BINARIES AND GLOBULAR CLUSTER DYNAMICS

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Abstract   We summarize the results of recent theoretical work on the dynamical evolution of globular clusters containing primordial binaries. Even a very small initial binary fraction (e.g., 10%) can play a key role in supporting a cluster against gravothermal collapse for many relaxation times. Inelastic encounters between binaries and single stars or other binaries provide a very significant energy source for the cluster. These dynamical interactions also lead to the production of large numbers of exotic systems such as ultracompact X-ray binaries, recycled radio pulsars, double degenerate systems, and blue stragglers. Our work is based on a new parallel supercomputer code implementing Hénon's Monte Carlo method for simulating the dynamical evolution of dense stellar systems in the Fokker-Planck approximation. This new code allows us to calculate very accurately the evolution of a cluster containing a realistic number of stars ($N \sim 10^5 - 10^6$) in typically a few hours to a few days of computing time. The discrete, star-by-star representation of the cluster in the simulation makes it possible to treat naturally a number of important processes, including single and binary star evolution, all dynamical interactions of single stars and binaries, and tidal interactions with the Galaxy.

1.   INTRODUCTION

The dynamical evolution of dense star clusters is a problem of fundamental importance in theoretical astrophysics, but many aspects of the problem have remained unresolved in spite of years of numerical work and improved observational data (see Meylan & Heggie 1997 for a recent review). The realization over the last 10 years that primordial binaries are present in globular clusters in dynamically significant numbers has completely changed our theoretical perspective on these systems (see, e.g., Gao et al. 1991; Hut et al. 1992; Sigurdsson & Phinney 1995). Most importantly, dynamical interactions between hard primordial binaries and other single stars or binaries are now thought to be the primary mechanism for supporting a globular cluster against gravothermal contraction and avoiding core collapse. In addition, exchange interactions

between primordial binaries and compact objects can explain very naturally the formation of large numbers of X-ray binaries and recycled pulsars in globular cluster cores (see, e.g., Camilo et al. 2000). Dynamical interactions involving primordial binaries can also result in dramatically increased collision rates in globular clusters. This is because the interactions are often *resonant*, with all the stars involved remaining together in a small volume for a long time ($\sim 10^2 - 10^3$ orbital times). For example, in the case of an interaction between two typical hard binaries with semi-major axes $\sim 1\,\mathrm{AU}$ containing $\sim 1\,M_\odot$ main-sequence stars, the effective cross section for a direct collision between any two of the four stars involved is essentially equal to the entire geometric cross section of the binaries (Bacon, Sigurdsson & Davies 1996; Cheung, Portegies Zwart, & Rasio 2001). This implies a collision rate ~ 100 times larger than for single stars. Direct observational evidence for stellar collisions and mergers in globular clusters comes from the detection of large numbers of blue stragglers concentrated in the dense cluster cores (see, e.g., Bailyn 1995).

2.   MONTE CARLO SIMULATIONS OF CLUSTER DYNAMICS

The first Monte Carlo methods for calculating the dynamical evolution of star clusters in the Fokker-Planck approximation were developed more than 30 years ago. They were first used to study the development of the gravothermal instability (Hénon 1971a,b; Spitzer & Hart 1971a,b). More recent implementations have established the Monte Carlo method as an important alternative to direct N-body integrations (see Spitzer 1987 for an overview). The main motivation for our recent work at MIT was our realization a few years ago that the latest generation of parallel supercomputers now make it possible to perform Monte Carlo simulations for a number of objects equal to the actual number of stars in a globular cluster (in contrast, earlier work was limited to using a small number of representative "superstars," and was often plagued by high levels of numerical noise). Therefore, the Monte Carlo method allows us to do right now what remains an elusive goal for N-body simulations (see, e.g., Aarseth 1999): perform realistic, star-by-star computer simulations of globular cluster evolution. Using the correct number of stars in a dynamical simulation ensures that the relative rates of different dynamical processes (which all scale differently with the number of stars) are correct. This is particularly crucial if many different dynamical processes are to be incorporated, as must be done in realistic simulations.

