binary destruction rate.

When all dynamical processes are taken into account, in case (v), we obtain a final binary fraction \( f_{b,c} = 6.9 \) per cent. While our semi-analytical estimate of the binary fraction provided an upper limit, the result obtained in this section is clearly a lower limit: in a real cluster, not all binaries will be exposed to the high interaction rates of the core at all times. However, as more massive objects are more likely to diffuse into the core, and as their destruction rate can be higher than for objects of average mass, this lower limit is only approximate, but we may expect it to be much closer to the real value than our previous, conservative upper limit.

Let us now consider in detail which mechanisms of binary destruction are most important. In Fig. 3 we show the rates of binary destruction (events per binary per Gyr) for a field population (with no interactions). Except for the interval between \( \sim 10^3 \) and \( \sim 10^4 \) yr, when black holes and neutron stars are formed, the binary destructions are mainly coming from mergers. The power-law behaviour observed for the overall destruction rate at late times is on the one hand imposed by the rate of orbital decay driven by magnetic braking, and on the other hand depends on the evolutionary time-scale for the companion to become a red giant.

In Fig. 4 we show the binary destruction rates again, but for the dense environment. During the first few Gyr the main destruction mechanism is the dynamical disruption of soft binaries. The rate of disruption through stellar evolution is smaller than in the
Initial conditions for all models are given in Table 1. The first group of models is used to cover the parameter space of initial conditions over fairly wide ranges. Our main reference model, model 1, has a core density of $n_c = 10^5$ pc$^{-3}$, a half-mass relaxation time of $t_{\text{bh}} = 10^6$ yr, an initial binary fraction of $f_{b,0} = 1$, a central velocity dispersion of $\sigma_{1D} = 10$ km s$^{-1}$ and a central escape speed of $v_c = 2.5 \sigma_{1D} = 43$ km s$^{-1}$. We then consider three models with different central densities (D3, D4 and D6), two with different half-mass relaxation time (T8 and T10) and one with an initial binary fraction decreased to 50 per cent (B05). The initial total number of stars is $N = 2.5 \times 10^5$ for all models, except for the ‘47 Tuc’ model (see below), where we used $N = 5 \times 10^4$. In all of these models the cluster core was assumed to contain $\sim 1$ per cent of the stars initially and the metallicity was fixed at Z = 0.001 (these two parameters have very little influence on our results).

Table 1. Initial conditions for all models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\log n_c$</th>
<th>$\log \rho_M^{\text{ob}}$</th>
<th>$\log t_{\text{bh}}$</th>
<th>$f_{b,0}$</th>
<th>$\sigma_{1D}$</th>
<th>$v_c$</th>
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<tr>
<td>1</td>
<td>5.0</td>
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<tr>
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<tr>
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<td>1.0</td>
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<td>1.0</td>
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<tr>
<td>T10</td>
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<td>1.0</td>
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<td></td>
</tr>
<tr>
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<td>0.5</td>
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<tr>
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<td>9.08</td>
<td>1.0</td>
<td>18.9</td>
<td>78.2</td>
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</table>

Notation: $n_c$ is the core number density in pc$^{-3}$ (assumed fixed), $\rho_M^{\text{ob}}$ is the observed core mass density in M$_\odot$ pc$^{-3}$, $t_{\text{bh}}$ is the half-mass relaxation time in yr, $f_{b,0}$ is the initial binary fraction, $\sigma_{1D}$ is the 1D velocity dispersion in km s$^{-1}$ and $v_c$ is the escape speed in km s$^{-1}$.

5 RESULTS

5.1 Overview of cluster models

Initial conditions for all of our models are given in Table 1. The first group of models is used to cover the parameter space of initial conditions over fairly wide ranges. Our main reference model, model 1, has a core density of $n_c = 10^5$ pc$^{-3}$, a half-mass relaxation time of $t_{\text{bh}} = 10^6$ yr, an initial binary fraction of $f_{b,0} = 1$, a central velocity dispersion of $\sigma_{1D} = 10$ km s$^{-1}$ and a central escape speed of $v_c = 2.5 \sigma_{1D} = 43$ km s$^{-1}$. We then consider three models with different central densities (D3, D4 and D6), two with different half-mass relaxation time (T8 and T10) and one with an initial binary fraction decreased to 50 per cent (B05). The initial total number of stars is $N = 2.5 \times 10^5$ for all models, except for the ‘47 Tuc’ model (see below), where we used $N = 5 \times 10^4$. In all of these models the cluster core was assumed to contain $\sim 1$ per cent of the stars initially and the metallicity was fixed at Z = 0.001 (these two parameters have very little influence on our results).

