Foreign Firms, Domestic Entrepreneurial Skills and Development*

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February 2007.

Abstract

In this paper I examine the impact of foreign firms for the accumulation of domestic skills in a developing country. The presence of foreign firms unleashes two countervailing forces. The first is a "diffusion effect" that boosts skill accumulation as local entrepreneurs are exposed to more advanced knowledge. The second is a "competition effect" that busts skill accumulation as local entrepreneurs foresee heightened competition in the future. I consider two models for diffusion. In the first, knowledge is a local public good and diffusion takes place via "spillovers". In the second, the costs and benefits of skill accumulation are fully internalized and diffusion is the result of efficient market transactions.

When diffusion takes place via spillovers, two stable steady states may arise and initial conditions determine the dynamics of the country. I show that opening to foreign firms can be welfare decreasing and that leapfrogging can occur because less backward countries converge more slowly or not at all while more backward countries always converge. When diffusion is fully internalized, opening is always welfare increasing. However, the presence of foreign firms leads to a reduction in the productivity of existing domestic firms, which is consistent with some empirical evidence.

*Along different stages, this paper greatly benefited from my conversations with Pol Antràs, Gadi Barlevi, Paco Buera, Hugo Hopenhayn, Boyan Jovanovic and Andrés Rodríguez-Clare. The usual disclaimer applies.
1. Introduction

Consider a country that is technologically lagging behind the rest of the world: Does opening to foreign firms accelerates the country’s accumulation of its own domestic level productive know-how? Does openness lead the country to fully catch up with the more advanced countries? Can openness instead lead the country to lag behind even further? Should the local government promote foreign firms? The purpose of this paper is to answer these questions using simple general equilibrium growth models.

The recent surge of FDI and multinational activity has motivated a renewed interest in the distributional and aggregate consequences of foreign firms. As shown by Antras, Garicano and Rossi-Hansberg (2006), importing skills from more advanced countries can have interesting redistributional implications, even within occupations, as the influx of skills change the assignment of individuals to tasks and the organization of production. At the aggregate level, Burstein and Monge-Naranjo (2007) quantify significant gains in the output and aggregate consumption of developing host countries, specially if individuals can be reallocated across occupations and if physical capital accumulation respond to the enhanced productivity. Ramondo (2006) shows that the gains are even larger if instead of rival skills, the productivity of firms is a non-rival factor that can be reproduced without abounds across countries. However, all these papers take skills and productivity as exogenously given, and hence are silent about their behavior over time.

The purpose of this paper is to study the impact of foreign firms on the accumulation of domestic entrepreneurial or managerial skills (hereafter “skills”) in a developing country. Opening to foreign firms unleashes two countervailing forces. The first force is a “diffusion effect.” The exposure to the more advanced know-how and the market ideas of foreign firms reduces the costs of local entrepreneurs to produce their own know-how. The second force is a “competition effect.” The more advance foreign competition reduces the return to accumulating skills for local entrepreneurs. With simple general equilibrium models, I will analyze how the combination of these two forces and initial conditions determine first, whether

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1 See, for example, Barba-Navarretti (2004), Markusen (2004)
2 The simple models considered in this paper cannot distinguish entrepreneurial from managerial abilities, skills and know-how. Examples of models in which the distinction is clear and important are Holmes and Schmitz (1991) and Chari, Golosov, and Tsyvinski, A. (2004).
a country would gain by opening to foreign firms, and second, whether the country would accelerate the accumulation of skills and even catch up with developed countries.

I study two alternative formalizations of “diffusion”. In the first model, the diffusion of productive knowledge is via "spillovers." Inside each country, knowledge is accumulated over time on the basis of the stock of ideas that are implemented in the production of goods within the geography of the country. Similar to the models of Romer (1986) and Lucas (1988), and as in the model of Stokey (1991), the knowledge of past generations is a local public good that impact the current generation of future entrepreneurs when they are forming their own skills. Under simple parameter restrictions, a closed economy exhibits constant growth. For an open economy, the impact of foreign firms on this local public good of knowledge circulating in the country depends on their quantity and their relative quality with respect to domestic firms.

