15

Steepening of Acoustic Waves Into Shock Waves

Reference: Whitham, *Linear and Nonlinear Waves.*

If nature used equations to move matter and radiation around (which I doubt), the method of characteristics—with its emphasis on the propagation of information at the natural speeds of the problem—would probably represent the way she would do it. Yet in our study of steady flow in a fluid, we found the characteristics to be real only for supersonic flow. For subsonic flow, the governing equations are elliptic, and the range of influence of any one part of the flow on the rest extends infinitely in all directions. How does this "action at a distance" jibe with modern perceptions that all physical influences have only a finite propagation speed?

The answer to this apparent paradox lies in the fact that we have assumed *steady state* conditions; i.e., we have allowed the fluid infinite duration to communicate with itself. If we consider instead the development of the flow in *time*, as we shall proceed to do below, we find the governing equations always to be hyperbolic, independent of whether the flow occurs supersonically or subsonically. The associated characteristics, which do not equal their counterparts for the steady flow problem, are always real. Physically, the difference arises because acoustic disturbances are always swept ahead by the passage of time, but they are swept downstream by steady flow only if that flow occurs supersonically (Figure 15.1).

SMALL-AMPLITUDE ACOUSTIC WAVES

In what follows we wish to consider the propagation of acoustic disturbances in an isentropic ideal gas. For an isentropic fluid, s = constant, implying pressure variations $P \propto \rho^\gamma$ with variations in density ρ. Thus, the gradient of the pressure can be calculated as

$$\nabla P = \left(\frac{\partial P}{\partial \rho}\right)_s \nabla \rho = a_s^2 \nabla \rho, \tag{15.1}$$

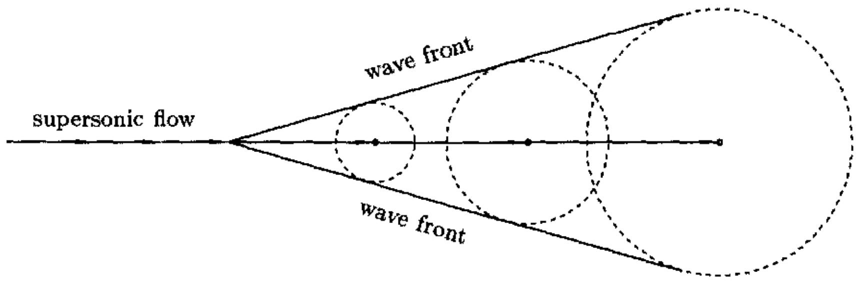

FIGURE 15.1
A small disturbance that occurs at a certain point in a gas in steady supersonic flow propagates away from that point (with respect to the moving fluid) at the speed of sound. The envelope of the wave front associated with this propagation forms a Mach cone with apex at the original point. In other words, knowledge of the original disturbance is swept downstream with the flow and cannot make its way upstream to the incident supersonic gas.

where a_s is the adiabatic speed of sound:

$$a_s = a_{s0} \left(\frac{\rho}{\rho_0} \right)^{(\gamma-1)/2}, \tag{15.2}$$

with ρ_0 and a_{s0} being the uniform density and acoustic speed of the static ambient background and $\gamma = c_P/c_v = 5/3$ for a perfect monatomic gas.

If we assume the disturbances to have infinitesimal amplitude, the linearized equations of motion in one dimension give rise to the following perturbation equations:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial u_1}{\partial x} = 0, \tag{15.3}$$

$$\rho_0 \frac{\partial u_1}{\partial t} = -a_{s0}^2 \frac{\partial \rho_1}{\partial x}. \tag{15.4}$$

If we subtract the space derivative of the second equation from the time derivative of the first, we get the homogeneous wave equation

$$\frac{\partial^2 \rho_1}{\partial t^2} - a_{s0}^2 \frac{\partial^2 \rho_1}{\partial x^2} = 0, \tag{15.5}$$

which has as its most general solution

$$\rho_1 = f(x - a_{s0}t) + g(x + a_{s0}t). \tag{15.6}$$

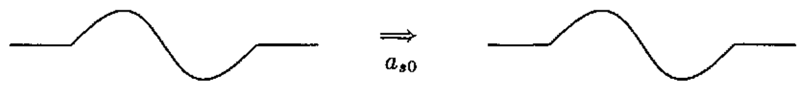

FIGURE 15.2
In the absence of dissipation and spatial inhomogeneities (or dispersion), the waveform of a disturbance governed by a linear wave equation maintains its size and shape forever, apart from propagation at a constant wave speed.