Our implementation of the Monte Carlo method is described in detail in the papers by Joshi, Rasio, & Portegies Zwart (2000), Joshi, Nave, & Rasio (2001), and Joshi, Portegies Zwart, & Rasio (2001). We adopt the usual assumptions of spherical symmetry (with a 2D phase space distribution function $f(E, J)$, i.e., we do *not* assume isotropy) and standard two-body relaxation in the weak scattering limit (Fokker-Planck approximation). In its simplest version, our code computes the dynamical evolution of a self-gravitating spherical cluster of N point masses whose orbits in the cluster are specified by an energy E and angular momentum J, with perturbations ΔE and ΔJ evaluated on a timestep that is a fraction of the local two-body relaxation time. The cluster is assumed to remain always very close to dynamical equilibrium (i.e., the relaxation time must remain much longer than the dynamical time). Our main improvements over Hénon's original method are the parallelization of the basic algorithm and the development of a more sophisticated method for determining the timesteps and for computing the two-body relaxation from representative encounters between neighboring stars. Our new method allows the timesteps to be made much smaller in order to resolve the dynamics in the cluster core more accurately.

We have performed a large number of test calculations and comparisons with direct N-body integrations, as well as direct integrations of the Fokker-Planck equation in phase space, to establish the accuracy of our basic treatment of two-body relaxation (Joshi et al. 2000). Fig. 1 shows the results from a typical comparison between Monte Carlo and N-body simulations. Fig. 2 shows the results obtained with our code for Heggie's Collaborative Experiment run (Heggie et al. 1998).