The second model considers an entirely different approach. Here, similar to Chari and Hopenhayn (1991) and Jovanovic and Nyarko (1995), and, as in the model of Prescott-Boyd (1987), I assume that entrepreneurial skills are formed in the interior of the firm. Direct involvement of experts is required for young agents to learn from them. Therefore, a well functioning market for the transfers of skills operates. Young workers fully anticipate the consequences of enrolling in firms with different characteristics and offering different compensation and learning opportunities. Entrepreneurs—the expert agents leading firms—also fully perceive the cost and benefits of forming skills. A competitive equilibrium is efficient. I provide the conditions for a closed economy to exhibit constant growth. In this environment, foreign firms diffuse skills and productivity to the host economy not via externality but instead by efficiently providing learning opportunities to the workers with entrepreneurial potential working for them.

The two models of diffusion are studied within a common, simple OLG environment. Production is carried out by teams of a manager and a set of workers. The output of goods of each team is determined by the number of labor units and the skills of the manager. There are two types of individuals, both of which live for two periods. The first type are “laborers”: they are workers in both peri-

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4I abstract from any contractual friction such as adverse selection that precludes the efficiency of the equilibrium in Jovanovic-Nyarko (1995).
ods. The second type are “potential managers.” When young, they are workers; when old, they have the option to become managers. Managers hire workers in a competitive labor market and become the residual claimants of their teams.\(^5\) Both types maximize the present value of their lifetime net income. To this end, however, potential managers have to make two choices: being worker or manager and the optimal investment in skill formation.

A closed country is one in which only national managers can lead production teams. An open country is one in which foreign managers can relocate their skills to the country and lead a team of local workers.\(^6\) Such a simple model conforms with the observation that multinational firms rely heavily on home expatriates—and home trained individuals—to manage their operations, specially in developing countries [see Chapters 5 and 6 of UNCTAD (1994).] It also conforms with the emphasis of the existent literature on firm specific intangible assets for multinational activity [e.g. Barba-Navarretti (2004), Markusen (2004).] Many other interesting aspects of multinational activity are omitted to focus on the diffusion and accumulation of skills.

Before analyzing the equilibria in my two models, and to put in context the results of this paper, I first consider as a benchmark the standard model of diffusion of Findlay (1978). Findlay’s model is the implicit theoretical framework motivating much of the empirical work on diffusion (discussed below). In this model, skill accumulation is exogenous and costless. The presence of foreign firms have spillovers that enhance the set of ideas, and therefore, the future productivity of domestic firms. It is easy to see that in the OLG environment studied here, an open developing country will asymptotically catch and foreign firms will eventually disappear. Opening is always welfare enhancing.

In my first model, I also assume that diffusion takes place via “spillovers”. However, as in Stokey (1991), skill formation is costly. Young agents invest to assimilate and improve upon the ideas to which they are being exposed to. Under suitable parameter restrictions, closed economies grow at a constant rate. Open economies, however, exhibit a more complex dynamics, exhibiting a form of predator-prey dynamics that can result in two steady states (of relative income). In one the economy converges in levels to the rest of the world. In the other, there is no full convergence. The occupation choice in the environment implies

\(^5\)That is why in this simple model “manager” and “entrepreneur” are synonyms.

\(^6\)In all cases, I assume that workers are fixed in their country. See Klein and Ventura (2004) for an analysis of cross country labor mobility.
that aggregate dynamics can be quite stark. Indeed, a developing catches up to the world leaders if and only if, for one period national domestic firms are entirely shut-down and all production is lead by multinational firms. Otherwise, inferior local know-how operates as a barrier that prevents foreign more productive know-how to fully operate in the economy and impact the skill formation of the young generation of potential managers.

This model can be used to isolate the impact of the diffusion effect from that of the competition effect. In this model, if spillovers are not present, regardless of the initial relative skills of the local managers, then in the limit the country converges to zero skills. All local agents are workers and all entrepreneurial skills are foreign. Moreover, open is welfare enhancing only if the country has very low initial levels of entrepreneurial skills. Otherwise, the country would be better-off remaining closed. The presence of spillovers introduces the force that push the economy to converge to the skill levels of advanced countries. However, if local skills are high enough, then the country might not converge. Two interesting implications arise. First, there is leapfrogging as the more backward countries catch up more quickly than relatively more advanced developing countries. Second, opening may be welfare reducing for more advanced developing countries.