The corresponding solution for the velocity perturbations reads

$$u_1 = \frac{a_{s0}}{\rho_0}\left[f(x - a_{s0}t) - g(x + a_{s0}t)\right]. \tag{15.7}$$

Notice that the individual terms maintain *forever* their original waveforms, $f(x)$ or $g(x)$, apart from a propagation to the right or left at the constant speed a_{s0} (Figure 15.2).

Waves can propagate without changing shape if their governing PDE is linear. Although linear equations are a basic feature in some parts of physics (e.g., Maxwell's equations in a vacuum; Schrödinger's equation in an external potential), the equations of fluid mechanics are fundamentally nonlinear, unless they have been forced to take a linear form by fiat [as we have done in equation (15.5)]. The interesting question then arises, can acoustic waves of finite amplitude also propagate without changes of the waveform? The answer, we shall find below, is no. Acoustic waves having finite amplitude of *any waveform* must always steepen (if we ignore the action of viscosity), until eventually they develop such steep wavefronts that they become shock waves. (Almost all nonlinear waves have a tendency to steepen. However, some dispersive waves, whose different Fourier components propagate at different speeds when they have small amplitudes, have special finite-amplitude solutions, called *solitons* or *solitary waves*, in which the nonlinear steepening tendency can be exactly balanced by the dispersive tendency in such a way that the soliton maintains its shape as it propagates. Acoustic waves, which are not dispersive at small amplitudes, do not have such solitary-wave solutions; to pursue this topic further, see Whitham's book.)

UNSTEADY FLOW OF AN ISENTROPIC IDEAL GAS IN ONE DIMENSION

Without the small-disturbance assumption, the full equations read

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x}\right) + \frac{\partial u}{\partial x} = 0, \tag{15.8}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{a_s^2}{\rho}\frac{\partial \rho}{\partial x}, \tag{15.9}$$

where we have made use of equation (15.1) to eliminate the pressure gradient. We now use equation (15.2) to eliminate ρ in favor of a_s (which we write henceforth as a for simplicity of notation):

$$\frac{d\rho}{\rho} = \frac{2}{\gamma - 1}\frac{da}{a}.$$

Equations (15.8) and (15.9) now become

$$\frac{\partial}{\partial t}\left(\frac{2}{\gamma - 1}a\right) + u\frac{\partial}{\partial x}\left(\frac{2}{\gamma - 1}a\right) + a\frac{\partial u}{\partial x} = 0, \tag{15.10}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + a\frac{\partial}{\partial x}\left(\frac{2}{\gamma - 1}a\right) = 0. \tag{15.11}$$

The coupled set of quasilinear PDEs of first order, equations (15.10) and (15.11), could be analyzed formally by the method of characteristics that we developed in Chapters 13 and 14. A faster route to the same answer follows the blazed trail by Riemann, who adopted the simple trick of adding and subtracting equations (15.10) and (15.11) to obtain

$$\left[\frac{\partial}{\partial t} + (u + a)\frac{\partial}{\partial x}\right]\left(u + \frac{2}{\gamma - 1}a\right) = 0, \tag{15.12}$$

$$\left[\frac{\partial}{\partial t} + (u - a)\frac{\partial}{\partial x}\right]\left(u - \frac{2}{\gamma - 1}a\right) = 0. \tag{15.13}$$

Now equations (15.12) or (15.13) individually represent a quasilinear PDE of first order in a single dependent variable Q or R, where

$$Q \equiv u + \frac{2}{\gamma - 1}a, \tag{15.14}$$

$$R \equiv u - \frac{2}{\gamma - 1}a. \tag{15.15}$$

The method of characteristics for such equations yields the ODEs

$$\frac{dt}{1} = \frac{dx}{u + a} = \frac{dQ}{0}, \tag{15.16}$$

$$\frac{dt}{1} = \frac{dx}{u - a} = \frac{dR}{0}. \tag{15.17}$$

In other words, we have the Riemann invariants,

$$Q \equiv u + \frac{2}{\gamma - 1}a = \text{constant} \quad \text{on the plus characteristic} \quad \frac{dx}{dt} = u + a, \tag{15.18}$$

$$R \equiv u - \frac{2}{\gamma - 1}a = \text{constant} \qquad \text{on the minus characteristic} \qquad \frac{dx}{dt} = u - a. \tag{15.19}$$

