Vector Semantics & Embeddings

Word Meaning

What do words mean?

N-gram or text classification methods we've seen so far

- Words are just strings (or indices w_i in a vocabulary list)
- That's not very satisfactory!

Introductory logic classes:

- The meaning of "dog" is DOG; cat is CAT

$$\forall x \; DOG(x) \longrightarrow MAMMAL(x)$$

Old linguistics joke by Barbara Partee in 1967:

- Q: What's the meaning of life?
- A: LIFE

That seems hardly better!

Desiderata

What should a theory of word meaning do for us?

Let's look at some desiderata

From lexical semantics, the linguistic study of word meaning

Lemmas and senses

lemma

mouse (N)

sense

1. any of numerous small rodents...
2. a hand-operated device that controls a cursor...

Modified from the online thesaurus WordNet

A sense or "concept" is the meaning component of a word

Lemmas can be polysemous (have multiple senses)

Relations between senses: Synonymy

Synonyms have the same meaning in some or all contexts.

- filbert / hazelnut
- couch / sofa
- big / large
- automobile / car
- vomit / throw up
- water / H_20

Relations between senses: Synonymy

Note that there are probably no examples of perfect synonymy.

- Even if many aspects of meaning are identical
- Still may differ based on politeness, slang, register, genre, etc.

Relation: Synonymy?

water/H_20

"H_20" in a surfing guide?

big/large

my big sister != my large sister

The Linguistic Principle of Contrast

Difference in form → difference in meaning

Abbé Gabriel Girard 1718

Re: "exact" synonyms

"je ne crois pas qu'il y ait de-
mot ſynonime dans aucune
Langue."

[I do not believe that there is a synonymous word in any language]

LA JUSTESSE
DE LA
LANGUE FRANÇOISE,
OU
LES DIFFERENTES SIGNIFICATIONS
DES MOTS QUI PASSENT
POUR
SYNONIMES.
Par M. l'Abbé GIRARD C. D. M. D. D. B.

A PARIS,
Chez LAURENT D'HOURY, Imprimeur-Libraire, au bas de la rue de la Harpe, vis-à vis la rue S. Severin, au Saint Eſprit.
M. DCC. XVIII.
Avec Approbation & Privilege du Roy.

Thanks to Mark Aronoff!

Relation: **Similarity**

Words with similar meanings. Not synonyms, but sharing some element of meaning

```
car, bicycle
cow, horse
```

Ask humans how similar 2 words are

word1	word2	similarity
vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

SimLex-999 dataset (Hill et al., 2015)

Relation: Word relatedness

Also called "word association"

Words can be related in any way, perhaps via a semantic frame or field

- `coffee, tea`: **similar**
- `coffee, cup`: **related**, not similar

Semantic field

Words that

- cover a particular semantic domain
- bear structured relations with each other.

hospitals

surgeon, scalpel, nurse, anaesthetic, hospital

restaurants

waiter, menu, plate, food, menu, chef

houses

door, roof, kitchen, family, bed

Relation: Antonymy

Senses that are opposites with respect to only one feature of meaning

Otherwise, they are very similar!

`dark/light    short/long fast/slow    rise/fall`
`hot/cold        up/down          in/out`

More formally: antonyms can

- define a binary opposition or be at opposite ends of a scale
  - `long/short, fast/slow`
- Be *reversives*:
  - `rise/fall, up/down`

Connotation (sentiment)

- Words have **affective** meanings
  - Positive connotations (*happy*)
  - Negative connotations (*sad*)
- Connotations can be subtle:
  - Positive connotation: *copy, replica, reproduction*
  - Negative connotation: *fake, knockoff, forgery*
- Evaluation (sentiment!)
  - Positive evaluation (*great*, *love*)
  - Negative evaluation (*terrible*, *hate*)

Connotation

Osgood et al. (1957)

Words seem to vary along 3 affective dimensions:

