Vector Semantics & Embeddings

Word2vec

Sparse versus dense vectors

tf-idf (or PPMI) vectors are

- **long** (length |V|= 20,000 to 50,000)
- **sparse** (most elements are zero)

Alternative: learn vectors which are

- **short** (length 50-1000)
- **dense** (most elements are non-zero)

Sparse versus dense vectors

Why dense vectors?

- Short vectors may be easier to use as **features** in machine learning (fewer weights to tune)
- Dense vectors may **generalize** better than explicit counts
- Dense vectors may do better at capturing synonymy:
  - *car* and *automobile* are synonyms; but are distinct dimensions
    - a word with *car* as a neighbor and a word with *automobile* as a neighbor should be similar, but aren't
  - **In practice, they work better**

Common methods for getting short dense vectors

"Neural Language Model"-inspired models
- Word2vec (skipgram, CBOW), GloVe

Singular Value Decomposition (SVD)
- A special case of this is called LSA – Latent Semantic Analysis

Alternative to these "static embeddings":
- Contextual Embeddings (ELMo, BERT)
- Compute distinct embeddings for a word in its context
- Separate embeddings for each token of a word

Simple static embeddings you can download!

Word2vec (Mikolov et al)

https://code.google.com/archive/p/word2vec/

GloVe (Pennington, Socher, Manning)

http://nlp.stanford.edu/projects/glove/

Word2vec

Popular embedding method

Very fast to train

Code available on the web

Idea: **predict** rather than **count**

Word2vec provides various options. We'll do:

skip-gram with negative sampling (SGNS)

Word2vec

Instead of **counting** how often each word *w* occurs near "*apricot*"

- Train a classifier on a binary **prediction** task:
  - Is *w* likely to show up near "*apricot*"?

We don't actually care about this task

- But we'll take the learned classifier weights as the word embeddings

Big idea: **self-supervision**:

- A word c that occurs near apricot in the corpus cats as the gold "correct answer" for supervised learning
- No need for human labels
- Bengio et al. (2003); Collobert et al. (2011)

Approach: predict if candidate word *c* is a "neighbor"

1. Treat the target word *t* and a neighboring context word *c* as **positive examples**.
2. Randomly sample other words in the lexicon to get negative examples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the learned weights as the embeddings

Skip-Gram Training Data

Assume a +/- 2 word window, given training sentence:

…lemon, a [tablespoon of apricot jam, a] pinch…
c1 c2 [target] c3 c4

Skip-Gram Classifier

(assuming a +/- 2 word window)

…lemon, a [tablespoon of apricot jam, a] pinch…
c1 c2 [target] c3 c4

Goal: train a classifier that is given a candidate (**w**ord, **c**ontext) pair
(apricot, jam)
(apricot, aardvark)
…
And assigns each pair a probability:
$P(+|w, c)$
$P(-|w, c) = 1 - P(+|w, c)$

Similarity is computed from dot product

Remember: two vectors are similar if they have a high dot product

- Cosine is just a normalized dot product

So:

- Similarity(w,c) $\propto w \cdot c$

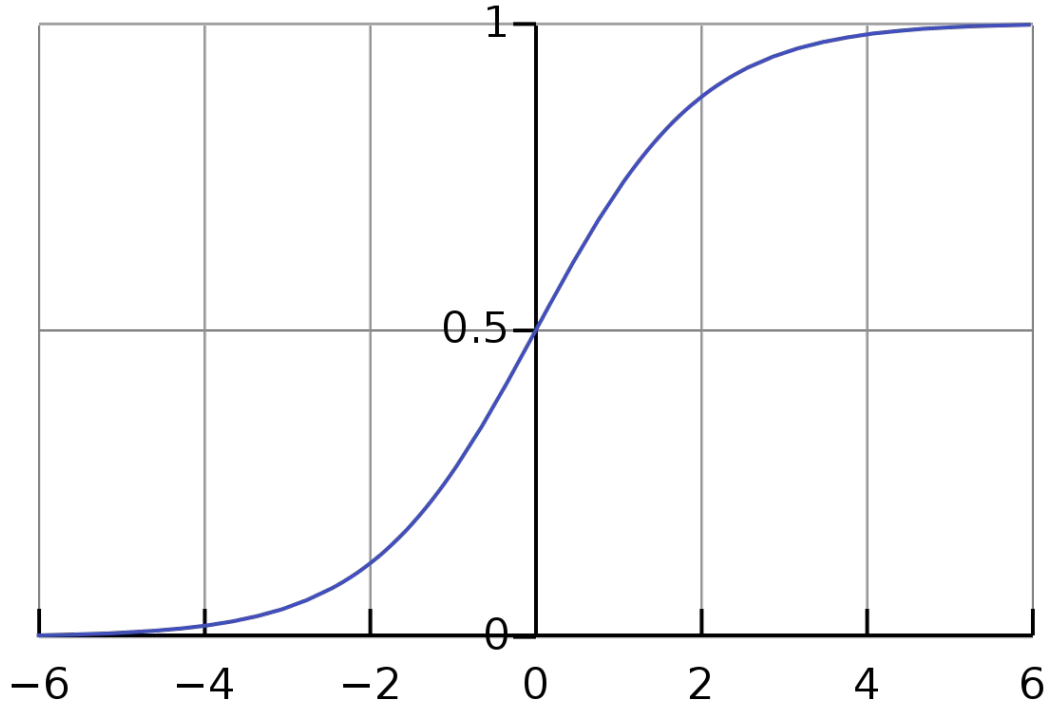
We'll need to normalize to get a probability

- (cosine isn't a probability either)

Turning dot products into probabilities

$\text{Sim}(w,c) \approx w \cdot c$ …to turn this into a probability we'll use the very useful sigmoid function:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