3.   SUMMARY OF RECENT RESULTS

Our recent work has focused on the addition of more realistic stellar and binary processes to the basic Monte Carlo code, as well as a simple but accurate implementation of a static tidal boundary in the Galactic field (Joshi et al. 2001a). As a first application, we have studied the dependence on initial conditions of globular cluster lifetimes in the Galactic environment. As in previous Fokker-Planck studies (Chernoff & Weinberg 1990; Takahashi & Portegies Zwart 1998), we include the effects of a power-law initial mass function (IMF), mass loss through a tidal boundary, and single star evolution, and we consider initial King models with varying central concentrations. We find that the disruption and core-collapse times of our models are significantly longer than those obtained with previous 1D (isotropic) Fokker-Planck calculations, but agree well with more recent results from direct N-body simulations
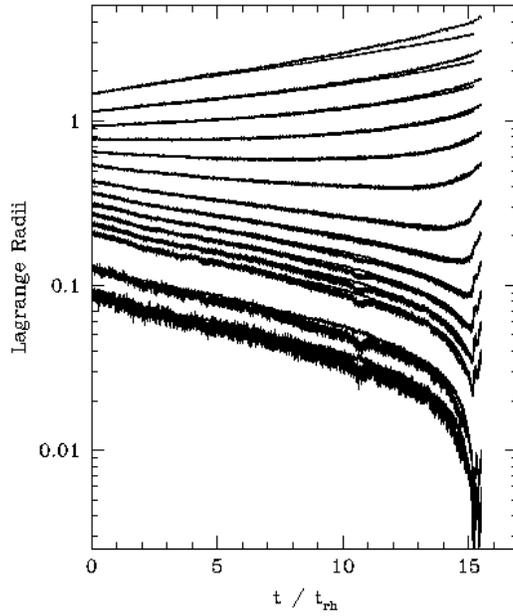
Figure 1 Evolution of the Lagrange radii for an isolated, single-component Plummer model (from bottom to top: radii containing 0.35%, 1%, 3.5%, 5%, 7%, 10%, 14%, 20%, 30%, 40%, 50%, 60%, 70%, and 80% percent of the total mass are shown as a function of time, given in units of the initial half-mass relaxation time). The results from a direct N-body integration with $N = 16,384$ (noisier lines) and from a Monte Carlo integration with $N = 10^5$ stars (smoother lines) are compared. The Monte Carlo simulation was completed in less than a day on a Cray/SGI Origin2000 parallel supercomputer, while the N-body integration ran for over a month on a dedicated GRAPE-4 computer. The agreement between the N-body and Monte Carlo results is excellent over the entire range of Lagrange radii and time. The small discrepancy in the outer Lagrange radii is caused mainly by a different treatment of escaping stars in the two models. In the Monte Carlo model, escaping stars are removed from the simulation and therefore not included in the determination of the Lagrange radii, whereas in the N-body model escaping stars are not removed. Note also that the Monte Carlo simulation is terminated at core collapse, while the N-body simulation continues beyond core collapse.
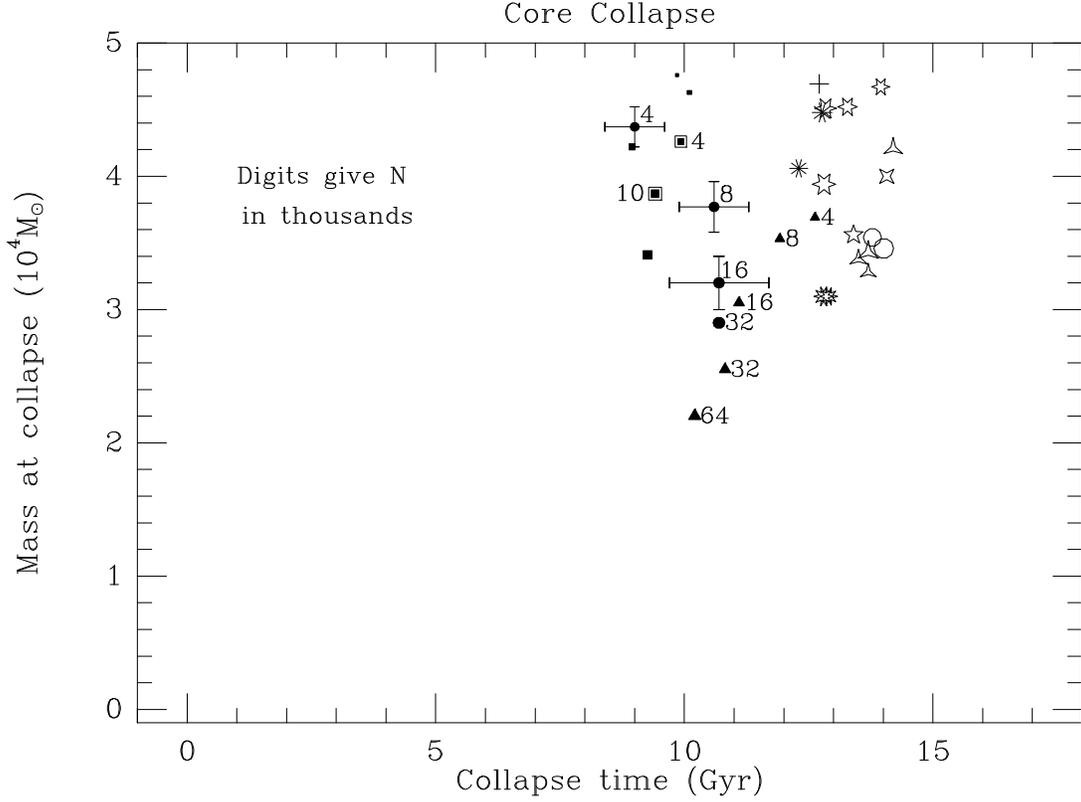
Figure 2   Core collapse time t_{cc} and cluster mass at core collapse M_{cc} as computed by many different numerical codes in Heggie's Collaborative Experiment, and with our Monte Carlo code. The solid round dots, triangles and squares are from various sets of direct N-body simulations. Open symbols are from Fokker-Planck simulations and the stars are from anisotropic gas models. Our data point is indicated by the plus symbol, corresponding to a core collapse time $t_{cc} = 12.86$ Gyr and a mass at core collapse $M_{cc} = 4.73 \times 10^4 \, M_\odot$. The initial condition is a King model with a dimensionless central potential $W_0 = 3$, a tidal radius $r_t = 30$ pc, and containing a mass $M = 6 \times 10^4 \, M_\odot$. The cluster contains single stars only with a power-law IMF of slope -2.35 (Salpeter mass function) between $0.1 \, M_\odot$ and $1.5 \, M_\odot$. All simulations are done assuming no stellar evolution. Heating by "3-body binaries" is not included in our Monte Carlo simulation (this only affects the evolution beyond core collapse). See Heggie et al. 1998 for more details.

and 2D Fokker-Planck integrations. In agreement with previous studies, our results show that the direct mass loss due to stellar evolution causes most clusters with a low initial central concentration to disrupt quickly in the Galactic tidal field. The disruption is particularly rapid for clusters with a relatively flat IMF. Only clusters born with high central concentrations or with very steep IMFs are likely to survive to the present and undergo core collapse.

