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Q in the Solar System

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Secular changes brought about by tidal friction in the solar system are reviewed. The presence or absence of specific changes is used to bound the values of Q (the specific dissipation function) appropriate for the planets and satellites. It is shown that the values of Q separate sharply into two groups. Values in the range from 10 to 500 are found for the terrestrial planets and satellites of the major planets. On the other hand, Q for the major planets is always larger than 6×10^4. Estimates of tidal dissipation in the atmospheres of the Jovian planets lead to values of Q which are consistent with those we have calculated on the basis of secular changes in the satellites' orbits. However, it is difficult to reconcile these large Q's with the much smaller values obtained in laboratory tests of solids. Lyttleton's hypothesis that Pluto is an escaped satellite of Neptune is critically examined. Using the Q's we obtain for the major planets and their satellites, we show that any eccentricity that Triton's orbit may have possessed after a near encounter with Pluto would have been subsequently damped, thus accounting for its present near-circular orbit.

I. Introduction

The departure of a tidally distorted body from perfect elasticity or fluidity is neatly summarized in terms of the parameter Q. The effective tidal dissipation function, Q^{-1}, is defined by

$$Q^{-1} = \frac{1}{2\pi E_0} \oint \left(- \frac{dE}{dt} \right) dt, \qquad (1)$$

where E_0 is the maximum energy stored in the tidal distortion and the integral over $-dE/dt$, the rate of dissipation, is the energy lost during one complete cycle.

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The mechanism of tidal friction is shown for a planet in Fig. 1. The potential field due to a satellite m raises a tidal distortion on the rotating planet M. If the planet were perfectly elastic, the tidal distortion would be symmetrical about the line of centers Mm, and the figure of the planet would rotate through it. But the presence of friction in the planet will produce a delay in the time of high tide. If the satellite's revolution period is longer than the planet's rotation period, the lagging tide is carried ahead of the satellite by an angle ϵ. The relation of Q to this lag angle ϵ is (MacDonald, 1964, Eq. 130)

$$1/Q = \tan 2\epsilon, \qquad (2)$$

or, since Q is generally large, $Q^{-1} \simeq 2\epsilon$.

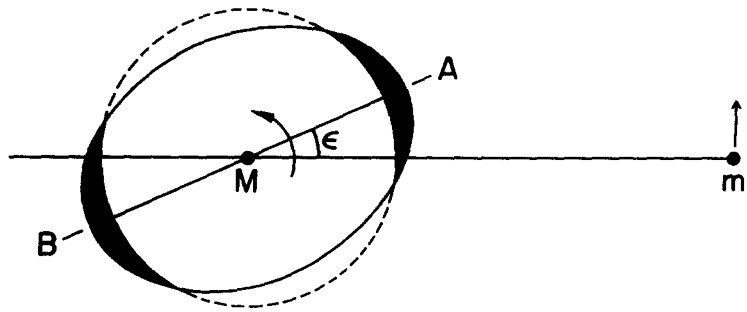

FIG. 1. The force of attraction between the satellite m and the nearer tidal bulge A exceeds that between m and B; a component of the net torque retards the rotation of the planet M and accelerates the satellite in its orbit.

The asymmetrical position of the tidal bulges with respect to the line of centers, Mm, introduces a net torque between the planet and satellite. Because the satellite is attracted more strongly by the near side bulge which is leading it in longitude, the torque acts to transfer angular momentum and energy from the planet's rotation into the satellite's orbital revolution. The excess planetary spin energy is dissipated as heat in the planet's interior. As a familiar example, the Earth's rotation is gradually slowing down while the lunar semimajor axis is expanding. In addition to affecting the satellite's semimajor axis, the frictionally retarded tides on the planet also produce secular changes in eccentricity, inclination, and obliquity. As we are particularly interested in the changes of eccentricity, we shall briefly describe the mechanism by which they are produced.

The tidal torque on a satellite which moves in an eccentric orbit is larger at pericenter than at apocenter. For this reason, we may approximate the total addition of angular momentum to the satellite orbit by one impulse at pericenter and by another, somewhat smaller impulse at apocenter. Due to the periodic nature of bound orbits in an inverse-square-law force field, it is evident that an impulse at pericenter increases the apocenter distance without altering the distance to pericenter. Similarly, an impulse at apocenter increases the pericenter distance but doesn't affect the distance to apocenter. Because the larger impulse occurs at pericenter, the net effect of the tidal torque is to increase the eccentricity, as well as the semimajor axis, of the satellite's orbit. The tangential component of force on the satellite, which is responsible for the tidal torque, is not the only component which affects the eccentricity. The radial component also plays a role in this process. Consider the situation where the satellite's orbital period just equals the planet's rotation period. High tide on a perfectly elastic planet would occur when the satellite was at pericenter. In reality, the maximum occurs some time after pericenter due to dissipation in these radial tides. Now consider the more usual case of relative rotation between the planet and satellite: the tides still retain a periodic radial component, provided $e \neq 0$. Although this component involves no net torques that transfer angular momentum between the planet and satellite, it nonetheless dissipates mechanical energy of the system. Because they decrease the orbital energy without changing the orbital angular momentum, the radial tides must diminish the eccentricity of the relative orbit. The net change in eccentricity, due to both the tidal torque and the radial forces, may be shown to be positive if the planet rotates much faster than the satellite revolves in its orbit. For constant Q, the planet's spin rate must be at least 50% faster than the satellite's mean motion in order that the eccentricity be increasing.

Up to now we have discussed the tides raised by a satellite on its planet. Tides raised on a satellite by its planet work to retard the satellite's spin (e.g., the Moon's

TABLE I
POSSIBLE TIDAL INTERACTIONS

Tides raised on	by	affect rotation of	and orbits[a] of
(1) planets	satellites	some planets	all close satellites
(2) planets	the Sun	the inner planets	
(3) satellites	planets	close satellites	close satellites
(4) satellites	the Sun		
(5) the Sun	satellites		
(6) the Sun	planets		

[a] Semimajor axis, eccentricity, and inclination.

synchronous rotation). These tides, by virtue of their radial character, also act to decrease the orbital eccentricity. Finally, tides raised on planets by the Sun are analogous to tides on satellites due to planets in that they tend to despin the distorted body.