In the second model spillovers are not present. Every period skills are accumulated inside the firm. The cost of producing those skills is in terms of foregone output of the firm. The return of accumulating skills is twofold: First, the skills enhance the ability of the future team in producing consumption goods. Second, those skills reduce the cost of producing skills in the next period. Besides the convexity of the profit function with respect to the manager’s skill in a the standard span-of-control of Lucas, the self-productivity of skills can easily lead to dynamic increasing returns unless the cost of producing skills is convex enough. I provide conditions upon which there is a unique constant growth rate that characterizes type-symmetric optimal allocations and equilibria. Under those conditions, type-symmetric closed economies are always in a balanced growth path. 7

When foreign firms enter a developing country, they use the transfer of skills to their workers as a means to compete with national firms and other multinational firms. In equilibrium they transfer the same level of skills as firms operating in developed countries. Along this process, a new sector of national managers (and firms) arise, each one with the same level of skills as their peers in developed

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7 The conditions in Boyd and Prescott (1987) and Prescott and Boyd (1987) are not enough to insure this result.
countries. The fraction of these new managers grows and in finite time fully takes over the entire economy. Thereafter the country would have the same level of income as developed countries. In this transition process, the sector of pre-existent national firms not only falls in size but also in relative productivity.

There is ample evidence that skill formation at the interior of the firm is an important form of productivity diffusion. For the U.S. car industry, Keppler (2001, 2002, 2006) documents that the genesis of the most important car makers can be traced to former employees of other important car makers. Agarwal et al (2004), Filson and Franco (2006) and Franco (2005) document the same for the rigid disk drive industry. In both of these cases, the authors provide ample information on the spin-offs and on the relationship between the characteristics and outcomes of parent firms with their progeny. At the international level there is also ample, albeit anecdotal, evidence. Perhaps the best known case is the build up of a textile sector in Bangladesh by a Korean firm. But, despite the evidence that some multinational firms spend significant resources in training their workers [see UNCTAD (1994)] little empirical work has been done on the skill formation investments by multinational firms. An exception is the recent paper by Poole (2006) who presents evidence on multinational firms in Brazil.

Instead, the empirical literature has been focused on establishing the existence and the magnitude of technology spillovers. Most of the attention has been placed on intraindustry spillovers. Interestingly, finding supportive evidence has proven to be quite an endeavor. While there is some evidence of positive spillovers for developed countries [e.g. Griffith et al (2002)], for developing countries most authors come back empty-handed when trying to revert the negative results of Aitken and Harrison (1999).9

What has been found is empirical evidence of inter-industry spillovers, in particular, to upstream industries [e.g.: Javorcik (2004) and Kugler (2005)]. However, Javorcik also reports direct involvement of the multinational firms in upgrading the productivity of their suppliers. Such transactions are better seen as voluntary transactions, not spillovers. In light of the third model, in this paper I argue that for productivity diffusion to materialize, foreign firms must have the incentives to get involved in building up the necessary skills. The model is consistent with the result that competing domestic firms will (optimally) reduce their investments in upgrading productivity.

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8 This case was popularized by Easterly (2001).
9 See Alfaro et al. (2006) for a detailed discussion of the empirical findings.
There is a vast literature on the diffusion of productivity across countries. For the most part, the attention has been directed at the diffusion of technological knowledge and the role played by the trade of goods.\textsuperscript{10} By its very nature, technological knowledge is a “non-rival” factor in the sense that its use by one party does not congest the use by others. Once it is produced, a non-rival factor could be used at negligible cost by any number of agents.\textsuperscript{11} The focus of this paper instead is on the diffusion of rival skills, which are complementary since technological knowledge requires properly skilled individuals to be productive. The scarcity of skills could explain why enormous income disparities persist in a world where most knowledge can be obtained—or at least be located—after a few clicks in Google. Moreover, it can also explain why productivity innovations seem to diffuse more easily to developed countries than to developing countries.\textsuperscript{12}

The rest of the paper is as follows. Section 2 lays out the basic environment and Section 3 characterizes the equilibrium of closed and open economies in the exogenous growth model. Section 4 considers the endogenous growth model with externalities. Section 5 develops the endogenous growth model, provide the conditions for existence and uniqueness of a balanced growth path and its efficiency examines the response of an economy that opens up. Section 6 concludes including some brief reflections on policy. An appendix contains the proofs of the main propositions.

