THE STABILITY AND DYNAMICS OF PLANETS IN TIGHT BINARY SYSTEMS

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ABSTRACT

Planets have been observed in tight binary systems with separations less than 20 AU. A likely formation scenario for such systems involves a dynamical capture, after which high relative inclinations are likely and may lead to Kozai oscillations. We numerically investigate the fate of an initially coplanar double-planet system in a class of binaries with separation ranging between 12 and 20 AU. Dynamical integrations of representative four-body systems are performed, each including a hot Jupiter and a second planet on a wider orbit. We find that, although such systems can remain stable at low relative inclinations (<40°), high relative inclinations are likely to lead to instabilities. This can be avoided if the planets are placed in a Kozai-stable zone within which mutual gravitational perturbations can suppress the Kozai mechanism. We investigate the possibility of inducing Kozai oscillations in the inner orbit by a weak coupling mechanism between the planets in which the coplanarity is broken due to a differential nodal precession. Propagating perturbations from the stellar companion through a planetary system in this manner can have dramatic effects on the dynamical evolution of planetary systems, especially in tight binaries and can offer a reasonable explanation for eccentricity trends among planets observed in binary systems. We find that inducing such oscillations into the orbit of a hot Jupiter is more likely in tight binaries and an upper limit can be set on the binary separation above which these oscillations are not observed.

Key words: binaries: close – celestial mechanics – methods: N-body simulations – planetary systems

1. INTRODUCTION

Among all extrasolar planets discovered to date, 20% are in multiple stellar systems (Desidera & Barbieri 2007; Eggenberger et al. 2007a). Binary stellar systems with circumstellar and circumbinary debris disks have been observed with an overall occurrence rate higher than that for debris disks around single stars (Trilling et al. 2007). Numerical simulations suggest that protoplanetary disks embedded in binary star systems should be able to form both terrestrial and giant planets (Barbieri et al. 2002; Quintana et al. 2002; Boss 2006).

Most binary and multiple stellar systems found to be harboring planets are wide, with separations larger than 100 AU (Eggenberger et al. 2004; Mugrauer et al. 2004). However, several systems with separations as low as ∼20 AU have been shown to harbor giant planets. These are the binary systems HD196885, γ Cephei, and Gliese 86, and the higher-order system HD41004 (Correia et al. 2008; Hatzes et al. 2003; Els et al. 2001; Zucker et al. 2004). There are two important things to note about these systems. First, since radial velocity surveys have always been biased against close binaries (Eggenberger et al. 2007a), this sample probably underrepresents the frequency of planets in such binaries. In particular, the lower limit for the separation of binaries that can harbor planets may be smaller than ∼20 AU. Second, such systems constitute a unique data set which can allow us to test theoretical models for planet formation and evolution. This is because the presence of a stellar companion at such proximity can greatly influence these processes. A stellar companion as far as ∼50 AU can weaken the chances for the formation of a giant planet by stirring, heating or truncating protoplanetary disks (Jang-Condell 2007; Boss 2006; Mayer et al. 2005; Kley 2001; Nelson 2000, Nelson et al. 2000). Therefore, the fact that systems as tight as 20 AU are harboring giant planets is puzzling and does seem to require further investigation. The most likely formation scenario for such systems may involve dynamical encounters in dense stellar systems where the binary companion is captured or moved inward from a wider orbit after the planet formation has taken place. A stellar companion can also affect the dynamical evolution of a planetary system, and, therefore, its long term survival, through secular perturbations. In the case of a large relative inclination between the planetary orbit and that of the stellar companion, Kozai oscillations can take place where angular momentum exchange between the orbits leads to large-amplitude synchronous oscillations in the eccentricity and inclination of the planet (Kozai 1962; Holman et al. 1997; Innanen et al. 1997; Ford et al. 2000; Takeda & Rasio 2005). This has the potential to disrupt the system if it results in close encounters, orbital crossings, or strong planet–planet interactions for systems with more than one planet.

A renewed interest in planetary dynamics in general and in the stability of planets in binary systems particularly has been triggered by the discovery of planets in binary and multiple stellar systems (Dvorak et al. 2003; Barnes & Raymond 2004; Raymond & Barnes 2005; Raymond et al. 2006a; Rivera & Haghighipour 2007). Looking into the dynamical history of planetary systems is key to understanding the processes involved in their formation and subsequent evolution. In fact, this problem has received special interest from dynamicists even before extrasolar planets were discovered, with many studies applying general results of the three-body problem to the special case of a binary stellar system harboring a planet (e.g., Dvorak & Henrard 1993; Innanen et al. 1997; Holman & Wiegert 1999, hereafter HW99). By performing numerical simulations on planetary orbits in binary systems, HW99 derived empirical expressions for the maximum semimajor axis of the planet as a function of both binary mass fraction and eccentricity, above which the system would become unstable. Their study considered the stability of binary systems harboring planets in the coplanar case only. Numerical simulations by David et al. (2003) of Earth-like planets in binary systems with different initial configurations led to the estimate that 50% of binary
systems allow an Earth-like planet to remain stable for 4.6 Gyr. The stability of Earth-like planets in binaries was also studied by Fatuzzo et al. (2006). With numerical simulations they calculated the survival time of an Earth-like planet orbiting a Sun-like star in the presence of a stellar companion. They explored a range of binary configurations including relative inclinations between the binary orbit and the orbit of the planet. Haghighipour (2005) performed numerical simulations to model the binary system γ Cephei. This is a tight binary with separation 18.5 AU and an eccentricity of 0.36. A Jovian planet with a minimum mass of 1.7 M_J was shown to be orbiting the primary of this system at 2.1 AU (Hatzes et al. 2003). The question of the presence of an Earth-like planet within the habitable zone (HZ) of the primary was addressed and a range of semimajor axes and eccentricities of the binary as well as orbital inclination of the Jupiter-like planet was explored. This study concludes that, within timescales in the range 10–100 Myr, such a planet can exist on a stable orbit but not within the HZ of the primary.