The characteristic velocities $dx/dt = u \pm a$ equal the velocities at which (nonlinear) acoustic disturbances propagate at speed a to the right or to the left with respect to the fluid motion u. In contrast with the infinitesimal-amplitude analysis, we do not here assume u to be negligible compared to a, nor do we approximate a by its undisturbed value a_0 ($\equiv a_{s0}$). For the finite-amplitude case, what remains conserved by propagation at the two characteristic speeds is not the shape of the initial waveform, but the Riemann invariants, Q and R.

PROPAGATION AND STEEPENING OF A SIMPLE WAVE

Consider the situation of a simple wave, in which R on left-propagating minus characteristics is a strict constant, but Q is a different constant on different right-propagating plus characteristics.

$$R = u - \frac{2}{\gamma - 1}a = \text{the same constant for all spacetime,} \tag{15.20}$$

$$Q = u + \frac{2}{\gamma - 1}a = \text{different constants on different plus characteristics.} \tag{15.21}$$

The physical situation, shown in Figure 15.3, arises because the minus characteristics all originate from a static region of uniform properties ahead of the wave on the right before the simple wave (propagating from left to right) has had a chance to disturb the medium.

By arguments similar to those used in Chapter 14, we easily show that the minus characteristics form curved trajectories in spacetime, whereas the plus characteristics yield straight lines. We already know the information, equation (15.20), carried by any minus characteristic; so we need not concern ourselves with plotting any of them. Indeed, knowing that all minus characteristics come from a static region of uniform properties allows us to identify the constant in equation (15.20):

$$R \equiv u - \frac{2}{\gamma - 1}a = -\frac{2}{\gamma - 1}a_0. \tag{15.22}$$

At time $t = 0$ we have assumed a sinusoidal profile that possesses a density ρ equal to the undisturbed value ρ_0 at three points, marked A, B, C. The disturbance has vanishing amplitude at these points, and therefore plus characteristics emanating from these points must propagate at the undisturbed sound speed a_0, making three parallel lines in space time. Consider a part of the wave that has a density excess; by equation (15.2) it has a

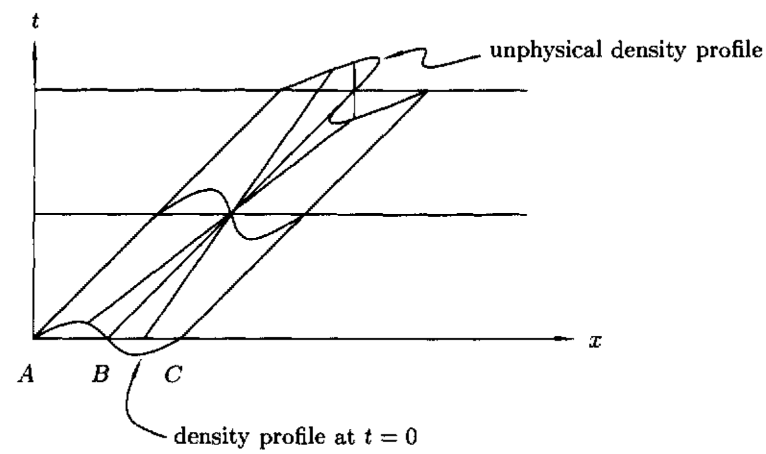

FIGURE 15.3
An acoustic wave of finite amplitude, even if it starts with a perfect sinusoidal shape and propagates in an undisturbed medium of exactly uniform properties, would inevitably steepen in its waveform.

sound speed $a > a_0$; therefore this effect tends to make a plus characteristic emanating from this part of the wave travel at a speed greater than average. There also exists the term u in the characteristic speed, $u + a$, which, by equation (15.22) $u = 2(a - a_0)/(\gamma - 1)$, is positive for parts of the wave that have density excesses, since $a > a_0$, and this just amplifies the effect. Now, consider a part of the wave that has a density deficit, where equations (15.2) and (15.22) imply $a < a_0$ and $u = 2(a - a_0)/(\gamma - 1) < 0$. Thus plus characteristics emanating from regions containing an initial density deficit travel slower than average. The net result is that the crest of the nonlinear wave tends to catch up with the trough, and the wave profile steepens. Nonlinear acoustic waves cannot maintain a constant waveform.