There are altogether six cases of solar system tidal interactions, as listed in Table I. The tidal torques are all of the same form whether the tide-raising body (m in Fig. 1) be the Sun, a planet, or a satellite. However the magnitude of the interaction is insignificant if the tide-raising body is too distant and/or too small. We indicate in Table I only those effects which may lead to significant changes over the age of the solar system. For example, tides raised on the Sun by a planet (Case 6) can be shown to have virtually no effect on the rotation and relative orbit of the Sun so these effects are not listed.

In general, we might expect Q to depend on the frequency and amplitude of the tidal oscillation, i.e., upon the relevant rotation period and the amplitude of the strain. The relevant range of rotation periods in the solar system is from 7^h40^m, for the presumed synchronous rotation of Phobos, to $\sim$244 days for the retrograde rotation of Venus. Strains to be considered range up to the order of 10^{-4} for Triton, Mimas, and some other close satellites, assuming they are about as rigid as ice ($\mu \sim 3 \times 10^{10}$ dynes/cm^2). However, laboratory and seismic investigations on the Q of rocks involve oscillations with periods always less than several minutes and strains smaller than 10^{-5} (Knopoff, 1964). These studies indicate that Q values for dry solids become increasingly frequency-independent in the lower frequency range, with little or no frequency dependence for the longest period oscillations. However, an increasing amplitude dependence is found for strains larger than 10^{-6}. Extrapolation from these data suggests that Q for solid body solar system tides is independent of frequency but may involve a nonlinear dependence on the amplitude of strain when the strains are larger than 10^{-6}.

Experimental values of Q ranging between 40 and several thousand have been determined for various rocks and metals, but ignorance of the outer composition and state of other planets and satellites precludes any confident assignment of Q's to these objects. The secular acceleration of the Moon is consistent with a tidal lag $\epsilon = 2\overset{\circ}{.}16$ for the Earth. At present it is not yet determined how much of the dissipation is due to ocean tides as opposed to solid body tides (MacDonald, 1964, Sec. 6). Much less is known about the Q's for dense atmospheres which are perhaps responsible for tidal dissipation in Venus and the major planets.

Much previous work on tidal evolution in the solar system has relied on conjectured values of tidal lag angles or Q values (e.g., Darwin, 1908; Jeffreys, 1962; Goldreich 1963). As a result the time scales for these processes are uncertain and in some cases even the direction is not known. For example, Earth tides work to increase the Moon's orbital eccentricity while tides on the Moon tend to decrease it; these two rates are of the same order of magnitude but they depend on the respective Q's for the Earth and Moon. Without a reliable Q for the Moon, it is uncertain whether the eccentricity is at present increasing or decreasing. (For an argument which implies that this

	Mass	Radius	Orbital semimajor axis	Axial angular velocity	Mean motion
Satellite s	m	a	r	ω	n
Planet p	M	A	R	Ω	

eccentricity was smaller in the past see Goldreich, 1965*c*.)

In this paper, we summarize the relevant information about Q in the solar system and examine those cases in which tidal evolution has been appreciable since the origin of the planets and satellites.

II. Mechanism of Tidal Evolution

The subscripts and notation to be used throughout are as shown above.

The satellite radii values to be used are those in Allen (1963). Most of the remaining data is from Blanco and McCuskey (1961).

We begin with tides raised on a planet by a satellite. From an integration of the satellite-produced tidal couple throughout the tidal distortion of a planet (Jeffreys, 1962, Chap. 8.02), the net effective torque is found to be

$$N_1 = \tfrac{8}{5}\pi(GmA^4/r^3)(\rho H \sin 2\epsilon)_p$$
$$= \tfrac{6}{5}(GMm/r^2)(A/r)(H \sin 2\epsilon)_p, \quad (3)$$

where G is the gravitational constant and ϵ the tidal lag angle. Strictly speaking, the density ρ should refer to the tidal bulge, but with the uncertainties involved, we can set it equal to the mean density of the planet.

The actual height of the tide is

$$H = \zeta h, \quad (4)$$

where

$$\zeta = \tfrac{3}{4}(m/M)(A^4/r^3) \quad (5)$$

is the equilibrium tidal height (disturbing potential divided by undisturbed surface gravity) and

$$h = \frac{5/2}{1 + 19\mu/2g\rho A} \quad (6)$$

is a correction factor for the rigidity of the planet and for a second degree disturbance of the tidal potential due to the deformation itself (Love, 1927, p. 259). The planet's rigidity and surface gravity are denoted by μ and g, respectively. For large planets, in which self-gravity far exceeds rigidity, h approaches 5/2.

From (2), for small tidal lag angle ϵ (large Q), we may set

$$\sin 2\epsilon = 1/Q. \quad (7)$$

Now let

$$Q' = Q(1 + 19\mu/2g\rho A). \quad (8)$$

With a substitution of these equations into (3), the torque due to tides raised on a planet by a satellite becomes

$$N_1 = \tfrac{9}{4}G(A^5/Q'_p)(m^2/r^6). \quad (9)$$

By analogous reasoning, the tides raised on a planet by the Sun give a torque of the same form,

$$N_2 = \tfrac{9}{4}G\mathfrak{M}^2(A^5/Q'_pR^6), \quad (10)$$

where $\mathfrak{M}$ is the mass of the Sun. Finally, tides raised on a satellite by a planet have an associated torque of

$$N_3 = \tfrac{9}{4}GM^2(a^5/Q'_sr^6), \quad (11)$$

where

$$Q'_s = Q_s(1 + 38\pi a^4\mu/3Gm^2). \quad (12)$$

This is the same as (8) but it eliminates having to use g and ρ for satellites. The effects of these three torques upon tidal evolution are now considered.

Despin of Planets and Satellites

The moment of inertia through the spin axis of a planet is

$$I_p = \alpha MA^2, \quad (13)$$

where α depends on internal structure. Then the retardation of a planet's rotation due to the tidal torque from a satellite is

$$-\frac{d\Omega_1}{dt}=\frac{N_1}{I_p}=\frac{9}{4}G\frac{A^3}{\alpha MQ'_p}\frac{m^2}{r^6},\quad (14)$$

while that due to the Sun is

$$-\frac{d\Omega_2}{dt}=\frac{N_2}{I_p}=\frac{9}{4}G\mathfrak{M}^2\frac{A^3}{\alpha MQ'_pR^6}.\quad (15)$$

Because of the R^{-6} factor, only the spins of the innermost planets are substantially retarded by the Sun. Indeed, the solar contribution to the Earth's rotational deceleration is considerably smaller than that due to the Moon.