In this study, we investigate the stability of double-planet systems in tight binaries. The tightest stellar system in which a planet was ever claimed is HD188753. This is a hierarchical triple-star system in which the primary is a Sun-like star with mass 1.06 M⊙. The binary companion is a double-star system with total mass 1.63 M⊙ orbiting the primary at separation ~ 12.3 AU and eccentricity 0.5 (Griffin 1977). A hot Jupiter of mass 1.14 M_J was claimed to be orbiting the primary star with a period of 3.35 d (Konacki 2005). Because of the proximity of the companion, the presence of a planet in this system is suggestive of a dynamical history in which the binary companion was either moved inward to a tighter orbit or captured after the claimed planet had already formed and reached its current position (Portegies Zwart & McMillan 2005; Pfahl 2005; Boss 2006; Jang-Condell 2007). Although the observation of this planet was refuted by Eggenberger et al. (2007b), we find this system to be a good representative of the lower limit (in separation) of a class of tight binary systems harboring giant planets. We test in this study the capacity of such systems to harbor two planets including a hot Jupiter. This class of binaries with separation ≤ 20 AU is of special interest since it is likely that such systems may have had active dynamical histories and are expected to induce large perturbative forces into planetary systems with dramatic results. Studying such systems in the presence of more than one planet, can uncover the role played by binary companions in shaping planetary systems. This is especially important since multiplanet systems seem to be common in nature (Fischer et al. 2003). The dynamics of double-planet systems in wide binaries was studied analytically by Takeda et al. (2008), where three different dynamical classes were identified as possible outcomes: (1) decoupled systems in which planetary orbits experience independent Kozai cycles due to their weak mutual interaction compared to perturbations from the companion; (2) weakly coupled systems in which mutual gravitational interactions become more dominant and relative nodal precession can cause mutual inclinations to grow and induce Kozai oscillations in the inner orbit; (3) dynamically rigid systems in which the orbital elements of both planets oscillate in concert. These possible outcomes depend on the initial conditions and therefore the relative strength of the different perturbative forces. Here we expand on this study by looking into systems with closer companions and test the possibility of inducing eccentricity oscillations in a hot Jupiter orbit through a weakly coupled system.

Since planets with orbital radii a ≲ 0.1 AU are expected to be tidally circularized, the recent observations of a few hot Jupiters on considerably eccentric orbits (such as XO-3, HAT-P-2, and HD185269 (Johns-Krull et al. 2008; Loeillet et al. 2008; Johnson et al. 2006)) does raise questions. Why are the orbits of these planets not circularized when the orbits of most hot Jupiters are? Matsumura et al. (2008) show that close-in planets on eccentric orbits can be explained by constraining their tidal Q factors under the assumption that these orbits are in the process of being circularized. They also explore the possibility that these eccentricities are induced by a perturber on a wider orbit for the system GJ436 and show that such a planet, if present, would cause a radial velocity amplitude above the current detection limit, and therefore, would have been observed.

Although such eccentricities can, in principle, be explained by the presence of a companion, whether stellar or substellar, one has to keep in mind that a planet on a close-in orbit experiences strong general relativistic (GR) forces. Therefore, a stellar companion as close as 12–20 AU cannot induce such eccentricities directly into a hot Jupiter orbit. We test here the possibility of inducing such eccentricities to a hot Jupiter through a second planet in the presence of a binary companion. We also investigate the dynamics and stability criteria of a double-planet system orbiting the primary of a tight binary system with separation in the range a_b = 12–20 AU. All systems we investigate in this study contain a hot Jupiter of mass 1 M_J and separation a_1 = 0.05 AU. The stability region for a second planet, placed on a wider orbit, that is initially circular and coplanar with a variable semimajor axis a_2, was explored for two different binary configurations. In the first, the binary companion orbits at a_b = 12 AU with an eccentricity e_b = 0.5, and in the second, a_b = 20 AU and e_b = 0. For the tightest binary separation (a_b = 12 AU), two different planetary masses m_2 = M_J and m_2 = M_b were investigated. In this class of systems, planets experience mutual gravitational perturbations in addition to perturbations from the stellar companion. A hot Jupiter orbit is strongly dominated by GR forces. Because of the tightness of these systems, the interplay between these different forces is expected to have a dramatic effect on the planetary system.

Throughout this paper, the subscripts 0, 1, 2, and b will be used to refer to the primary star, the inner planet, the outer planet, and the binary companion, respectively. We use a semianalytical approach to predict the stability region of the outer planet m_2. Our numerical simulations incorporate a wide range of parameters including mutual inclinations of the orbits and perturbative forces to the Keplerian motion of each planet including planet–planet interactions and GR effects. Such numerical simulations can reveal to us the likelihood of a compact stellar system to maintain multiple planets and can be a useful guide for future observational efforts such as HARPS and the Kepler Mission.

An overview of secular perturbations affecting this class of systems is presented in Section 2. In Section 3, we discuss the analytical background for our predicted stability region for the outer planet in a stellar system resembling HD188753. The numerical techniques used in this study are presented in Section 4, results and discussion in Section 5, and finally, Section 6 summarizes our conclusions and suggestions for future work.
2. SECULAR PERTURBATIONS IN HIERARCHICAL TRIPLE SYSTEMS

Hierarchical triple systems can be studied using classical Hamiltonian perturbation techniques, which show that the secular evolution of the orbits takes the form of eccentricity oscillations coupled with precession of the longitudes of pericenter (Brouwer & Clemence 1961; Kozai 1962; Harrington 1968; Rasio et al. 1997; Ford et al. 2000). Such perturbations are highly dependent on the initial relative inclination between the two orbits. The classical planetary perturbation theory (Brouwer & Clemence 1961) applies to the case of low relative inclination and low eccentricities for planets around a central star and is applicable to all orders in the ratio of the semimajor axes $\alpha = a_1/a_2$, where $a_1$ and $a_2$ represent the semimajor axes of the inner and the outer orbits, respectively (e.g., Rasio 1994, 1995). The secular evolution of a triple system with large relative inclination was derived by Kozai (1962) for the limit of small $\alpha$ using the quadrupole approximation of secular perturbation theory. In the following, we discuss and calculate precession rates associated with these perturbations in addition to GR forces as they apply to the type of systems investigated in this study (see Section 1).