Figure 3. Binary destruction rates as a function of time for a field population (i.e. without dynamical interactions). Rates are given as numbers of events per binary per Gyr for mergers and disruptions (following a supernova explosion). Note the peak in evolutionary disruptions at $t \sim 10^5 - 10^6$ yr, which is due to supernovae.

Figure 4. Same as in Fig. 3, but now for the same binary population evolved in a high-density environment with $10^5$ binaries pc$^{-3}$. Here disruptions through binary–binary (BB) and binary–single (BS) interactions dominate for the first $\sim 5$ Gyr.
We performed several simulations with parameters that attempt to match those of specific globular clusters in the Galaxy (the bottom part of Table 1). All of these clusters are classified observationally as ‘non-core-collapsed’, meaning that they are well fitted by standard King models. These are precisely the kinds of clusters that, theoretically, we expect to be in the ‘binary burning’ thermal equilibrium state. For this set, we tried to consider the maximum range of dynamical parameters, while concentrating on clusters at relatively small distances and/or very well studied observationally, so that current or future observations could test our predictions. For our most important model, 47 Tuc, we also considered additional models in which we varied the initial binary fraction or introduced a time-dependent core density (see Section 5.5). For all of our models of specific observed globular clusters, the central number density was chosen in order to provide (at the actual age of the cluster, ∼11–13 Gyr) the best fit to the observed mass density ρ⊙M⊙/pc³ (Pryor & Meylan 1993). Metallicities for these models are taken from Harris (1996).

### 5.2 Main reference model

First, we present our results for a typical dense cluster, represented by model 1. With 100 per cent binaries initially, the final core binary fraction (at 14 Gyr), f b,c, is only 9.5 per cent. This is strikingly low, given that the cluster started with all stars in binaries, and that binaries should concentrate more into the core through mass segregation, but it is expected from our estimates in Section 4. Decreasing the initial binary fraction, f b,0, to a more reasonable (but still large) 50 per cent reduces f b,c further to 7 per cent, as shown in model B05 (see Table 2). The dependence of f b,c on f b,0 is not linear. This is mainly due to mass segregation: decreasing f b,0 also increases the ratio of mean binary mass to mean stellar mass in the cluster, thereby resulting in a higher concentration of binaries in the core.

The majority (∼75 per cent) of destroyed binaries were disrupted by close dynamical encounters (or, rarely, following a supernova explosion). Note that some binaries that are initially hard eventually become soft after undergoing significant mass loss due to stellar evolution. About 20 per cent of the destroyed binaries experienced mergers, typically after significant hardening through interactions. A few per cent of the binaries lost were actually not destroyed but instead were ejected from the cluster as a result of recoil in strong encounters. Tidal capture did not play a significant role: the total number of tidal-capture binaries formed during the cluster lifetime is less than 1 per cent of the final number of binaries in the core.

The total core mass during most of the cluster evolution (the last ∼10 t c) did not vary by more than a factor of 2.

We also performed several simulations with parameters that attempt to match those of specific globular clusters in the Galaxy (the bottom part of Table 1). All of these clusters are classified observationally as ‘non-core-collapsed’, meaning that they are well fitted by standard King models. These are precisely the kinds of clusters that, theoretically, we expect to be in the ‘binary burning’ thermal equilibrium state. For this set, we tried to consider the maximum range of dynamical parameters, while concentrating on clusters at relatively small distances and/or very well studied observationally, so that current or future observations could test our predictions. For our most important model, 47 Tuc, we also considered additional models in which we varied the initial binary fraction or introduced a time-dependent core density (see Section 5.5). For all of our models of specific observed globular clusters, the central number density was chosen in order to provide (at the actual age of the cluster, ~11–13 Gyr) the best fit to the observed mass density ρ⊙M⊙/pc³ (Pryor & Meylan 1993). Metallicities for these models are taken from Harris (1996).

### 5.3 Different densities and relaxation times

Let us now compare results for different central densities, in models 1, D3, D4 and D6. The evolution of f b,c for these models is shown in Fig. 6. As expected, the core binary fraction decreases as ρ increases. The dependence is steeper at high densities, where dynamical interaction rates play a more dominant role compared with stellar evolution.