In dealing with satellites, we shall assume a rotational moment of inertia given by

$$I_s=\tfrac{2}{5}ma^2.\quad (16)$$

Thus the rate of despin of a satellite due to tides raised on it by a planet is given by

$$-\frac{d\omega}{dt}=\frac{N_3}{I_s}=\frac{45}{8}GM^2\frac{a^3}{mQ'_sr^6},\quad (17)$$

which is analogous to (15).

Secular Acceleration of Satellites

The satellite torque N_1 acting on a planet produces an almost negligible despin of the planet (except for the Earth–Moon system). However, there is an equal and opposite tidal torque working on the smaller orbital angular momentum of the satellite which may produce a significant secular acceleration. The orbital angular momentum of the satellite is

$$\begin{aligned}L&=mr^2n(1-e^2)^{1/2}\\&=m(GM)^{1/2}r^{1/2}(1-e^2)^{1/2}.\end{aligned}\quad (18)$$

For a circular orbit

$$dL/dt=\tfrac{1}{2}m(GM)^{1/2}r^{-1/2}(dr/dt),\quad (19)$$

but this must be the same torque as N_1 in (9). Equating them, we can solve for

$$\frac{dr}{dt}=\frac{9}{2}\left(\frac{G}{M}\right)^{1/2}\frac{A^5}{Q'_p}\frac{m}{r^{11/2}}.\quad (20)$$

Differentiation of Kepler's third law, $n^2r^3 = GM$, gives an equivalent expression for (20) in terms of the satellite's mean motion,

$$\frac{dn}{dt}=-\frac{27}{4}G\frac{A^5}{Q'_p}\frac{m}{r^8}.\quad (21)$$

Therefore, tides raised on a planet by a satellite cause expansion of the satellite's orbit, but with two exceptions: (a) Phobos is unique as the only satellite with an orbit period shorter than the rotation period of its planet, i.e., with $n > \Omega$. The tidal bulge it raises must lie on the trailing side of the line of centers (reverse both direction arrows in Fig. 1) and the resulting torque acts to retard the satellite. An equal and opposite torque adds angular momentum to the planet, but the effect of this is negligible due to the insignificant mass of Phobos. (b) Triton (Neptune I) is the only close satellite with a retrograde orbit. In this case (change m's direction in Fig. 1) the transfer of angular momentum by the tidal torque works to retard both the satellite and the planet. It is clear that both Phobos and Triton are approaching their primaries due to secular tidal deceleration, and a change of sign in (20) and (21) will allow for this.

Change in Satellite Eccentricity

Darwin (1908) developed the tidal disturbing function into Fourier components and then worked out the equations of variation for the orbital elements. In reviewing this solution, Jeffreys (1961) indicated how tides raised on planets by satellites would usually cause a secular increase in orbital eccentricity. Goldreich (1963) showed that tides raised on satellites tend to decrease eccentricity and may often succeed in conteracting the effect of tides raised on the planet, i.e.,

$$\frac{de}{dt}=\left(\frac{de}{dt}\right)_p+\left(\frac{de}{dt}\right)_s\quad (22)$$

is frequently negative. Simplifying the results in the latter paper (Goldreich, 1963, pp. 259, 261) by assigning the same lag angle to all the component tides we can write

$$\frac{1}{e}\left(\frac{de}{dt}\right)_p=\frac{171}{16}\left(\frac{G}{M}\right)^{1/2}\frac{A^5}{Q'_p}\frac{m}{r^{13/2}}\sigma,\quad (23)$$

where

$$\sigma=\operatorname{sign}(2\Omega-3n)\quad (24)$$

and

$$\frac{1}{e}\left(\frac{de}{dt}\right)_s=-\frac{63}{4}(GM^3)^{1/2}\frac{a^5}{mr^{13/2}Q'_s}.\quad (25)$$

The sign condition (24) is analogous to the $n > \Omega$ criterion following (21) which determines that $\dot{r} < 0$ for Phobos. It is not the same condition however, because tidal components other than the simple diurnal bulge are of importance in affecting eccentricity. As a result, (23) is negative for the close satellites Deimos and Jupiter V as well as for Phobos and Triton.

For tides raised on satellites, it is the radial component that works to decrease eccentricity since this involves energy loss with angular momentum conservation. Thus (25) is always negative for satellites which rotate synchronously because this behavior assures the dominance of radial tides. Since it may be shown (Sec. V) that any satellite close enough to its primary to have undergone tidal orbital evolution will have suffered a despinning to synchronous or nearly synchronous rotation, it is therefore safe to say that $(de/dt)_s$ is negative in every case where it is not negligible.

III. Tides on Terrestrial Planets and Satellites

Mercury

Tides raised on Mercury by the Sun have obviously worked to despin the planet to its present rotation rate. These tides would have brought Mercury into a synchronous 88-day rotation *if* the orbit were circular. But with a sizeable eccentricity (of 0.2056), the R^{-6} tidal torque (10) exerts its maximum drag effect at perihelion and tends to impart a rotation period of close to 56.6 days, characteristic of the orbital angular velocity at perihelion. Therefore, the final spin state will not be synchronous rotation but instead will possess a period between 56.6 and 88 days. The precise value of the period is determined by the condition that the time-averaged (over each orbit) tidal torque about Mercury's spin axis should equal zero (Peale and Gold, 1965). Indeed this new approach was prompted by the recent radar discovery of a 59 ± 5-day rotation period for Mercury (Pettengill and Dyce, 1965).

