

Vector  
Semantics &  
Embeddings

Word Meaning

# What do words mean?

N-gram or text classification methods we've seen so far

- Words are just strings (or indices  $w_i$  in a vocabulary list)
- We can do some cleanliness and de-sparsifying, e.g. lemmatization
- ... but still, that's not very satisfactory!

Introductory logic classes:

- The meaning of "dog" is DOG; cat is CAT  
$$\forall x \text{ DOG}(x) \rightarrow \text{MAMMAL}(x)$$

Old linguistics joke by Barbara Partee in 1967:

- Q: What's the meaning of life?
- A: LIFE

That seems hardly better!

# Desiderata

What should a theory of word meaning do for us?

Let's look at some desiderata

From **lexical semantics**, the linguistic study of word meaning

# Lemmas and senses

lemma

mouse (N)

1. any of numerous small rodents...

2. a hand-operated device that controls  
a cursor...

sense

Modified from the online thesaurus WordNet

A **sense** or “**concept**” is the meaning component of a word  
Lemmas can be **polysemous** (have multiple senses)

# Relations between senses: Synonymy

Synonyms have the same meaning in some or all contexts.

- filbert / hazelnut
- couch / sofa
- big / large
- automobile / car
- vomit / throw up
- water / H<sub>2</sub>O

# Relations between senses: Synonymy

## Note!

There are probably no examples of perfect synonymy.

- Even if many aspects of meaning are identical
- Still may differ based on politeness, slang, register, genre, connotation, group identity signaling, etc.

# Relation: **Synonymy?**

water/H<sub>2</sub>O

"H<sub>2</sub>O" in a surfing guide?

big/large

my big sister != my large sister

# The Linguistic Principle of Contrast

Difference in form → difference in meaning

(See great paper by Clark 1987  
in relevant readings)



# Abbé Gabriel Girard 1718

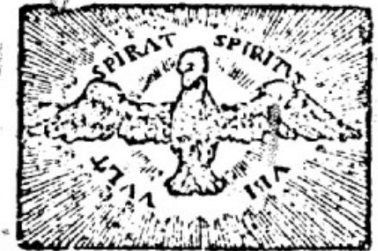
Re: "exact" synonyms

"je ne crois pas qu'il y ait de  
mot synonyme dans aucune  
Langue."

[I do not believe that there  
is a synonymous word in any  
language]

LA JUSTESSE  
DE LA  
LANGUE FRANÇOISE,  
OU  
LES DIFFERENTES SIGNIFICATIONS  
DES MOTS QUI PASSENT  
POUR  
SYNONIMES.

Par M. l'Abbé GIRARD C. D. M. D. D. B.



A PARIS,  
Chez LAURENT D'HOURY, Imprimeur-  
Libraire, au bas de la rue de la Harpe, vis-  
à vis la rue S. Severin, au Saint-Esprit.

M. DCC. XVIII.

Avec Approbation & Privilège du Roy.

# Relation: **Similarity**

Words with similar meanings.

Not synonyms, but sharing some element(s) of meaning.

car, bicycle

cow, horse

# Ask humans how similar 2 words are

word1	word2	similarity
vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

# Relation: Word relatedness

Also called "word association"

Words can be related in any way, perhaps via a semantic frame or field

- coffee, tea: **similar**
- coffee, cup: **related**, not similar

# Semantic field

Words that

- cover a particular semantic domain
- bear structured relations with each other.

## **hospitals**

*surgeon, scalpel, nurse, anaesthetic, hospital*

## **restaurants**

*waiter, menu, plate, food, menu, chef*

## **houses**

*door, roof, kitchen, family, bed*

# Relation: Antonymy

Senses that are opposites

... with respect to (usually) **only one** feature of meaning

Otherwise, they are very similar!

dark/light

short/long

fast/slow

rise/fall

hot/cold

up/down

in/out

# Relation: Antonymy

More formally: antonyms can

- define a non-scalar binary opposition
  - open/closed, mortal/immortal
- define opposite ends of a scale
  - long/short, fast/slow
- be *reversives*, denoting opposing processes:
  - rise/fall, up/down
- denote *relational* opposition relative to point of view:
  - parent/child, come/go

... and more! Can be challenging to handle computationally.