In another recent study, we have used our Monte Carlo code to examine the development of the Spitzer "mass stratification instability" in simple two-component clusters (Watters, Joshi, & Rasio 2000). We have performed a large number of dynamical simulations for star clusters containing two stellar populations with individual masses m_1 and $m_2 > m_1$, and total masses M_1 and $M_2 < M_1$. We use both King and Plummer model initial conditions and we perform simulations for a wide range of individual and total mass ratios, m_2/m_1 and M_2/M_1, in order to determine the precise location of the stability boundary in this 2D parameter space. As predicted originally by Spitzer (1969) using simple analytic arguments, we find that unstable systems never reach energy equipartition, and are driven to rapid core collapse by the heavier component. These results have important implications for the dynamical evolution of any population of primordial black holes or neutron stars in globular clusters. In particular, primordial black holes with $m_2/m_1 \sim 10$ are expected to undergo very rapid core collapse independent of the background cluster, and to be ejected from the cluster through dynamical interactions between single and binary black holes (see Portegies Zwart & McMillan 2000 and references therein). We have also used Monte Carlo simulations of simple two-component systems to study the evaporation (or retention) of *low-mass* objects in globular clusters, motivated by the surprising recent observations of planets and brown dwarfs in several clusters (Fregeau et al. 2001).