How Skip-Gram Classifier computes $P(+|w, c)$

$$P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

This is for one context word, but we have lots of context words.
We'll assume independence and just multiply them:

$$P(+|w,c_{1:L}) = \prod_{i=1}^{L} \sigma(c_i \cdot w)$$

$$\log P(+|w,c_{1:L}) = \sum_{i=1}^{L} \log \sigma(c_i \cdot w)$$

Skip-gram classifier: summary

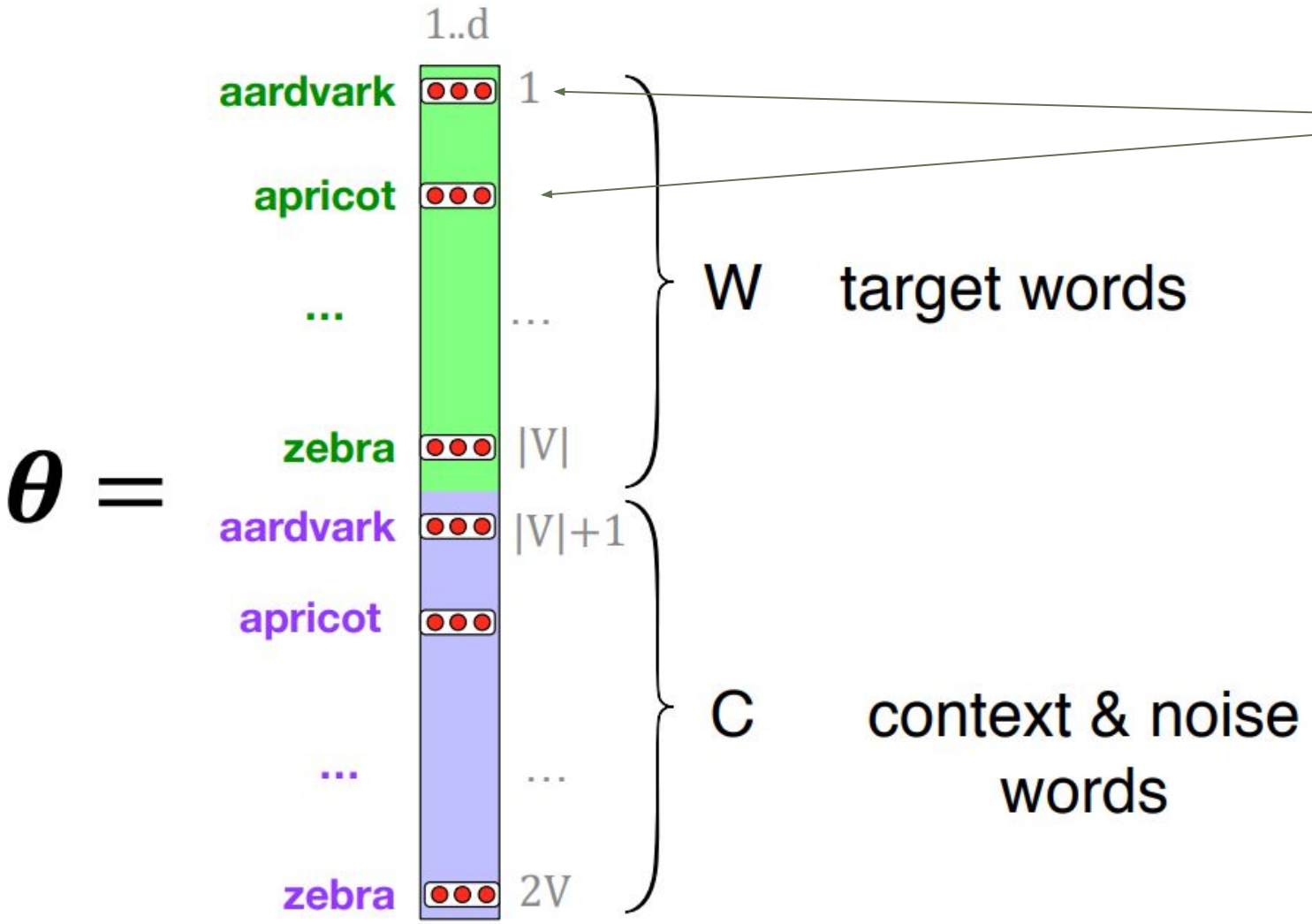
A probabilistic classifier, given

- a test target word *w*
- its context window of *L* words $c_{1:L}$

Estimates probability that w occurs in this window based on similarity of w (embeddings) to $c_{1:L}$ (embeddings).

To compute this, we just need embeddings for all the words.

These embeddings we'll need: a set for w, a set for c



Each of these embeddings is a dense "word vector"

Each composed of *d* dimensions of numbers which we hope to make (very) abstractly represent its meaning / semantics

Vector Semantics & Embeddings

Word2vec

Vector Semantics & Embeddings

Word2vec: Learning the embeddings

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

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positive examples +

t	c
apricot	tablespoon
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For each positive example we'll grab k negative examples, sampling by frequency

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

t	c	t	c
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

Word2vec: how to learn vectors

Given the set of positive and negative training instances, and an initial set of embedding vectors

The goal of learning is to adjust those word vectors such that we:

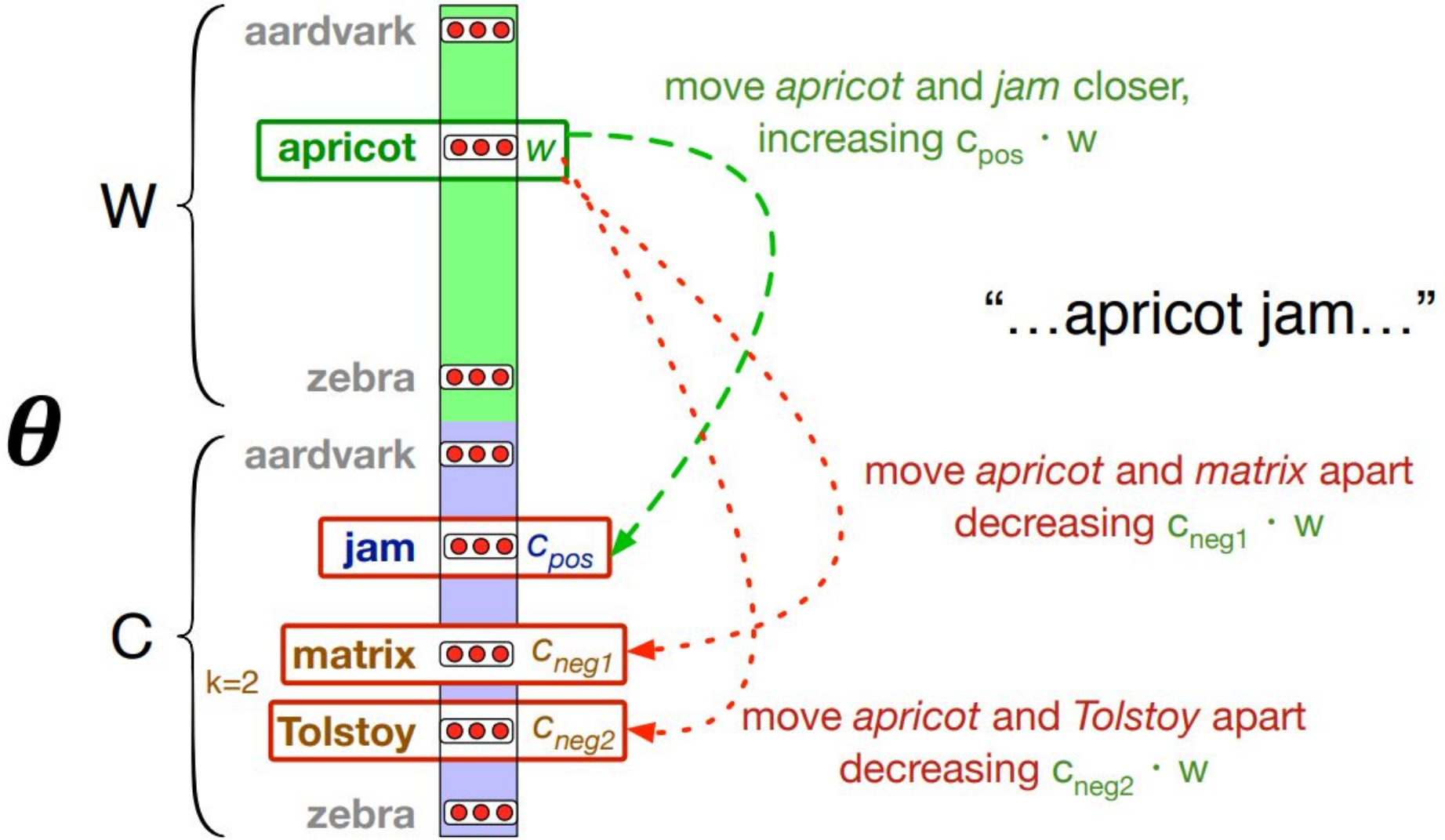
- **Maximize** the similarity of the target word, context word pairs (w , c_{pos}) drawn from the positive data
- **Minimize** the similarity of the (w , c_{neg}) pairs drawn from the negative data.