Eventually, if we were to follow this analysis far enough, we would find plus characteristics (belonging the same family) trying to cross. When this happens, the flow attempts to take on multiple values for the density, velocity, etc. This can not happen, of course, and the steepening must be halted by viscous forces (and have its thermal energy redistributed by heat conduction), so that the wave profile cannot steepen beyond the point where it suffers essentially one discontinuous jump (Figure 15.4).

At this stage, we have a shock wave, and although the characteristic analysis begins to break down, we describe what happens subsequently in

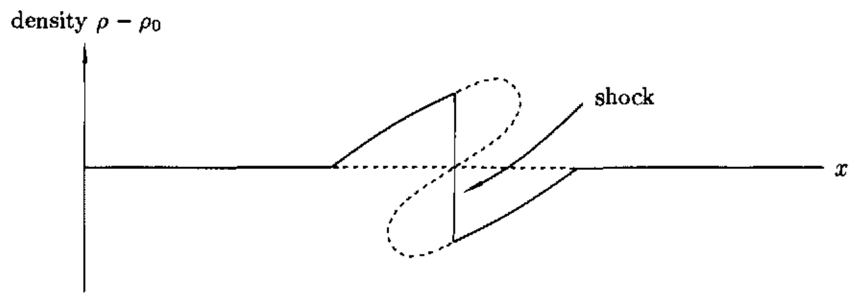

FIGURE 15.4
The tendency for nonlinearities to steepen the wave profile, which would produce multiple values for fluid properties such as gas density and velocity, must be eventually offset by the onset of strong viscous forces. The balance of the viscous forces and the steepening tendency mediates a shock, which is approximated in ideal fluid flow as a discontinuous jump of gas properties across the front.

a qualitative fashion so that we may form a relatively complete physical picture for the phenomenon. (See Whitham or Landau and Lifshitz for quantitative calculations.)

The high-density and high-pressure region just behind the shock wave must continue to push the front ahead faster than the undisturbed speed of sound a_0, so that eventually the shock front completely overruns the trough of the wave, and begins to propagate at supersonic speeds through the undisturbed medium. In the meantime, the tail of the wave, whose end propagates only at speed a_0, begins to lag behind the front, so that the wave profile gets stretched out along its length (Figure 15.5).

The redistribution of the density and pressure excess to more and more fluid must damp the disturbance, so that the height of the discontinuity (the strength of the shock) progressively weakens with time. Eventually, the wave will dissipate itself and become a small amplitude acoustic disturbance. Unlike a steady shock wave driven by a constant source of momentum and energy input (e.g., an airplane), in the current problem, we have the resources of only the initial input to draw on, and these resources must inevitably degrade once viscosity and heat conduction come into play.

THE STRUCTURE OF VISCOUS SHOCKS

In this section we wish to examine the structure of viscous shocks. We will verify first that the deceleration and compression of the fluid (in a frame

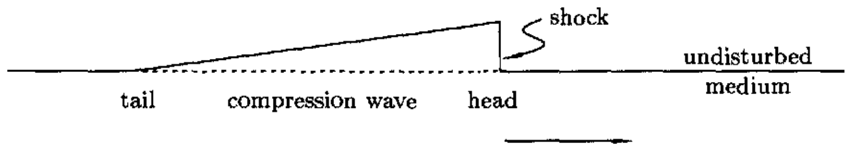

FIGURE 15.5
Since a shock front propagates at supersonic speeds with respect to the gas ahead of the front, the front eventually overruns the trough of a wave that initially had one cycle of a sinusoid, and the front increasingly also leaves behind the part of the wave that forms its tail. Thus, after shock formation, the initial single-sinusoid wave (indeed, any waveform consisting of no more than a single compression peak) increasingly takes on the shape of a triangular waveform.

which travels with the shock front) occur in a very thin layer. To the extent that the details of this transition do not interest us, we may approximate the entire process as a single discontinuous jump. We have as another goal, therefore, the derivation of the shock jump conditions.