Much of our current work concerns the treatment of dynamical interactions with primordial binaries. We are in the process of completing a first study of globular cluster evolution with primordial binaries (Joshi et al. 2001b), based on the same set of approximate cross sections and recipes for dynamical interactions used in the Fokker-Planck simulations of Gao et al. (1991). Typical results are illustrated in Figs. 3–6. The heating of the cluster core generated by a small population of primordial binaries can support the cluster against core collapse for very long times, although the details of the evolution depend sensitively both on the number of stars N and on the initial binary fraction $f_{\rm b}$. Clusters with smaller N tend to evolve faster towards deeper core collapse (compare Figs. 3 and 4, and note that the main dependence on N through
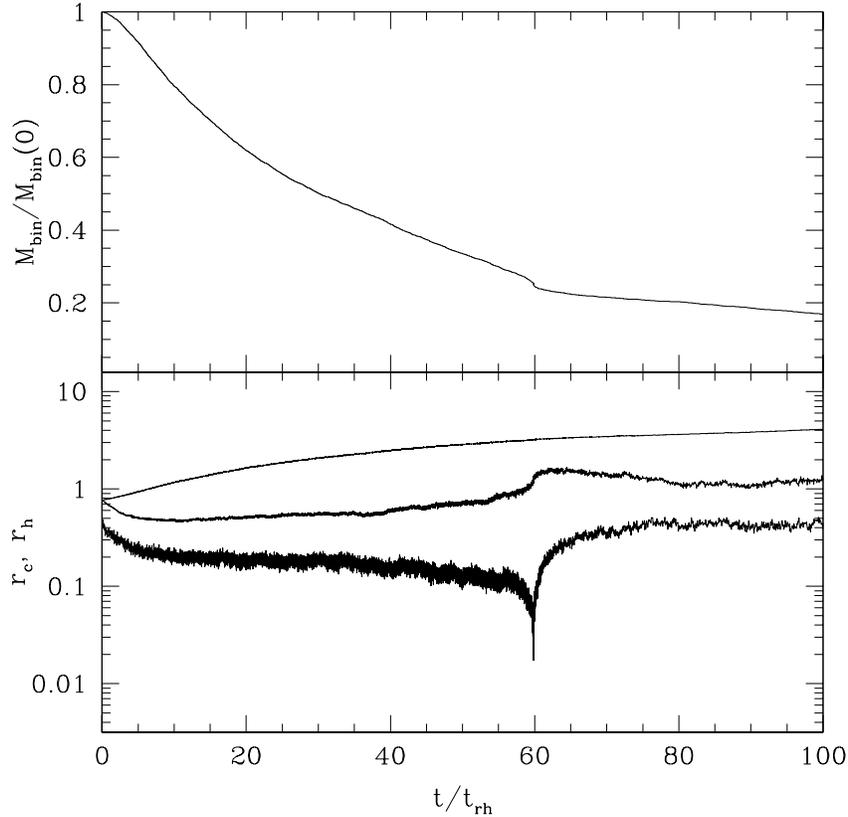
Figure 3   Results of a Monte Carlo simulation for the evolution of an isolated Plummer model containing $N = 3 \times 10^5$ equal-mass stars, with 10% of the stars in primordial binaries. The binaries are initially distributed uniformly throughout the cluster, and with a uniform distribution in the logarithm of the binding energy (roughly between contact and the hard-soft boundary, i.e., no soft binaries are included). The simulation includes a treatment of energy production and binary destruction through binary-single and binary-binary interactions. Stellar evolution and tidal interactions with the Galaxy are not included. Time is given in units of the initial half-mass relaxation time $t_{\rm rh}$. The upper panel shows the evolution of the total mass (or number) of binaries. The lower panel shows, from top to bottom