Let us assume then that Mercury has reached its final state of stable rotational angular velocity, and let this be $\Omega = 1.23 \times 10^{-6}$ sec^{-1}, corresponding to $P = 59$ days. Further, let Ω_0 be the initial spin imparted to the planet at its origin and let Δt be the time interval during which this spin was retarded to Ω. Then integrating (15), it is clear that

$$\Omega_0 - \Omega = \frac{9}{4} G\mathfrak{M}^2 \frac{A^3}{\alpha M Q'_p R^6} \Delta t, \qquad (26)$$

with α and R both roughly constant in time and thus taken outside the integral. From (8) and (26), the Q_p for despin tidal attenuation is written as

$$Q_p = \frac{9}{4} G\mathfrak{M}^2 \frac{A^3}{\alpha M(1 + 19\mu/2g\rho A)_p R^6} \frac{\Delta t}{\Omega_0 - \Omega}. \qquad (27)$$

It now becomes possible to place an upper bound on Q_p because

$$\Delta t \leq \Delta T = 4.5 \times 10^9 \text{ years.} \qquad (28)$$

In other words, although we do not know how long it has taken the planet to achieve the (presumed final) rotation rate presently observed, we can at least bound the despin time within the age of the solar system. An evaluation of (27) requires that the moment of inertia factor α, the initial rotation Ω_0, and the internal rigidity μ of the planet be known.

We will assume for Mercury, as for satellites (16), that a possible indication as to the value of Mercury's long-vanished initial rotation is found in the striking empirical relationship of spin angular momentum density to planet mass first pointed out by MacDonald (1963), as shown in Fig. 2.

It is easily shown that Mars and the major planets must have maintained roughly their initial rotational angular momenta, perhaps only very slightly diminished by solar- and/or satellite-induced tides. Now notice that these planets lie nearly on a straight line in Fig. 2, the slope of which is given approximately by

$$\alpha A^2\Omega \propto M^{0.87}, \qquad (29)$$

where $\alpha A^2\Omega$ is the rotational angular momentum per unit mass of the planet. What is significant here is that Mercury and Venus fall well below this line of presumed initial angular momentum density. Also, the Earth, which has been somewhat retarded by the

Moon and Sun, falls measurably below the line. Thus those planets, and only those planets, that could have suffered substantial tidal despin fail to conform to (29). Assuming that the position marked 1 in Fig. 2 actually represents the original angular momentum density of Mercury, then the corresponding initial rotation velocity is $\Omega_0 \sim 9.1 \times 10^{-5}$ sec^{-1}, so $P_0 \sim 19$ hr.

Now the coefficient of rigidity μ in (27) is not known for Mercury. On the basis of the values determined for rocks and for the interior of the Earth we suspect μ to be at least 10^{11} dynes/cm^2. For convenience, we shall write

$$\mu = b \times 10^{11} \text{ dynes/cm}^2. \qquad (30)$$

Surface granite has $b \sim 3$. The calculated rigidity for the Earth's mantle ranges monotonically from $b \sim 7$ at 200 km to $b \sim 28$ at 2800 km, and then drops off sharply at the outer core (Gutenberg, 1959, Chap. 8.6). Assuming that the effect of lower internal pressure more than compensates for the probable absence of a viscous core in Mercury, we choose as a rough lower limit to the planet's rigidity $b \sim 7$.

Using this value of rigidity and assuming that Fig. 2 gives a reasonable initial spin Ω_0, (27) and (28) then set the upper bound for the Q_p of Mercury at

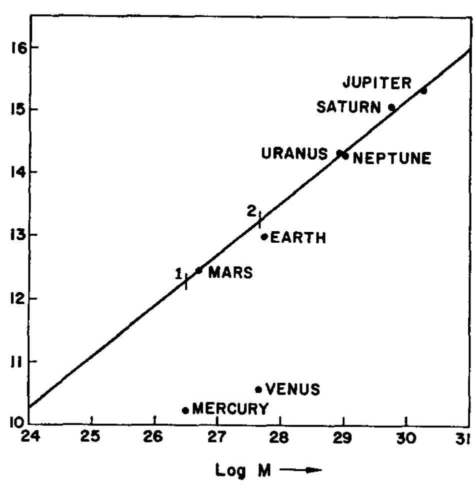

FIG. 2. Logarithmic plot of rotational angular momentum density vs. mass among the planets (cgs units).

$$Q_{☿} \leqslant 190, \qquad (31)$$

which corresponds to a tidal lag angle of not less than 9 arc minutes.

Venus

If the initial angular momentum density of Venus were indeed that suggested by location 2 in Fig. 2, then Ω_0 is approximately 1.34×10^{-4} sec^{-1}, corresponding to $P_0 \sim 13.5$ hr. Since the orbital eccentricity is small, we would expect tides raised by the Sun to despin Venus to a synchronous rotation with $P \sim 225$ days ($\Omega = 3.23 \times 10^{-7}$ sec^{-1}). But recent observations with radar ascribe to Venus a (e.g., Carpenter, 1966) retrograde rotation of $P \sim 244$ days. Possibly tidal drag was sufficient to bring Venus into synchronous spin long ago and then some other source of torque (perhaps atmospheric tidal effects as suggested by MacDonald, 1964) continued the retardation into retrograde rotation.

Assuming for Venus an internal structure similar to the Earth's (including a core), we use $\alpha = 1/3$ and set a lower limit on the mean rigidity at $b \sim 7$. Then (27) and (28) set an upper bound for the Q_p of Venus at

$$Q_{♀} \leqslant 17, \qquad (32)$$

which corresponds to a tidal lag angle $\epsilon \geqslant 1.^\circ 7$.

Earth and Moon

Tidal effects in the Earth–Moon system are by far the most complex in the solar system. In this case three separate interactions are of importance. They are the tides produced on the Earth by the Sun and Moon and the tides raised on the Moon by the Earth. Part of the complexity in this case arises from the large solar perturbations of the Moon's orbit. In addition, the Earth–Moon system is unique in that in this case the satellite's orbital angular momentum is not only comparable with, but actually exceeds, the planet's spin angular momentum. Those features of the Earth–Moon system lead to complications in the motion of the Moon's orbit plane and in the precession of the Earth's axis which are not encountered in other cases of solar system tidal evolution. Nevertheless, our presence on the Earth has enabled us to obtain far more detailed information about its Q than about the Q of any other solar system body. This knowledge comes from many sources, most of which have received ample treatment in the current literature. Thus, we shall only briefly mention those of greatest importance.

The most direct measure of the tidal effective Q for the Earth is obtained from astronomical observations of the Moon's secular acceleration. These observations lead to a tidal phase lag of $\epsilon = 2\overset{\circ}{.}16$ and a corresponding $Q = 13$ (MacDonald, 1964). The phase lag may also be estimated (but with considerably less accuracy) from direct observations of the body tides. These values possess a large scatter but are consistent with the astronomical value of $2\overset{\circ}{.}16$. An excellent brief summary, with references, of other lines of evidence bearing on the Earth's Q is given in MacDonald (1964).