Because the shock transition occurs in a thin layer, the time required for fluid to traverse the layer will be small compared to the time for the exterior conditions to change appreciably. In this case, if we transform to a frame which moves at the local velocity of the shock front, we may approximate the flow to be steady and to vary appreciably along only the one dimension perpendicular to the surface of the shock front. In addition, we assume that we may ignore any variation of the large-scale gravitational force field across the thin layer, as well as any radiative losses that might occur because the gas gets strongly heated behind the shock. (We shall treat radiative shocks in Chapter 16.) For sake of definiteness, we could pretend that we are blowing up the sharp part of the steepened wave profile of the previous section for detailed examination. In the frame of the wave front, preshock gas in region 1 rushes toward the front from the right (at minus the shock wave speed in the frame used in the previous section) and suffers a sudden compression and deceleration to become postshock gas of region 2 (Figure 15.6).

On the length scale of interest for investigating the shock layer, we can not see any of the smooth variations in the structure of region 1 (undisturbed gas) or of region 2 (gas inside the compression wave). In this approximation (which can be made mathematically rigorous by the method of matched asymptotic expansions), we may regard 1 and 2 as two regions of uniform flow, and the shock layer as the mechanism by which the fluid

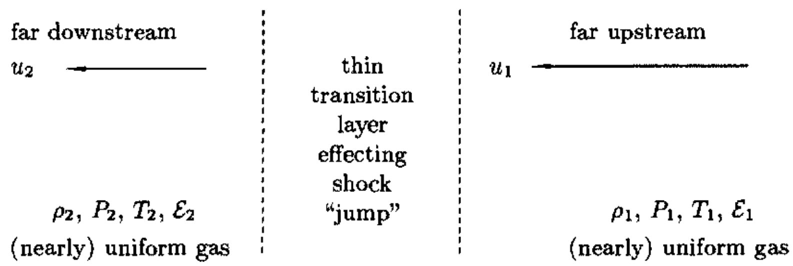

FIGURE 15.6
Schematic representation of the narrow transition region in a shockwave whereby upstream gas properties get transformed to downstream gas properties.

effects a transformation from 1 to 2. From this point of view, the macroscopic flow environment which brought about the production of the shock does not matter; once the shock exists, the structure would be the same if some completely different set of circumstances had led to the same upstream conditions 1. Thus, for example, our analysis would also apply to the part of the standing bow shock illustrated in Figure 15.7 that develops when supersonic gas flows past a blunt body.

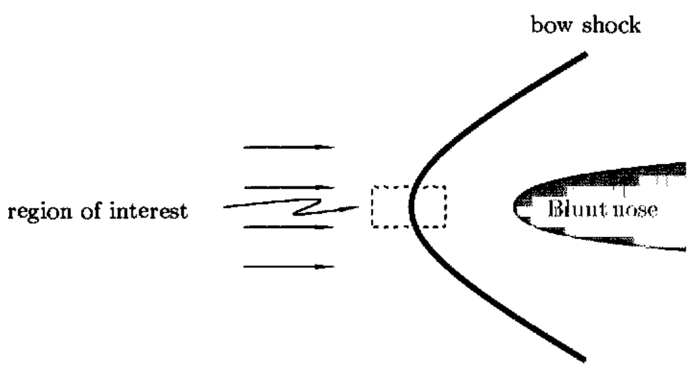

FIGURE 15.7
The part of the bow shock near the nose of a blunt object has the geometry of a normal shock.

We begin by writing the steady-state fluid equations with variations in one spatial dimension in their conservative form when we ignore any external sources for changing the mass, momentum, or energy content of the gas [see equations (4.16), (4.19), and (4.21)]:

$$\frac{d}{dx}(\rho u) = 0, \tag{15.23}$$

$$\frac{d}{dx}\left(\rho u^2 + P - \frac{4}{3}\mu\frac{du}{dx}\right) = 0, \tag{15.24}$$

$$\frac{d}{dx}\left[\rho\left(\frac{1}{2}u^2 + \mathcal{E}\right)u + \left(P - \frac{4}{3}\mu\frac{du}{dx}\right)u - \mathcal{K}\frac{dT}{dx}\right] = 0, \tag{15.25}$$

where we have written the xx-component of the viscous stress tensor as [cf. equation (4.28) with zero bulk viscosity]