2. The Model

In this section I lay out the simple environment and provide some preliminaries on the equilibrium that I will later use for alternative accumulation models and assumptions on openness.

2.1. The environment.

Consider an infinite horizon, discrete time, overlapping generations economy. Individuals live for two periods, youth and maturity. Agents born in period $t$ value

\textsuperscript{10} For instance see Eaton and Kortum (2004), Grossman and Helpman (1991), Klenow-Rodriguez Clare (2005), and Rodriguez-Clare (2006).

\textsuperscript{11} Jones (2006), Klenow (1998) and Romer (1990) discuss non-rival factors in the literature of growth.

\textsuperscript{12} For instance, positive intraindustry spillovers of multinational firms are found for developed countries but not for developing countries. See Alfaro et al. (2006).
linearly their own consumption during periods \( t \) and \( t + 1 \). Their utility is

\[
U_t^t = c_t^t + \beta c_{t+1}^t,
\]

where \( c_t^t \) is the consumption during period \( t \) of an individual born in period \( t \) and \( 0 < \beta < 1 \) is a discount factor.

Each period the cohort of newborns is a continuum of size one. All individuals have an endowment of one unit of time in the periods in which they are alive. Each cohort is composed of two types of agents: a fraction \( \omega \in (0, 1] \) of potential firm leaders or “managers” and a fraction of \( 1 - \omega \) of individuals that are laborers. Laborers are workers in both periods of their life. Potential managers are also workers during their youth, but in the second period they have the choice of being managers or workers.

There is a single consumption good and production is carried out by teams of a manager and workers. A manager with skills \( z \) commanding \( n \) units of labor services produce \( y \) units of consumption goods according to the production function

\[
y = zn^\alpha,
\]

where \( \alpha \in (0, 1) \), is the “span-of-control” parameter that dictates the degree of decreasing returns to the labor units controlled by the manager. As in Lucas (1978) I call these teams “firms” and assume that the skills of the manager determine the productivity of the firm by limiting the technologies that the firm can efficiently use and the available market opportunities.

2.2. Equilibrium Preliminaries

As in Lucas, I assume that managers are the residual claimant of the firm. I will consider perfect foresight competitive equilibria under alternative models for the accumulation of skills and openness (to be defined below) to foreign firms. In all the cases financial and labor markets are frictionless and competitive. In each case, the price system is solely determined by the market clearing sequence \( \{w_t\}_{t=0}^\infty \) of wages to workers because the linearity of preferences pins down the interest rate is \( \beta^{-1} \), the inverse of the discount factor.

Because it is common to the different cases, it is convenient to characterize here the optimization problem of active managers, taking as given their level of skills. Facing a wage \( w \), a manager with productive skills \( z \) maximizes his rents
\[ \pi(z, w) = \max_{n} \{zn^\alpha - wn\}. \]

This is a simple and standard problem. The optimal hiring of labor is

\[ n^*(z, w) = \left[ \frac{\alpha z}{w} \right]^{\frac{1}{1-\alpha}}, \quad (2.1) \]

and the rents attained by the manager are

\[ \pi(z, w) = \theta z^{\frac{1}{1-\alpha}} w^{\frac{\alpha}{1-\alpha}}. \quad (2.2) \]

where \( \theta \equiv [\alpha]^{\frac{\alpha}{1-\alpha}} \) \((1 - \alpha) > 0\).

Moreover, since a potential manager has the option to be a worker, he would choose to be a manager if and only if his net earnings as manager are higher than his earnings as workers, i.e. \( \pi(z, w) > w \).

In the case that all potential managers choose to be active firm leaders, then, the total mass of workers is given by \( 2 - \omega \), which is composed of the mass of 1 young agents plus the mass of \( 1 - \omega \) old laborers. In such case, the ratio of workers-per-managers is given by

\[ \eta \equiv \frac{2 - \omega}{\omega} \geq 1. \]

If in addition, the level of skills \( z \) of each national manager is equal to a level \( Z > 0 \) then all managers hire the same number of units of labor and from (2.1) the market clearing wage is

\[ w = \alpha Z \eta^{\alpha-1}, \quad (2.3) \]

and the managers’ rents would be

\[ \pi = (1 - \alpha) Z \eta^{\alpha}. \quad (2.4) \]

Notice that \( Z \) shifts both options proportionally, as more skillful managers bid up the wages for worker.

