Reading Material
Chapter 7: § 7.2 and 7.3.3
Chapter 2: § 2.1– 2.4

Problem Set 6
Due Date: May 24, 2004

6.1 Consider a 3 M\(_\odot\) pre-main-sequence star forming through gravitational contraction and shining due to release of gravitational energy. Part of its evolution occurs at constant radius equal to 8 R\(_\odot\), constant effective temperature 5300 K, and hence constant luminosity, while its structure is changing from a polytrope of index 3/2 to a polytrope of index 3. Calculate the lifetime of this phase.

6.2 Evolution on the red giant branch. Once a low-mass star leaves the main sequence and begins to ascend the giant branch, its luminosity is provided by hydrogen burning in a thin shell surrounding a degenerate He core. As hydrogen is burned in the shell, the mass of the He core increases. From evolutionary calculations we have the two empirical relations for the luminosity \(L\) and radius \(R\) of the giant as functions of the He core mass \(M_c\),

\[
L \simeq 2 \times 10^5 L_\odot \left(\frac{M_c}{M_\odot}\right)^6
\]

\[
R \simeq 3700 R_\odot \left(\frac{M_c}{M_\odot}\right)^4.
\]

(a) First derive a relation for the core mass as a function of time. Use the result to compute the evolution of a star \((L\) and \(R\) as functions of time\) as it ascends the giant branch. Take the initial core mass value to be \(M_c = 0.1 M_\odot\) and the final value (when the star reaches the tip of the giant branch) to be \(M_c = 0.45 M_\odot\).

(b) Plot the radius, luminosity, and effective temperature of the star as function of time while it is on the giant branch. Also plot the track of the evolving red giant in an HR diagram. All plots should be in a log-log scale.

6.3 (extra undergraduate credit) Consider a star with a core mass \(M\) and core radius \(R\) surrounded by a fully convective, ideal gas envelope of primarily ionized hydrogen. The envelope has a thickness \(\Delta r \ll R\) and mass \(\Delta m \ll M\) (i.e., the self-gravity of the envelope is negligible). Use the appropriate polytrope to derive the radial profiles of pressure and temperature within the the envelope. Use the above approximations to show that the temperature depends linearly on the inward radial distance from the envelope’s surface.