The Moon's Q is very uncertain. However, extrapolation from the available data on the Q of the Earth's upper mantle (Knopoff, 1964) strongly suggests that its value is less than about 150. Indeed, if Q really decreases with amplitude as MacDonald (1964) has suggested, then Q for the Moon may be comparable to that for the Earth, the tidal strains on the Moon being somewhat larger than the terrestrial ones. Finally, by means of an indirect argument we can derive a lower bound for the Moon's Q. Our argument is based on the obvious synchronous rotation of the Moon. As is discussed by Goldreich (1965*c*), the Moon could not have attained synchronous rotation in an orbit having the present value of eccentricity (at least if its initial spin angular velocity were direct and of greater magnitude than its orbital mean motion). However, the Moon could have reached synchronous rotation if its orbital eccentricity was smaller in the past (by about 25% or more below the present value). As we have previously mentioned, tides raised on the Earth by the Moon tend to increase the eccentricity while tides raised on the Moon by the Earth act to decrease it. Using Eqs. (23) and (25), we find that these two effects just cancel, i.e., $de/dt = (de/dt)_p + (de/dt)_s = 0$, for $Q_s/Q_p = 0.79$, where we have set $b = 7$ in estimating μ for the Moon. If we believe that the Moon's orbital eccentricity is increasing, then $Q_s > 0.79\ Q_p = 10$. Thus we estimate

$$10 < Q_m < 150. \tag{33}$$

Mars and Its Satellites

The planet Mars is effectively outside the Sun's range of tidal influence. Further, the minute tidal bulges raised by Phobos and Deimos have not materially affected the planet's spin. However, these latter tides do affect the orbits of the two satellites, particularly that of the closer and larger Phobos. Since the "month" of Phobos is shorter than the Martian "day," the conventional tidal effects are reversed and the satellite should be gradually spiralling in toward the planet. Indeed a secular acceleration coefficient in its longitude of $+0\overset{\circ}{.}001882 \pm 0.000171\ \text{year}^{-1}$ has been reported (Sharpless, 1945). Assuming that this change is real and that it is caused entirely by tidal friction, it can be used in (21) to estimate Q_p for Mars, although torque terms due to the second order tidal potential ought to be retained because of the close proximity of Phobos.

This has been done by Redmond and Fish (1964) and by Fish (1965). From density and albedo considerations, they set an upper bound for the mass of Phobos at $<7.6 \times 10^{18}$ gm. Then estimating the tidal effective rigidity of Mars at $b < 8$, they show that the minimum tidal lag angle that would account for the observed secular acceleration in longitude is $1\overset{\circ}{.}1$. This corresponds to a Q_p for Mars of

$$Q_{♂} < 26, \tag{34}$$

comparable to the values for Venus and the Earth.

IV. Tides on the Major Planets

Jupiter

Tidal transfer of angular momentum is working to drive the satellites of Jupiter outward into everexpanding orbits. The very existence of close satellites supplies a lower bound to the Q_p of Jupiter, if we assume that they are about as old as the solar system. For if Q_p were too small, the orbits' evolutions would be too rapid and, tracing them back in time, the satellites would have been at the surface of the planet less than 4.5×10^9 years ago.

Thus, integrating (20) for a satellite,

$$\frac{2}{13}(r^{13/2} - r_0^{13/2}) = \frac{9}{2}\left(\frac{G}{M}\right)^{1/2} \frac{A^5}{Q'_p} m(t - t_0), \tag{35}$$

and the lower bound for Q_p is given by

$$Q_p \geqslant \frac{117}{4}\left(\frac{G}{M}\right)^{1/2} \frac{A^5 \Delta T}{(1 + 19\mu/2g\rho A)_p} \times \frac{m}{r^{13/2} - r_0^{13/2}} \tag{36}$$

where ΔT is the age of the solar system. Strictly speaking, r_0 ought to be set at the Roche limit for the satellite, but it will be neglected altogether since $r^{13/2} \gg r_0^{13/2}$.

The greatest lower bound to the Q_p of Jupiter is supplied by the satellite Io. It is $Q_{♃} \geqslant (8.05 \times 10^5)/(1 + 0.04b)$, where b is the tidal effective rigidity factor for the planet. Since the atmosphere of Jupiter may account for a substantial fraction of the total tidal dissipation, and since b for the outer layers of the planet is probably small for any reasonable model in any case, we will neglect rigidity here and for the other major planets as well.

An additional consideration affects the lower limit on Q_p for Jupiter. As a satellite orbit evolves, its mean motion may come into a low order commensurability with that of another (assumed independently evolving) satellite. Several such commensurabilities are known to be stable against disruption. Although the tides continue to transfer angular momentum independently to the satellites, this angular momentum is distributed between them by mutual perturbations in such a way as to preserve the commensurability (Goldreich, 1965*b*). Now Io is involved in a stable commensurability with Europa and Ganymede. Therefore, some of the angular momentum it receives from Jupiter is transferred to these other two satellites. Thus the rate at which its semimajor axis expands is somewhat smaller than $\dot{r}$ given by (20). Allowing for the possibility that the commensurability has existed for an appreciable fraction of the age of the solar system, the calculated lower bound to Q_p must be reduced by a factor of between 5 and 7.5 (Goldreich, 1965*b*). The lower limit to Q_p for Jupiter we arrive at then becomes

$$Q_{♃} \simeq (1 \text{ to } 2) \times 10^5. \tag{37}$$

The lower bound we have obtained for Jupiter's Q is at least three orders of magnitude greater than the values of Q we derived for the terrestrial planets. Unfortunately, our ignorance of the internal structure of Jupiter makes it impossible to estimate an independent Q value for the planet. However, we can investigate the rate of turbulent tidal dissipation in Jupiter's atmosphere and show that it is not inconsistent with the lower bound we have derived for Jupiter's Q. Our estimate of the tidal dissipation in Jupiter's atmosphere will lead to an estimate of an upper bound to Jupiter's Q (since we neglect dissipation in the planet's solid interior).

