COALESCING BINARY NEUTRON STARS

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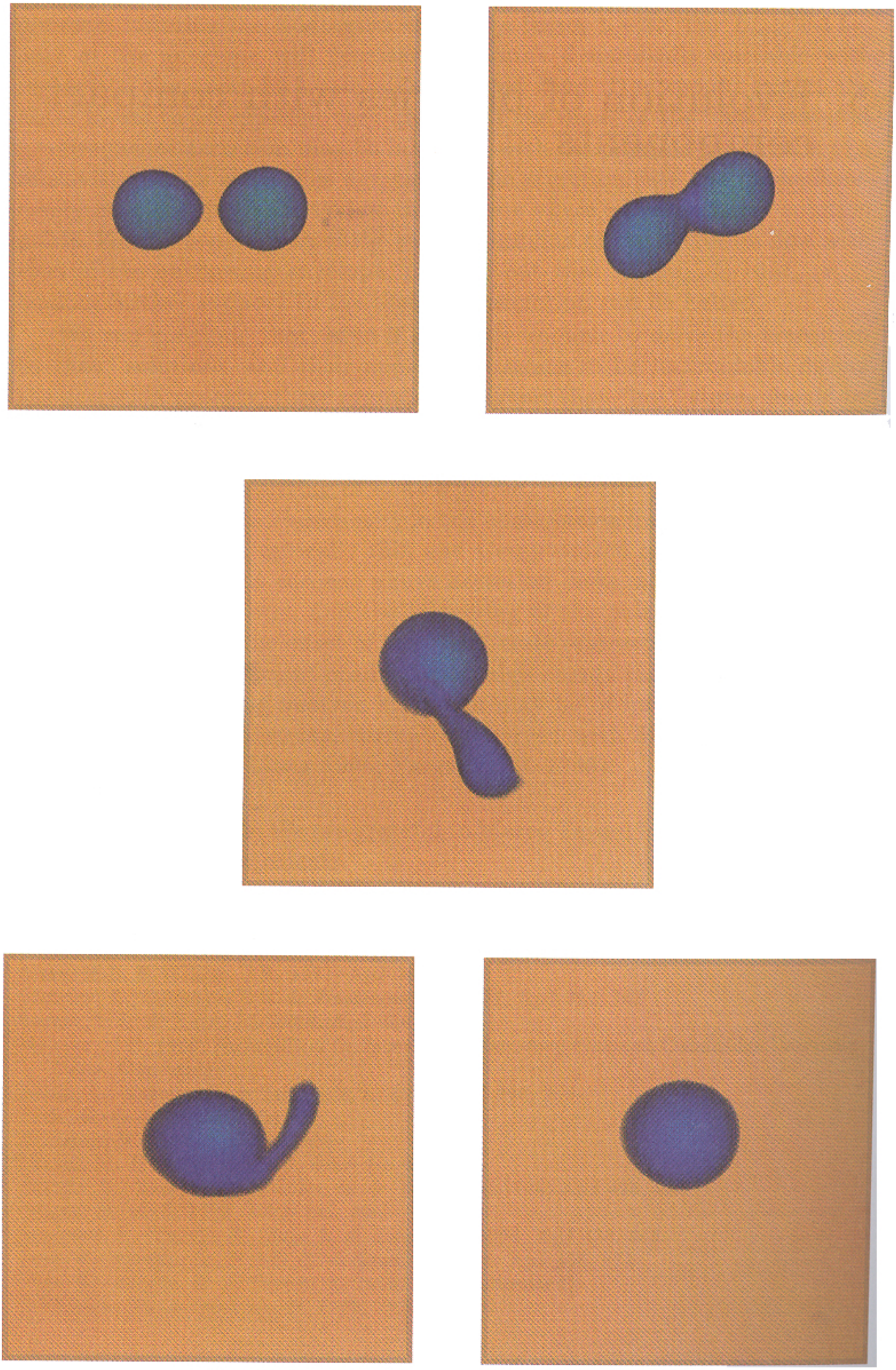
1. Introduction

The coalescence and merging of two stars into a single object is the almost inevitable end point of close-binary evolution. Dissipation mechanisms such as friction in common envelopes, tidal dissipation, or the emission of gravitational radiation, are always present and cause the orbits of close binary systems to decay. Examples of the binary coalescence process that are of great current interest include the formation of blue stragglers in globular clusters from mergers of contact main-sequence binaries, and the nuclear explosion or gravitational collapse of white dwarf mergers with total masses above the Chandrasekhar limit (for other examples and discussions, see, e.g., Bailyn 1993; Chen & Leonard 1993; Iben, Tutukov & Yungelson 1996; Rasio 1995; Segretain, Chabrier & Mochkovitch 1997).