initially, the half-mass radius of the entire cluster, the half-mass radius of the binaries, and the cluster core radius. These quantities are in units of the virial radius of the cluster. Note the long, quasi-equilibrium phase of "binary burning" lasting until $t \simeq 60\,t_{\rm rh}$, followed by a brief episode of core contraction and re-expansion to an even longer quasi-equilibrium phase with an even larger core. By $t \sim 100\,t_{\rm rh}$, only about 15% of the initial population of binaries remains in the cluster, but this is enough to support the cluster against core collapse for another $\sim 100\,t_{\rm rh}$. For most globular clusters, $t_{\rm rh} \sim 10^9$ yr, and this is well beyond a Hubble time. The evolution shown here should be contrasted to that of an identical cluster, but containing single stars only (Fig. 1), where core collapse is reached at $t \simeq 15\,t_{\rm rh}$.
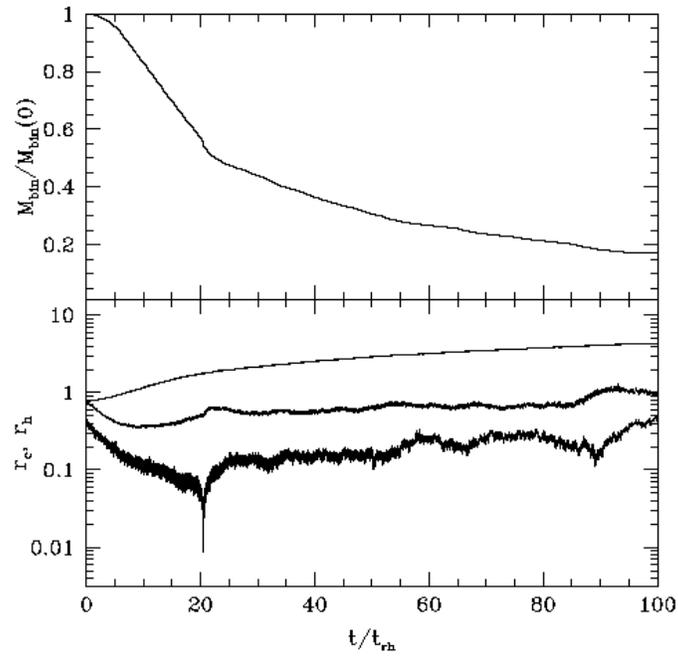
Figure 4   Same as Fig. 3 but for $N = 10^5$ equal-mass stars with 10% binaries. Note the somewhat faster and deeper initial contraction, but still followed by re-expansion into a long-lived, quasi-equilibrium phase of binary burning.
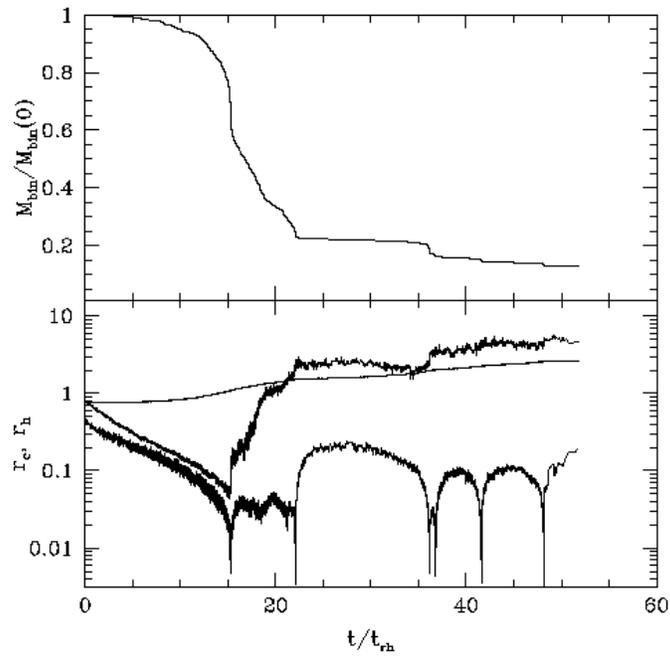
Figure 5 Same as Fig. 3 but for $N = 10^5$ equal-mass stars with only 1% binaries. Here the initial core contraction is followed by more rapid gravothermal oscillations, but the time-averaged core size remains similar to what is seen in Figs. 3 and 4.
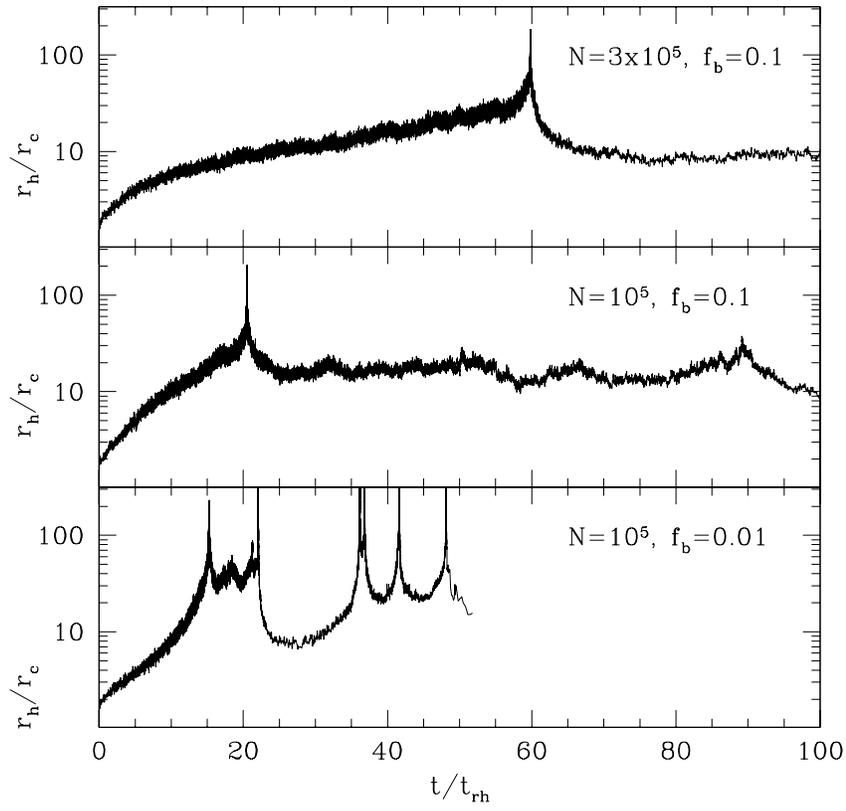
Figure 6 Evolution of the half-mass radius to core radius ratio for the three cases illustrated in Figs. 3–5. For comparison, most observed globular clusters with resolved cores have $r_{\rm h}/r_{\rm c} \simeq 2 - 10$.