# Relation: Antonymy

Fun and relevant sidenote -  
have you ever noticed *auto-antonyms*?

Words for which two diff senses are antonyms, e.g.:

- fast (moving quickly / fixed in place)
- sanction (prohibition / permission)
- clip (attach / cut off)
- peruse (consider in detail / look over cursorily)
- dust (remove dust / add dust, like on a cake)

Why would these occur?



# Connotation (sentiment)

- Words have **affective** meanings
  - Positive connotations (*happy*)
  - Negative connotations (*sad*)
- Connotations can be subtle:
  - Positive connotation: *copy, replica, reproduction*
  - Negative connotation: *fake, knockoff, forgery*
- Evaluation (sentiment!)
  - Positive evaluation (*great, love*)
  - Negative evaluation (*terrible, hate*)

# Connotation

Osgood et al. (1957)

Words seem to vary along 3 affective dimensions:

- **valence**: the pleasantness of the stimulus
- **arousal**: the intensity of emotion provoked by the stimulus
- **dominance**: the degree of control exerted by the stimulus

	Word	Score		Word	Score
<b>Valence</b>	love	1.000		toxic	0.008
	happy	1.000		nightmare	0.005
<b>Arousal</b>	elated	0.960		mellow	0.069
	frenzy	0.965		napping	0.046
<b>Dominance</b>	powerful	0.991		weak	0.045
	leadership	0.983		empty	0.081

Values from NRC VAD Lexicon (Mohammad 2018)

# So far

## Concepts or word senses

- Have a complex many-to-many association with **words** (homonymy, multiple senses)

## Have multifaceted relations with each other

- Synonymy
- Antonymy
- Similarity
- Relatedness
- Connotation

Lexical semantics is fun and interesting and deep!

Keep on a lookout for fun  
examples in your life!

Here's one from my commute.



Vector  
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Embeddings

Word Meaning

# Vector Semantics & Embeddings

## Vector Semantics

# Computational models of word meaning

Can we build a theory of how to represent word meaning, that accounts for at least some of these desiderata?

We'll introduce **vector semantics**

The standard model in language processing!

Handles many of our goals!

# Ludwig Wittgenstein

## Philosophical Investigations #43:

"For a large class of cases—though not for all—in which we employ the word 'meaning' it can be defined thus: the meaning of a word is its use in the language"



# Let's define words by their usages

One way to define "usage":

words are defined by their environments  
(the words around them)

Zellig Harris (1954):

**If A and B have almost identical environments we say that they are synonyms.**

# What does recent English borrowing *ongchoi* mean?

Suppose you see these sentences:

- Ong choi is delicious **sautéed with garlic**.
- Ong choi is superb **over rice**
- Ong choi **leaves** with salty sauces

And you've also seen these:

- ...spinach **sautéed with garlic over rice**
- Chard stems and **leaves** are **delicious**
- Collard greens and other **salty** leafy greens

Conclusion:

- Ongchoi is a leafy green like spinach, chard, or collard greens
- We could conclude this based on words like "leaves" and "delicious" and "sauteed"

# Ongchoi: *Ipomoea aquatica* "Water Spinach"

空心菜

*kangkong*

rau muống

...



## Idea 1: Defining meaning by linguistic distribution

Let's define the meaning of a word by its distribution in language use, meaning its neighboring words or grammatical environments.

# Idea 2: Meaning as a point in space (Osgood et al. 1957)

## 3 affective dimensions for a word

- **valence**: pleasantness
- **arousal**: intensity of emotion
- **dominance**: the degree of control exerted

	Word	Score		Word	Score
<b>Valence</b>	love	1.000		toxic	0.008
	happy	1.000		nightmare	0.005
<b>Arousal</b>	elated	0.960		mellow	0.069
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	leadership	0.983		empty	0.081

NRC VAD Lexicon  
(Mohammad 2018)

Hence the connotation of a word is a vector in 3-space

Idea 1: Defining meaning by linguistic distribution

Idea 2: Meaning as a point in multidimensional space

# Defining meaning as a point in space based on distribution

Each word = a vector (not just "good" or " $w_{45}$ ")