2.1. The Low-Inclination Regime

Mutual gravitational perturbations among planets can lead to a precession of the periastron and eccentricity oscillations of both planets. In the case of low eccentricities and low relative inclinations, the periods of these oscillations can be calculated using the formulation of the Laplace–Lagrange secular theory in which the disturbing potentials of the planets are simplified by eliminating short-period terms (dependent on the mean longitudes). Following the classical planetary perturbation theory (Brouwer & Clemence 1961; see also Murray & Dermott 1999) for a two-planet system, the disturbing functions for the two planets, second order in eccentricity, are

$$R_1 = n_1 a_1^2 \left[ \frac{1}{2} A_{11} e_1^2 + A_{12} e_1 e_2 \cos(\omega_1 - \omega_2) \right]$$

and

$$R_2 = n_2 a_2^2 \left[ \frac{1}{2} A_{22} e_2^2 + A_{21} e_1 e_2 \cos(\omega_1 - \omega_2) \right],$$

where $n, n_1, a_1, a_2, \omega_1, \omega_2$ refer to the mean motions, semimajor axes, and longitudes of pericenter of the planets, respectively. The matrix elements

$$A_{11} = \frac{1}{4} n_1^2 \frac{m_2}{m_0 + m_1} a^{2(1)}_{3/2} \alpha,$$

$$A_{22} = \frac{1}{4} n_2^2 \frac{m_1}{m_0 + m_2} a^{2(1)}_{3/2} \alpha,$$

and

$$A_0 = \frac{A_{11}}{A_{12}} \approx -\frac{5}{6} \left( 1 - \frac{1}{8 \alpha^2} \right),$$

are functions of the planetary masses and semimajor axes, where $\alpha = a_1/a_2$, and

$$a^{(1)}_{3/2}(\alpha) = \frac{1}{\pi} \int_0^{2\pi} \frac{\cos \psi d\psi}{(1 - 2\alpha \cos \psi + \alpha^2)^{3/2}}$$

The eigenvalues of the matrix $A$ represent the characteristic frequencies of the system with which the planetary orbital elements will evolve. These eigenfrequencies are given by

$$g^+ = \frac{1}{2} \left[ (A_{11} + A_{22}) + \sqrt{(A_{11} - A_{22})^2 + 4A_0^2 A_{11} A_{22}} \right],$$

and

$$g^- = \frac{1}{2} \left[ (A_{11} + A_{22}) - \sqrt{(A_{11} - A_{22})^2 + 4A_0^2 A_{11} A_{22}} \right].$$

Pericenter precession rates are, in general, nonlinear combinations of these eigenvalues and can be estimated for different planetary systems (Takeda et al. 2008). The periods of oscillations can then be calculated from these rates. In this study, we use the period of orbital eccentricity oscillations given by $P_e = \frac{2\pi}{g^+}$.

2.2. The High-Inclination Regime

Kozai (1962) showed that in a hierarchical system where the ratio of semimajor axes, $\alpha$, is sufficiently small and a large relative inclination ($\gtrsim 40^\circ$) between the orbits exists, exchange of angular momentum between the orbits results in simple periodic oscillations in both the inner eccentricity and the mutual inclination of the orbits $i$ such that they are coupled by the integral of motion $(1 - e^2) \cos^2 i$. Under these conditions, the amplitude of eccentricity oscillations depends only on the relative inclination $i$ (Innanen et al. 1997) and is given at quadrupole order by

$$e_{\text{max}} = \sqrt{1 - \frac{5}{3} \cos^2 i}. $$

The period of oscillations is a function of $\alpha$, the masses, and the eccentricity of the outer orbit (Kiseleva et al. 1998)

$$P_{\text{Kozai}} \approx P_2 \left( \frac{m_0 + m_2}{m_b} \right) \frac{a_0}{a_2} \left( 1 - e_b^2 \right)^{3/2},$$

where $P_2$ is the orbital period of the planet.

2.3. GR Forces

For close-in planets, general relativistic effects become dominant and can cause the periastron of the planet’s orbit to precess on very short timescales (Eggleton & Kiseleva-Eggleton 2001),

$$P_{\text{GR}} \approx 3011 \text{yr} \left( \frac{a}{0.05 \text{ AU}} \right)^{5/2} \left( \frac{m_0}{1.0 \text{ M}_\odot} \right)^{3/2},$$

where $a$ is the semimajor axis of the planet and $m_0$ is the mass of the central star. This precession can lead to the suppression of Kozai oscillations by averaging out the torques responsible for it if they act on shorter timescales.

2.4. Comparison of Timescales

We compare in Figure 1 the precession rates due to all three perturbations acting on both planets in the case of a double-planet system in a binary with separation $a_b = 12$ AU and eccentricity $e_b = 0.5$. The planets, with masses $m_1 = m_2 = M_J$, are placed initially on circular and coplanar orbits with $a_1 = 0.05$ AU.