$t_{\rm rh} \propto N/\log N$ has been scaled out), and so do clusters with smaller initial binary fractions (Fig. 5). After the initial core collapse, gravothermal oscillations powered by primordial binaries can continue for very long times $\sim 50 - 100\,t_{\rm rh}$ even with an initial binary fraction as small as 1%. With an initial binary fraction of 10%, we observe a single, very moderate phase of core collapse during which the core shrinks by a factor $\sim 5 - 10$ in radius and then re-expands rapidly on a timescale of a few central relaxation times (this occurs at $t/t_{\rm rh} \simeq 60$ in Fig. 3 and at $t/t_{\rm rh} \simeq 20$ in Fig. 4). The cluster then enters a second quasi-equilibrium phase of primordial binary burning with the core radius *increasing slowly* until well beyond $\sim 100\,t_{\rm rh}$.

Since $t_{\rm rh} \sim 10^9\,{\rm yr}$ for most clusters, if the type of evolution illustrated in Fig. 3 applied to all globular clusters, there would be no "core-collapsed" clusters in the Galaxy (10–15% of all Galactic globular clusters are classified observationally as "core collapsed"). However, the timescale on which real clusters will exhaust their primordial binary supply and undergo (deeper) core collapse depends on a number of factors not considered here, including the orbit of the cluster in the Galaxy (the simulations are for an isolated cluster, but mass loss and tidal shocking can accelerate the evolution dramatically), and the stellar IMF (the clusters shown here contain all equal-mass stars and binaries; a more realistic mass spectrum will also accelerate the evolution). In addition, some clusters may be formed with much fewer primordial binaries than even the 1% considered in Fig. 5. However, the simple picture that emerges from these simulations may well, to first approximation, describe the dynamical state of most Galactic globular clusters observed today. Indeed, for a cluster in the stable "binary burning" phase, the ratio of half-mass radius to core radius $r_{\rm h}/r_{\rm c} \sim 2 - 10$ (Fig. 6), which is precisely the range of values observed for the $\sim 80\%$ of globular clusters that have a well-resolved core and are well-fitted by King models (see, e.g., Djorgovski 1993). Some of these clusters may have gone in the past through a brief episode of "moderate core collapse" (as shown around $t \simeq 60\,t_{\rm rh}$ in Fig. 3). Yet, they should not be called "core-collapsed" (nor would they be classified as such by observers). Unfortunately some theorists will even call "core-collapsed" clusters that have just reached the *initial phase* of binary burning ($t \simeq 10 - 50\,t_{\rm rh}$ in Fig. 3). Since the core has just barely contracted by a factor $\sim 2 - 3$ by the time it reaches this phase, it seems hardly justified to speak of a "collapsed" state.

The addition of binary stellar evolution processes to our simulations will allow us to study in detail the dynamical formation mechanisms for many exotic objects such as X-ray binaries and millisecond radio pulsars, which have been detected in large numbers in globular clusters.

For example, exchange interactions between neutron stars and primordial binaries can lead to common-envelope systems and the formation of short-period neutron-star / white-dwarf binaries that can become visible both as ultracompact X-ray binaries and binary millisecond pulsars with low-mass companions (see, e.g., Camilo et al. 2000, on observations of 20 such millisecond radio pulsars in 47 Tuc). Rasio, Pfahl, & Rappaport (2000) present a preliminary study of this formation scenario, based on simplified dynamical Monte Carlo simulations. In a dense cluster such as 47 Tuc, most neutron stars acquire binary companions through exchange interactions with primordial binaries. The resulting systems have semimajor axes in the range $\sim 0.1 - 1\,\mathrm{AU}$ and neutron star companion masses $\sim 1 - 3\,M_\odot$. For many of these systems it is found that, when the companion evolves off the main sequence and fills its Roche lobe, the subsequent mass transfer is dynamically unstable. This leads to a common envelope phase and the formation of short-period neutron-star / white-dwarf binaries. For a significant fraction of these binaries, the decay of the orbit due to gravitational radiation will be followed by a period of stable mass transfer driven by a combination of gravitational radiation and tidal heating of the companion. The properties of the resulting short-period binaries match well those of observed binary pulsars in 47 Tuc (Fig. 7). A similar dynamical scenario involving massive CO white dwarfs in place of neutron stars could explain the recent detections of double degenerate binaries containing He white dwarfs concentrated in the core of a dense globular cluster (Hansen et al. 2001; see Edmonds et al. 1999 and Taylor et al. 2001 on observations of He white dwarfs in the core of NGC 6397).

