

Monte Carlo Simulations of Globular Cluster Dynamics

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Abstract. We have developed a new parallel supercomputer code based on Hénon’s Monte Carlo method for simulating the dynamical evolution of globular clusters. This new code allows us to calculate the evolution of a cluster containing a realistic number of stars ($N \sim 10^5 - 10^6$) in about a day of computing time. The discrete, star-by-star representation of the cluster in the simulation allows us to treat naturally a number of important processes, including single and binary star evolution, all dynamical interactions of single stars and primordial binaries, and tidal interactions with the Galaxy.

1. Introduction

The first Monte Carlo methods for calculating the dynamical evolution of star clusters in the Fokker-Planck approximation were developed more than 30 years ago. They were first used to study the development of the gravothermal instability (Hénon 1971a,b; Spitzer & Hart 1971a,b). More recent implementations have established the Monte Carlo method as an important alternative to direct N -body integrations (see Spitzer 1987 for an overview, and the article by Giersz in this volume). The main motivation for our recent work at MIT was our realization a few years ago that the latest generation of parallel supercomputers now make it possible to perform Monte Carlo simulations for a number of objects equal to the actual number of stars in a globular cluster (in contrast, earlier work was limited to using a small number of representative “superstars,” and was often plagued by high levels of numerical noise). Therefore, the Monte Carlo method allows us to do right now what remains an elusive goal for N -body simulations (see, e.g., Aarseth 1999, and the article by Makino in this volume): perform realistic, star-by-star computer simulations of globular cluster evolution. Using the correct number of stars in a dynamical simulation ensures that the relative rates of different dynamical processes (which all scale differently with the number of stars) are correct. This is particularly crucial if many different dynamical processes are to be incorporated, as must be done in realistic simulations (cf. the article by Heggie in this volume).

Our implementation of the Monte Carlo method is described in detail in the papers by Joshi, Rasio, & Portegies Zwart (2000), Joshi, Nave, & Rasio (2000), and Joshi & Rasio (2000). We adopt the usual assumptions of spherical symmetry (with a 2D phase space distribution function $f(E, J)$, i.e., we do *not* assume isotropy) and standard two-body relaxation in the weak scattering limit (Fokker-Planck approximation). In its simplest version, our code computes the

dynamical evolution of a self-gravitating spherical cluster of N point masses whose orbits in the cluster are specified by an energy E and angular momentum J , with perturbations ΔE and ΔJ evaluated on a timestep that is a fraction of the local two-body relaxation time. The cluster is assumed to remain always very close to dynamical equilibrium (i.e., the relaxation time must remain much longer than the dynamical time). We have performed a large number of test calculations and comparisons with direct N -body integrations, as well as direct integrations of the Fokker-Planck equation in phase space, to establish the accuracy of our basic treatment of two-body relaxation (Joshi et al. 2000a). Fig. 1 shows the results from a typical comparison between Monte Carlo and N -body simulations. Our main improvements over Hénon’s original method are the parallelization of the basic algorithm and the development of a more sophisticated method for determining the timesteps and for computing the two-body relaxation from representative encounters between neighboring stars. Our new method allows the timesteps to be made much smaller in order to resolve the dynamics in the cluster core more accurately.

2. Summary of Recent Results

Our recent work has focused on the addition of more realistic stellar and binary processes to the basic Monte Carlo code, as well as a simple but accurate implementation of a static tidal boundary in the Galactic field (Joshi et al. 2000b). As a first application, we have studied the dependence on initial conditions of globular cluster lifetimes in the Galactic environment. As in previous Fokker-Planck studies (Chernoff & Weinberg 1990; Takahashi & Portegies Zwart 1998), we include the effects of a power-law initial mass function (IMF), mass loss through a tidal boundary, and single star evolution, and we consider initial King models with varying central concentrations. We find that the disruption and core-collapse times of our models are significantly longer than those obtained with previous 1D (isotropic) Fokker-Planck calculations, but agree well with more recent results from direct N -body simulations and 2D Fokker-Planck integrations (see also the article by Takahashi in this volume). In agreement with previous studies, our results show that the direct mass loss due to stellar evolution causes most clusters with a low initial central concentration to disrupt quickly in the Galactic tidal field. The disruption is particularly rapid for clusters with a relatively flat IMF. Only clusters born with high central concentrations or with very steep IMFs are likely to survive to the present and undergo core collapse.