The dynamics of the outer planet are expected to be dominated by the Kozai effect as long as the semimajor axis of its orbit is larger than $a_2 \sim 0.3$ AU. Eccentricity oscillations with large amplitudes can cause this planet to experience close encounters either with the companion or with the hot Jupiter, leading to instabilities. Below $a_2 \approx 0.3$ AU, mutual gravitational interactions among the planets dominate, and Kozai oscillations are gradually suppressed. Therefore, a Kozai-stable zone exists
below this point, within which this second planet can maintain
stability even when relative initial inclinations are larger than
the Kozai limit $i_{\text{init}} \approx 40^\circ$. The inner orbit at $a_1 = 0.05$ AU is
strongly affected by GR forces such that $\dot{\omega}_{\text{GR}} > \dot{\omega}_{\text{Koz}}$ and Kozai
oscillations are suppressed in this planet. On the other hand,
multiple planetary interactions can dominate the dynamics of
the system as the outer planetary orbit gets tighter than $a_2 \lesssim 0.2$ AU
and periastron precession rates associated with this force $\dot{\omega}_{\text{pp}}$
exceed those associated with GR forces $\dot{\omega}_{\text{GR}}$. This should be
revealed in the form of mild eccentricity oscillations in both
orbits.

We look into the possibility of inducing Kozai oscillations in
the inner orbit by propagating perturbations from the companion
through the outer planet in the system. Kozai oscillations can be
induced in the inner orbit if a relative nodal precession occurs,
splitting the planes of the planetary orbits and producing relative
inclinations larger than the Kozai critical angle. For these
oscillations to occur two conditions must be satisfied: (1) the
outer planet’s orbit must experience nodal precession causing
periodic growth in the mutual inclination angle between the
planets. The outer planet can experience nodal precession even
if it is not experiencing Kozai oscillations since this precession
is a natural consequence of secular interactions with the stellar
companion (Takeda et al. 2008); (2) precession rates associated
with the induced Kozai oscillation of the inner orbit must exceed
GR precession rates. Whether both conditions can be satisfied
simultaneously depends on the choice of initial conditions. In
our systems the inner planetary orbit is fixed at 0.05 AU. The
outer planet must be placed in a region where it is close enough
for the induced Kozai effect to compete with GR forces. We
show in Figure 2 the timescales for precession in the inner orbit
calculated from Equation (10) caused by GR forces (Equation (11))
and those caused by the mutual planetary Kozai effect calculated from Equation (10) by
replacing $m_b$ with $m_2$ as the perturber and replacing $m_2$ with
$m_1$. Two different perturber masses are tested, $m_2 = M_J$ and
$m_2 = M_{\oplus}$. It shows that while an Earth-mass planet cannot
compete with the strength of GR forces acting on such a tight
inner orbit, a Jupiter-mass planet can produce this effect when
$a_2 \lesssim 0.8$ AU.

3. ANALYTICAL PREDICTIONS FOR STABILITY OF A
DOUBLE-PLANET SYSTEM IN A TIGHT BINARY

We investigate the stability criteria of a double-planet system
in a binary with separation $a_b \leq 20$ AU. The inner planet is a hot Jupiter with semimajor axis $a_1 = 0.05$ AU and mass
$m_1 = M_J$. An approximate stability region for the outer planet is
derived analytically within the two-dimensional space $[i_{\text{init}}, a_2]$ where $a_2$ is its semimajor axis and $i_{\text{init}}$ is the initial relative
inclination between the common planetary orbit and that of
the binary. This stability region is derived analytically for two
binary separations: $a_b = 12$ AU and $a_b = 20$ AU. In the case
$a_b = 12$ AU, the stability region is compared for two different
planetary masses $m_2 = M_J$ and $m_2 = M_{\oplus}$ in the outer orbit. For
a given value of $i_{\text{init}}$, a planetary orbit is expected to be stable
if $a_2$ lies between two limits, $a_{2,\text{min}}$ and $a_{2,\text{max}}$. The lower limit,
$a_{2,\text{min}}$, is defined as the minimum semimajor axis below which
strong perturbations with the inner orbit can lead to instabilities,
while the upper limit, $a_{2,\text{max}}$, is the maximum semimajor axis of
the planet beyond which the planet enters a region of instability
due to the presence of the stellar companion.

3.1. Upper Limit

The upper limit for the semimajor axis of a planet in a binary
system was determined numerically by HW99 for initially
circular planetary orbits which are coplanar with the binary
orbit. This was done for a range of binary mass fractions
$\mu_b = m_b/(m_0 + m_b)$ (where $m_0$ is the mass of the primary
and $m_b$ is the mass of the perturber) and eccentricities $e_b$ of the binary. For each set of values [$\mu_b$, $e_b$], the binary system was numerically integrated with test particles representing the planet, and a critical semimajor axis $a_c$ was determined beyond which a planetary orbit may become unstable. An analytical expression for $a_c$ was derived from these numerical results such that,

$$a_c \approx (0.464 - 0.380 \mu_b - 0.631 e_b + 0.586 e_b \mu_b + 0.150 e_b^2 - 0.198 e_b^2 \mu_b) a_b,$$

(12)

where $a_b$ is the semimajor axis of the binary.

Applying Equation (12) to the outer planet in the binary systems we consider gives a value of $a_c \approx 1.23$ AU for the stellar system HD188753, and $a_c \approx 5.48$ AU for the equal-mass binary with separation 20 AU, both in the coplanar case. When large relative inclinations ($i \geq 40^\circ$) are present, we can estimate for an elliptical orbit the upper limit on the planet’s semimajor axis using,

$$a_{2, \text{max}} \approx \frac{a_c}{(1 + e_{\text{max}})},$$

(13)

where $e_{\text{max}}$ is the maximum eccentricity gained during Kozai oscillations (Equation (1)), and assuming the value of $a_c$ calculated from Equation (12) is the distance from the primary at apastron.