For binary neutron stars (hereafter NS), the terminal stage of orbital decay is always hydrodynamic in nature, with the final merging of the two stars taking place on a time scale comparable to the orbital period. Indeed, in addition to the angular momentum loss to gravitational radiation, *global hydrodynamic instabilities* will drive the binary system to rapid coalescence once the tidal interaction between the two stars becomes sufficiently strong. The existence of these global instabilities for close-binary equilibrium configurations containing a compressible fluid was demonstrated for the first time by Rasio & Shapiro (1992, 1994, 1995; hereafter RS1–3) using numerical hydrodynamic calculations. In addition, the classical analytic work for

Figure 1. Final coalescence of two neutron stars. The initial binary configuration is in quasi-hydrostatic equilibrium at the onset of dynamical instability. The binary mass ratio is $q = 0.85$. The total time elapsed between the first and last frames is roughly 5 ms. See §4 for details.

R. Buccheri et al. (eds.), The Many Faces of Neutron Stars, 475-488.



close binaries containing an incompressible fluid (Chandrasekhar 1969) was recently extended to compressible fluids in the work of Lai, Rasio & Shapiro (1993a,b, 1994a,b,c, hereafter LRS1–5). This new analytic study confirmed the existence of dynamical and secular instabilities for sufficiently close binaries containing polytropes. Although the simplified analytic studies have given us much physical insight into difficult questions of global fluid instabilities, fully numerical calculations remain essential for establishing the stability limits of close binaries accurately and for following the non-linear evolution of unstable systems all the way to complete coalescence. Given the absence of any underlying symmetry in the problem, these calculations must be done in 3D and therefore require supercomputers.

A number of different groups have now performed such calculations, using a variety of numerical methods and focusing on different aspects of the problem. Nakamura and collaborators (see Nakamura 1994, and references therein) were the first to perform 3D hydrodynamic calculations of binary NS coalescence, using a traditional Eulerian finite-difference code. Instead, RS have been using the Lagrangian method SPH (Smoothed Particle Hydrodynamics). They focused on determining the stability properties of initial binary models in strict hydrostatic equilibrium and calculating the emission of gravitational waves from the coalescence of unstable binaries. Many of the results of RS have now been independently confirmed in the work of New & Tohline (1997), who used completely different numerical methods but also focused on stability questions, and by Zhuge, Centrella & McMillan (1994, 1996), who also used SPH. Zhuge et al. (1996) also explore in details the dependence of the gravitational-wave signals on the initial NS spins. Davies et al. (1994) and Ruffert et al. (1996, 1997) have incorporated a treatment of the nuclear physics in their hydrodynamic calculations (done using SPH and PPM codes, respectively), motivated by cosmological models of γ-ray bursts.

In close NS binaries, general-relativistic effects combine non-linearly with Newtonian tidal effects so that close binary configurations can become dynamically unstable earlier during the spiral-in phase (i.e., at larger binary separation and lower orbital frequency) than predicted by Newtonian hydrodynamics alone. The combined effects of relativity and hydrodynamics on the stability of close compact binaries have only very recently begun to be studied. Preliminary results have been obtained using both analytic approximations (basically, post-Newtonian generalizations of LRS; see Lai 1996; Taniguchi & Nakamura 1996; Lai & Wiseman 1997; Lombardi, Rasio & Shapiro 1997) as well as numerical hydrodynamics calculations in 3D incorporating simplified treatments of relativistic effects (Wilson & Mathews 1995; Shibata 1996; Baumgarte et al. 1997; Mathews & Wilson 1997). A NASA Grand Challenge project is under way (Seidel 1997; Swesty &

Saylor 1997) that will ultimately attempt a fully relativistic calculation of the final coalescence, combining the techniques of numerical relativity and numerical hydrodynamics in 3D.