In another recent study, we have used our Monte Carlo code to examine the development of the Spitzer “mass stratification instability” in simple two-component clusters (Watters, Joshi, & Rasio 2000). We have performed a large number of dynamical simulations for star clusters containing two stellar populations with individual masses m_1 and $m_2 > m_1$, and total masses M_1 and $M_2 < M_1$. We use both King and Plummer model initial conditions and we perform simulations for a wide range of individual and total mass ratios, m_2/m_1 and M_2/M_1 , in order to determine the precise location of the stability boundary in this 2D parameter space. As predicted originally by Spitzer (1969) using simple analytic arguments, we find that unstable systems never reach energy equipar-

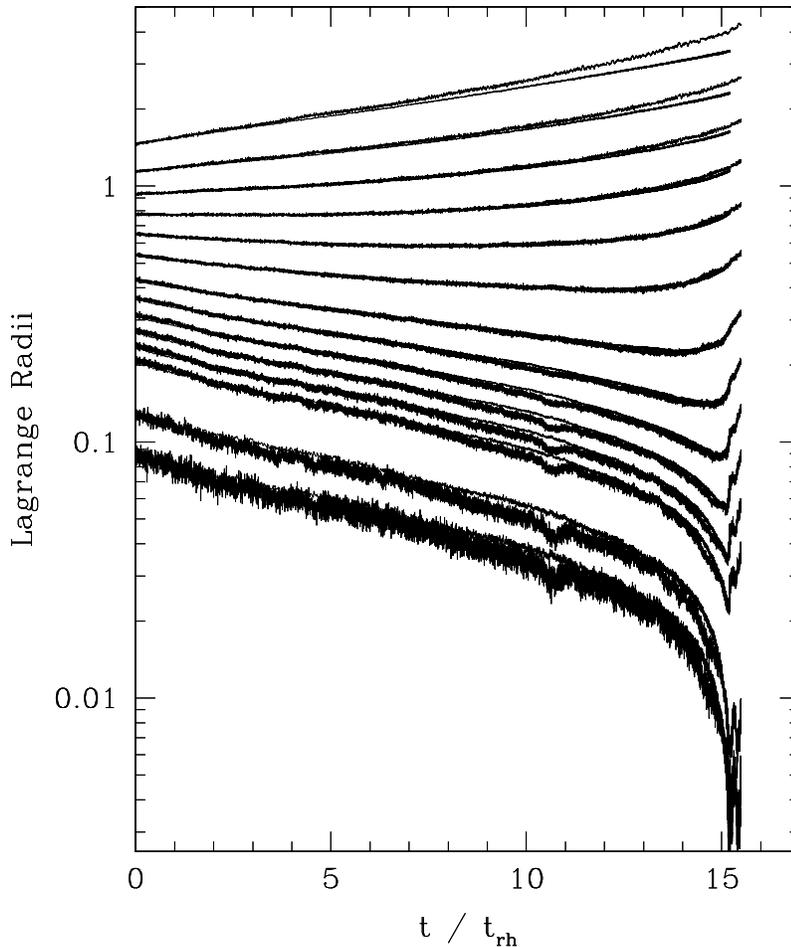


Figure 1. Evolution of the Lagrange radii for an isolated, single-component Plummer model (from bottom to top: radii containing 0.35%, 1%, 3.5%, 5%, 7%, 10%, 14%, 20%, 30%, 40%, 50%, 60%, 70%, and 80% percent of the total mass are shown as a function of time, given in units of the initial half-mass relaxation time). The results from a direct N -body integration with $N = 16,384$ (noisier lines) and from a Monte Carlo integration with $N = 10^5$ stars (smoother lines) are compared. The Monte Carlo simulation was completed in less than a day on a Cray/SGI Origin2000 parallel supercomputer, while the N -body integration ran for over a month on a dedicated GRAPE-4 computer. The agreement between the N -body and Monte Carlo results is excellent over the entire range of Lagrange radii and time. The small discrepancy in the outer Lagrange radii is caused mainly by a different treatment of escaping stars in the two models. In the Monte Carlo model, escaping stars are removed from the simulation and therefore not included in the determination of the Lagrange radii, whereas in the N -body model escaping stars are not removed. Note also that the Monte Carlo simulation is terminated at core collapse, while the N -body simulation continues beyond core collapse.