Similar words are "**nearby in semantic space**"

We build this space automatically by seeing which words are **nearby in text**



# We define meaning of a word as a vector

Called an "embedding" because it's embedded into a space (see textbook)

The standard way to represent meaning in NLP

**Every modern NLP algorithm uses embeddings as the representation of word meaning**

Fine-grained model of meaning for similarity



# Intuition: why vectors?

Consider sentiment analysis:

- With **words**, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires **exact same word** to be in training and test
- With **embeddings**:
  - Feature is a word vector
  - 'The previous word was vector [35,22,17...]
  - Now in the test set we might see a similar vector [34,21,14]
  - We can generalize to **similar but unseen** words!!!

# We'll discuss 2 kinds of embeddings

## tf-idf

- Information Retrieval workhorse!
- A common baseline model
- **Sparse** vectors
- Words are represented by (a simple function of) the **counts** of nearby words

## Word2vec

- **Dense** vectors
- Representation is created by training a classifier to **predict** whether a word is likely to appear nearby
- Later we'll discuss extensions called **contextual embeddings**

# From now on: Computing with meaning representations instead of string representations

荃者所以在鱼，得鱼而忘荃    Nets are for fish;  
Once you get the fish, you can forget the net.  
言者所以在意，得意而忘言    Words are for meaning;  
Once you get the meaning, you can forget the words  
庄子(Zhuangzi), Chapter 26

# Vector Semantics & Embeddings

## Vector Semantics

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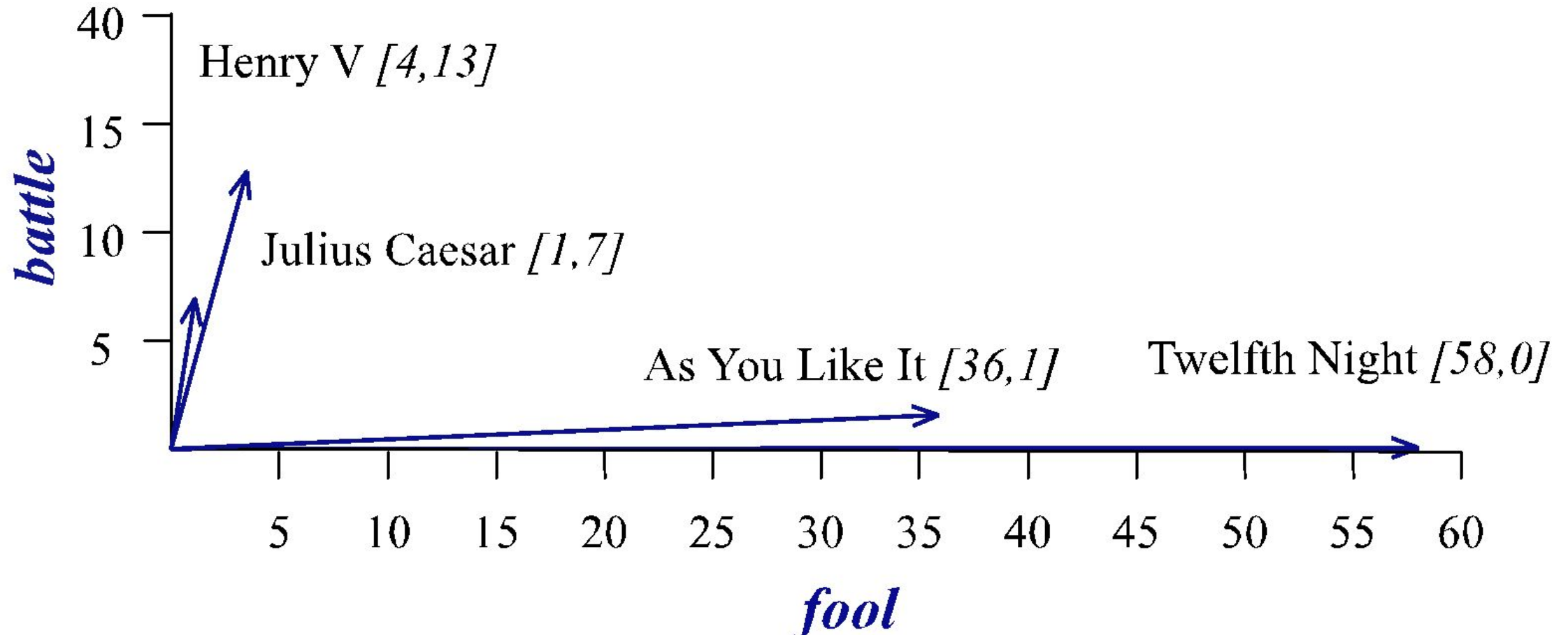
# Words and Vectors

