1. Measurements of the radial recession velocities of five galaxies in a cluster give velocities of 9700, 8600, 8200, 8500, and 10,000 km s\(^{-1}\).

   a. What is the distance to the cluster if the Hubble parameter is \( H_0 = 72 \) km s\(^{-1}\) Mpc\(^{-1}\)?
      
      *Hint: Use the Hubble law for the average velocity of the members in the cluster.*

   b. Estimate, to an order of magnitude, the mass of the cluster if every galaxy is projected roughly half a degree from the cluster center. *Hint: Use the virial theorem*

2. **The age of the Universe** For a Hubble constant of \( H_0 = 72 \) km s\(^{-1}\) Mpc\(^{-1}\), calculate the Hubble time \( t_H = H_0^{-1} \).

3. **The critical density**

   a. Consider the Friedmann equation: \( \left( \frac{\dot{R}}{R} \right)^2 = H^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{R^2} \). What is the critical density \( \rho_c \) that gives a marginally bound Universe. Assume: \( H_0 = 72 \) km s\(^{-1}\) Mpc\(^{-1}\) and flat Universe.

   b. Estimate the stellar mass density \( \rho_* \). Assume that the density of galaxies is \( 2 \times 10^{-2} \) Mpc\(^{-3}\) and that in each galaxy there are \( 5 \times 10^{10} \) stars, and that each star has an average mass of \( 0.5 \) M\(_\odot\). What is the ratio \( \frac{\rho_*}{\rho_c} \).

4. The proper distance to a source is \( rR_0 \) where \( r \) is the comoving distance and \( R \) is the scale factor.

   a. Use the relation between redshift and the scale factor, i.e., \( 1 + z = \frac{R}{R(t)} \) and show that \( \frac{1}{R(t)} = \frac{1}{R_0} - \frac{1}{R_0} H_0 (t - t_0) \). *Hint: Use Taylor series about the point \( t = t_0 \) (the age of the Universe today) to the first order*

   b. Consider the geodesic expression: \( c dt = R(t) \frac{dx}{\sqrt{1 - k x^2}} \), and assume a flat Universe. Calculate to the first order the physical distance today, \( rR_0 \), in terms of \( z \) and Hubble time \( t_0 \). *Hint: Use the above approximation, and approximate to the first order.*