For all potential managers to become active ones, the latter option has to be greater or equal than the former. Throughout the paper I will assume the following condition holds:

**Condition 1:** \( \eta > \frac{\alpha}{1-\alpha} \).
It is immediate to verify that under this condition implies that $\pi > w$, and all potential managers prefer to be active managers.\footnote{I impose this condition for simplicity. If the direction of the inequality is reverted, then in equilibrium potential managers would be indifferent between the two choices and only a fraction would become active managers.}

These simple static equilibrium conditions are later used to characterize the equilibrium accumulation of skills $\{Z_t : t \geq 0\}$ under alternative assumptions. The main purpose is to analyze the implications of “foreign firms” in the formation and diffusion of skills. To this end I compare the equilibria of closed economies –in which only local managers can hire local workers– with open economies –in which foreign managers can lead and control teams of local workers.

I use the subscript $n$ to indicate a national variable of the “home” country, the subscript $f$ to indicate a variable of the “foreign” country and the subscript $g$ to indicate a “geographic” variable in the home country. I use lower cases for individual variables and upper cases for aggregates or averages.

### 3. A Benchmark

As in a standard Solow growth model, I will start assuming that the engine of growth is a recurrent and exogenous improvement in the productive know-how available in the economy. In this environment, this know-how is embedded in the skills of potential managers. This process can be thought of as the young cohort learning, revising and improving the productive knowledge of the old cohort.

Specifically, if the set of managers operating in the country has an average $Z_g$ of skills, then, each member of the next generation of national managers will have skills $Z'_n$ given by

$$Z'_n = GZ_g,$$

where $G > 1$ is the gross growth rate. Closed and open economies will differ in the value of $Z_g$, the set of productive ideas surrounding the agents in the young (formative) stage of life.

#### 3.1. A Closed Economy

Assume that all potential managers have skills $z_n$ equal to $Z_n$. Under Condition 1 all those who can become active managers and hire $\eta$ workers. Market clearing
wages and managers earnings are given by (2.3) and (2.4) where \( Z_n \) takes the place of \( Z \).

In a closed economy, the geographically available productive knowledge is equal to the knowledge accumulated by nationals, i.e. \( Z_g = Z_n \). With the assumed process for accumulation, for the next period we have \( z_n' = Z_n' = GZ_n \).

Domestic output \( (Y_g) \) is equal to national output \( (Y_n) \) and given by

\[
Y_g = Z_n \omega^{1-\alpha} (2 - \omega)^\alpha.
\]  

(3.2)

Aggregate national consumption \( (C_n) \) is also equal to output since the accumulation of skills requires no real resources. These expressions characterize the unique Balanced-Growth-Path (BGP) since they follow the law of motion of \( Z_n \).

### 3.2. A Small Open Economy

Consider now an economy that is open in the sense that foreign managers can frictionless hire workers inside the country. I will consider only the small economy case. This is, in solving for the equilibrium of the home country, I take as given all the present and future variables of the foreign country, which follows the same path \( Z_f' = GZ_f \) and equilibrium conditions as discussed above. Also, I assume that \( Z_n < Z_f \), i.e. the country starts lagging behind the rest of the world in the sense

#### 3.2.1. Labor Market Clearing

With free entry, foreign managers must be indifferent between operating in the foreign country or in country \( n \). Since the home home country is small, the rents of foreign managers operating therein are

\[
\pi_f = (1 - \alpha) \eta^\alpha Z_f.
\]

(3.3)

The entry of foreign managers takes place until the national wage \( w_n \) is equal to the foreign wage

\[
w_f = \alpha \eta^{\alpha-1} Z_f.
\]

(3.4)

Facing this wage rate, each foreign manager would hire

\[
n_f = \eta,
\]
units of labor while active national managers—using (2.1)—would hire

\[ n_n = \eta \left( \frac{Z_n}{Z_f} \right)^{\frac{1}{1-\alpha}}, \]

and attain net earnings of

\[ \pi_n = (1 - \alpha) \eta^\alpha Z_n \left( \frac{Z_n}{Z_f} \right)^{\frac{\alpha}{1-\alpha}}. \]

Regardless of their skills \( Z_n \), potential managers have the option of earning \( w_f \) working for foreign managers. It is easy to verify that they would choose to be managers if the relative level of skills is above a threshold level \( R_S \):

\[ \frac{Z_n}{Z_f} > R_S \equiv \left[ \frac{\alpha}{\eta (1 - \alpha)} \right]^{1-\alpha}. \]

Under Condition 1, \( R_S \) is strictly less than one. Therefore, even if domestic potential managers are less productive than foreign managers, if their level of productivity is sufficiently high, they become active managers.