$$\pi_{xx} = \frac{4}{3}\mu\frac{du}{dx}. \tag{15.26}$$

Integration of equations (15.23)–(15.25) gives the constancy of the fluxes of mass, momentum, and energy:

$$\rho u = \text{constant}, \tag{15.27}$$

$$\rho u^2 + P - \frac{4}{3}\mu\frac{du}{dx} = \text{constant}, \tag{15.28}$$

$$\rho u\left(\frac{1}{2}u^2 + \mathcal{E} + \frac{P}{\rho}\right) - \frac{4}{3}\mu u\frac{du}{dx} - \mathcal{K}\frac{dT}{dx} = \text{constant}. \tag{15.29}$$

In the shock transition layer (see Figure 15.8), we expect the frictional momentum flux $-(4/3)\mu du/dx$ to have an order of magnitude comparable to that of any other term in equation (15.28), for example, the momentum flux advected through the layer, ρu^2 (which diminishes as the gas suffers rapid deceleration). This requirement allows us to estimate the shock thickness, Δx, over which a jump in fluid velocity Δu is made. (In Problem Set 3, you are asked to perform a detailed integration to make a more quantitative analysis.) If $(4/3)\mu du/dx \sim \rho\nu\Delta u/\Delta x$ is to be comparable to ρu^2, we require

$$\Delta x \sim \nu\Delta u/u^2.$$

For a strong shock Δu is comparable to u itself, whereas u jumps (as we shall see) from a supersonic value to a subsonic one; so an estimate $u \sim v_T$ represents a good compromise. On the other hand, for a neutral gas, the kinematic viscosity satisfies, $\nu \sim \ell v_T$. Collecting expressions, we estimate that the thickness of the transition layer in a strong shock,

$$\Delta x \sim \ell,$$

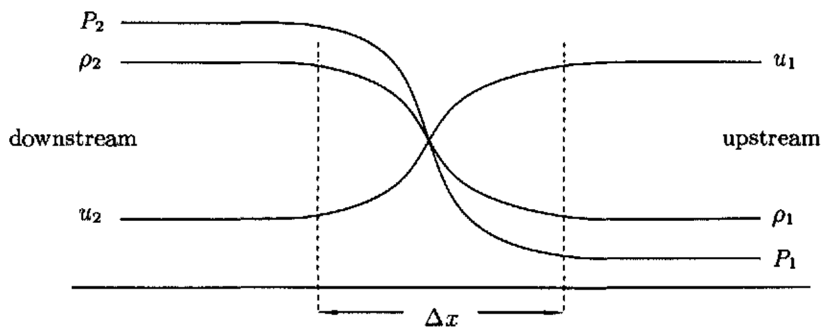

FIGURE 15.8
Across a viscous shock, the pressure and density increase and the velocity decreases as the gas flows from the upstream state to the downstream state. The transition is made in a characteristic distance Δx that equals a few mean free paths ℓ for the elastic scattering of the gas particles.

occupies a distance comparable to only a single mean free path ℓ for elastic collisions!

We can get the same conclusion from equation (15.29). A shock wave takes preshock gas at a low temperature and transforms it to postshock gas at a high temperature. In the transition layer, the heat flux by thermal conduction $\mathcal{K}dT/dx$ must become comparable to the enthalpy flux ρhu, where

$$h \equiv \mathcal{E} + \frac{P}{\rho}, \tag{15.30}$$

is the specific enthalpy, as defined in thermodynamics. (This quantity equals $\int dP/\rho$ for a barotropic fluid, which we have also denoted by the symbol h, only if we restrict ourselves to an isentropic fluid where $ds = 0$.) In order for $\mathcal{K}dT/dx \sim \mathcal{K}\Delta T/\Delta x$ to be comparable to $\rho hu \sim nkTu$, we require

$$\Delta x \sim \mathcal{K}\Delta T/nkTu.$$

For a strong shock, $\Delta T \sim T$, while $u \sim v_T$. On the other hand, for a neutral gas, the heat conduction coefficient satisfies $\mathcal{K} \sim c_v\mu \sim knv_T\ell$, so that we again get

$$\Delta x \sim \ell.$$

Of course, when gases begin to acquire structure on the scale of a few mean free paths, we must question whether a fluid treatment remains viable, or whether we need a kinetic analysis based on a solution of the