tion, and are driven to rapid core collapse by the heavier component. These results have important implications for the dynamical evolution of any population of primordial black holes or neutron stars in globular clusters. In particular, primordial black holes with $m_2/m_1 \sim 10$ are expected to undergo very rapid core collapse independent of the background cluster, and to be ejected from the cluster through dynamical interactions between single and binary black holes (see Portegies Zwart & McMillan 2000 and references therein). We have also used Monte Carlo simulations of simple two-component systems to study the evaporation (or retention) of *low-mass* objects in globular clusters, motivated by the surprising recent observations of planets and brown dwarfs in several clusters (Fregeau et al. 2000).

Much of our current work concerns the treatment of dynamical interactions with primordial binaries. We are in the process of completing a first study of globular cluster evolution with primordial binaries (Joshi & Rasio 2000), based on the same set of approximate cross sections and recipes for dynamical interactions used in the Fokker-Planck simulations of Gao et al. (1991). Typical results are illustrated in Fig. 2. The heating of the cluster core generated by a small population of primordial binaries can support the cluster against core collapse for very long times.

The addition of binary stellar evolution processes will allow us to study in detail the dynamical formation mechanisms for many exotic objects, such as X-ray binaries, millisecond radio pulsars, and cataclysmic variables, which have been detected in large numbers in globular clusters. For example, exchange interactions between neutron stars and primordial binaries can lead to common-envelope systems and the formation of short-period neutron-star / white-dwarf binaries that can become visible both as ultracompact X-ray binaries and binary millisecond pulsars with low-mass companions (see, e.g., Camilo et al. 2000, on observations of 20 such millisecond radio pulsars in 47 Tuc; Rasio, Pfahl, & Rappaport 2000 present a preliminary study of this dynamical formation scenario, based on simplified Monte Carlo simulations).

We are also currently working on incorporating a more realistic treatment of tidal interactions, and, in particular, tidal shocking through the Galactic disk (based on Gnedin, Lee, & Ostriker 1999). Tidal shocks can accelerate significantly both core collapse and the evaporation of globular clusters, reducing their lifetimes in the Galaxy (Gnedin & Ostriker 1997).

Future work will include a fully dynamical treatment of all strong binary-single and binary-binary interactions (exploiting the parallelism of the code to perform separate numerical 3- or 4-body integrations for all dynamical interactions) as well as a fully dynamical treatment of tidal shocking (performing short, N -body integrations for each passage of the cluster through the Galactic disk or bulge).

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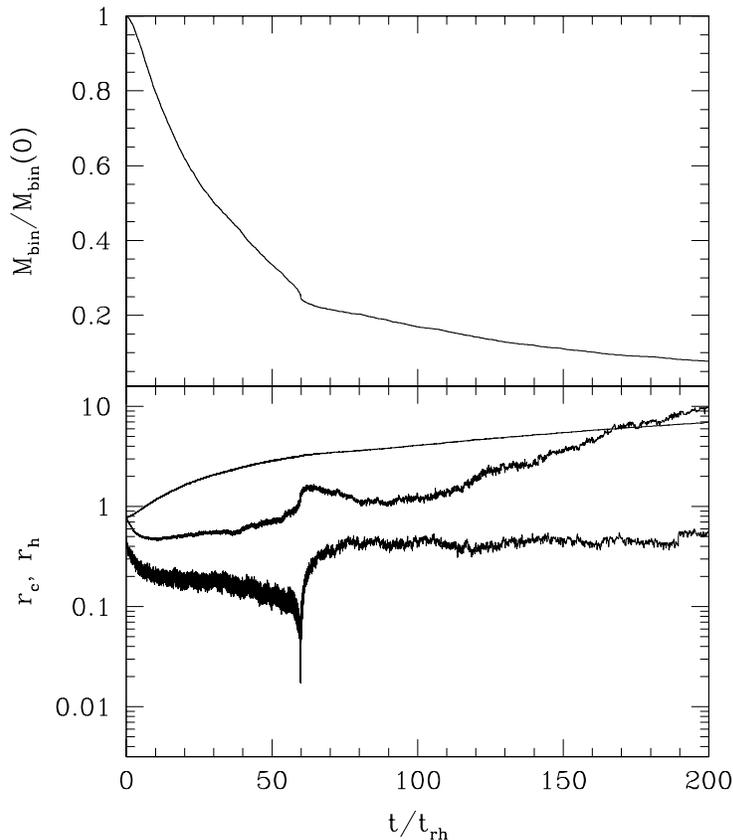