It is left to determine the mass of foreign managers operating in the country. Define \( m \in [0, 1] \) to be the fraction of the national workforce that works for foreign managers. The mass of foreign managers is \( m/\eta \) times the mass of the labor force since each foreign manager hires \( \eta \) local workers. Notice that if \( Z_n/Z_f < R_S \), the mass of the national workforce is 2 because it is the entire population; otherwise its mass is \( 2 - \omega \), as all potential managers are active.

Therefore, if \( Z_n/Z_f < R_S \), the entire national population works in multinational firms, \( m = 1 \), and the mass of foreign managers is \( 2/\eta \). If instead \( Z_n/Z_f \geq R_S \), then the fraction \( m \) is given by the national labor market clearing condition

\[ m (2 - \omega) + \omega \eta \left( \frac{Z_n}{Z_f} \right)^{\frac{1}{1-\alpha}} = (2 - \omega), \]

where the terms in the left side are, respectively, the aggregate demand for labor by foreign and national managers. Using the definition of \( \eta \), the previous condition implies that \( m \) is given by

\[ m = 1 - \left( \frac{Z_n}{Z_f} \right)^{\frac{1}{1-\alpha}}. \] (3.5)
The presence of multinational firms is a decreasing function of the productivity of the domestic competition. When $Z_n/Z_f < R_S$, all the geographic output of the country is generated by multinational firms. When $Z_n/Z_f > R_S$ geographic output is generated by a combination of national and multinational firms. After some algebra, it can be shown that $Y_g$ is given by:

$$Y_g = \begin{cases} 2\eta^\alpha \frac{Z_f}{Z_n} & \text{if } Z_n/Z_f < R_S \\ \omega \eta^\alpha Z_f & \text{if } Z_n/Z_f \geq R_S. \end{cases}$$

Notice that in the second case, the output of the home country is the same as the output in the foreign country. Moreover, the geographic output is higher when there are no national firms operating. However, the level of national income (and aggregate consumption) is higher when national firms operate. After removing the aggregate net earnings of foreign managers, it can be verified that

$$C_n = Y_n = \begin{cases} \alpha Y_g & \text{if } Z_n/Z_f < R_S \\ Y_g \left[ \alpha + (1 - \alpha) \left( \frac{Z_n}{Z_f} \right)^{\frac{1}{1-\alpha}} \right] & \text{if } Z_n/Z_f \geq R_S, \end{cases}$$

The first branch follows directly from the fact that, in this case, nationals of the country are all workers. In the second case, national agents are both workers and managers; the fraction of the income depends on the share of the total rents of managers that accrue to national managers which is a direct function of $Z_n/Z_f$. Clearly, $Y_n < Y_g$ and it can be directly verified that $Y_n$ is higher in the case $Z_n/Z_f \geq R_S$, i.e. when domestic firms operate.

### 3.2.2. Isolated skill formation

If we assume that the skills in the country and in the rest of the world follow autonomous the laws of motion (??), then the ratio $Z_n/Z_f$ remains constant over time. The static equilibrium described above also describes a steady state.

### 3.2.3. Spillovers in skill formation

Consider instead that as the presence of foreign firms brings know-how to the productive activities in the country, it also enhances the quantity and quality of the ideas to which young agents are exposed to. Similar assumptions can be find in the infinitely lived agents models of Romer (1986) and Lucas (1988).
particular, in a model of multinational firms, Findlay (1978) directly assumes that the superior technology of foreign firms spills over the productivity of local firms. Findlay’s paper is the theoretical foundation of the large empirical literature on spillovers using data on Foreign Direct Investment (FDI).