Learning the classifier

Learning is the process by which we incrementally adjust the word weights to

- make the positive pairs more likely,
- and the negative pairs less likely,
- over the entire training set.

Intuition of one step of learning



This reminds of the Perceptron

… but a bit trickier.

There we only did updates when incorrect

Here, as a raw prediction task, we will almost always "guess wrong" (but we don't mind)

What we actually want is to incrementally raise the probabilities of the "right" answers

Another problem: sigmoid gives a probabilistic interpretation, but introduces a non-linearity

Two New Learning Components

A loss function:

- **cross-entropy loss**

An optimization algorithm:

- **stochastic gradient descent**

Loss functions

We know the true label **y** - in the case of word2vec, e.g.:

positive example, y = 1 (apricot, jam)

negative example, y = 0 (apricot, matrix)

Given some setting of weights / embeddings **w** and **c**,
we can calculate **ŷ** - the probability of the positive class:

Maximize: $P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$

We want one equation to quantify "how wrong we were" -

the loss function **L(ŷ, y)**

Cross-entropy Loss

Defined as the negative log-likelihood of the probability we want to maximize

Notice $\hat{\mathbf{y}}$ must incorporate the negative examples as well, have to think about "maximizing" their probability, therefore need to define by flipping positive class:

$$P(-|w,c) = 1 - P(+|w,c)$$

$$= \sigma(-c \cdot w) = \frac{1}{1+\exp(c \cdot w)}$$

Loss function for one *w with* c_{pos}, c_{neg1} …c_{negk}

Maximize the similarity of the target with the actual context words, and minimize the similarity of the target with the *k* negative sampled non-neighbor words.

$$L_{CE} = -\log\left[P(+|w,c_{pos})\prod_{i=1}^{k}P(-|w,c_{neg_i})\right]$$

This total quantity is relatively smaller if our probabilities are closer to 1 for **+** and 0 for **-**, bigger if they're further away

$$= -\left[\log P(+|w,c_{pos}) + \sum_{i=1}^{k}\log P(-|w,c_{neg_i})\right]$$

$$= -\left[\log P(+|w,c_{pos}) + \sum_{i=1}^{k}\log\left(1-P(+|w,c_{neg_i})\right)\right]$$

$$= -\left[\log \sigma(c_{pos}\cdot w) + \sum_{i=1}^{k}\log\sigma(-c_{neg_i}\cdot w)\right]$$

Our goal: minimize the loss

Let's make explicit that the loss function is parameterized by weights θ=(w,b)

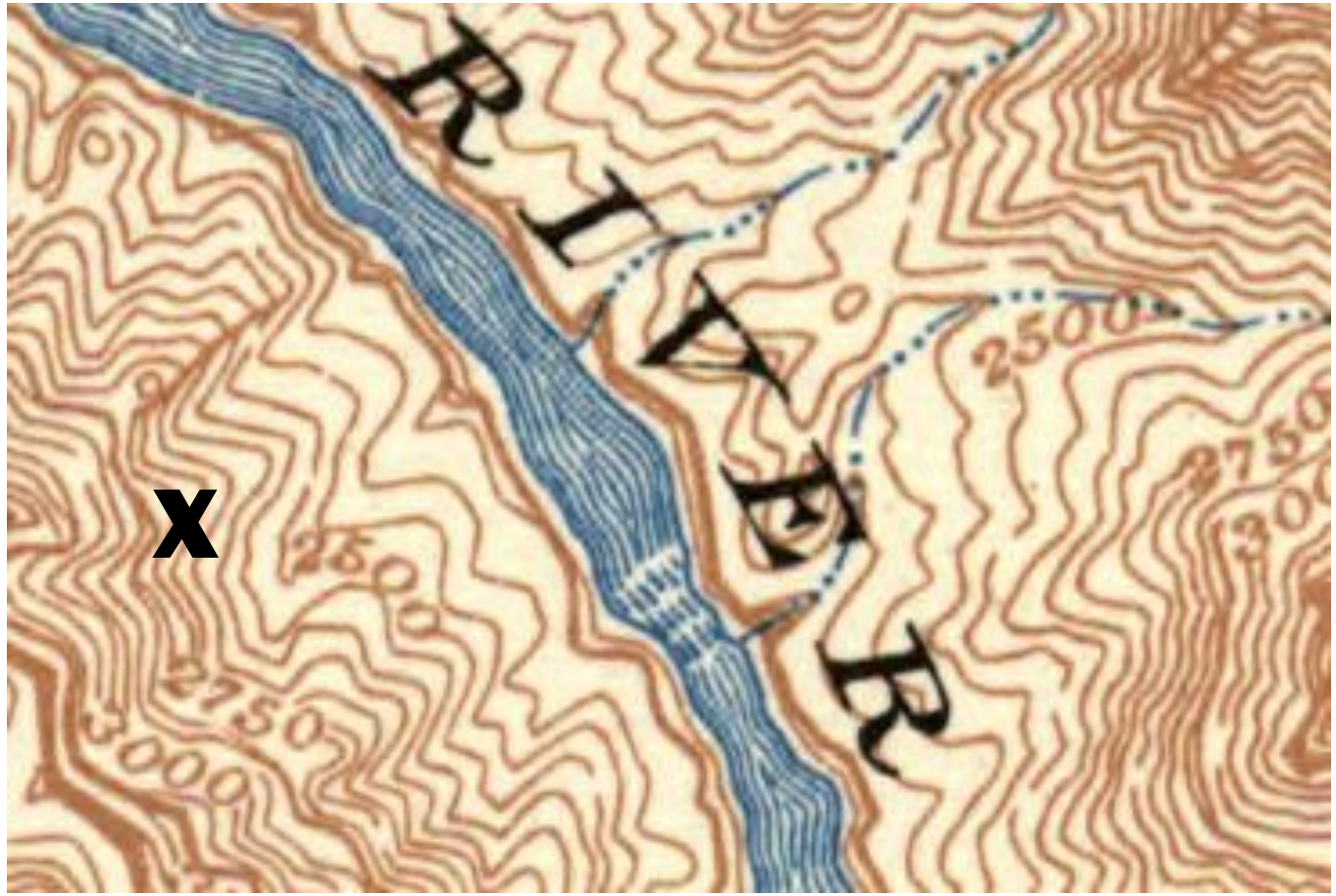
- And we'll represent $\hat{y}$ as $f(x; \theta)$ to make the dependence on θ more obvious