Let D be the depth of the atmosphere beneath the observed surface of radius A, and let ζ be the height of its fluid tidal distortion over and above its normal level (see

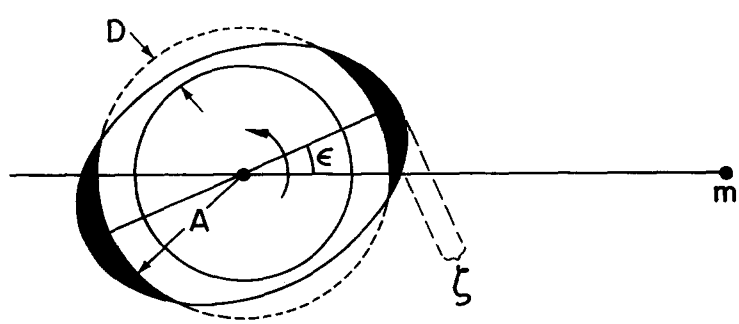

FIG. 3. Atmospheric tides of height ζ above a planetary atmosphere of depth D, where $\zeta \ll D$.

Fig. 3). Taking v as the velocity of the tidal current, the boundary layer thickness at the solid–fluid interface would be $\delta \simeq (\nu/2\Omega)^{1/2}$, if the boundary layer flow were laminar. Here ν is the fluid's kinematic viscosity and Ω is the planet's angular velocity (π/Ω is approximately the tidal period). The Reynolds number in the boundary layer is given by

$$Re \simeq v\delta/\nu \simeq v/(2\Omega\nu)^{1/2}. \tag{38}$$

Since most of the energy dissipation is due to the semidiurnal tide (shown in Fig. 3) the tidal period is approximated by one-half the planet's rotation period. The velocity v of the tidal current is less than the velocity of the tidal bulge, $2\pi A/P = \Omega A$, because the material contributing to the bulge flows under it from all depths in the atmosphere. In other words, the bulge is not a separate quantity of material racing along the surface of the atmosphere, but is instead continually replenished by deeper tidal currents of velocity v. The volume available to the currents is, per unit surface area, simply equal to the atmospheric depth D. Thus v equals the surface velocity of the bulge reduced by the ratio of the height of the bulge to the atmospheric depth D, or, approximately

$$v = \Omega A(\zeta/D), \tag{39}$$

where ζ is the equilibrium height of the fluid bulge, as defined in (5). Using the values for Io we estimate the velocity of the tidal current to be $v \simeq 0.17A/D$. The kinematic viscosity will, in any case, be less than 10^{-3} cm^2 sec^{-1} so that $Re \gtrsim 300$ A/D, where A/D is always greater than unity and in general will contribute at least one to two orders of magnitude to Re. Thus the Reynolds number criterion for turbulence in the boundary layer will be satisfied; whether turbulence actually arises is undoubtedly dependent on the degree to which the atmosphere is stably stratified at these depths. In any case, the rate of dissipation calculated on the assumption of a turbulent boundary layer will represent an upper bound to the actual dissipation rate.

Assuming a turbulent boundary layer, the energy dissipated by the tides per unit area is just $k\rho v^3$ (Jeffreys, 1962, Chap. 8.06), where $k = 0.002$ is the coefficient of skin friction, ρ is the density of the atmosphere, and v is the velocity of the tidal current. The total energy loss per tidal period becomes

$$\Delta E = (\pi/\Omega)k\rho v^3 \text{ ergs cm}^{-2}. \tag{40}$$

The potential energy stored per unit area in the tidal distortion is (Jeffreys, 1962, Chap. 8.08).

$$E_0 = \tfrac{1}{2}g\rho\zeta^2 \text{ ergs cm}^{-2}. \tag{41}$$

Then, using the definition of Q in (1), we find

$$Q \simeq \frac{2\pi E_0}{\Delta E} = \frac{4}{3}\frac{GM^2D^3r^3}{k\Omega^2A^9m}. \tag{42}$$

Evaluation in the case of tides raised by Io gives $Q_♃ = 1.4 \times 10^{-19}\, D^3$, and since this should be greater than or equal to the Q_p found by examining the orbital evolution of Io in (37), we can solve for a very rough lower limit to the depth of the Jovian atmosphere. The result is

$$D \geq 1000 \text{ km}, \tag{43}$$

which is comparable to recent independently deduced values, e.g., the upper bound of ~2800 km suggested by Hide (1962) from hydrodynamic considerations concerning the Great Red Spot.

An upper bound for Jupiter's Q_p may be estimated if we assume a tidal origin for the commensurabilities. Then the very fact that Io is found to be involved in a commensurability indicates that a considerable tidal evolution of its mean motion has occurred. Thus, the upper bound for Q must be low enough for this evolution to have taken place. Applying Eq. (20) we find $Q_♃ \leq 10^6$.

Saturn

The analysis for Saturn is similar to that for Jupiter. The innermost satellite Mimas provides the greatest lower bound on Q_p, and since Mimas has achieved a commensurability with the mean motion of another satellite, the upper bound is probably not far removed. We have (Goldreich, 1965b)

$$Q_♄ \sim (6 \text{ to } 7) \times 10^4, \tag{44}$$

and by the reasoning following (42), this supplies a lower bound to the depth of Saturn's atmosphere,

$$D_{♄} \geqslant 180 \text{ km}, \tag{45}$$

where we again assume that the boundary layer flow is turbulent.

Uranus

The existence of the innermost and non-commensurate satellite Miranda sets a lower bound for the Q_p of Uranus at

$$Q_{♅} \geqslant 7.2 \times 10^4. \tag{46}$$

Neptune and Pluto

Neptune has two known satellites: Nereid is tiny and distant while Triton, though massive and close, has a retrograde orbit and is thus approaching the planet. So neither satellite orbit yields information on the Q_p of Neptune by the method previously discussed.

A possible explanation for the retrograde motion of Triton was provided by Lyttleton (1936). If both Pluto and Triton were originally direct satellites of Neptune, then their separate tidal orbital evolutions could have eventually arranged a near collision. Provided that Pluto's mass is not too much greater than Triton's, such an encounter could have reversed the motion of Triton and discharged Pluto into its present orbit.