- **valence**: the pleasantness of the stimulus
- **arousal**: the intensity of emotion provoked by the stimulus
- **dominance**: the degree of control exerted by the stimulus

	Word	Score		Word	Score
Valence	love	1.000		toxic	0.008
	happy	1.000		nightmare	0.005
Arousal	elated	0.960		mellow	0.069
	frenzy	0.965		napping	0.046
Dominance	powerful	0.991		weak	0.045
	leadership	0.983		empty	0.081

Values from NRC VAD Lexicon (Mohammad 2018)

So far

Concepts or word senses

- Have a complex many-to-many association with **words** (homonymy, multiple senses)

Have relations with each other

- Synonymy
- Antonymy
- Similarity
- Relatedness
- Connotation

Vector Semantics & Embeddings

Word Meaning

Vector Semantics & Embeddings

Vector Semantics

Computational models of word meaning

Can we build a theory of how to represent word meaning, that accounts for at least some of the desiderata?

We'll introduce **vector semantics**

The standard model in language processing!

Handles many of our goals!

Ludwig Wittgenstein

PI #43:

"The meaning of a word is its use in the language"

Let's define words by their usages

One way to define "usage":

words are defined by their environments (the words around them)

Zellig Harris (1954):

If A and B have almost identical environments we say that they are synonyms.

What does recent English borrowing *ongchoi* mean?

Suppose you see these sentences:

- Ong choi is delicious **sautéed with garlic**.
- Ong choi is superb **over rice**
- Ong choi **leaves** with salty sauces

And you've also seen these:

- …spinach **sautéed with garlic over rice**
- Chard stems and **leaves** are **delicious**
- Collard greens and other **salty** leafy greens

Conclusion:

- Ongchoi is a leafy green like spinach, chard, or collard greens
  - We could conclude this based on words like "leaves" and "delicious" and "sauteed"

Ongchoi: *Ipomoea aquatica "Water Spinach"*

空心菜

kangkong

rau muống

…

Yamaguchi, Wikimedia Commons, public domain

Idea 1: Defining meaning by linguistic distribution

Let's define the meaning of a word by its distribution in language use, meaning its neighboring words or grammatical environments.

Idea 2: Meaning as a point in space (Osgood et al. 1957)

3 affective dimensions for a word

- **valence**: pleasantness
- **arousal**: intensity of emotion
- **dominance**: the degree of control exerted

	Word	Score		Word	Score
Valence	love	1.000		toxic	0.008
	happy	1.000		nightmare	0.005
Arousal	elated	0.960		mellow	0.069
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Dominance	powerful	0.991		weak	0.045
	leadership	0.983		empty	0.081

NRC VAD Lexicon (Mohammad 2018)

Hence the connotation of a word is a vector in 3-space

Idea 1: Defining meaning by linguistic distribution

Idea 2: Meaning as a point in multidimensional space

Defining meaning as a point in space based on distribution

Each word = a vector (not just "good" or "w_{45}")

Similar words are "**nearby in semantic space**"

We build this space automatically by seeing which words are **nearby in text**

We define meaning of a word as a vector

Called an "embedding" because it's embedded into a space (see textbook)

The standard way to represent meaning in NLP

Every modern NLP algorithm uses embeddings as the representation of word meaning

Fine-grained model of meaning for similarity

Intuition: why vectors?

Consider sentiment analysis:

- With **words**, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires **exact same word** to be in training and test
- With **embeddings**:
  - Feature is a word vector
  - 'The previous word was vector [35,22,17…]
  - Now in the test set we might see a similar vector [34,21,14]
  - We can generalize to **similar but unseen** words!!!