We want the weights that minimize the loss, averaged over all examples:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} L_{\text{CE}}(f(x^{(i)}; \theta), y^{(i)})$$

Stochastic Gradient Descent: Intuition

How do I get to the bottom of this river canyon?



Look around me 360°

Find the direction of steepest slope down

Go that way

Our goal: minimize the loss

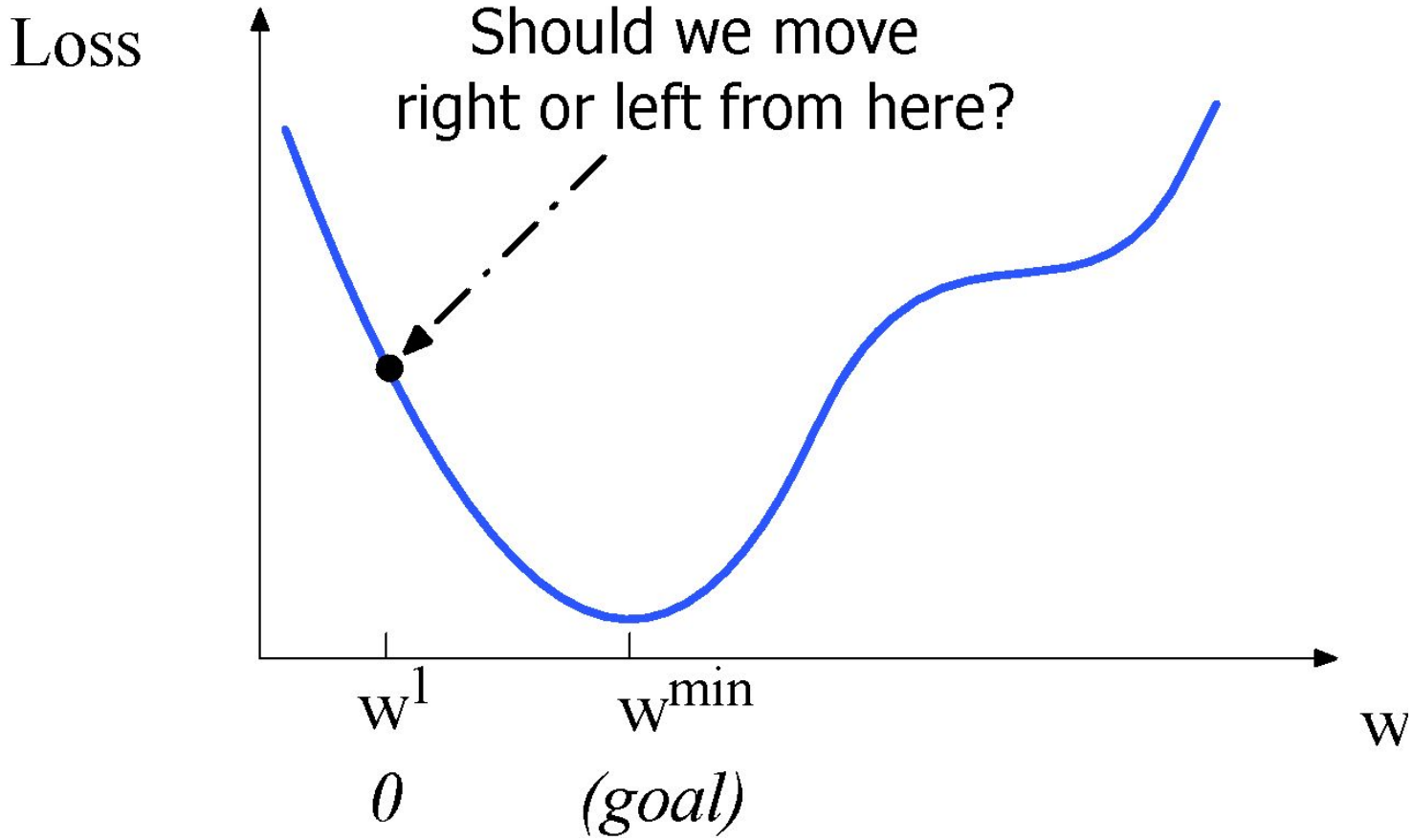
For logistic regression, loss function is **convex**

- A convex function has just one minimum
- Gradient descent starting from any point is guaranteed to find the minimum
  - (Loss for neural networks is non-convex)

Let's first visualize for a single scalar w

Q: Given current w, should we make it bigger or smaller?