Now it may be shown (Sec. V) that all but the more distant satellites must relax into synchronous or near-synchronous rotation about a planet in a time short with respect to the age of the solar system. Thus if Pluto was once a satellite of Neptune, close enough for an encounter with Triton, it would have possessed a roughly synchronous rotation period. It is doubtful that the spin could have adjusted to the short-lived eccentricity induced by perturbations prior to the actual encounter. Pluto's present spin should then equal its mean motion near the time of its ejection by Triton, provided that this did not actually involve a physical collision.

Lyttleton's hypothesis has been enhanced by the subsequent determination of Pluto's rotation period at 6.39 days (Walker and Hardie, 1955). If this was once Pluto's synchronous rotation period about Neptune, this would place Pluto at the time of ejection just outside the present orbit of Triton which has a period of 5.88 days.

Such an ejection of Pluto may also account for Triton's presently observed large inclination ($\sim 20°$ to Neptune's equator in a retrograde sense). Triton's low eccentricity (zero to three figures) may have been damped from a larger value by tidal friction. We assume that prior to the encounter, both Pluto and Triton had direct orbits in or very near Neptune's equator plane. Dynamical considerations strongly favor low inclinations for close satellites of an oblate planet (Goldreich, 1965*a*). In conserving momentum, any encounter sufficient to eject Pluto would presumably reverse Triton and impart sizeable eccentricities and inclinations to both bodies. Subsequently, tides raised on Neptune and on the retrograde Triton would dissipate orbital energy and reduce the satellite's orbital eccentricity. From (23) and (24), we have for tides raised on Neptune by Triton,

$$(1/e)(de/dt)_p = -2.4 \times 10^{-18} \text{ sec}^{-1}, \tag{47}$$

where we have used $Q_p = 7.2 \times 10^4$, the lower limit obtained for Uranus.

This alone would be insufficient to damp out any reasonable eccentricity given Triton by the encounter. But from (25), tides raised on Triton by Neptune give

$$\frac{1}{e}\left(\frac{de}{dt}\right)_s = -\frac{9.4 \times 10^{-13}}{Q_s(1 + 5.6b_s)} \text{ sec}^{-1}. \tag{48}$$

If we assume for Triton that $Q_s b < 300$, an upper bound which is close to the one we derive for Iapetus (cf. Sec. V), then

$$\frac{1}{e}\left(\frac{de}{dt}\right)_s = -5.0 \times 10^{-16} \text{ sec}^{-1}, \tag{49}$$

which predicts a marked damping of Triton's eccentricity in times on the order of 10^{17} sec. In any case, the orbital inclination would not be strongly affected by the cumulative drag of tides, since it is not as sensitive to energy.

Even if Triton was *captured* into a retrograde orbit, the tides could still damp its eccentricity.

The radius of Pluto has been determined at ~ 3000 km from disc meter readings by Kuiper and Humason with the 200-inch telescope (Kuiper, 1950). Using this value, the mass of Pluto as calculated from perturbations of Neptune and Uranus would yield a density of ~ 50 gm cm^{-3}, so the mass remains in question (Brouwer, 1955). These perturbations are perhaps caused by a postulated trans-Neptunian cometary belt for which there exists independent evidence (Whipple, 1964). We shall see that the enhanced plausibility of the Lyttleton hypothesis lends credence to a less anomalous value for the density of Pluto. The hypothesis requires that Triton and Pluto have comparable masses. If we assume that the mass of Pluto is not more than twice that of Triton, then the density of Pluto will be less than ~ 2.3 gm cm^{-3}, a value which is consistent with those found for other satellites of the major planets.

V. Tides on Satellites of Major Planets

It is easily shown from (17) and (20) that, since the values of the intrinsic spin angular momenta of satellites are so small in comparison to their orbital angular momenta, the satellites would relax to synchronous or near-synchronous rotations before the tides could substantially evolve their orbits, even if the satellites had values of Q as large as those of their planets. The eccentricities of close satellite orbits are small, consistent with the general dominance of $(de/dt)_s$ over $(de/dt)_p$ as suggested by Goldreich (1963). But even with a sizeable eccentricity, a close satellite would achieve near-synchroneity (as has Mercury).

All satellites for which rotations have been observed exhibit synchroneity. This is inferred from the equivalence of any regular magnitude variation period and the satellite's orbit period. But since most of these satellites achieved synchroneity relatively soon after they were formed, this tells us little about their Q values. We can obtain a useful upper bound on Q only for those more distant satellites which are known to have achieved synchronous rotation. Iapetus (Saturn VIII) is the most distant satellite from its planet for which there exist reliable photometric observations, and fortuitously, it exhibits a regular sinusoidal light variation of about two magnitudes (by far the largest for any planet or satellite) which corresponds to its orbital period (Widorn, 1950). This secure evidence of synchronous rotation for Iapetus sets a meaningful upper bound on its Q, providing we know roughly its rigidity and the initial rotational angular velocity from which it has been retarded.

If we simply plot Iapetus by mass in Fig. 2 and read off the initial spin angular momentum density, the corresponding initial rotation period is about 5 days. Now this angular momentum density line for planets may have a slope and intercept entirely different from a similar curve for a family of satellites. However, it may be an improvement over mere guess work to obtain rough initial rotation rates for satellites from Fig. 2. In any case, it seems reasonable that Iapetus was originally spinning at only a fraction of its present synchronous period of 79.33 days, and so we provisionally adopt an initial period of 5 days.

Integrating (17), we obtain an upper bound on Q_s analogous to that in (27) and (28)

$$Q_s \leq \tfrac{45}{8}\, GM^2 \frac{a^3}{m(1 + 38\pi a^4 b \times 10^{11}/3Gm^2)r^6} \times \frac{\Delta T}{\omega_0 - \omega}. \quad (50)$$

Using ω_0 corresponding to 5 days for Iapetus, we obtain for that satellite

$$Q_s b \leq 290. \quad (51)$$

The rigidity for small satellites is unknown except that it is probably not very great, say $b < 5$, because of the relatively low internal pressure. In any case, $Q_s < 500$.

VI. Discussion and Summary

Let us begin by taking a critical look at each of the Q values we have obtained in order to assess its reliability. We start with Mercury.