We'll discuss 2 kinds of embeddings

tf-idf

- Information Retrieval workhorse!
- A common baseline model
- **Sparse** vectors
- Words are represented by (a simple function of) the **counts** of nearby words

Word2vec

- **Dense** vectors
- Representation is created by training a classifier to **predict** whether a word is likely to appear nearby
- Later we'll discuss extensions called **contextual embeddings**

From now on:
Computing with meaning representations instead of string representations

荃者所以在鱼，得鱼而忘荃	Nets are for fish; Once you get the fish, you can forget the net.
言者所以在意，得意而忘言	Words are for meaning; Once you get the meaning, you can forget the words

庄子(Zhuangzi), Chapter 26

Vector Semantics & Embeddings

Vector Semantics

Vector Semantics & Embeddings

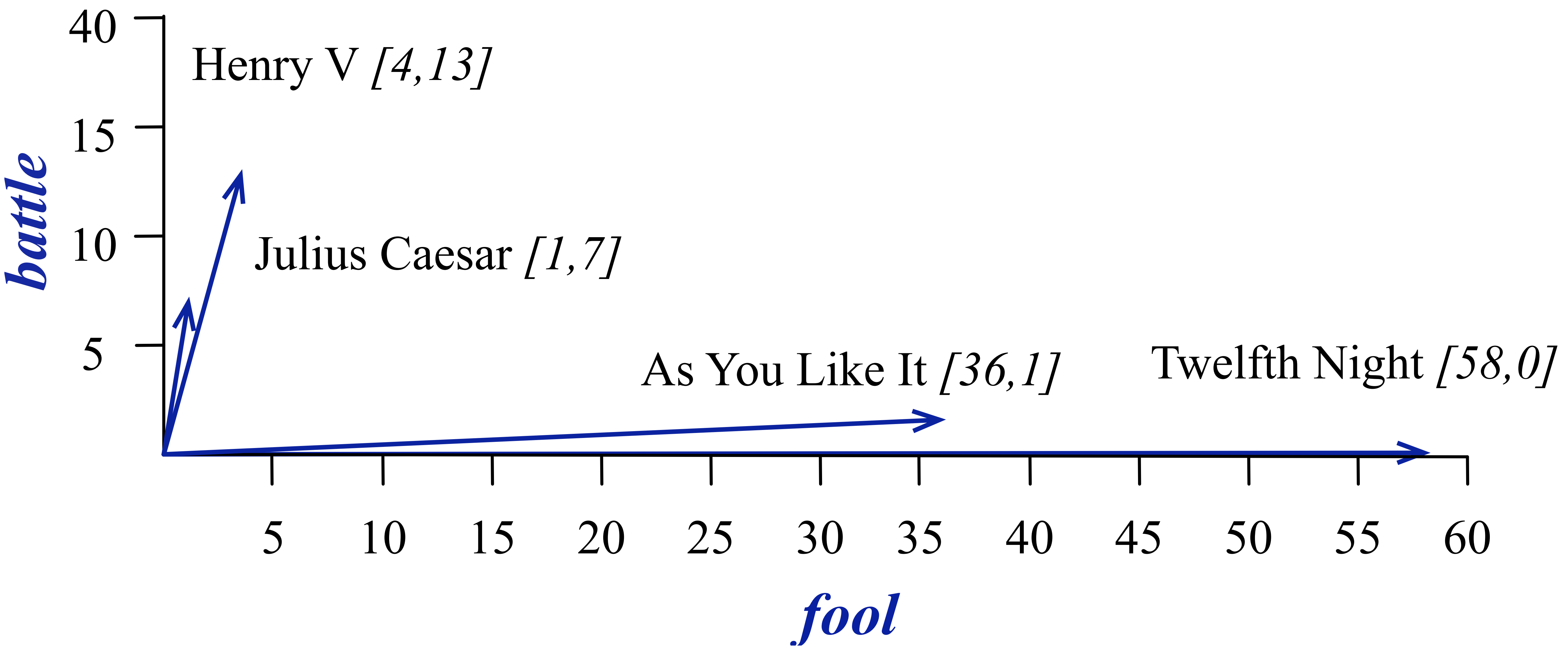
Words and Vectors

Term-document matrix

Each document is represented by a vector of words

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Visualizing document vectors



Vectors are the basis of information retrieval

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Vectors are similar for the two comedies

But comedies are different than the other two

Comedies have more *fools* and *wit* and fewer *battles*.