# Term-document matrix

Each document is represented by a vector of words

	<b>As You Like It</b>	<b>Twelfth Night</b>	<b>Julius Caesar</b>	<b>Henry V</b>
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

# Visualizing document vectors



# Vectors are the basis of information retrieval

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

Vectors are similar for the two comedies

But comedies are different than the other two

Comedies have more *fools* and *wit* and fewer *battles*.



# Idea for word meaning: Words can be vectors too!!!

	<b>As You Like It</b>	<b>Twelfth Night</b>	<b>Julius Caesar</b>	<b>Henry V</b>
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

*battle* is "the kind of word that occurs in Julius Caesar and Henry V"

*fool* is "the kind of word that occurs in comedies, especially Twelfth Night"

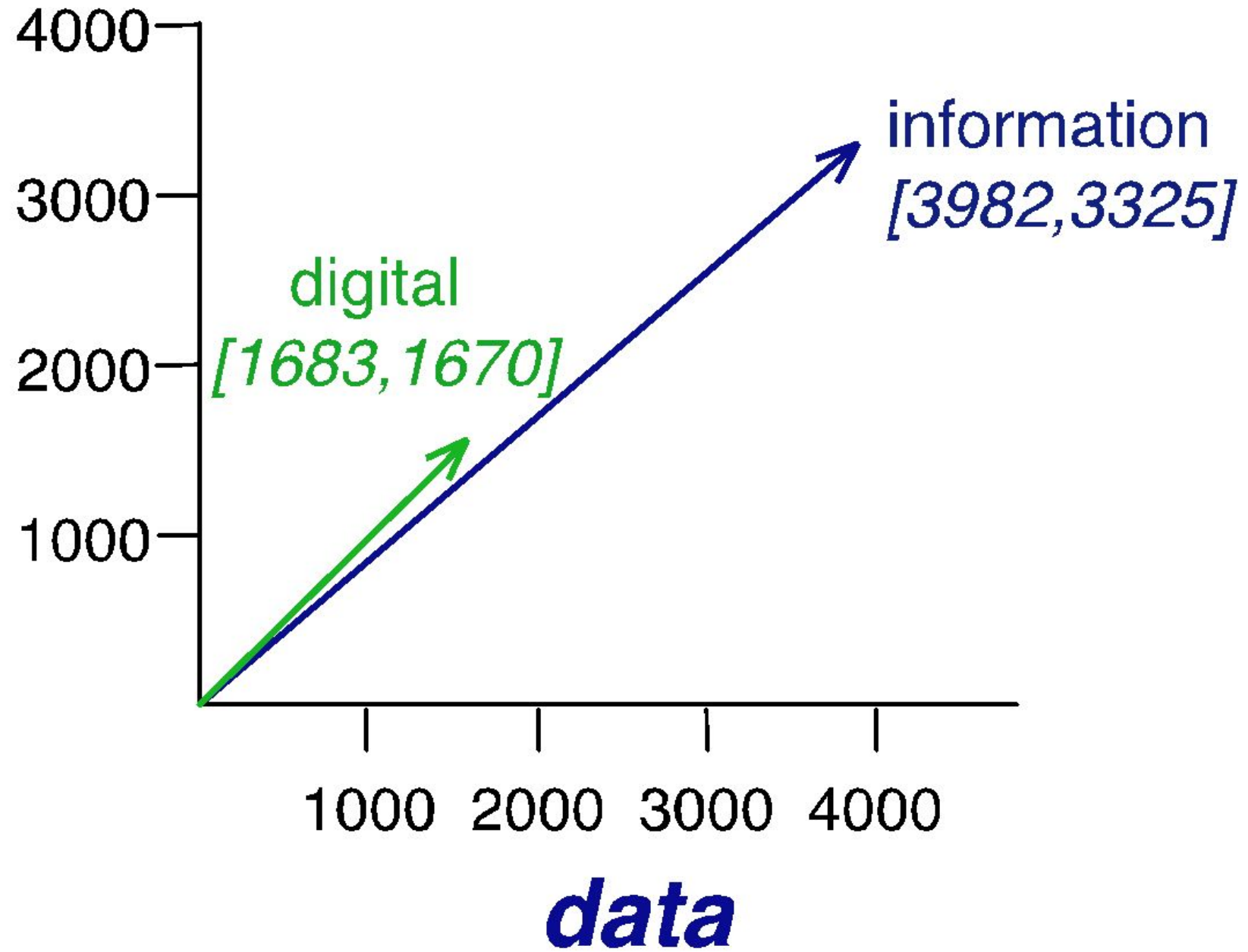
# More common: word-word matrix (or "term-context matrix")