The explanation of Mercury's rotation put forth by Peale and Gold (1965) proves beyond reasonable doubt that Mercury is very close to its final spin state and that this was brought about by tidal friction. The upper bound of $Q = 190$ that we have derived for Mercury involves uncertain values for the planet's rigidity and for its initial rate of rotation. Of these two factors the latter is undoubtedly the more uncertain. However, it is hard to believe that our estimate of the initial spin rate could be in error by more than a factor of 5, so we consider the derived upper limit for Q to be quite reliable. In addition, tests of laboratory rocks lead to Q values of this order or smaller, which gives us an added reason for confidence.

The case for Venus is much more uncertain. The slow retrograde rotation of the planet could not have resulted purely from conventional tidal torques (if the planet were originally rotating in the direct sense). Thus, we must invoke the presence of another source of torque, which must dominate the ordinary tidal torque for at least some fraction of the time. The question then arises as to whether this other torque might not always have been dominant on Venus. This suspicion is heightened by the low upper bound we have derived for the Q of Venus. In particular, we suspect that tides of thermal origin in the planet's dense atmosphere may be responsible for the observed rotation of Venus. Thus we conclude that the upper bound of Q for Venus, which we set at 17, must be regarded as uncertain.

For the Earth we consider the present value of Q to be very well determined (to within factors of a few) from observations of the Moon's secular acceleration. In addition, values of Q derived from seismic observations and from the damping of the Chandler wobble give us additional, somewhat larger, values for processes which involve smaller strains.

The case for Mars is again rather uncertain. The low value for Q (<26), derived from the secular acceleration of Phobos, is rather surprising in view of the small tidal strains that are involved. This small value must be viewed with distrust compared with the larger ones obtained from laboratory tests of rocks (at the same values of strain). The reality of the reported secular acceleration is, however, in doubt.

We have no direct evidence bearing on the Q's of the Moon or of the satellites of Mars. However, based on the values obtained for Mercury and the Earth, together with those obtained in laboratory tests of rocks, we conclude that these objects probably possess Q's in the range from 10 to 500.

For the planets Jupiter, Saturn, and Uranus we have derived lower limits for Q of 1.0×10^5, 6.0×10^4, and 7.2×10^4, respectively. These values are quite secure, being based merely on the existence of close satellites. For Jupiter and Saturn, the presence of commensurate satellites indicates that the lower bounds for Q are probably not far removed from the actual values.

As regards values of Q for the satellites of the major planets, we have two separate pieces of information. The first, involving the synchronous rotation of Iapetus is quite straightforward. The large magnitude variation displayed by Iapetus makes it clear that we are dealing with an example of synchronous rotation. Because of its great distance from Saturn we find that this implies that Iapetus must have a Q which is smaller than about 500, a value which is derived by choosing a very low value for the satellite's rigidity. As a more probable upper limit we would estimate $Q \simeq 150$. The second, indirect piece of evidence involves the low (zero to three places) eccentricity of Triton's orbit. This satellite moves on a retrograde orbit inclined by 20° to the plane of Neptune's equator. Its unusual orbit (compare with the orbits of the larger satellites of Jupiter and Saturn, all of which are direct and have nearly zero inclinations) makes it appear that Triton has been the victim of a near collision with another object at some past date. It would be truely remarkable if such an interaction left Triton with a zero orbital eccentricity. We expect then, that the eccentricity of Triton's orbit was subsequently damped to its present low value. When this damping was investigated in more detail (in Sec. IV) it was found that the tides raised on Neptune by Triton would have been unable to produce sufficient damping in 4.5×10^9 years. On the other hand, tides raised on Triton by Neptune could have produced the requisite damping, if Triton had a Q of several hundred or less. Thus, we conclude that Triton probably possesses a $Q \lesssim 200$, which is comparable to the value that we derive for Iapetus.

The values of Q that we have obtained divide sharply into two classes. The first class, containing values from ten up to several hundred (the larger values are upper limits, not actual values) includes the Q's of the terrestrial planets, their satellites, and the satellites of the major planets. The second class, consisting of the Q's of the major planets, has as lower bound the value of 6×10^4 derived for Saturn. Larger values still are obtained for Uranus and Jupiter.

There is a fundamental difficulty involved in comparing the Q's of different bodies. We expect the amount of energy dissipated in the distortion of a body to be proportional to the square of the amplitude of strain. The energy stored during the distortion will be of two kinds, elastic and gravitational. The former will depend only on the strains, while the latter will be a function of the size of the body as well. Thus, for equal strains in two bodies of identical material the stored energy per unit volume will be larger in the larger body. Since the energy dissipated per unit volume will be the same for both bodies, the larger body will have the larger Q. This difficulty is only of importance when the stored gravitational energy is comparable to, or exceeds, the stored elastic energy. Thus, for the terrestrial planets and satellites it is only of slight relevance. However, for the major planets it may be somewhat more important. In particular, distortion of Jupiter's presumed solid interior will involve storing comparable amounts of gravitational and elastic energy if its rigidity $\mu \simeq 2.5 \times 10^{12}$, a value slightly greater than twice the tidal rigidity of the Earth. The problem of comparison is further complicated by the varied nature of fluid portions of the different planets. In spite of these difficulties a very real difference has been demonstrated between the properties of terrestrial planets, their satellites, and the satellites of the major planets on the one hand and, on the other hand, the properties of the major planets themselves. For similar tidal strains, the rate of energy dissipation in the former objects exceeds that in the latter group by a factor of at least 10^3. Also, while the magnitudes of the Q's that we have derived for the first group are consistent with values obtained from laboratory tests of solids, the values obtained from the second group are much higher than the experimentally determined ones. Perhaps a clue to the interior structure of the major planets is to be found from their large values of Q.

As a final remark, we mention that use of the Q's that we have obtained for the major planets and Iapetus enables us to check the consistency of Lyttleton's hypothesis on the origin of Pluto. We find that tides raised on Triton by Neptune would have been able to damp Triton's eccentricity, provided the satellite's Q is comparable to that of Iapetus. Furthermore, the predicted rate of change of semimajor axis of Triton's orbit is sufficiently small (provided Neptune has a Q similar to the one for Uranus), so that the similarity of Triton's orbital period (5.9 days) with Pluto's rotational period (6.4 days) suggests that Pluto may once have been a satellite of Neptune, in an orbit close to Triton's. Finally, the requirement that Pluto have a mass comparable to that of Triton, together with the observed diameter of Pluto, implies a reasonable value for its density.

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