With respect to the half-mass relaxation time (models T8 and T10), we find that, surprisingly, the model with shorter relaxation time has a higher core binary fraction. There are two competing
Evolution of binary fractions in clusters

5.4 Binary period distribution

Through dynamical interactions, we would expect that the initial period distribution of binaries should be depleted above the boundary between hard and soft binaries. Stellar evolution should deplete a fraction of hard binaries, especially at very short periods, and dynamical encounters should further deplete some of the wider hard binaries. Indeed, for lower-density clusters, we find that the distribution remains much flatter in log $P$ (see Figs 8 and 9), with more and more of the wider hard binaries disappearing as the density increases.

This period distribution can be used to better extrapolate the observed binary fractions, which are usually limited to rather narrow ranges of masses and periods. In particular, it is clear that the usual assumption of a flat distribution in log $P$ for hard binaries at present in a cluster core (e.g. Albrw et al. 2001) can be very misleading. This will be shown in Section 5.5 for our models of several real clusters.

5.5 Comparison with observations

We performed several simulations with parameters that attempt to match those of specific globular clusters in the Galaxy (Tables 1 and 3). For these models, the total core masses we compute at present match well those of King models fitted to the observed cluster parameters, and the corresponding core radii of our models (from total core mass and density) are all $\sim 0.5$ pc, matching the observed values. In all cases the initial binary fraction is assumed to be 100 per cent, so our results for final core binary fractions represent upper limits. As in all reference models, we predict low values for $f_{b,c}$ in all globular clusters, with smaller values in higher-density cores.

We compare our derived binary fraction for M4 with the results from Cote & Fischer (1996). Using a Monte Carlo modelling of the binary population, they found that the best fit to the observed binaries (in the period range from 2 d to 3 yr) is $f_{b,c} \simeq 15$ per cent. This is in good agreement with our overall predicted $f_{b,c} = 11.5$ per cent, and also with our prediction for heavier main-sequence binaries, $f_{0.5} = 17.5$ per cent.

In the 47 Tuc model (for which we increased the total number of stars initially to a more realistic $N = 5 \times 10^5$), $f_{b,c}$ is only 8 per cent at an age of 11 Gyr and $\sim 7$ per cent at 14 Gyr (we provide this value at different ages given the uncertainty in observationally
The ratio between the number of BY Dra systems and the present binary fraction can be considered a lower limit, as it is much lower than the estimate based on eclipsing binaries. This estimate was based on 31 BY Dra binaries (observed in the period range 0.4–40 d) and five eclipsing binaries (period range 4–16 d). We analysed the core binary population in our model in order to identify BY Dra systems and eclipsing binaries, and considering the ratio of the two. We adopted the standard definition for a BY Dra binary: primary mass in the range 0.3–0.7 M⊙ (see, e.g., Bopp & Fekel 1977) and period in the range 0.4–10 d (as for the observed sample in 47 Tuc). For eclipsing binaries we adopted the observed period range 4–16 d with each star older than 0.25 M⊙. The ratio between the number of BY Dra systems and the number of eclipsing binaries is found to be 5.9, 6.7, 7.2 and 3.5 for models with f_{b,0} = 1.0, 0.75, 0.5 and 0.25, respectively. Therefore, a large initial binary fraction greater than 75 per cent is most likely.

Of the quantities that we explicitly assume in our cluster model to be constant, the central (core) density is that with the largest dynamic range in models that provide for dynamical evolution. Hence it is also the quantity that is most likely to significantly affect our final results. To test the sensitivity of our results to a changing central density, we have run our model of 47 Tuc with a central density assumed to increase by a factor of 10 from t = 0 to the present. Specifically, we still match the currently observed core density while ramping up the value either exponentially or linearly in time, starting with a value 10 times smaller than at present. This could represent qualitatively the gradual core contraction observed in some N-body models of star clusters with binaries (see Aarseth & Heggie 1993, and, for a steeper core contraction, see Giersz & Spurzem 2003). These models predict a present-day core binary fraction that is only slightly higher, by ~1–2 per cent. To increase the binary fraction more significantly, the time-averaged core density in the cluster would have had to be many orders of magnitude lower than what is observed today. There is no reasonable dynamical history that would produce such an unusual result. In contrast, recent N-body simulations show that the presence of just 10 per cent hard primordial binaries leads to core radius expansion and therefore the core density might be higher in the past (Wilkinson et al. 2003). Though we did not run another 47 Tuc model with a central density decreasing with time, we can predict that the present-day binary fraction would then be smaller by ~1–2 per cent.