Two **words** are similar in meaning if their context vectors are similar

is traditionally followed by **cherry** pie, a traditional dessert  
often mixed, such as **strawberry** rhubarb pie. Apple pie  
computer peripherals and personal **digital** assistants. These devices usually  
a computer. This includes **information** available on the internet

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...

***computer***



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# Words and Vectors

# Cosine for computing word similarity

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# Computing word similarity: Dot product and cosine

The dot product between two vectors is a scalar:

$$\text{dot product}(\mathbf{v}, \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^N v_i w_i = v_1 w_1 + v_2 w_2 + \dots + v_N w_N$$

The dot product tends to be high when the two vectors have large values in the same dimensions

Dot product can thus be a useful similarity metric between vectors

# Problem with raw dot-product

Dot product favors long vectors

Dot product is higher if a vector is longer (has higher values in many dimension)

Vector length:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^N v_i^2}$$

Frequent words (of, the, you) have long vectors (since they occur many times with other words).

So dot product overly favors frequent words

# Alternative: cosine for computing word similarity

$$\text{cosine}(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

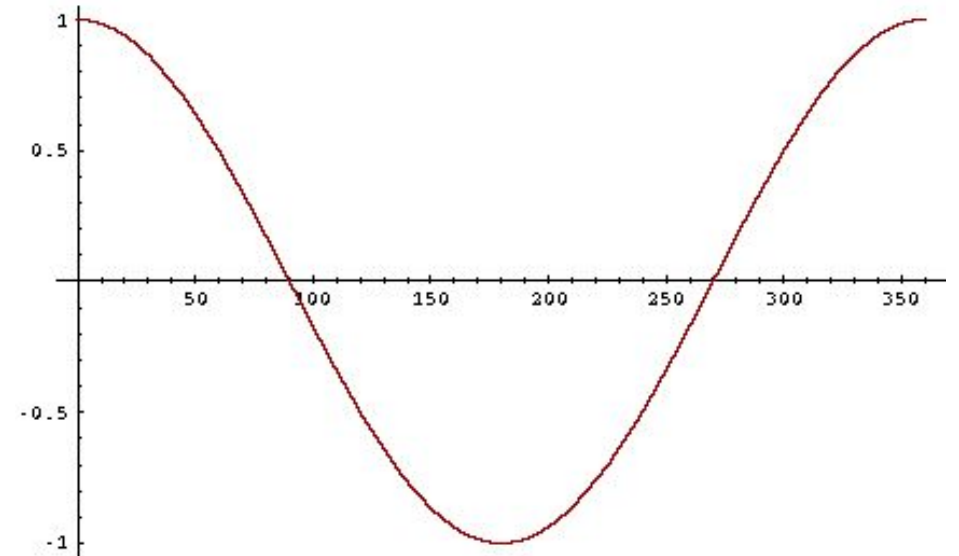
Based on the definition of the dot product between two vectors  $\mathbf{a}$  and  $\mathbf{b}$