2. Astrophysical Motivation

Coalescing compact binaries are the most promising known sources of gravitational radiation that could be detected by the new generation of laser interferometers now under construction. These include the Caltech-MIT LIGO (Abramovici et al. 1992; Cutler et al. 1992) and the European projects VIRGO (Bradaschia et al. 1990) and GEO (Danzmann 1997). In addition to providing a major new confirmation of Einstein's theory of general relativity, including the first direct proof of the existence of black holes (Flanagan & Hughes 1997; Lipunov et al. 1997), the detection of gravitational waves from coalescing binaries at cosmological distances could provide accurate independent measurements of the Hubble constant and mean density of the Universe (Schutz 1986; Chernoff & Finn 1993; Marković 1993). For a recent review on the detection and sources of gravitational radiation, see Thorne (1996).

Expected rates of NS binary coalescence in the Universe, as well as expected event rates in forthcoming laser interferometers, have now been calculated by many groups. Although there is some disparity between various published results, the estimated rates are generally encouraging. Statistical arguments based on the observed local population of binary radio pulsars with probable NS companions lead to an estimate of the rate of NS binary coalescence in the Universe of order $10^{-7}\,\mathrm{yr}^{-1}\,\mathrm{Mpc}^{-3}$ (Narayan et al. 1991; Phinney 1991). Using this estimate, Finn & Chernoff (1993) predict that an advanced LIGO detector could observe as many as 70 events per year. These numbers are based on a Galactic merger rate $R \simeq 10^{-6}\,\mathrm{yr}^{-1}$ derived from radio pulsar surveys. More recently, however, Van den Heuvel & Lorimer (1996) revised this number to $R \simeq 0.8 \times 10^{-5}\,\mathrm{yr}^{-1}$, using the latest galactic pulsar population model of Curran & Lorimer (1995). This value is consistent with the upper limit of $10^{-5}\,\mathrm{yr}^{-1}$ for the Galactic binary NS birth rate derived by Bailes (1996) on the basis of very general statistical considerations about pulsars. In addition, theoretical models of the binary star population in our Galaxy also suggest that the NS binary coalescence rate may be as high as $\gtrsim 10^{-6}\,\mathrm{yr}^{-1}\,\mathrm{Mpc}^{-3}$ (Tutukov & Yungelson 1993; see also the more recent studies by Portegies Zwart & Spreeuw 1996, and by Lipunov et al. 1997).

Most recent calculations of the gravitational radiation waveforms from coalescing binaries have focused on the signal emitted during the last few thousand orbits, as the frequency sweeps upward from about 10 Hz to

1000 Hz. The waveforms in this regime can be calculated fairly accurately by performing high-order post-Newtonian (hereafter PN) expansions of the equations of motion for two *point masses* (Lincoln & Will 1990; Junker & Schäfer 1992; Kidder, Will & Wiseman 1992; Wiseman 1993; Will 1994; Blanchet et al. 1996). High accuracy is essential here because the observed signals will be matched against theoretical templates. Since the templates must cover $\sim 10^3 - 10^4$ orbits, a phase error as small as $\sim 10^{-4}$ could in principle prevent detection (Cutler et al. 1993; Cutler & Flanagan 1994; Finn & Chernoff 1993).

Near the end of the inspiral, when the binary separation becomes comparable to the stellar radii, hydrodynamic effects become important and the character of the waveforms will change. Special-purpose narrow-band detectors that can sweep up frequency in real time will be used to try to catch the corresponding final few cycles of gravitational waves (Meers 1988; Strain & Meers 1991; Danzmann 1997). In this terminal phase of the coalescence, the waveforms contain information not just about the effects of general relativity, but also about the internal structure of the stars and the nuclear equation of state (hereafter EOS) at high density. Extracting this information from observed waveforms, however, requires detailed theoretical knowledge about all relevant hydrodynamic processes.

Many theoretical models of γ-ray bursts (GRB) have postulated that the energy source for the bursts could be coalescing NS binaries at cosmological distances (Paczyński 1986; Eichler et al. 1989; Narayan, Paczyński & Piran 1992). The isotropic angular distribution of the bursts detected with the BATSE experiment on the Compton GRO satellite (Meegan et al. 1992) strongly suggests a cosmological origin, as does the distribution of number versus intensity of the bursts. In addition, the rate of GRBs detected with BATSE, of order one per day, is in rough agreement with theoretical predictions for the rate of NS binary coalescence in the Universe (cf. above). In the past few months the first optical counterparts of several GRBs have been identified (Groot et al. 1997; Van Paradijs et al. 1997; Bond 1997), after their positions were measured accurately with the BeppoSAX satellite (e.g., Costa et al. 1997). As of this writing, in one case (GRB 970508), absorption features corresponding to a redshift of $z = 0.84$ have been reported (Metzger et al. 1997). If confirmed, these observations would clearly establish that at least some GRBs originate at cosmological distances.