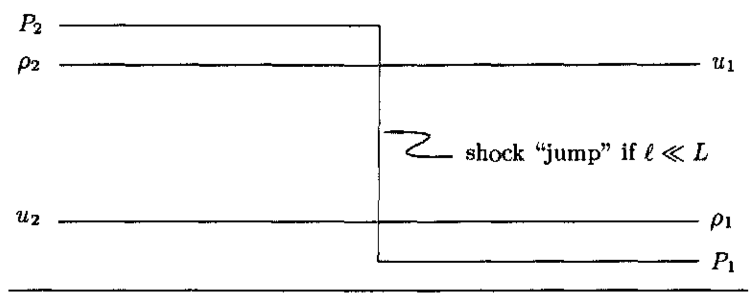

FIGURE 15.9
On macroscopic scales, shock transitions may be approximated as single discontinuous jumps.

Boltzmann transport equation. From a kinetic viewpoint, a strong shock wave consists of two interpenetrating Maxwellians (gas in LTE moving at some high upstream mean speed slamming into gas in LTE moving at some low downstream speed). The gas in the interaction layer is unlikely to have a good representation in terms of an expansion about LTE (the basis of the Chapman-Enskog procedure by which we obtained the Navier-Stokes equations from the moment equations of the Boltzmann equation—see Chapter 3). Nevertheless, detailed calculation using the Boltzmann equation *does* show that, apart from details on the upstream side, the viscous shock structure given by a fluid analysis yields a reasonable representation of the actual state of affairs (Problem Set 3). In particular, the transition layer thickness in a strong shock really does occupy only a few mean free paths for elastic scattering. (That's all it takes for elastic collisions to transform one state of local thermodynamic equilibrium to another.) If we ignore structure on the scale of ℓ, we may approximate a shock transition as a discontinuous jump (Figure 15.9).

RANKINE-HUGONIOT JUMP CONDITIONS

On a scale much larger than a mean free path ℓ, we may ignore the diffusive terms proportional to μ and $\mathcal{K}$ in equations (15.28) and (15.29). This allows us to calculate the "constants" on the right-hand sides of equations (15.27)–(15.29) as equal to the sum of the rest of the terms evaluated *either* far upstream (region 1) or far downstream (region 2) from the transition layer. In other words, the net transition satisfies the *jump conditions*:

$$\rho_2 u_2 = \rho_1 u_1, \tag{15.31}$$

$$\rho_2 u_2^2 + P_2 = \rho_1 u_1^2 + P_1, \tag{15.32}$$

$$\frac{1}{2}u_2^2 + h_2 = \frac{1}{2}u_1^2 + h_1. \tag{15.33}$$

In equation (15.33), we have used equation (15.31) to eliminate a factor of $\rho_2 u_2 = \rho_1 u_1$. Equations (15.31)–(15.33) give the downstream conditions (2) if we know the upstream ones (1).

On both sides of equation (15.33), h equals the specific enthalpy, which for a perfect gas satisfies the constitutive relations

$$h = \frac{\gamma}{\gamma - 1}\frac{P}{\rho} = \frac{\gamma}{\gamma - 1}\frac{kT}{m}. \tag{15.34}$$

Far upstream (on a scale of ℓ) and far downstream from the shock front, we may use thermodynamic relations of the type given by equation (15.34) because the gas exists in LTE (at least insofar as its translational degrees of freedom are concerned; see Chapter 9 of Volume I).

There are many different ways to algebraically solve the (Rankine-Hugoniot) jump conditions (15.31)–(15.33) supplemented by the constitutive relations (15.34). The most useful of these relations may be the expressions that give the ratios $\rho_2/\rho_1 = u_1/u_2$, P_2/P_1, and T_2/T_1 in terms of the upstream Mach number $M_1 \equiv u_1/a_s$, where $a_s^2 \equiv \gamma P/\rho$. The quantity M_1 is known as the Mach number of the shock. (Recall that we are in the frame of the shock front; so the shock front moves at velocity $-u_1$ with respect to the upstream gas that is often regarded by a laboratory observer as being at rest). For a perfect gas, we find

