Reading Material
Chapter 4: § 4.1 – 4.3 and your class notes

Problem Set 3
Due Date: April 23, 2004

3.1 For an axisymmetric and nearly isotropic field one can write: \( I_\nu(\mu) \simeq I_\nu^{(0)} + \mu I_\nu^{(1)} \), where \( I_\nu^{(0)} \) and \( I_\nu^{(1)} \) are independent of \( \mu \) and hence the angle \( \theta \). Relate these quantities to the energy flux and energy density of this radiation field. Under what conditions on the ratio of the energy density over the flux is the (isotropic) relation \( F_\nu^+ = \pi I_\nu^{(0)} \) likely to be valid?

3.2 Consider the solution for the emergent (i.e., \( \mu > 0 \)) specific intensity \( I(\tau = 0, \mu) \) (from eq. 4.12). Assuming that the source function \( S(\tau) = (3\sigma/4\pi) T_{\text{eff}}^4 (\tau + 2/3) \) (true for a simple stellar atmosphere model) derive the quantity \( I(\mu)/I(\mu = 1) \) (this ratio is usually called the limb darkening law - note that \( \mu > 0 \)). If we consider the Earth receiving radiation from the Sun viewed as a solar disk, then \( \mu = 1 \) corresponds to the disk center and \( \mu \to 0 \) corresponds to the edges of the disk. Based on the limb darkening law you derive what can you say about the brightness of the disk edges relative to the center? Is this difference observed? Why is it so?

3.3 Consider a star at a distance \( D \) much larger than its radius \( R \), so that the rays arriving to the observer (on Earth) may be considered to be all parallel to the line of sight. The energy received per unit time per unit area perpendicular to the line of sight is equal to \( df_\nu = I_\nu d\Omega \), where \( d\Omega \) is the solid angle subtended by the unit surface element and \( I_\nu \) is the emergent specific intensity (i.e., consider only outgoing \( \mu \) values). Also consider the stellar disk as a collection of differential annuli with area \( dS \) and radius \( r = R \sin \theta \) (where \( \mu = \cos \theta \)), so that \( dS = 2\pi r dr \). Calculate the energy flux received on Earth integrated over the stellar disk as a function of the stellar angular diameter. (Hint: use the relation \( d\Omega = dS/D^2 \)).

3.4 (extra undergraduate credit)
(a) Consider the phase space number density of photons \( dN/(d^3p \ d^2x) \) and the definition of specific intensity \( I_\nu \) and derive the relationship between these two physical quantities.
(b) Prove that the specific intensity \( I_\nu \) in a pencil of radiation propagating through vacuum (i.e., no absorption, emission, or scattering) is conserved (i.e., does not drop off with distance).