Idea for word meaning: Words can be vectors too!!!

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

battle is "the kind of word that occurs in Julius Caesar and Henry V"

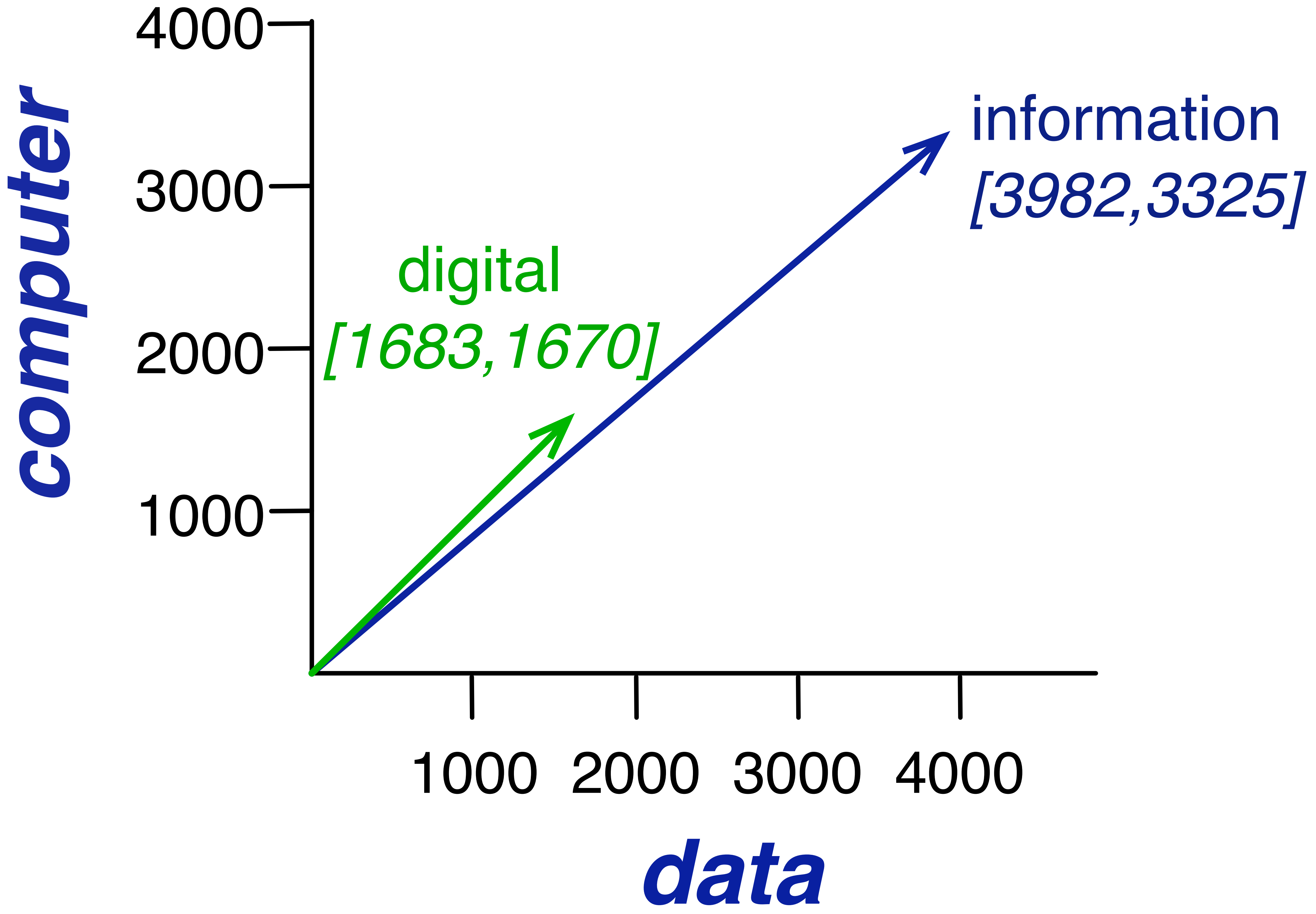
fool is "the kind of word that occurs in comedies, especially Twelfth Night"