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}| |\mathbf{b}| \cos \theta \\ \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} &= \cos \theta \end{aligned}$$



# Cosine as a similarity metric

- 1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0–1

# Cosine examples

$$\text{cosine}(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

$$\text{cos}(\text{cherry}, \text{information}) =$$

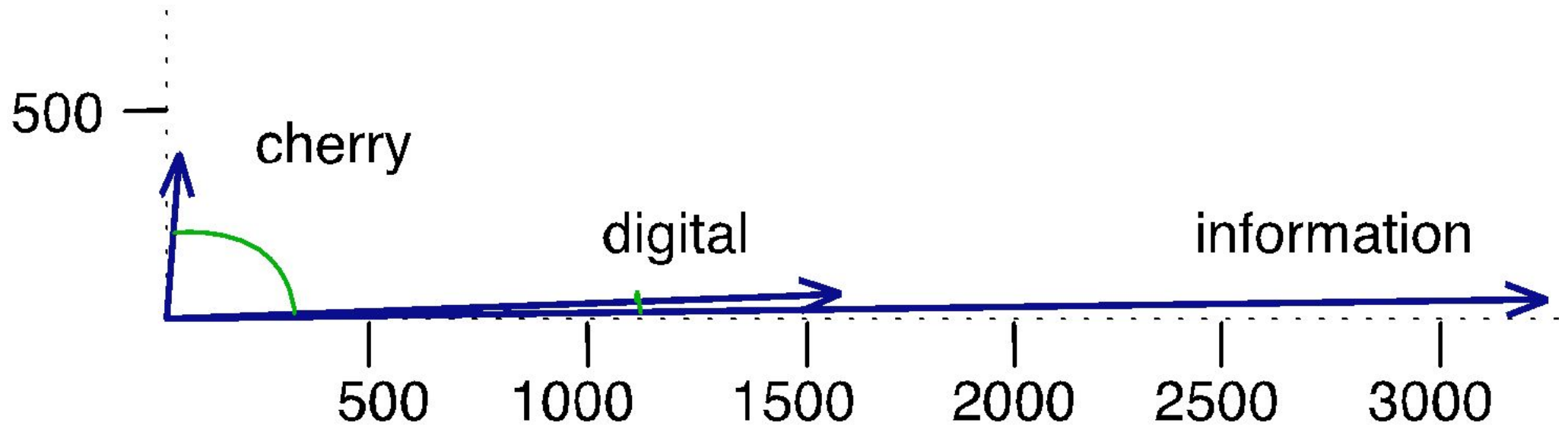
$$\frac{442 * 5 + 8 * 3982 + 2 * 3325}{\sqrt{442^2 + 8^2 + 2^2} \sqrt{5^2 + 3982^2 + 3325^2}} = .017$$

$$\text{cos}(\text{digital}, \text{information}) =$$

$$\frac{5 * 5 + 1683 * 3982 + 1670 * 3325}{\sqrt{5^2 + 1683^2 + 1670^2} \sqrt{5^2 + 3982^2 + 3325^2}} = .996$$

# Visualizing cosines (well, angles)

*Dimension 1: 'pie'*



*Dimension 2: 'computer'*

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Cosine for computing word  
similarity

Vector  
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## TF-IDF and PPMI

# But raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies.
- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it*, or *they* are not very informative about the context
- It's a paradox! How can we balance these two conflicting constraints?

# Two common solutions for word weighting

**tf-idf:** tf-idf value for word  $t$  in document  $d$ :

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Words like "the" or "it" have very low idf

**PMI:** (Pointwise mutual information)

- $\text{PMI}(w_1, w_2) = \log \frac{p(w_1, w_2)}{p(w_1)p(w_2)}$

See if words like "good" appear more often with "great" than we would expect by chance

# tf-idf - Term frequency (tf)

$$tf_{t,d} = \text{count}(t,d)$$

Instead of using raw count, we squash a bit:

$$tf_{t,d} = \log_{10}(\text{count}(t,d)+1)$$



# tf-idf - Document frequency (df)

$df_t$  is the number of documents  $t$  occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	<b>Collection Frequency</b>	<b>Document Frequency</b>
Romeo	113	1
action	113	31

# tf-idf - Inverse document frequency (idf)

$$\text{idf}_t = \log_{10} \left( \frac{N}{\text{df}_t} \right)$$

N is the total number of documents in the collection

<b>Word</b>	<b>df</b>	<b>idf</b>
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

# What is a document?

Could be a play or a Wikipedia article

But for the purposes of tf-idf, documents can be **anything**; we often call each paragraph a document!