We are also currently working on incorporating into our Monte Carlo simulations a more realistic treatment of tidal interactions, and, in particular, tidal shocking through the Galactic disk (based on Gnedin, Lee, & Ostriker 1999). Tidal shocks can accelerate significantly both core collapse and the evaporation of globular clusters, reducing their lifetimes in the Galaxy (Gnedin & Ostriker 1997). Future work may include a fully dynamical treatment of all strong binary-single and binary-binary interactions (exploiting the parallelism of the code to perform separate numerical 3- or 4-body integrations for all dynamical interactions) as well as a fully dynamical treatment of tidal shocking (performing short, N-body integrations for each passage of the cluster through the Galactic disk or near the bulge).
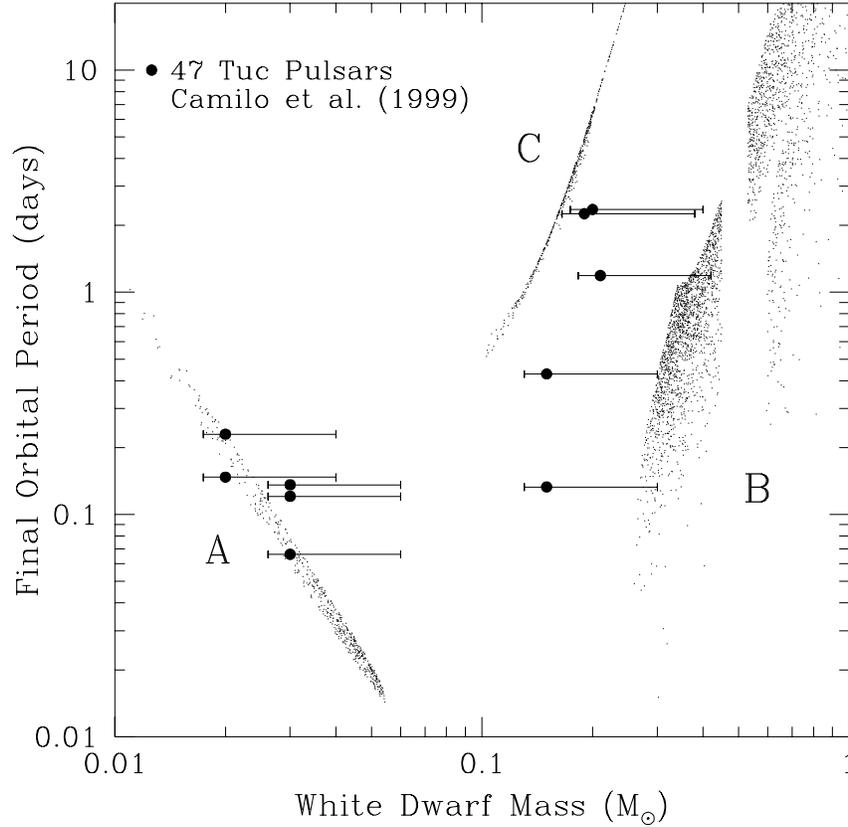
Figure 7 Results of our initial Monte Carlo study of binary millisecond pulsar formation in a dense globular cluster such as 47 Tuc. Each small dot represents a binary system in our simulation, while the filled circles are the 10 binary pulsars in 47 Tuc with well measured orbits (the error bars extend from the minimum companion mass to the 90% probability level for random inclinations). There are 3 principal groups of simulated binaries. Systems in the diagonal band on the left (A) are neutron star – white dwarf binaries that decayed via gravitational radiation to very short orbital periods ($\sim$ mins), then evolved with mass transfer back up to longer periods under the influence of both gravitational radiation and tidal heating. The large group labeled B on the right contains neutron star – white dwarf binaries which had insufficient time to decay to Roche-lobe contact via the emission of gravitational radiation. The neutron stars in this group are not likely to be recycled since they may not have accreted much mass during the common envelope phase. Finally, the systems lying in the thin diagonal band toward longer periods (C) are those in which the mass transfer from the giant or subgiant to the neutron star in the initial binary would be stable. These have not been evolved through the mass transfer phase; the mass plotted is simply that of the He core of the donor star when mass transfer commences. There are many more systems in this category that have longer periods but lie off the graph. See Rasio, Pfahl, & Rappaport 2000 for more details.

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Acknowledgments

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