More common: word-word matrix (or "term-context matrix")

Two **words** are similar in meaning if their context vectors are similar

is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

	aardvark	...	**computer**	**data**	**result**	**pie**	**sugar**	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...



4000
3000
2000
1000
computer
information
[3982,3325]
digital
[1683,1670]
1000
2000
3000
4000
data

Vector Semantics & Embeddings

Words and Vectors

Vector Semantics & Embeddings

Cosine for computing word similarity

Computing word similarity: Dot product and cosine

The dot product between two vectors is a scalar:

$$\text{dot product}(\mathbf{v},\mathbf{w}) = \mathbf{v}\cdot\mathbf{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

The dot product tends to be high when the two vectors have large values in the same dimensions

Dot product can thus be a useful similarity metric between vectors

Problem with raw dot-product

Dot product favors long vectors

Dot product is higher if a vector is longer (has higher values in many dimension)

Vector length:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

Frequent words (of, the, you) have long vectors (since they occur many times with other words).

So dot product overly favors frequent words

Alternative: cosine for computing word similarity

$$\text{cosine}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

Based on the definition of the dot product between two vectors a and b

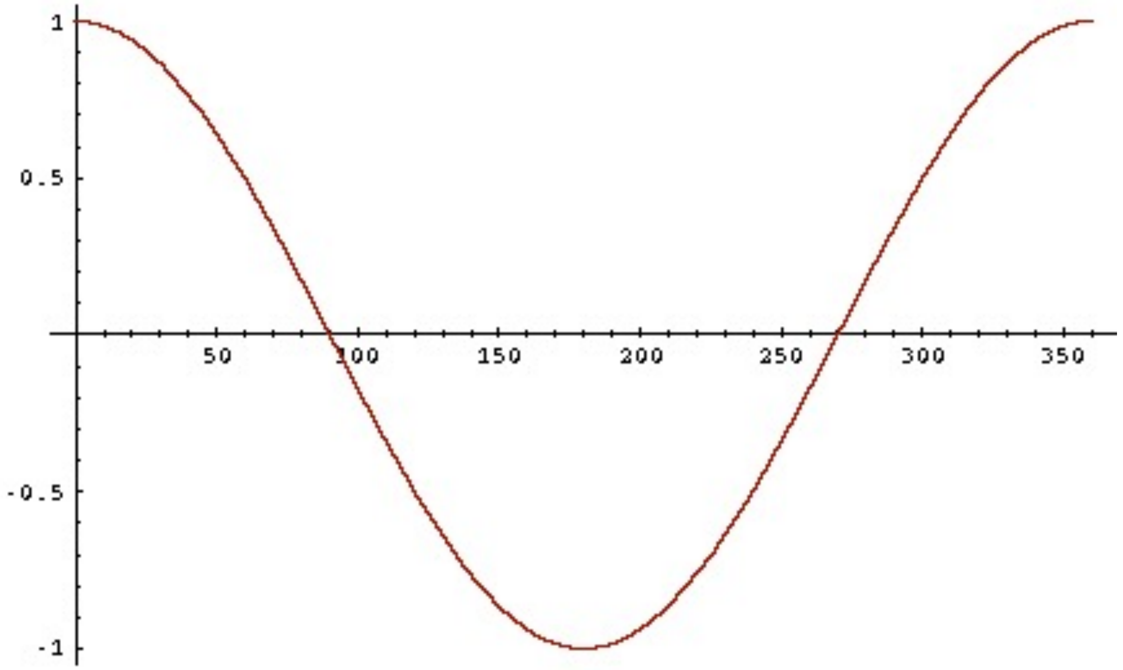
$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta \\ \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} &= \cos\theta\end{aligned}$$