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma+1) + (\gamma-1)(M_1^2 - 1)} = \frac{u_2}{u_1}, \tag{15.35}$$

$$\frac{P_2}{P_1} = \frac{(\gamma+1) + 2\gamma(M_1^2 - 1)}{(\gamma+1)}, \tag{15.36}$$

$$\frac{T_2}{T_1} = \frac{[(\gamma+1) + 2\gamma(M_1^2-1)][(\gamma+1) + (\gamma-1)(M_1^2-1)]}{(\gamma+1)^2 M_1^2}. \tag{15.37}$$

Notice that $P_2 \geq P_1$, $\rho_2 \geq \rho_1$ ($u_2 \leq u_1$), and $T_2 \geq T_1$ if $M_1 \geq 1$ (supersonic upstream) with equality if $M_1 = 1$ (no shock at all). In the limit of a very strong shock, $M_1 \to \infty$, the density jump is bounded by a finite value, $(\gamma+1)/(\gamma-1)$, which equals 4 if $\gamma = 5/3$. In the same limit, the pressure and temperature jumps have no bound. In any case the deceleration of a gas from supersonic speeds to subsonic values in a shock produces compression and heating, outcomes which have observational consequences in astronomical objects if the process results in enhanced radiation. (It is not directly obvious from the above relations that $M_2 < 1$ when $M_1 > 1$, but Problem Set 3 shows it to be true.)

The equations (15.35)–(15.37) formally allow "expansive shocks" in which $M_1 < 1$ and $M_2 > 1$, because the jump conditions themselves do not prevent a reversal of roles for regions 1 and 2. Rarefaction shocks, in which subsonically moving hot gas suddenly expands and accelerates to become supersonically moving cool gas, do not arise in nature. It is possible to show that the entropy jump, $s_2 - s_1$, has positive values for compressive shocks, and negative ones for rarefaction shocks. The second law of thermodynamics forbids internal processes by which a gas in one uniform state can spontaneously lower its specific entropy to become gas in another uniform state. In other words, viscosity can transform the energy of bulk motion into heat, but it can not do the reverse. From another point of view—that which began this chapter—nonlinear compression waves steepen into shocks; expansion waves remain smooth as they propagate. For these reasons, the word "shocks" always refers to "compressive shocks," and we never need consider adding either modifier, "expansive" or "compressive."

Finally, notice that viscosity and heat conduction are the microscopic mechanisms which counteract the nonlinear tendency for compressive waves to get ever steeper. The balance between spreading by diffusion and steepening by nonlinearity gives the wave a semi-permanent profile (that of a shock wave). Yet, despite this fundamental role for viscosity and conductivity, the end values of quantities like s, ρ, P, T, etc., at 1 and 2 do not depend on any of the detailed properties of the transport coefficients μ and $\mathcal{K}$. From a thermodynamic point of view, quantities like s, etc. represent *state variables*; they cannot depend on the path of the transformation taken to reach state 2 from state 1. What happens on a mechanistic level is that the shock thickness *automatically adjusts* itself to whatever value is necessary so that the level of viscosity and heat conduction available can balance the nonlinear steepening tendency. [Notice the counterintuitive result: The smaller the viscosity, the thinner the shock transition layer (if a shock arises at all).] The end states of the transformation are completely determined by the joint conservation of mass, momentum, and energy.

ARTIFICIAL VISCOSITY

Because the magnitude of the viscous term does not affect the net shock-jump conditions, many numerical schemes implicitly or explicitly incorporate the trick of *artificial viscosity* for halting the ever-growing steepening tendency produced by nonlinear effects, thereby gaining the automatic insertion of shock waves wherever they are needed (in time-dependent calculations). The central idea involves the introduction of a numerical viscous term large enough to spread out shock transitions over a few zones of the computational grid, making the inclusion of shock waves resolvable by a finite-difference calculation. As long as the grid size remains small compared to interesting macroscopic lengths (essential for reliable results), no harm results from the smearing of the shock wave over a few zones instead of a few mean free paths. However, do not blindly adopt artificial viscosity schemes for astrophysical applications that involve radiative transfer. If you spread out shocks over more than a fraction of a photon mean free path, you may obtain spurious flows of radiant energy (wrong direction as well as wrong magnitude). Fortunately, photon mean free paths usually greatly exceed particle mean free paths (review Chapter 1), so one can often achieve adequate numerical resolution for the former even if one cannot for the latter.