### 3.2. Lower Limit

The stability of three-body systems was discussed analytically in the literature in more than one context. Some aimed at the stability limits for a system composed of a central star orbited by two planets (e.g., Gladman 1993, hereafter G93), while others were mainly focused on triple stellar systems (e.g., Mardling & Aarseth 2001, hereafter MA01). G93 provides stability criteria for a system of one star with two planets. He derived a minimum separation between the two planets “$\delta$” below which the system cannot maintain Hill stability. A system is considered Hill stable if the planets cannot experience close encounters at any time. For systems in which the central star $m_0$ is much more massive than both planets, $m_1$ and $m_2$ ($\mu_1 = m_1/m_0 \ll 1$ and $\mu_2 = m_2/m_0 \ll 1$), and for initially circular and nearly coplanar orbits, the minimum separation $\delta$, expressed in units such that the semimajor axis of the inner planet is unity, is given to lowest order in the masses as

$$\delta \simeq 2.40 (\mu_1 + \mu_2)^{1/3}.$$  

(14)

For equal-mass planets this reduces to $\delta \simeq 3\mu^{1/3}$. For noncircular orbits, but small eccentricities ($e \leq \mu^{1/3}$) and equal-mass planets, this becomes

$$\delta \simeq \sqrt{\frac{8}{3} (e_1^2 + e_2^2) + 9 \mu^{2/3}}.$$  

(15)

This reduces to the previous equal-mass case for zero eccentricities. G93 also derived the Hill stability criterion for the case of equal-mass planets and equal and arbitrarily large eccentricities ($e_1 = e_2 = e$), giving

$$\delta \simeq \left(\frac{3 + e^2}{2(1 - e^2)} - \frac{1}{2} \sqrt{\frac{(9 - e^2)}{(1 - e^2)}} + \frac{1}{2} \sqrt{(9 - e^2)} - \frac{3}{2} \right)^2 - 1 + O(\mu^{1/3}),$$

(16)

This expression is valid only for eccentricities larger than $e \simeq \mu^{1/3}$, in which case the mass terms become negligible compared to the large eccentricity terms. Therefore, at large eccentricities, $\delta$ becomes independent of planetary masses.

One can apply this stability criterion to the two planets in our systems. Eccentric orbits are expected if the binary orbit is highly inclined relative to the common planetary plane. In this case, $\delta$ is calculated from Equation (16) by substituting $e_{\text{max}}$ from Equation (1) for the eccentricity. This sets a lower limit on the minimum separation allowed between the planets, since in some cases, the outer planetary orbit may experience Kozai oscillations while the inner planet is shielded from this effect due to stronger perturbations (Section 2.4). To calculate $\delta$ for systems with relative inclinations less than 40°, assuming $e_1 = e_2 \approx 0$, we use Equation (15) in the case of equal mass planets ($m_1 = m_2 = M_J$) and Equation (14) in the case of a second planet of Earth-mass ($m_2 = M_\oplus$). Once $\delta$ is estimated, the minimum semimajor axis allowed for the second planet can be calculated as $a_{2, \text{min}} = a_1 (1 + \delta),$ where $a_1$ is the semimajor axis of the inner planet. The difference in the value of $a_{2, \text{min}}$ obtained when applying the G93 criterion to the two different values of $m_2$ used in this study, is $\sim 0.01$ AU. This is small enough that in practice here we ignore it when considering our predicted stability region.

The stability of a hierarchical triple configuration was discussed in MA01, who describe hierarchical coordinates for an inner binary with masses $m_0$ and $m_1$, and orbit described by $e_{\text{in}}$ and $e_{\text{in}}$ (the inner eccentricity and semimajor axis). A more distant object $m_2$ orbits around the center of mass of the inner binary with eccentricity $e_{\text{out}}$ and periastron separation $R_{\text{per}}$. They derive a critical value for $R_{\text{per}}/a_{\text{in}}$ below which the triple configuration can become unstable and may experience the escape of one object,

$$R_{\text{per}}/a_{\text{in}} = 2.8 \left(\frac{(1 + q_{\text{out}})(1 + e_{\text{out}})}{(1 - e_{\text{out}})^{1/2}}\right)^{2/5},$$

(17)

where $q_{\text{out}} = m_2/(m_0 + m_1)$ is the outer mass ratio. This equation gives an upper limit, which represents coplanar systems. Numerical simulations by Mardling & Aarseth (1999) show that noncoplanar systems tend to be more stable, and obtain from their simulations a reduction factor $f = 1 - 0.3i/\pi$ (where $i$ is the relative inclination between the inner and outer orbit) to account for the increased stability of inclined orbits.

This stability criterion can also be applied to the two planets around the primary star in HD188753 to place a lower limit on the distance of periastron of the second planet. One can again substitute for $e_{\text{out}}$ the value of $e_{\text{max}}$ obtaining a lower limit on the semimajor axis of the second planet. This gives

$$a_{2, \text{min}} = 2.8 \times a_1 \left(\frac{(1 + q_{\text{out}})(1 + e_{\text{max}})^{2/5}}{(1 - e_{\text{max}})^{1/5}}\right).$$

(18)

The MA01 criterion is weakly dependent on the mass of the outer planet $m_2$ through the ratio $q_{\text{out}} = m_2/(m_0 + m_1)$ in Equation (18). Therefore, the difference in the estimated value of $a_{2, \text{min}}$ obtained when using the two different values of $m_2$ ($M_J$; $M_\oplus$) is very small and therefore is neglected.