Cosine as a similarity metric

-1: vectors point in opposite directions

+1: vectors point in same directions

0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0–1

Cosine examples

$$\cos(\vec{v},\vec{w}) = \frac{\vec{v} \bullet \vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|} \bullet \frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

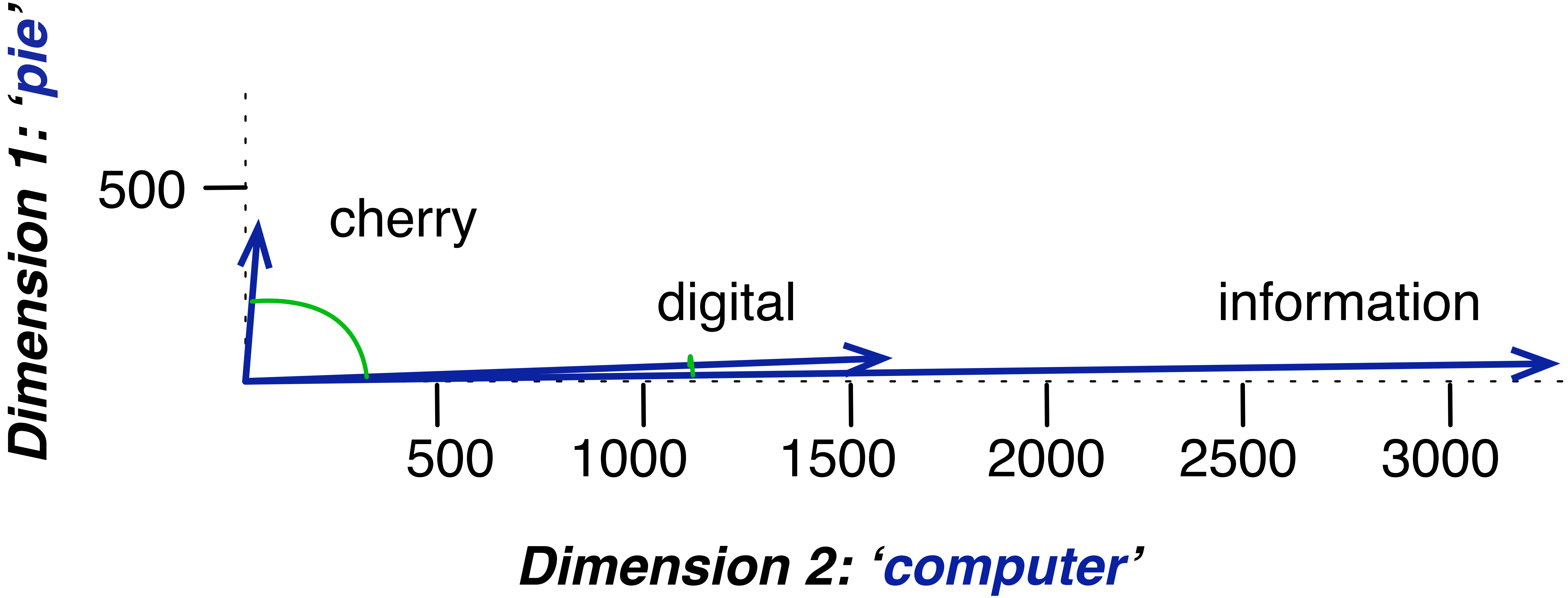
$$\cos(\text{cherry}, \text{information}) = $$

$$\frac{442 * 5 + 8 * 3982 + 2 * 3325}{\sqrt{442^2 + 8^2 + 2^2}\sqrt{5^2 + 3982^2 + 3325^2}} = .017$$

$$\cos(\text{digital}, \text{information}) = $$

$$\frac{5 * 5 + 1683 * 3982 + 1670 * 3325}{\sqrt{5^2 + 1683^2 + 1670^2}\sqrt{5^2 + 3982^2 + 3325^2}} = .996$$

Visualizing cosines (well, angles)



Vector Semantics & Embeddings

Cosine for computing word similarity

Vector Semantics & Embeddings

TF-IDF

But raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies.
- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it,* or *they* are not very informative about the context
- It's a paradox! How can we balance these two conflicting constraints?