To model the γ-ray emission realistically, the complete hydrodynamic and nuclear evolution during the final merging of the two NS, especially in the outermost, low-density regions of the merger, must be understood in details. This is far more challenging than understanding the emission of gravitational waves, which is mostly sensitive to the bulk motion of the fluid, but is totally *insensitive* to nuclear processes taking place in low-density regions. Numerical calculations of NS binary coalescence including some treatment of the nuclear physics have been performed by Davies et al. (1994) and Ruffert et al. (1996, 1997). The most recent results from these calculations indicate that, even under the most favorable conditions, the energy provided by $\nu\bar{\nu}$ annihilation is too small by at least an order of magnitude, and more probably two or three orders of magnitude, to power typical γ-ray bursts at cosmological distances (Janka & Ruffert 1996).

3. Hydrodynamic Instabilities

Hydrostatic equilibrium configurations for binary systems with sufficiently close components can become *dynamically unstable* (Chandrasekhar 1975; Tassoul 1975). The physical nature of this instability is common to all binary interaction potentials that are sufficiently steeper than $1/r$ (see, e.g., Goldstein 1980, §3.6). It is analogous to the familiar instability of test particles in circular orbits sufficiently close to a black hole (Shapiro & Teukolsky 1983, §12.4). Here, however, it is the *tidal interaction* that is responsible for the steepening of the effective interaction potential between the two stars and for the destabilization of the circular orbit (LRS3). The tidal interaction exists of course already in Newtonian gravity and the instability is therefore present even in the absence of relativistic effects. For sufficiently compact binaries, however, the combined effects of relativity and hydrodynamics lead to an even stronger tendency towards dynamical instability (see §7).

The stability properties of close NS binaries depend sensitively on the NS EOS. Close binaries containing NS with stiff EOS (adiabatic exponent $\Gamma \gtrsim 2$ if $P = K\rho^{\Gamma}$, where P is pressure and ρ is density) are particularly susceptible to a dynamical instability. This is because tidal effects are stronger for stars containing a less compressible fluid (i.e., for larger Γ). As the dynamical stability limit is approached, the secular orbital decay driven by gravitational wave emission can be dramatically accelerated (LRS2, LRS3). The two stars then plunge rapidly toward each other, and merge together into a single object in just a few rotation periods. This dynamical instability was first identified in RS1, where the evolution of Newtonian binary equilibrium configurations was calculated for two identical polytropes with $\Gamma = 2$. It was found that when $r \lesssim 3R$ (r is the binary separation and R the radius of an unperturbed NS), the orbit becomes unstable to radial perturbations and the two stars undergo rapid coalescence. For $r \gtrsim 3R$, the system could be evolved dynamically for many orbital periods without showing any sign of orbital evolution (in the absence of dissipation). Many of the results derived in RS and LRS concerning the stability properties of NS binaries have been confirmed recently in completely independent work by New & Tohline (1997) and by Zhuge, Centrella & McMillan (1996). New & Tohline (1997) used completely different numerical methods (a combination of a 3-D Self-Consistent Field code for constructing equilibrium configurations and a grid-based Eulerian code for following the dynamical evolution of the binaries), while Zhuge et al. (1996) used SPH, as did RS.