### 3.3. The Predicted Stability Region

In Figure 3, we show the analytically predicted stability region for a double-planet system with $m_1 = m_2 = M_J$ in HD188753 where a hot Jupiter at 0.05 AU represents the inner
planet. The upper limit \(a_{2,\text{max}}\) is constrained by the Holman & Wiegert (1999) criteria from which a value of \(\sim 1.23\) AU was obtained in the coplanar case. For the lower limit on \(a_{2}\), both the G93 and MA01 stability criterion were applied to the central star in HD188753 with two planetary orbits. The maximum eccentricity of the outer planet was calculated from Equation (1) for every value of the initial inclination \(i_{\text{init}}\). For the range \(i_{\text{init}} < 40^\circ\), the value \(e_{\text{max}} = 0\) was adopted. Therefore, this range represents the expected stability criterion for initially circular orbits. The minimum semimajor axis allowed for the second planet derived from G93 is \(a_{2,\text{min}} \approx 0.065\) AU, while the MA01 sets a more stringent limit of \(a_{2,\text{min}} \approx 0.14\) AU. Applying the reduction factor \(f = 1 - 0.3i/\pi\) for \(i = 70^\circ\) reduces this value to \(\sim 0.12\) AU. Figure 3 also shows the Kozai Suppression Limit (KSL). The shaded area below this line is the \textit{predicted Kozai-stable zone} within which a second planet is expected to survive Kozai oscillations even in the presence of high relative inclinations \((i_{\text{initial}} \geq 40^\circ)\).

4. NUMERICAL METHODS

The dynamics of a double-planet system in a tight binary with separation in the range \(a_b = 12–20\) AU is studied. We look into the effect a second planet may have on the orbit of a hot Jupiter in the presence of a close-in binary companion. We also investigate the stability criteria of this class of systems for two different binary configurations. The first, taken as a representative of the tightest binary in this class, resembles the stellar system HD188753 with separation 12 AU and eccentricity 0.5. The second is a 20 AU equal-mass binary \((m_0 = m_b = M_\odot)\) with zero eccentricity. In all our integrated systems a hot Jupiter of mass \(m_1 = M_J\) is placed on an initially circular orbit at 0.05 AU. We consider two cases for a second planet: a Jupiter-mass planet (equal-mass planets), and an Earth-mass planet. We examine the possibility of a habitable planet in HD188753 in the light of these results. For integrations resembling HD188753, the semimajor axis of the second planet was taken in the range 0.055 AU < \(a_2\) < 1.8 AU in bins of 0.1 or 0.2 AU in all regions excluding the closest to the inner orbit where we examine numerically the cases \(a_2 = 0.055, 0.06, 0.063, 0.066\) and 0.1 AU. In the case of the 20 AU binary, \(a_2\) was varied between the values 0.06 AU and 6.0 AU in bins of 1 or 0.5 AU. In all of our simulations, the initial planetary orbits are circular and coplanar. The common planetary orbital plane is chosen initially to make an angle with the binary orbit \(i_{\text{max}}\), which was varied between 0° and 70° in steps of 10° in most cases.

We perform a total of 5460 simulations representing 273 different initial configurations. For each configuration, many realizations were constructed with different initial relative phases between the orbits, which were chosen randomly. The number of realizations varied between 5 and 100 depending on the configuration in hand. Integrations were performed using a modified version of the integration package MERCURY6.2 (Chambers 1999). The code was modified to include GR effects as an additional force. We adopted for this force the standard textbook \(1\over r^3\) term (e.g., Misner et al. 1973, Equation (40.3)). Since our integrations are expected to involve close encounters, we use for all of our integrations the Bulirsch–Stoer integrator. We monitor conservation of total energy and angular momentum for the duration of the integration. Energy degeneration was always maintained to less than \(10^{-5}\) and fractional angular momentum change was always smaller than \(10^{-7}\), excluding runs performed with the second planet’s orbit close to the inner orbit \((a_2 \lesssim 0.1\) AU\). In these cases, the fractional angular momentum change would fall in the range \(10^{-6}\)–\(10^{-5}\).

Since direct integrations of such tight systems involve many close encounters, and are computationally expensive, and since we are not interested in following the dynamical evolution of these systems after they become unstable, we optimize our computations by stopping them as soon as instability develops. We test the timescale for the onset of instability by integrating a system just outside the expected stability region. The system chosen for this test is one with a Jupiter mass planet placed on an orbit with semimajor axis \(a_2 = 1.4\) AU and a relative initial inclination of 40°. We show the instability timescale histogram for this system in Figure 4, which includes 363 different realizations. We find that 99% of these simulations hit instability by \(6.6 \times 10^5\) yr. Based on these results we chose for our integrations the integration time \(6.6 \times 10^5\) yr to decide stability for a Jupiter-mass planet. This timescale corresponds to more than \(10^4\) binary cycles. We also integrate systems with a Jupiter-mass planet for as long as 6.6 Myr when this planet is near the Kozai suppression limit, since these systems tend to survive longer than their counterparts.

5. RESULTS AND DISCUSSION

We test the stability region within a tight binary for a double-planet system composed of a Jupiter-mass planet at 0.05 AU and a second planet on a wider orbit. Two binary separations are investigated, 12 AU and 20 AU. The stability region is tested for a Jupiter-mass planet in the outer orbit for both binary separations. In the case of 12 AU separation, the stability region for a potentially habitable planet is also tested where \(m_2 = M_\oplus\).
5.1. A 12 AU Binary: The Case of HD188753

5.1.1. Equal-Mass Planets

The stability region of the double-planet system described above in a binary system resembling HD188753 is investigated. Figure 5 summarizes the results of all our simulations of this system spanning the whole expected region of stability for a second planet of Jupiter mass. It shows that as long as the orbits are highly inclined ($\gtrsim 40^\circ$), Kozai oscillations will always lead to instabilities in this system. For small relative inclinations, $i_{\text{ini}} \lesssim 40^\circ$, the simulations give a lower limit for the semimajor axis of this planet in the range $0.063 \text{ AU} < a_{2,\text{min}} < 0.066 \text{ AU}$, which is consistent with the G93 predicted value of 0.065 AU. The upper limit lies in the range $1.3 \text{ AU} < a_{2,\text{max}} < 1.4 \text{ AU}$ for the coplanar case, larger than the value predicted by HW99. These results are obtained using full four-body simulations which include planetary interactions with all other particles in the system and therefore, are expected to set more realistic limits on stability boundaries than do studies that treat planets as massless point particles.