A: Move *w* in the reverse direction from the slope of the function



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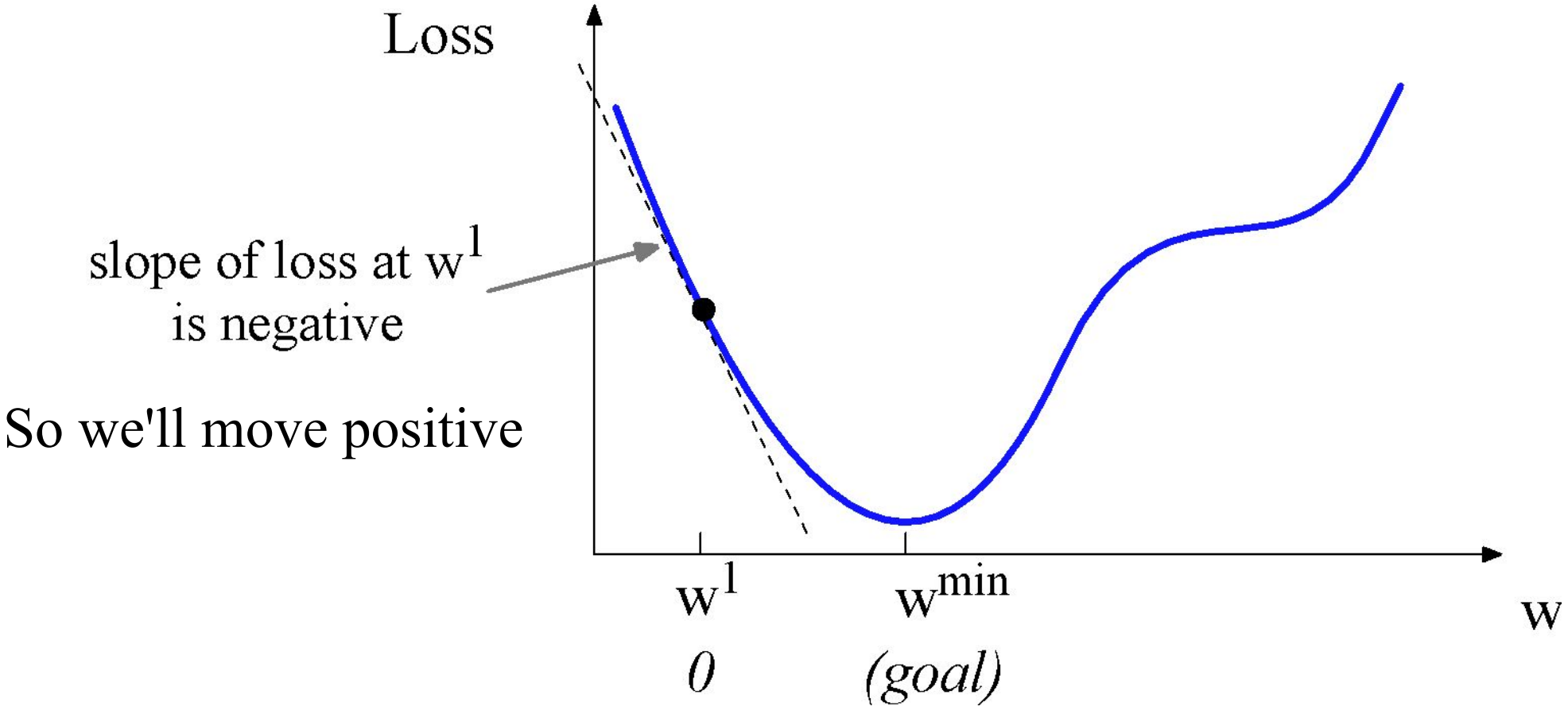

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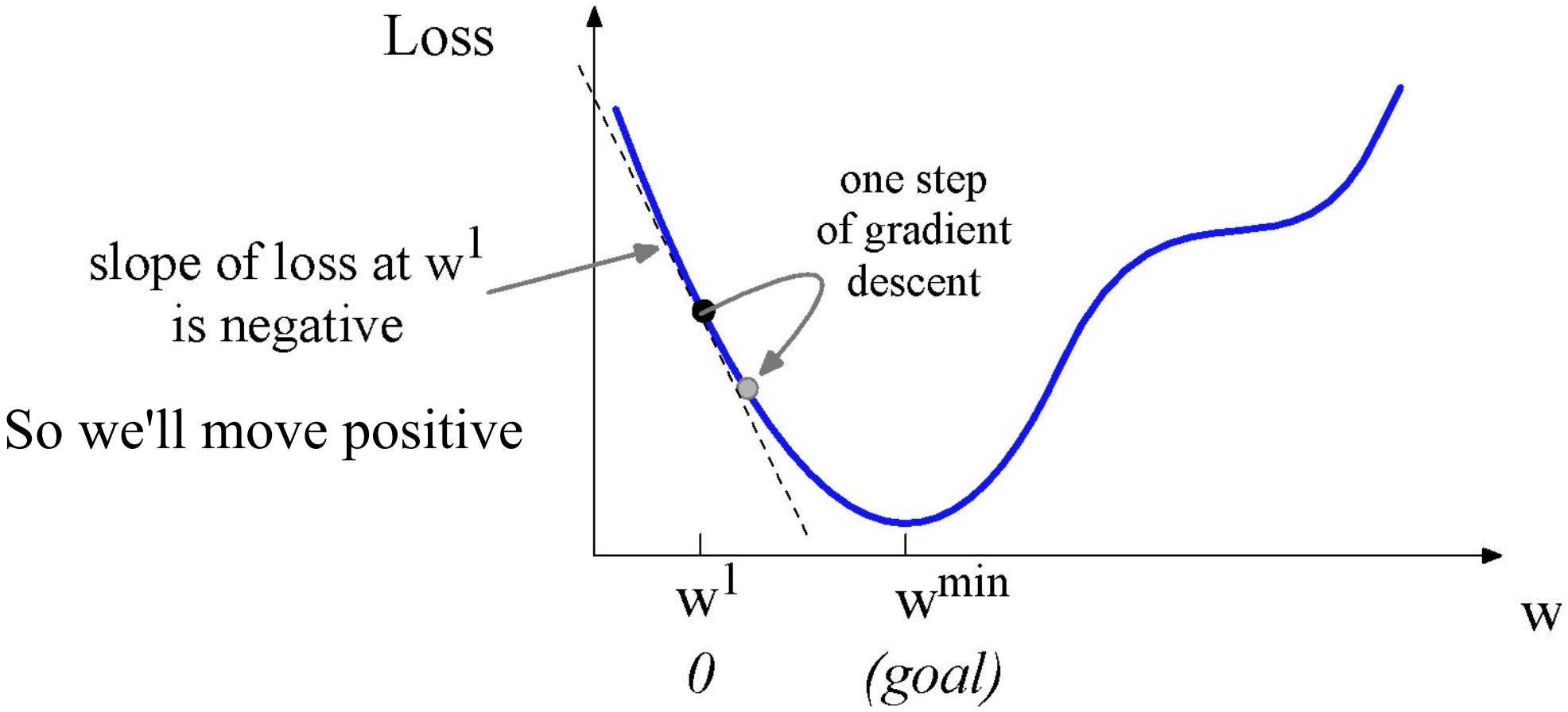


Gradients

The **gradient** of a function of many variables is a vector pointing in the direction of the greatest increase in a function.