The dynamical evolution of an unstable, initially synchronized (i.e., rigidly rotating) binary containing two identical stars can be described typically as follows (RS1, RS2). During the initial, linear stage of the instability, the two stars approach each other and come into contact after about one orbital revolution. In the corotating frame of the binary, the relative velocity remains very subsonic, so that the evolution is adiabatic at this stage. This is in sharp contrast to the case of a head-on collision between two stars on a free-fall, radial orbit, where shocks are very important for the dynamics (RS1). Here the stars are constantly being held back by a (slowly receding) centrifugal barrier, and the merging, although dynamical, is much more gentle. After typically two orbital revolutions the innermost cores of the two stars have merged and the system resembles a single, very elongated ellipsoid. At this point a secondary instability occurs: *mass shedding* sets in rather abruptly. Material is ejected through the outer Lagrange points of the effective potential and spirals out rapidly. In the final stage, the spiral arms widen and merge together. The relative radial velocities of neighboring arms as they merge are supersonic, leading to some shock-heating and dissipation. As a result, a hot, nearly axisymmetric rotating halo forms around the central dense core. The halo contains about 20% of the total mass and the rotation profile is close to a pseudo-barotrope (Tassoul 1978, §4.3), with the angular velocity decreasing as a power-law $\Omega \propto \varpi^{-\nu}$ where $\nu \lesssim 2$ and ϖ is the distance to the rotation axis (RS1). The core is rotating uniformly near break-up speed and contains about 80% of the mass still in a cold, degenerate state. If the initial NS had masses close to $1.4\,M_\odot$, then most recent stiff EOS would predict that the final merged configuration is still stable and will not immediately collapse to a black hole, although it might ultimately collapse to a black hole as it continues to lose angular momentum (see Cook, Shapiro & Teukolsky 1994).

The emission of gravitational radiation during dynamical coalescence can be calculated perturbatively using the quadrupole approximation (RS1). Both the frequency and amplitude of the emission peak somewhere during the final dynamical coalescence, typically just before the onset of mass shedding. Immediately after the peak, the amplitude drops abruptly as the system evolves towards a more axially symmetric state. For an initially synchronized binary containing two identical polytropes, the properties of the waves near the end of the coalescence depend very sensitively on the stiffness of the EOS.

When $\Gamma < \Gamma_{\rm crit}$, with $\Gamma_{\rm crit} \simeq 2.3$, the final merged configuration is perfectly axisymmetric. Indeed, a polytropic fluid with $\Gamma < 2.3$ (polytropic index $n > 0.8$) cannot sustain a non-axisymmetric, uniformly rotating configuration in equilibrium (see, e.g., Tassoul 1978, §10.3). As a result, the amplitude of the waves drops to zero in just a few periods (RS1). In contrast, when $\Gamma > \Gamma_{\rm crit}$, the dense central core of the final configuration remains *triaxial* (its structure is basically that of a compressible Jacobi ellipsoid; cf. LRS1) and therefore it continues to radiate gravitational waves. The amplitude of the waves first drops quickly to a non-zero value and then decays more slowly as gravitational waves continue to carry angular momentum away from the central core (RS2). Because realistic NS models have effective Γ values precisely in the range 2–3 (LRS3), i.e., close to $\Gamma_{\rm crit} \simeq 2.3$, a simple determination of the absence or presence of persisting gravitational radiation after the coalescence (i.e., after the peak in the emission) could place a strong constraint on the stiffness of the EOS.

4. Mass Transfer and the Dependence on the Mass Ratio

Clark & Eardley (1977) suggested that secular, *stable* mass transfer from one NS to another could last for hundreds of orbital revolutions before the lighter star is tidally disrupted. Such an episode of stable mass transfer would be accompanied by a secular *increase* of the orbital separation. Thus if stable mass transfer could indeed occur, a characteristic "reversed chirp" would be observed in the gravitational-wave signal at the end of the inspiral phase (Jaranowski & Krolak 1992).

The question was later reexamined by Kochanek (1992) and Bildsten & Cutler (1992), who both argued against the possibility of stable mass transfer on the basis that very large mass transfer rates and extreme mass ratios would be required. Moreover, in LRS3 it was pointed out that mass transfer has in fact little importance for most NS binaries (except perhaps those containing a very low-mass NS). This is because for $\Gamma \gtrsim 2$, dynamical instability always arises *before the Roche limit* along a sequence of binary configurations with decreasing separation r. Therefore, by the time mass transfer begins, the system is already in a state of dynamical coalescence and it can no longer remain in a nearly circular orbit. Thus stable mass transfer from one NS to another appears impossible.