6 CONCLUSIONS

We have considered in detail processes of binary destruction (and formation) in dense stellar systems. In particular, we have shown that the present binary fraction in cluster cores should be relatively small.
We considered several models that attempted to match observed globular clusters. For those with good available data on binaries (M4 and 47 Tuc), we found our predicted binary fractions to be in good agreement with observations once we take into account the correct binary period distribution. The main conclusion we derive from these calculations and our semi-analytic estimates is that the currently observed binary fractions in cluster cores suggest very high initial (primordial) binary fractions – close to 100 per cent.

In addition to their implications for the interpretation of observed binary fractions in cluster cores, our results have important consequences for the theoretical modelling of globular clusters using N-body simulations. Indeed, it is clear that `realistic' dynamical simulations of globular cluster evolution should include large populations of primordial binaries, with initial binary fractions in the range ~50–100 per cent (similar to what is usually assumed for the field; see, e.g., Duquennoy & Mayor 1991). This poses a particular challenge for direct N-body simulations, where the treatment of even relatively small numbers of binaries can add enormous computational costs. For this reason, current direct N-body simulations of star clusters with large initial binary fractions typically have N too small to be considered representative of globular clusters (see, e.g., Wilkinson et al. 2003; Portegies Zwart et al. 2004). Other methods, such as Monte Carlo simulations, do not suffer from the same limitations, and routinely simulate clusters with reasonably large N (∼106–108) and binary fractions (∼30 per cent), but have not yet included advanced treatments of binary star evolution (see, e.g., Fregeau et al. 2003, and references therein).

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Heggie D. C., 1974, Celestial Mech., 10, 217
First, we consider the rate (per star) at which three objects come together within a region of size $b_{\text{max}}$. Effectively, it reflects velocity at which the population at the vicinity $b$ is increasing. We do not consider here the population decrease, assuming that if the object was in the neighbourhood, a three-body encounter has happened.

The final result then takes the form given in Binney & Tremaine (1987, Section 8), but with an additional mass- and hardness-dependent factor $f$,

$$f(m_1, m_2, m_3, \eta) = \frac{n_c^2 G^3 (m)^5}{\sigma^9} \left( \frac{m_1}{n_c} \right) \left( \frac{m_2}{(m)^5} \right) \eta^{-5} (1 + 2\eta) \left( 1 + \frac{v_3}{\sigma} \right)^{-1/2} \sqrt{2 \frac{m_1 m_2}{(m_1 + m_2)(m)}} v_{12}. \quad (A7)$$

It is clear that the rate is highly dependent on the masses of the participating stars and decreases steeply with increasing hardness of the binary that is formed.

If all encounters resulted in the formation of a binary with hardness $\eta$, then the rate of binary formation would be completely described by equation (A7).

Using the equation (A7), the time-scale for three-body binary formation can also be written as

$$T_{\text{th}} = 2.08 \times 10^{10} \text{(yr)} \left( \frac{10^3}{n_c} \right)^2 \left( \frac{M_\odot}{(m)} \right)^5 \frac{(\sigma)}{(10 \text{ km s}^{-1})^9} \times \frac{n_c n_c}{n_2 n_3} \left( \frac{m_1}{m_2} \right)^5 \left( \frac{m_1}{m_2} \right)^5 \frac{\sigma}{v_{12}} \times \eta^9 (1 + 2\eta)^{-1} \left[ 1 + \frac{v_3}{\sigma} \sigma^{-1/2} \sqrt{2 \frac{m_1 m_2}{(m_1 + m_2)(m)}} \right]^{-1}. \quad (A9)$$

Even in the core of a large, very dense cluster, with density $\sim 10^{10}$ pc$^{-3}$ and containing $\sim 10^6$ stars, no binaries with $\eta > 1$ will be formed from $\sim 1$ M$_\odot$ stars in a Hubble time. It has also been shown that many three-body binary formation events will lead to physical collisions for stars as small as white dwarfs (Chernoff & Huang 1996). We therefore neglect all three-body interactions in our cluster simulations.