Gradient Descent: Find the gradient of the loss function at the current point and move in the **opposite** direction.

How much do we move in that direction ?

- The value of the gradient (slope in our example) $\frac{d}{dw} L(f(x;w),y)$ weighted by a **learning rate** η
- Higher learning rate means move *w* faster

$$w^{t+1} = w^t - \eta \frac{d}{dw} L(f(x;w),y)$$

Now let's consider N dimensions

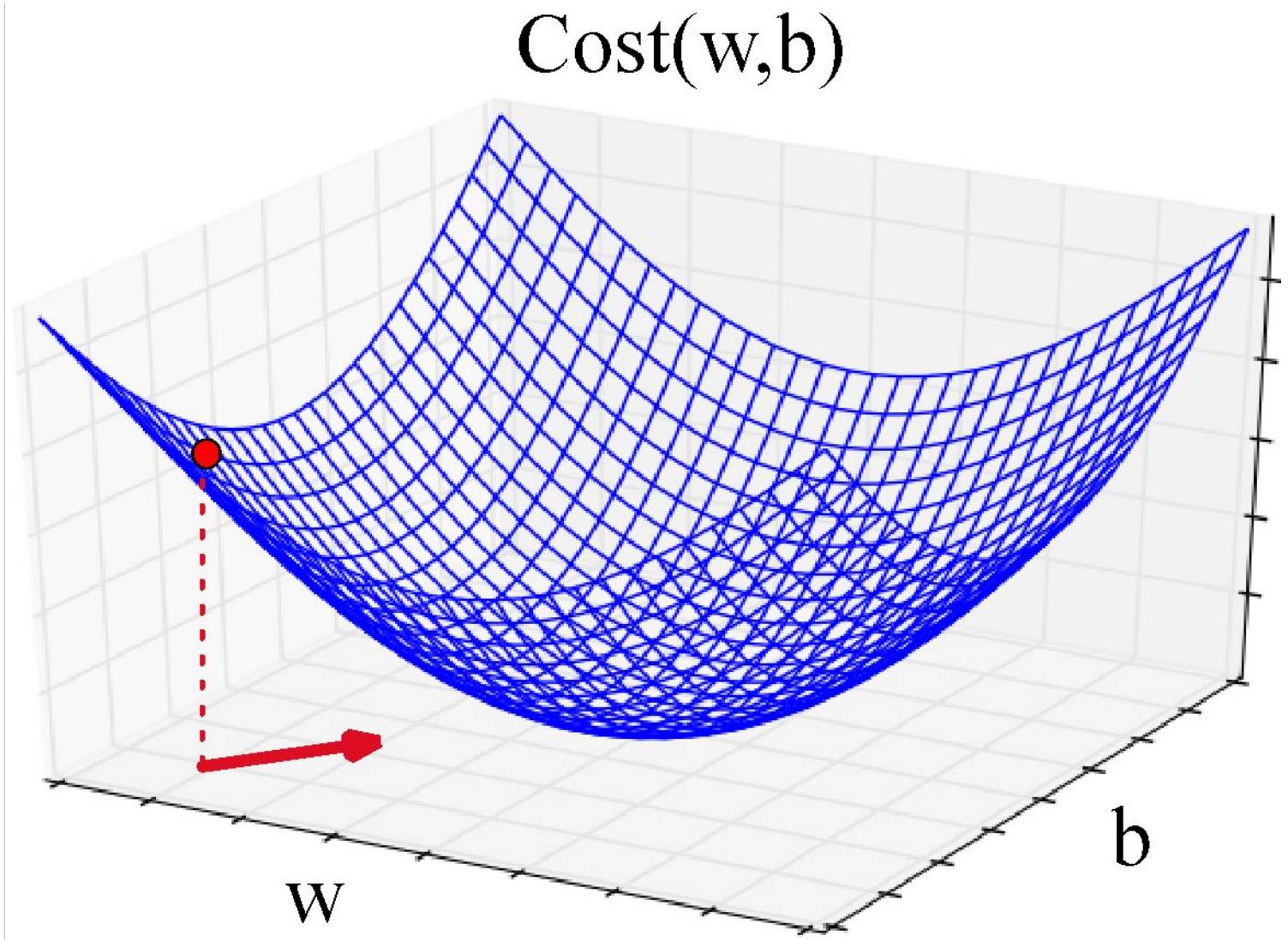
We want to know where in the N-dimensional space (of the *N* parameters that make up θ) we should move.

The gradient is just such a vector; it expresses the directional components of the sharpest slope along each of the *N* dimensions.

Imagine 2 dimensions, w and b

Visualizing the gradient vector at the red point

It has two dimensions shown in the x-y plane



Real gradients

Are much longer; lots and lots of weights

For each dimension w_i the gradient component *i* tells us the slope with respect to that variable.

- "How much would a small change in w_i influence the total loss function *L*?"
- We express the slope as a partial derivative ∂ of the loss ∂w_i

The gradient is then defined as a vector of these partials.

same dimensionality as the original weights

Hyperparameters

The learning rate η is a **hyperparameter**

- too high: the learner will take big steps and overshoot
- too low: the learner will take too long

Hyperparameters:

- A special kind of parameter for an ML model
- Instead of being learned by algorithm from supervision (like regular parameters), they are chosen by algorithm designer.

The derivatives of the loss function

$$L_{CE} = -\left[\log \sigma(c_{pos} \cdot w) + \sum_{i=1}^{k} \log \sigma(-c_{neg_i} \cdot w)\right]$$

$$\frac{\partial L_{CE}}{\partial c_{pos}} = [\sigma(c_{pos} \cdot w) - 1]w$$

$$\frac{\partial L_{CE}}{\partial c_{neg}} = [\sigma(c_{neg} \cdot w)]w$$

$$\frac{\partial L_{CE}}{\partial w} = [\sigma(c_{pos} \cdot w) - 1]c_{pos} + \sum_{i=1}^{k} [\sigma(c_{neg_i} \cdot w)]c_{neg_i}$$

Update equation in SGD

Start with randomly initialized C and W matrices, then incrementally do updates

$$
\begin{aligned}
c_{pos}^{t+1} &= c_{pos}^{t} - \eta\left[\sigma\left(c_{pos}^{t} \cdot w^{t}\right) - 1\right] w^{t} \\
c_{neg}^{t+1} &= c_{neg}^{t} - \eta\left[\sigma\left(c_{neg}^{t} \cdot w^{t}\right)\right] w^{t} \\
w^{t+1} &= w^{t} - \eta\left[\left[\sigma\left(c_{pos} \cdot w^{t}\right) - 1\right] c_{pos} + \sum_{i=1}^{k}\left[\sigma\left(c_{\mathrm{neg}_i} \cdot w^{t}\right)\right] c_{neg_i}\right]
\end{aligned}
$$

Two sets of embeddings

SGNS learns two sets of embeddings

- Target embeddings matrix W
- Context embedding matrix C

It's common to just add them together, representing word *i* as the vector $w_i + c_i$

Summary: How to learn word2vec (skip-gram) embeddings

Start with V random d-dimensional vectors as initial embeddings

Train a classifier based on embedding similarity

- Take a corpus and take pairs of words that co-occur as positive examples
- Take pairs of words that don't co-occur as negative examples
- Train the classifier to distinguish these by slowly adjusting all the embeddings to improve the classifier performance
- Throw away the classifier code and keep the embeddings.