In RS2 a complete dynamical calculation was presented for a system containing two polytropes with $\Gamma = 3$ and a mass ratio $q = 0.85$. This value corresponds to the most likely mass ratio for the binary pulsar PSR 2303+46 (Thorsett et al. 1993) and represents the largest observed departure from $q = 1$ in any known binary pulsar with likely NS companion. For comparison, $q = 1.386/1.442 = 0.96$ in PSR 1913+16 (Taylor & Weisberg 1989) and $q = 1.32/1.36 = 0.97$ in PSR 1534+12 (Wolszczan 1991). For the system with $q = 0.85$, RS2 found that the dynamical stability limit is at $r/R \simeq 2.95$, whereas the Roche limit is at $r/R \simeq 2.85$. The dynamical evolution turns out to be quite different from that of a system with $q = 1$. The Roche limit is quickly reached while the system is still in the linear stage of growth of the instability. Dynamical mass transfer from the less massive to the more massive star begins within the first orbital revolution. Because of the proximity of the two components, the fluid acquires very little velocity as it slides down from the inner Lagrange point to the surface of the other star. As a result, relative velocities of fluid particles remain largely subsonic and the coalescence proceeds quasi-adiabatically, just as in the $q = 1$ case. In fact, the mass transfer appears to have essentially no effect on the dynamical evolution. After about two orbital revolutions the smaller-mass star undergoes complete tidal disruption. Most of its material is quickly spread on top of the more massive star, while a small fraction of the mass is ejected from the outer Lagrange point and forms a single-arm spiral outflow. The more massive star, however, remains little perturbed during the entire evolution and simply becomes the inner core of the merged configuration. This type of dynamical evolution, which is probably the most typical one for the final merging of two NS with slightly different masses, is illustrated in Figure 1.

The dependence of the peak amplitude $h_{\max}$ of gravitational waves on the mass ratio q appears to be very strong, and non-trivial. In RS2 an approximate scaling $h_{\max} \propto q^2$ was derived. This is very different from the scaling obtained for a detached binary system with a given binary separation. In particular, for two point masses in a circular orbit with separation r the result would be $h \propto \Omega^2 \mu r^2$, where $\Omega^2 = G(M + M')/r^3$ and $\mu = MM'/(M + M')$. At constant r, this gives $h \propto q$. This linear scaling is obeyed (only approximately, because of finite-size effects) by the wave amplitudes of the various systems at the *onset* of dynamical instability. For determining the *maximum* amplitude, however, hydrodynamics plays an essential role. In a system with $q \neq 1$, the more massive star tends to play a far less active role in the hydrodynamics and, as a result, there is a rapid suppression of the radiation efficiency as q departs even slightly from unity. For the peak luminosity of gravitational radiation RS found approximately $L_{\max} \propto q^6$. Again, this is a much steeper dependence than one would expect based on a simple point-mass estimate, which gives $L \propto q^2(1 + q)$ at constant r. The results of RS are all for initially synchronized binaries, but very similar results have been obtained more recently by Zhuge et al. (1996) for binaries containing initially nonspinning stars with unequal masses.

5. What Can We Learn about Neutron Stars with LIGO/VIRGO?

The most important parameter that enters into quantitative estimates of the gravitational wave emission during the final coalescence is the ratio M/R for a NS (here we take $G = c = 1$). In particular, for two identical point masses we know that the wave amplitude h obeys $(r_O/M)h \propto (M/R)$, where r_O is the distance to the observer, and the total luminosity $L \propto (M/R)^5$. Similarly the wave frequency $f_{\rm max}$ during final merging should satisfy approximately $f_{\rm max} \propto (M/R)^{3/2}$ since it is roughly twice the Keplerian frequency for two NS in contact (binary separation $r \simeq 2-3R$). Thus one expects that any quantitative measurement of the emission near maximum should lead to a direct determination of the NS radius R, assuming that the mass M has already been determined from the low-frequency inspiral waveform (Cutler & Flanagan 1994). Most current NS EOS give $M/R \sim 0.1$, with $R \sim 10\,{\rm km}$ nearly independent of the mass in the range $0.8M_\odot \lesssim M \lesssim 1.5M_\odot$ (see, e.g., Baym 1991; Cook et al. 1994; LRS3).