Our results also confirm the existence of a Kozai-stable zone within which stability is maintained even in the presence of large relative inclinations. This is seen in the region $a_2 = 0.1-0.3$ AU. At $a_2 = 0.3$ AU, we can see a transitional region between stability and instability as the system approaches the edge of the Kozai-stable zone. We compare in Figure 6 the time evolution of both planets in two different cases. In the first the outer planet is within the Kozai-stable zone at $a_2 = 0.2$ AU. Although its orbit is highly inclined relative to the orbit of the companion, we see stable orbits with no sign of growth. The second case represents the outer planet outside the Kozai-stable zone at $a_2 = 1.0$ AU, where mutual gravitational perturbations become too weak to suppress Kozai oscillations. This case leads to the ejection of the planet within less than 80,000 years.

Figure 5 also shows a region of the explored space ($0.35 < a_2 < 0.8$ AU, and $i_{\text{ini}} \gtrsim 40^\circ$) within which the planets are weakly coupled and therefore Kozai oscillations are induced in the inner orbit by interaction with the outer planet. Large eccentricities are induced into the orbit of the companion, eventually bringing it to the surface of the star. We discuss this effect in detail in Section 5.3.
of an Earth-mass planet at low relative inclinations (\(i_{\text{init}}\)). Our results show that the upper limit for stability can survive down to a region between 0.06 AU and 0.063 AU such a planet will be extremely hot and inhabitable.

In addition to dynamical stability, an important condition for habitability is climate stability which can be compromised if the planet’s orbit becomes eccentric. This has the potential to drive the planet’s orbit outside the HZ at apastron or/and periastron, or may lead to extreme heat variations on the surface of the planet. We show in Figure 8 the orbital elements of an Earth-mass planet in this system with low relative inclination and initially on a circular orbit at \(a_2\) \(\sim\) 1.0 AU. It experiences mild eccentricity oscillations with an amplitude of \(e\) \(\sim\) 0.1, with no sign of growth. Still, habitability of this planet can become questionable since such an eccentricity can drive the planet near the edges of the HZ at periastron and apastron. Therefore, the presence of a perturber to the Keplerian motion of a potentially habitable planet can alter the positions of the outer and the inner edges of the HZ through their effect on the orbit. In this case, the outer edge of the HZ is expected to move inward to \(a_2\) \(\sim\) 1.24 AU from the primary while the inner edge move from \(a_2\) \(\sim\) 0.95 AU to \(\sim\) 1.0 AU, altering the width of the HZ by \(\sim\) 40%.

5.2. A 20 AU Equal-Mass Binary

It has been shown by Jang-Condell (2007) that a stellar system as tight as HD188753 is unlikely to support a disk massive enough to form a Jovian planet in situ. Therefore, the presence of a planet in such a system would be indicative of a dynamical history. On the contrary, the truncated disk around a binary with a slightly larger separation such as \(γ\) Cephei (separation 20 AU and eccentricity 0.4) has been shown (Jang-Condell et al. 2008) to have sufficient mass to form such a planet. Such wider binaries may host giant planets and still have no active dynamical histories, and therefore, maintain dynamical stability with multiple planets which are coplanar with the binary orbit. Therefore, a binary with separation 20 AU, represents an upper limit in this study to a class of binaries with low separation in which the presence of a hot Jupiter is questionable.
We show in this section, our results for the stability region of a double-planet system in a binary with two stars of solar mass orbiting at 20 AU. The planetary system used in this test is the same system used in Section 5.1 in which two coplanar, equal-mass planets of Jupiter size are placed on circular orbits. The inner planet is a hot Jupiter at 0.05 AU, while the semimajor axis of the second planet, $a_2$, is varied along the expected range. The stability region is also tested here in the two-dimensional space $[i_{\text{ini}}, a_2]$ where $i_{\text{ini}}$ is the initial relative inclination between the binary orbit and the common planetary plane. We apply the same criteria as in Section 3 to predict the stability region for the outer planet when the inner orbit is fixed. These results are shown in Figure 9 where one can see that the stability region for this system is similar to that of HD188753 in its structure but varies in scale. The larger binary separation has allowed more space for the outer planet to survive. In some cases the outer planet can experience Kozai oscillations (with moderate eccentricities) for as long as 1 Myr without disrupting the system as long as it does not reach the upper limit for stability at apastron. The Kozai stable zone expands with the binary separation in general since mutual planetary interactions become more dominant relative to the Kozai effect. In this system the edge of the Kozai stable zone is at $a_2 = 0.5$ AU. We also see that the two planets are placed in the “weakly coupled” dynamical class in a small region of the shown parameter space ($0.5 < a_2 < 0.8$ AU and $i_{\text{ini}} = 50^\circ$) within which the inner planet experiences Kozai oscillations induced by the outer planet. The width of this region is narrower than that observed in the case $a_b = 12$ AU (see Section 5.3 for a more detailed discussion).

5.3. Induced Eccentricity Oscillations in a Hot Jupiter

We test the possibility of inducing Kozai oscillations in the hot Jupiter orbit in our systems by propagating perturbations from the stellar companion through a second planet of equal mass. We find that inducing large eccentricities in the hot Jupiter orbit is more likely in binaries with smaller separation. In the case of a 12 AU binary separation this effect is observed within the region of the explored phase space defined by $0.35 < a_2 < 0.8$ AU and $i_{\text{ini}} > 40^\circ$). This behavior is not observed within the Kozai-stable zone $a_2 < 0.3$ AU, and shows its strongest behavior in the region $0.4$ AU $\leq a_2 < 0.5$ AU. As the distance between the planets increases, this effect gets weaker until it disappears beyond $a_2 > 0.8$ AU. Figure 10 shows the time evolution of the orbital elements of a system representative of this behavior with the outer planet at $a_2 = 0.5$ AU and $i_{\text{ini}} = 50^\circ$. The outer planet is experiencing Kozai oscillations induced by the stellar companion. The nodal precession of this planet creates a split in the planetary orbits and the mutual inclination between the planets, $I_{12}$, grows and oscillates with the same period as $\Omega_2$. The amplitude of these oscillations grows to a value twice the common initial inclination relative to the binary plane, $i_{\text{ini}}$, causing the inner planet’s eccentricity to grow chaotically.
inclination between the planetary orbits and the eccentricities of both planets in a system similar to that shown in Figure 10, but once with the GR forces removed from the calculation (top two panels) and second using an Earth-mass planet in the outer orbit (lower two panels). The former shows the faster growth of eccentricity when compared to Figure 10 and therefore demonstrates the goal played by GR forces in stabilizing the inner orbit against other perturbations, while the latter confirms the dependency of this mutual planetary Kozai effect on the planetary masses.