Vector Semantics & Embeddings

Word2vec: Learning the embeddings

Vector Semantics & Embeddings

Useful and Interesting Properties of Embeddings

The kinds of neighbors depend on window size

Small windows (C= +/- 2) : nearest words are syntactically similar words in same taxonomy

- *Hogwarts* nearest neighbors are other fictional schools
  - *Sunnydale, Evernight, Blandings*

Large windows (C= +/- 5) : nearest words are related words in same semantic field

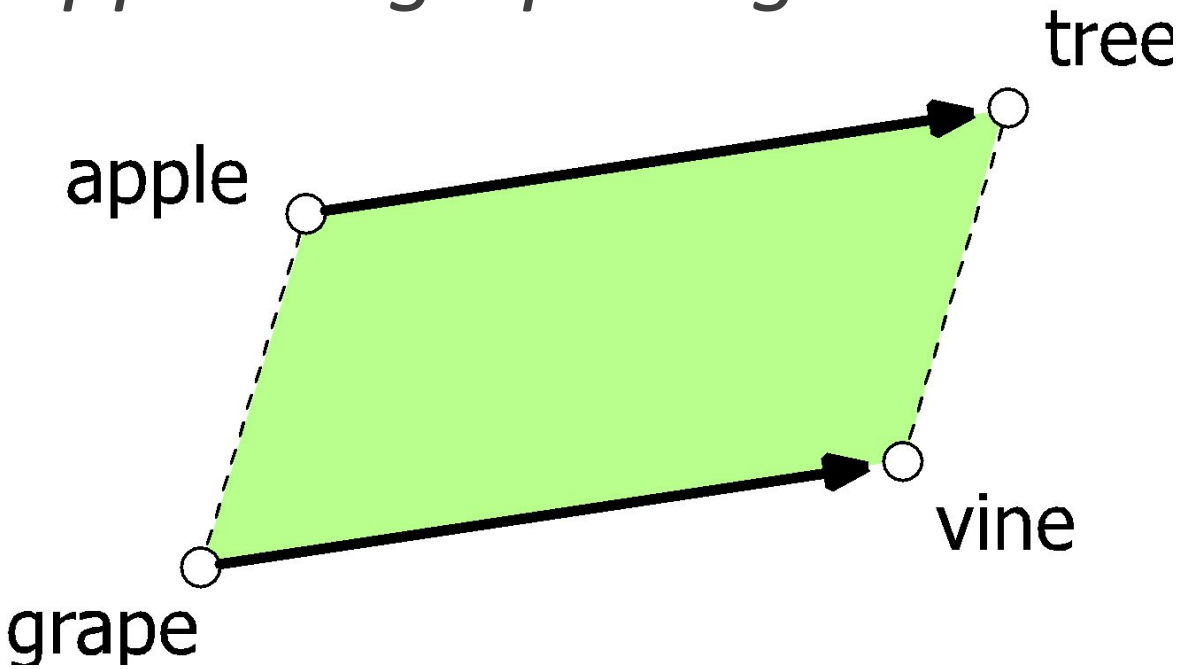
- *Hogwarts* nearest neighbors are Harry Potter world:
  - *Dumbledore, half-blood, Malfoy*

Analogical relations

The classic parallelogram model of analogical reasoning (Rumelhart and Abrahamson 1973)

To solve: *"apple is to tree as grape is to _____"*

Add $\overrightarrow{tree}$ – $\overrightarrow{apple}$ to $\overrightarrow{grape}$ to get $\overrightarrow{vine}$



Analogical relations via parallelogram

The parallelogram method can solve analogies with both sparse and dense embeddings (Turney and Littman 2005, Mikolov et al. 2013b)

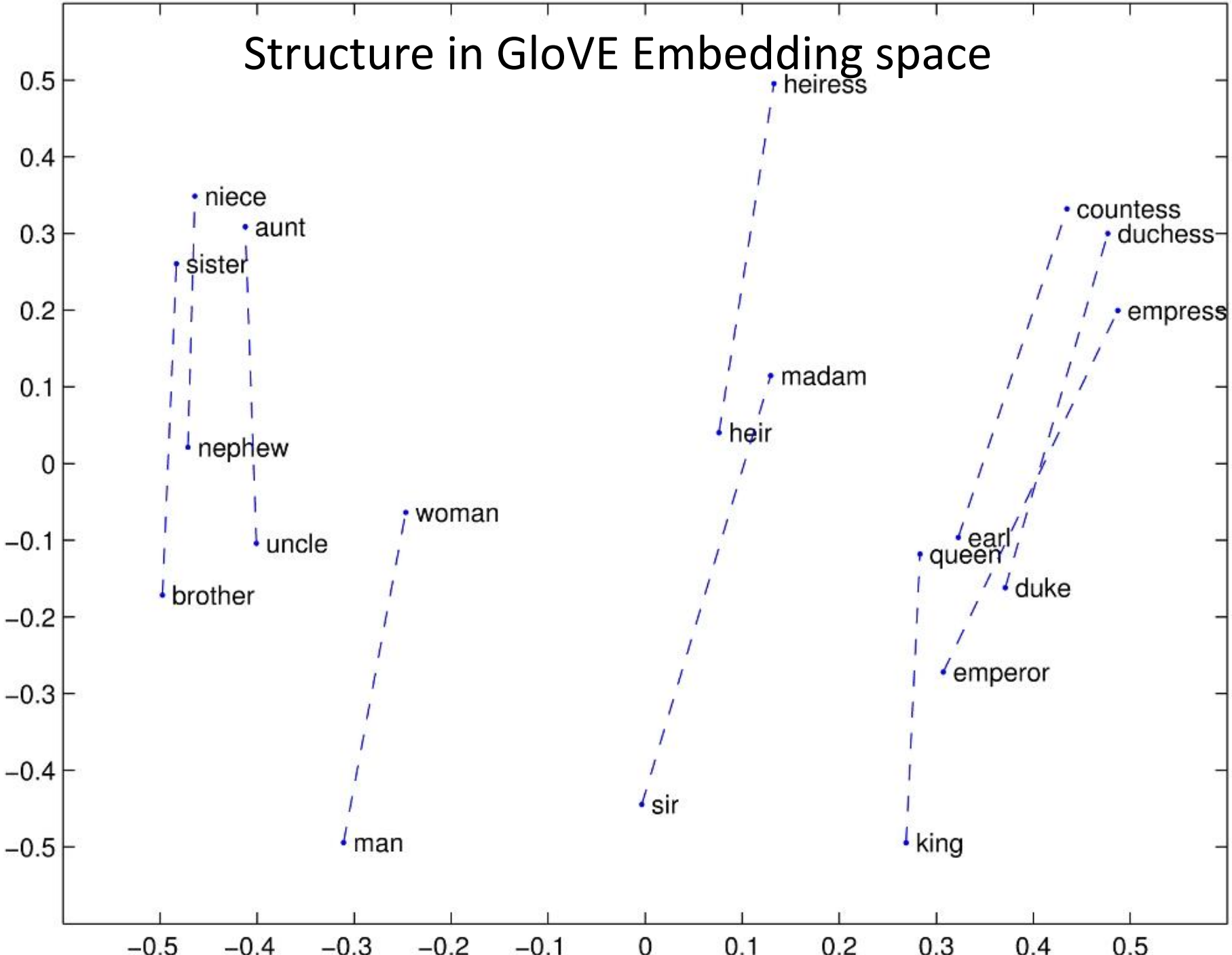
$\overrightarrow{\text{king}} - \overrightarrow{\text{man}} + \overrightarrow{\text{woman}}$ is close to $\overrightarrow{\text{queen}}$