However, the details of the hydrodynamics also enter into this determination. The importance of hydrodynamic effects introduces an explicit dependence of all wave properties on the EOS (which we represent here by a single dimensionless parameter Γ), and on the mass ratio q. If relativistic effects were taken into account for the hydrodynamics itself, an additional, non-trivial dependence on M/R would also be present. This can be written conceptually as

$$\left(\frac{r_O}{M}\right) h_{\rm max} \equiv \mathcal{H}(q,\Gamma,M/R) \times \left(\frac{M}{R}\right) \quad (1)$$

$$\frac{L_{\rm max}}{L_o} \equiv \mathcal{L}(q,\Gamma,M/R) \times \left(\frac{M}{R}\right)^5 \quad (2)$$

where $L_o \equiv c^5/G = 3.6 \times 10^{59}\,{\rm erg\,s^{-1}}$. Combining all the results of RS, we can write, in the limit where $M/R \to 0$ and for q not too far from unity,

$$\mathcal{H}(q,\Gamma,M/R) \simeq 2.2\,q^2 \qquad \mathcal{L}(q,\Gamma,M/R) \simeq 0.5\,q^6, \quad (3)$$

essentially independent of Γ in the range $\Gamma \simeq 2$–3 (RS2). The results of RS were for the case of synchronized spins. Zhuge et al. (1996) have performed calculations for non-synchronized binaries and obtained very similar results (but see §6 below). For example, for the coalescence of two *non-spinning* stars with $q = 1$ they found $\mathcal{H} \simeq 1.9 - 2.3$ and $\mathcal{L} \simeq 0.29 - 0.59$, where the range of values corresponds to Γ between $5/3$ and 3. Note that the calculations of Zhuge et al. (1996) included an approximate treatment of PN effects by setting up an initial inspiral trajectory for two NS of mass $M = 1.4\,{\rm M}_\odot$ and radius in the range $R = 10 - 15\,{\rm km}$. Varying the radius of the stars in this range appears to leave the coefficients $\mathcal{H}$ and $\mathcal{L}$ practically unchanged within their approximation. Zhuge et al. (1994, 1996) also compute frequency spectra for the gravitational wave emission and discuss various ways of defining precisely the characteristic frequency $f_{\max}$.

6. Non-synchronized Binaries

It is very likely that the synchronization time in close NS binaries always remains longer than the orbital decay time due to gravitational radiation (Kochanek 1992; Bildsten & Cutler 1992). In particular, Bildsten & Cutler (1992) show with simple dimensional arguments that one would need an implausibly small value of the effective viscous time, $t_{\rm visc} \sim R/c$, in order to reach complete synchronization just before final merging. In the opposite limiting regime where viscosity is completely negligible, the fluid circulation in the binary system is conserved during the orbital decay and the stars behave approximately as Darwin-Riemann ellipsoids (Kochanek 1992; LRS3). Of particular importance are the irrotational Darwin-Riemann configurations, obtained when two initially non-spinning (or, in reality, slowly spinning) NS evolve in the absence of significant viscosity. Compared to synchronized systems, these irrotational configurations exhibit smaller deviations from point-mass Keplerian behavior at small r. However, as shown in LRS3 and RS4, irrotational configurations for binary NS with $\Gamma \gtrsim 2$ can still become dynamically unstable near contact. Thus the final coalescence of two NS in a non-synchronized binary system is also driven by hydrodynamic instabilities.

The details of the hydrodynamics are very different, however. Because the two stars appear to be counter-spinning in the corotating frame of the binary, a *vortex sheet* (where the tangential velocity jumps discontinuously by $\Delta v = |v_+ - v_-| \simeq \Omega r$) appears when the stellar surfaces come into contact. Such a vortex sheet is Kelvin-Helmholtz unstable on all wavelengths and the hydrodynamics is therefore extremely difficult to model accurately given the limited spatial resolution of 3D calculations. The breaking of the vortex sheet generates a large turbulent viscosity so that the final configuration may no longer be irrotational. In numerical simulations, however, vorticity is generated mostly through spurious shear viscosity introduced by the spatial discretization (Lombardi, Rasio & Shapiro 1998).

An additional difficulty is that non-synchronized configurations evolving rapidly by gravitational radiation emission tend to develop significant tidal lags, with the long axes of the two components becoming misaligned (LRS5). This is a purely dynamical effect, present even if the viscosity is zero, but its magnitude depends on the entire previous evolution of the system. Thus the construction of initial conditions for hydrodynamic calculations of non-synchronized binary coalescence must incorporate the gravitational radiation reaction *self-consistently*. Instead, previous calculations of non-synchronized binary coalescence by Shibata et al. (1992), Davies et al. (1994), and Zhuge et al. (1994, 1996) used very crude initial conditions consisting of two *spherical* stars placed on an inspiral trajectory calculated for two point masses.