Figure 2. Results of a Monte Carlo simulation for the evolution of an isolated Plummer model containing $N = 3 \times 10^5$ equal-mass stars, with 10% of the stars in primordial binaries. The binaries are initially distributed uniformly throughout the cluster, and with a uniform distribution in the logarithm of the binding energy (roughly between contact and the hard-soft boundary, i.e., no soft binaries are included). The simulation includes a treatment of energy production and binary destruction through binary-single and binary-binary interactions. Stellar evolution and tidal interactions with the Galaxy are not included. Time is given in units of the initial half-mass relaxation time t_{rh} . The upper panel shows the evolution of the total mass (or number) of binaries. The lower panel shows, from top to bottom initially, the half-mass radius of the entire cluster, the half-mass radius of the binaries, and the cluster core radius. These quantities are in units of the virial radius of the cluster. Note the long, quasi-equilibrium phase of “binary burning” lasting until $t \simeq 60 t_{rh}$, followed by a brief episode of core contraction and re-expansion to an even longer quasi-equilibrium phase with an even larger core. By $t \sim 100 t_{rh}$, only about 15% of the initial population of binaries remains in the cluster, but this is enough to support the cluster against core collapse for another $\sim 100 t_{rh}$. For most globular clusters, $t_{rh} \sim 10^9$ yr, and this is well beyond a Hubble time. The evolution shown here should be contrasted to that of an identical cluster, but containing single stars only (Fig. 1), where core collapse is reached at $t \simeq 15 t_{rh}$.

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Ivan King: You showed that binaries stabilize a cluster for much longer than a Hubble time. Yet we see 15–20% of clusters in a state of core collapse. How do you reconcile this?

Fred Rasio: Indeed, if the type of evolution shown in Fig. 2 applied to all globular clusters, there would be no “core-collapsed” clusters in the Galaxy. However, the timescale on which real clusters will exhaust their primordial binary supply and undergo (deep) core collapse depends on a number of factors not considered here: the initial primordial binary fraction (some clusters may have much fewer binaries than the 10% assumed in Fig. 2), the orbit of the cluster in the Galaxy (the simulation of Fig. 2 is for an isolated cluster, but mass loss and tidal shocking can accelerate the evolution dramatically), the stellar IMF (the cluster shown in Fig. 2 contains all equal-mass stars and binaries; a more realistic mass spectrum will also accelerate the evolution), etc.

However, the simple picture that emerges from Fig. 2 may well, to first approximation, describe the dynamical state of most Galactic globular clusters observed today. Note that, for a cluster in the stable “binary burning” phase of Fig. 2, the ratio of half-mass radius to core radius $r_h/r_c \simeq 2 - 10$ (for $t \sim 10$ Gyr and $t_{\text{rh}} \sim 0.1 - 10$ Gyr), which is precisely the range of values observed for the $\sim 80\%$ of clusters that have a well-resolved core and are well-fitted by King models. Some of these clusters may have gone in the past through a brief episode of “moderate core collapse” (as shown around $t \simeq 60 t_{\text{rh}}$ in Fig. 2). Yet, I do not believe that they should be called “core-collapsed” or “post-core-collapse” (nor would they be classified as such by observers). Unfortunately some theorists will even call “core-collapsed” clusters that have just reached the *initial phase* of binary burning ($t \simeq 10 - 50 t_{\text{rh}}$ in Fig. 2). Since the core has just barely contracted by a factor $\sim 2 - 3$ by the time it reaches this phase, it seems hardly justified to speak of a “collapsed” state.