# Final tf-idf weighted value for a word

Raw counts:  $w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	1	0	7	13
<b>good</b>	114	80	62	89
<b>fool</b>	36	58	1	4
<b>wit</b>	20	15	2	3

tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
<b>battle</b>	0.074	0	0.22	0.28
<b>good</b>	0	0	0	0
<b>fool</b>	0.019	0.021	0.0036	0.0083
<b>wit</b>	0.049	0.044	0.018	0.022

# Pointwise Mutual Information

Intuition from probability/information theory, quantifies: do events  $x$  and  $y$  co-occur more than we would expect if they were independent?

$$\text{PMI}(X, Y) = \log_2 \frac{P(x, y)}{P(x)P(y)}$$

**PMI between two words:** (Church and Hanks 1989)

Do two words co-occur with one another more than if they were independent?

$$\text{PMI}(\text{word}_1, \text{word}_2) = \log_2 \frac{P(\text{word}_1, \text{word}_2)}{P(\text{word}_1)P(\text{word}_2)}$$

# Positive Pointwise Mutual Information

- PMI ranges from  $-\infty$  to  $+\infty$
- But the negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine  $w_1$  and  $w_2$  whose probability is each  $10^{-6}$
    - Hard to be sure  $p(w_1, w_2)$  is significantly different than  $10^{-12}$
    - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (**PPMI**) between word1 and word2:

$$\text{PPMI}(word_1, word_2) = \max\left(\log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0\right)$$

# Computing PPMI on a term-context matrix

Matrix  $F$  with  $W$  rows (words) and  $C$  columns (contexts)

$f_{ij}$  is # of times  $w_i$  occurs in context  $c_j$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad p_{i*} = \frac{\sum_{j=1}^C f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}} \quad p_{*j} = \frac{\sum_{i=1}^W f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*} p_{*j}} \quad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$P_{ij} = \frac{f_{ij}}{\sum_{i=1}^W \sum_{j=1}^C f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

$$p(w=\text{information}, c=\text{data}) = 3982/11716 = .3399$$

$$p(w=\text{information}) = 7703/11716 = .6575$$

$$p(c=\text{data}) = 5673/11716 = .4842$$

$$p(w_i) = \frac{\sum_{j=1}^C f_{ij}}{N} \quad p(c_j) = \frac{\sum_{i=1}^W f_{ij}}{N}$$

	p(w,context)					p(w)
	computer	data	result	pie	sugar	p(w)
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	



$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_i * p_j}$$

	p(w,context)					p(w)
	computer	data	result	pie	sugar	p(w)
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

$$pmi(\text{information}, \text{data}) = \log_2 (.3399 / (.6575 * .4842)) = .0944$$

Resulting PPMI matrix (negatives replaced by 0)

	computer	data	result	pie	sugar
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

# Weighting PMI

PMI is biased toward infrequent events

- Very rare words have very high PMI values

Two solutions:

- Give rare words slightly higher probabilities
- Use add-one smoothing (which has a similar effect)

# Weighting PMI: Giving rare context words slightly higher probability

Raise the context probabilities to  $\alpha = 0.75$ :

$$\text{PPMI}_\alpha(w, c) = \max\left(\log_2 \frac{P(w, c)}{P(w)P_\alpha(c)}, 0\right)$$

$$P_\alpha(c) = \frac{\text{count}(c)^\alpha}{\sum_c \text{count}(c)^\alpha}$$

This helps because  $P_\alpha(c) > P(c)$  for rare  $c$

Consider two events,  $P(a) = .99$  and  $P(b) = .01$

$$P_\alpha(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97 \quad P_\alpha(b) = \frac{.01^{.75}}{.01^{.75} + .01^{.75}} = .03$$