7. General-Relativistic Effects on the Stability of Compact Binaries

Over the last two years, various efforts have started to calculate the stability limits for NS binaries including both hydrodynamic finite-size (tidal) effects and relativistic effects. Note that, strictly speaking, equilibrium circular orbits do not exist in general relativity because of the emission of gravitational waves. However, the stability of quasi-circular orbits can still be studied in the framework of general relativity by truncating the radiation-reaction terms in a PN expansion of the equations of motion (Lincoln & Will 1990; Kidder et al. 1992; Will 1994). Alternatively, one can solve the full Einstein equations numerically in the $3+1$ formalism on time slices with a spatial 3-metric chosen to be conformally flat (Wilson & Mathews 1989, 1995; Wilson et al. 1996; Baumgarte et al. 1997). This effectively minimizes the gravitational wave content of space-time. The field equations then reduce to a set of coupled elliptic equations (for the $3+1$ lapse and shift functions and the conformal factor).

Several groups are now working on PN generalizations of the semi-analytic Newtonian treatment of LRS based on ellipsoids. Taniguchi & Nakamura (1996) consider NS-BH binaries and adopt a modified version of the pseudo-Newtonian potential of Paczyński & Wiita (1980) to mimic general-relativistic effects near the BH. Lai & Wiseman (1997) concentrate on NS-NS binaries and the dependence of the results on the NS EOS. They add a restricted set of PN orbital terms to the dynamical equations given in Lai & Shapiro (1995) for a binary system containing two NS modeled as Riemann-S ellipsoids (cf. LRS), but they neglect relativistic corrections to the fluid motion, self-gravity and tidal interaction. Lombardi, Rasio & Shapiro (1997) include PN corrections affecting both the orbital motion and the interior structure of the stars and explore the consequences not only for orbital stability but also for the stability of each NS against collapse. The most important result, on which these various studies all seem to agree, is that neither the relativistic effects nor the Newtonian tidal effects can be neglected if one wants to obtain a quantitatively accurate determination of the stability limits. In particular, the critical frequency corresponding to the onset of dynamical instability can be much lower than the value obtained when only one of the two effects is included. This critical frequency for the "last stable circular orbit" is potentially a measurable quantity (with LIGO/VIRGO) and can provide direct information on the NS EOS (cf. §5).

A surprising result coming from the numerical 3 + 1 relativistic calculations of Wilson & Mathews (1995; Wilson, Mathews & Marronetti 1996; Mathews & Wilson 1997) is the appearance of a "binary-induced collapse instability" of the NS, with the central density of each star increasing by an amount proportional to $1/r$. This must be a purely relativistic effect, since the Newtonian tidal effects in fact tend to *stabilize* the NS against collapse (making the central density *decrease* by an amount proportional to $1/r^6$; cf. Lai 1996). In effect, the maximum stable mass of a NS in a relativistic close binary system could be slightly lower than that of a NS in isolation. An initially stable NS close to the maximum mass could then collapse to a black hole well before getting to the final phase of binary coalescence! It should be noted, however, that the numerical results of Wilson & Mathews (hereafter WM) have yet to be confirmed independently by other studies. Even if it is real, the WM effect would be of importance only if the NS EOS is very soft, and the maximum stable mass for a NS in isolation is not much larger than $1.4\,M_\odot$. In addition, the numerical results of WM have been criticized by many authors on theoretical grounds. Brady & Hughes (1997) show analytically that, in the limit where the NS companion becomes a test particle of mass m, the central density of the NS remains unchanged to linear order in m/R, in contrast to what would be expected from the WM results. Lombardi, Rasio & Shapiro (1997) and Wiseman (1997) argue that there should be no destabilizing relativistic effect to first PN order. In contrast, WM claim that their effect is at least partially caused by a non-linear first PN order enhancement of the gravitational potential. Lombardi et al. (1997) also find that, to first PN order, the *maximum equilibrium mass* of a NS in a binary *increases* as the binary separation r decreases, in agreement with the fully relativistic numerical calculations of Baumgarte et al. (1997).

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