The program given below show how a very simple formula can be used to calculate the volume of a shell of a redshift shell. This formula is accurate to 10% at Omega matter = 0.1 and is perfect at Omega = 1. These calculations only apply for a lambda = 0, matter dominated universe.

I am now going to write the relavent volume formula as given by V. Padmenabhan on page 357 (v. Padmenabhan, "Cosmology and Astronphysics Through Problems," Cambridge University Press)

$$\frac{dN}{d\Omega_\phi dz} = \frac{n(z)dV}{d\Omega_\phi dz} = \frac{n(z)a_o^2 r^2 d_H}{(1+z)^2} \tag{1}$$

$\Omega_\phi \equiv$ the solid angle "Ω" rather than the critical density parameter ("Ω").

$$d_H = \frac{c}{H_o}[\Omega_o(1+z)^3 + (1-\Omega_o)(1+z)^2]^{-1/2} \tag{2}$$

$$a_o r = \frac{c}{H_o}\frac{[2\Omega_o z + 2(\Omega_o - 2)(\sqrt{(\Omega_o z + 1)} - 1)]}{H_o \Omega_o^2 (1+z)} \tag{3}$$

Note in the program below I've assumed that $n(z) = n_o(1+z)^3$ and that we're assumed that we're cacluationg the volume as if there is no other effect going on beyound the normal shrinking of the the universe as we go back in time. In other words, if you use this formula and you find that the number of objects per unit volume does change with redshift shell, you've got evidence for "true" evolution.

The simple formula that works for $\Omega = 1$, is $\mathbf{V} = d_L^3/(1+z)^3$, so the volume of shell is just the difference of the Volumes calulated for z1 and z2 (say). Where, d_L is the luminosity distance $= (1+z) \times a_o r$ in the $(a_o r)$ notation that V. Padmenabhan uses. Also note in the program below the names of the variables do not excatly correspond to the above, but the end result is correct, and see the examples at the end of the program. The program is written in IDL. For reasons that are not entirely clear, the QSIMP program provided by IDL tries to take integration steps that are too small if the defualt accuracy is used in the Omega = 0.1 case, hence, the comment next to the QSIM call.

The program follows:

```
function dist_plus,z
common dd, omega
; this is funxtion for volume intergral Assume matter evolves as (1+z)^3
; can't run commn so make kludge
;q = Omega/2 anything greater than 0)
;omega=0.1
h = 50.d
c = 2.997d5
d_h= 1./sqrt( Omega*(1.+z)^3 + (1-Omega)*(1+z)^2 )
r_a = (2.*Omega*z + 2*(omega-2)* ( sqrt(1+omega*z) -1  ) )/( (Omega^2)*(1.+z))
```

```
d_p=r_a^2*(c/h)^3*d_h

return,d_p
end
function lum_dist,z
;from /home/brad/sims/idl
;standard cosmology
common dd,  omega
;q = .5
h = 50.d
c = 2.997d5
; h is the hubble constant over c
;omega = 0.1

if(z lt 0.) then begin
 print,'A negative redshift is meaningless.'
 return,0.
endif

r_a = (2.*omega*z + 2*(omega-2.)* ( sqrt(1+omega*z) -1  ) )* (c/(h*omega^2))
;r_a = r_a
;r_a = c*r_a/(h*omega*(1+z))

d_l = r_a

return,d_l
end

pro volume_mel_v0, z1, z2, ome
common dd, omega
;omega=0.1
omega=ome
; This procedure compares the volume for a shell done two ways and the input
; z1 and z1 different values of z for shell
;
; This is all for FRW universe, i.e no non-zero Lambda

print, ' Omega = ', Omega , '  z1 =', z1, '  z2 =', z2

d_l1=lum_dist(z1)
d_l2=lum_dist(z2)
vol= qsimp('dist_plus',z1,z2,eps=0.0003); purposely reduced accruacy so
;program doesn't blow up
print,'easy way value is =' , ((d_l2/(1+z2))^3-(d_l1/(1+z1))^3)/3.
print,'integral value is =' , vol
return
end
;some sample runs
;IDL> volume_mel_v0,0.6,0.7,1.
; Omega =       1.00000  z1 =     0.600000  z2 =     0.700000
```

```
;easy way value is =   1.9922483e+09
;integral value is =  1.99225e+09
;IDL> volume_mel_v0,0.1,0.3,1.
; Omega =       1.00000  z1 =     0.100000  z2 =     0.300000
;easy way value is =   1.0092554e+09
;integral value is =  1.00926e+09
;IDL> volume_mel_v0,0.1,0.3,.1
; Omega =      0.100000  z1 =     0.100000  z2 =     0.300000
;easy way value is =   1.2466229e+09
;integral value is =  1.22304e+09
;IDL> volume_mel_v0,0.6,0.7,.1
; Omega =      0.100000  z1 =     0.600000  z2 =     0.700000
;easy way value is =   3.6818225e+09
;integral value is =  3.31191e+09
;IDL>
```