Inducing such oscillations into the orbit of a hot Jupiter requires specific initial conditions to be satisfied (see Section 2.4). We find that the region in parameter space that allows these conditions to be satisfied is narrower for wider binaries. We show in Figure 12 the range in semimajor axis of the outer planet that will allow induced Kozai oscillations in the inner orbit as a function of binary separation. This is shown for an inner planet on an initially circular orbit at \( a_1 = 0.05 \, \text{AU} \), and a binary orbit with inclination \( i_{\text{init}} = 50^\circ \) relative to the common planetary plane. All binaries in this figure share the following parameters \( m_0 = m_b = M_\odot, m_1 = m_2 = M_\star, e_b = 0 \). These results show that tighter binaries are more likely to produce this effect and that a cut-off binary separation \( a_{b, \text{cut}} \) exists beyond which it is not observed. For a hot Jupiter of mass \( M_\star \) and semimajor axis \( a_2 = 0.05 \, \text{AU} \), this cut-off binary separation lies in the range \( 30 < a_{b, \text{cut}} < 40 \, \text{AU} \). As the inner planet’s orbit is made wider, mutual planetary interactions grow larger which should allow a wider binary to induce eccentricities through weak coupling between the planets into the inner orbit.

6. SUMMARY AND DIRECTION FOR FUTURE WORK

The presence of a giant planet in a compact stellar system with separation \( \lesssim 20 \, \text{AU} \) is indicative of an active dynamical history in which the binary companion is either captured or moved inward to form a harder binary during an encounter after the planet has already formed. Such an event is likely to result in high relative inclinations between the orbits of the planet and the companion and may lead to Kozai oscillations in the planetary orbit/s. Since multiplanet systems seem to be common, it is likely that the primary in such a system may be harboring more than one planet at the time of its encounter with a stellar companion. In such a scenario, Kozai oscillations can easily lead to instabilities including collisional events and ejections. Therefore, such encounters which may occur in dense stellar systems when planetary systems are in their infancy, can be very important events in shaping the final outcome of planetary systems and leading to the large diversity observed in their characteristics.

We investigate the dynamical evolution of a double-planet system in a class of tight binaries with separation in the range 12–20 AU assuming relative inclinations between the common planetary plane and that of the binary in the range 0°–70°. Our simulations reveal the large parameter space involved with a double-planet system in a binary such that the final outcome of a system is highly sensitive to the initial conditions. The stability criteria are tested for two systems with binary separations 12 and 20 AU. We find that if the orbit of the stellar companion is highly inclined (\( \gtrsim 40^\circ \)) relative to the common planetary plane, the system is likely to experience instabilities unless the outer planet is situated in a narrow Kozai-stable zone within which gravitational perturbations with the inner planet are
strong enough to suppress the Kozai mechanism. At inclinations smaller than the critical Kozai angle, we find that a binary as tight as 12 AU can remain stable and an Earth-like planet can survive within the HZ of the central star up to 6 million years.

The eccentric hot Jupiter phenomenon is a puzzling observation since hot Jupiters are expected to be the result of a migration process during which circularization of the orbit occurs. We suggest that eccentricities can be pumped up into a hot Jupiter by inducing Kozai oscillations into its orbit through a weak coupling mechanism with a second planet in the presence of a close-in binary companion. A fine tuning of the initial conditions can produce this effect allowing perturbations from the stellar companion to propagate through the planetary system into the inner orbit. We find that tighter binaries are more likely to produce this effect, and that for a given hot Jupiter orbit, a cut-off binary separation can be found beyond which inducing Kozai oscillation into its orbit becomes impossible. The final outcome of such a scenario as the inner planet gains large eccentricities cannot be predicted accurately without the inclusion of tidal torques. These forces become important as the planet approaches the primary at periastron during cycles of large eccentricities. Circularization into a tighter orbit is one possibility that seems unlikely if the outer planet remains in the system undergoing nodal precession and inducing oscillations in the inner orbit. This is since the timescales associated with these oscillations are orders of magnitude shorter than tidal circularization timescales. If the outer planet reaches the edge of the Hill’s sphere of the primary at apastron and is ejected from the system, the inner orbit may have enough time to experience tidal circularization during the lifetime of the star. Explaining eccentric hot Jupiters by such a mechanism requires the inclusion of tidal circularization effects in more comprehensive studies of binaries harboring double-planet systems. Such studies may require numerical testing over longer timescales. This mechanism can also be used to explain the higher eccentricities among planets hosted by binary systems in general and the relative paucity of planets in binaries with near-circular orbits (for periods greater than 40 days; Eggenberger et al. 2004). This is because the wider the orbit of the inner planet, the easier it is to induce eccentricities into it by weak coupling with another planet on a wider orbit.

Tight binary systems with planets constitute a unique class of binaries which can allow us to test theoretical models of planetary formation and evolution. Therefore, more comprehensive studies including multiple planets and stellar collisions are necessary for a better understanding of the forces that were responsible for shaping planetary systems with a large diversity in characteristics. Finally, we can benefit greatly from better observational constraints on these systems, especially orbital inclinations.

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