$\overrightarrow{\text{Paris}} - \overrightarrow{\text{France}} + \overrightarrow{\text{Italy}}$ is close to $\overrightarrow{\text{Rome}}$

For a problem a:a*::b:b*, the parallelogram method is:

$$\hat{b}^* = \underset{x}{\text{argmax}}\ \text{distance}(x, a^* - a + b)$$

Structure in GloVE Embedding space



Caveats with the parallelogram method

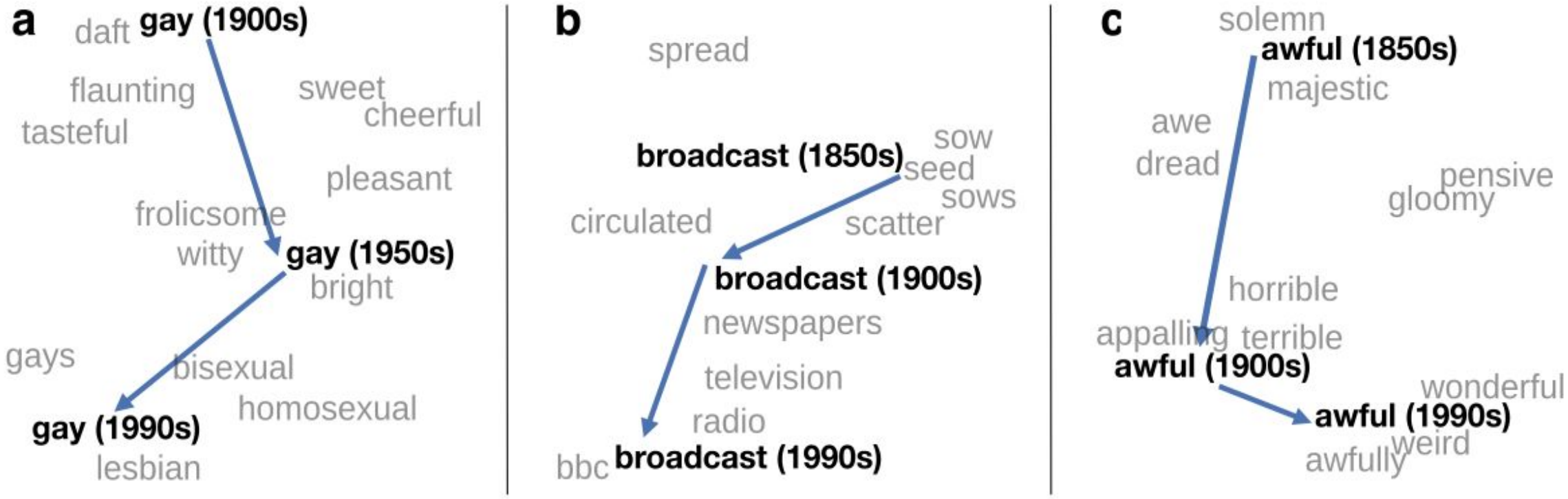
It only seems to work for frequent words, small distances and certain relations (relating countries to capitals, or parts of speech), but not others. (Linzen 2016, Gladkova et al. 2016, Ethayarajh et al. 2019a)

Understanding analogy is an open area of research (Peterson et al. 2020)

Embeddings as a window onto historical semantics

Train embeddings on different decades of historical text to see meanings shift

~30 million books, 1850-1990, Google Books data



William L. Hamilton, Jure Leskovec, and Dan Jurafsky. 2016. Diachronic Word Embeddings Reveal Statistical Laws of Semantic Change. Proceedings of ACL.

Embeddings reflect cultural bias!

Bolukbasi, Tolga, Kai-Wei Chang, James Y. Zou, Venkatesh Saligrama, and Adam T. Kalai. "Man is to computer programmer as woman is to homemaker? debiasing word embeddings." In *NeurIPS*, pp. 4349-4357. 2016.

Ask “Paris : France :: Tokyo : x”

- x = Japan

Ask “father : doctor :: mother : x”

- x = nurse

Ask “man : computer programmer :: woman : x”

- x = homemaker

Algorithms that use embeddings as part of e.g., hiring searches for programmers, might lead to bias in hiring

Historical embedding as a tool to study cultural biases

Garg, N., Schiebinger, L., Jurafsky, D., and Zou, J. (2018). Word embeddings quantify 100 years of gender and ethnic stereotypes. Proceedings of the National Academy of Sciences 115(16), E3635–E3644.

- Compute a **gender or ethnic bias** for each adjective: e.g., how much closer the adjective is to "woman" synonyms than "man" synonyms, or names of particular ethnicities
  - Embeddings for **competence** adjectives (*smart, wise, brilliant, resourceful, thoughtful, logical)* are biased toward men, a bias slowly decreasing 1960-1990
  - Embeddings for **dehumanizing** adjectives (barbaric, monstrous, bizarre) were biased toward Asians in the 1930s, bias decreasing over the 20th century.
- These match the results of old surveys done in the 1930s

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