Two common solutions for word weighting

tf-idf: tf-idf value for word t in document d:

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Words like "the" or "it" have very low idf

PMI: (Pointwise mutual information)

- PMI$(w_1, w_2) = log \frac{p(w_1,w_2)}{p(w_1)p(w_2)}$

See if words like "good" appear more often with "great" than we would expect by chance

Term frequency (tf)

$$\mathrm{tf}_{t,d} = \mathrm{count}(t,d)$$

Instead of using raw count, we squash a bit:

$$\mathrm{tf}_{t,d} = \log_{10}(\mathrm{count}(t,d)+1)$$

Document frequency (df)

df_t *is* the number of documents *t* occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	Collection Frequency	**Document Frequency**
Romeo	113	1
action	113	31

Inverse document frequency (idf)

$$\mathrm{idf}_t = \log_{10}\left(\frac{N}{\mathrm{df}_t}\right)$$

N is the total number of documents in the collection

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

What is a document?

Could be a play or a Wikipedia article

But for the purposes of tf-idf, documents can be **anything**; we often call each paragraph a document!

Final tf-idf weighted value for a word

$$w_{t,d} = \mathrm{tf}_{t,d} \times \mathrm{idf}_t$$

Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Vector Semantics & Embeddings

TF-IDF

Vector Semantics & Embeddings

PPMI

Pointwise Mutual Information

Pointwise mutual information:
Do events x and y co-occur more than if they were independent?

$$\mathrm{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

PMI between two words: (Church & Hanks 1989)
Do words x and y co-occur more than if they were independent?

$$\mathrm{PMI}(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

Positive Pointwise Mutual Information

- PMI ranges from $-\infty$ to $+\infty$
- But the negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure p(w1,w2) is significantly different than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (**PPMI**) between word1 and word2:

$$\text{PPMI}(word_1, word_2) = \max\left(\log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0\right)$$

Computing PPMI on a term-context matrix

Matrix F with W rows (words) and C columns (contexts)

f_{ij} is # of times w_{i} occurs in context c_{j}

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

p(w=information,c=data) = 3982/111716 = .3399

p(w=information) = 7703/11716 = .6575

p(c=data) = 5673/11716 = .4842

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

	p(w,context)					p(w)
	computer	**data**	**result**	**pie**	**sugar**	**p(w)**
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

	p(w,context)					p(w)
	computer	**data**	**result**	**pie**	**sugar**	**p(w)**
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

pmi(information,data) = $\log_2$ (.3399 / (.6575*.4842)) = .0944

Resulting PPMI matrix (negatives replaced by 0)

	computer	**data**	**result**	**pie**	**sugar**
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

Weighting PMI

PMI is biased toward infrequent events

- Very rare words have very high PMI values

Two solutions:

- Give rare words slightly higher probabilities
- Use add-one smoothing (which has a similar effect)

Weighting PMI: Giving rare context words slightly higher probability

Raise the context probabilities to $\alpha = 0.75$:

$$\text{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)}, 0)$$

$$P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

This helps because $P_\alpha(c) > P(c)$ for rare *c*

Consider two events, P(a) = .99 and P(b)=.01

$$P_\alpha(a) = \frac{.99^{.75}}{.99^{.75}+.01^{.75}} = .97 \quad P_\alpha(b) = \frac{.01^{.75}}{.01^{.75}+.01^{.75}} = .03$$