I will assume that the set of ideas \( Z_g \) within the geographic boundaries of the country is a local public good and is given by a geometric average of the know-how of local and foreign firms:

\[
Z_g = (Z_f)^m (Z_n)^{1-m}.
\]

where, as before, \( m \) is the fraction of all young agents that work in multinational firms. I assume that \( m \) also represents the fraction of potential managers that work for multinational firms.

Under this assumption, there is an externality in the formation of know-how since neither local nor foreign managers receive compensation for their contribution to the ideas available to young workers.\(^{14}\) While ad-hoc, this formulation has three desirable properties: (a) \( Z_g \) is increasing in both \( Z_n \) and \( Z_f \) (b) the relative importance of \( Z_f \) increases with \( m \) reflecting the intensity in which the country is exposed to foreign ideas; and (c) \( Z_g \) is homogeneous of degree one in \( Z_n \) and \( Z_f \) and there are no scale effects driven by the total mass of managers operating in the country.

Since all potential entrepreneurs are surrounded by the same set of ideas, next period each one will command skills of the form:

\[
z'_{n} = Z'_{n} = GZ_g.
\]

Using the expressions (3.6), it can be shown that when \( 0 < m < 1 \) the growth rate of the skills of local managers follows:

\[
\frac{Z'_{n}}{Z_n} = G \left( \frac{Z_f}{Z_n} \right)^{1-(Z_n/Z_f)^{1-m}}.
\]

\(^{14}\)It might be helpful to think of this model as a happy hour diffusion: Imagine that everyday, after work, all the young workers go to a bar for a happy hour. Among more other things—and with objectives different than training—they tell each other the specifics of what they do and how they do it. After repeated happy hours, the set of ideas in the brain of each and every one is \( Z_g \).
Therefore, if a country is lagging behind (low $Z_n/Z_f$), foreign firms would find it profitable to operate in that country. As foreign firms bring new ideas to the country they accelerate the formation of skills for the future national firms.

If $Z_n/Z_f < R_S$, the entry of foreign firms drive away the presence of national firms as the old cohort of potential managers are better off being workers. On the other hand, as $m = 1$, the younger cohort of national potential managers are only exposed to foreign ideas. Since there are no country specific distortions to the formation of skills, then, next period the cohort of national managers will exhibit the same level of know-how as their counterparts in the foreign country. A simple manipulation of (3.7) renders the transition function for $Z_n/Z_f$, the relative national know-how with respect to the rest of the world:

$$
\frac{Z'_n}{Z'_f} = \begin{cases} 
1 & \text{if } Z_n/Z_f < R_S \\
\left( \frac{Z_n}{Z_f} \right)^{1-(Z_n/Z_f)} & \text{if } Z_n/Z_f \geq R_S.
\end{cases}
$$

Clearly, if $R_S < Z_n/Z_f < 1$, then $Z_n/Z_f < Z'_n/Z'_f < 1$ and countries lagging behind catch up over time monotonically. For any initial condition above $R_S$, the system converges to $Z_n/Z_f = 1$ and $m = 0$. Foreign firms breed the competition that would eventually have them disappear from the country. Clearly, if $R_S > Z_n/Z_f$, the convergence takes only one generation.

To summarize this simple result:

**Proposition 3.1.** Use the superscript $t = 0, 1, 2, ..$ to index time periods. If (a) $R_S < Z^0_n/Z^0_f < 1$, then, the sequences $\{Z'_n/Z'_f, Y^t_n/Y^t_g\}_{t=0}^\infty$ are strictly increasing and $\{m^t\}_{t=0}^\infty$ with limiting points decreasing and $\lim_{t\to\infty} \left( \frac{Z'_n}{Z'_f} \right) = \lim_{t\to\infty} \left( \frac{Y^t_n}{Y^t_g} \right) = 1$, and $\lim_{t \to \infty} m^t = 0$. If instead (b) $Z^0_n/Z^0_f < R_S$ then $m^0 = 1$, $m^t = 0$ and $(Z'_n/Z'_f) = (Y^t_n/Y^t_g) = 1$ all $t \geq 1$.

A similar model to this one, Findlay (1978), has motivated a large body of empirical research trying to quantify the spillovers of multinational firms and is perhaps also motivating policy-makers that actively pursue the multinational corporations to locate in their countries.

I will now explore the impact of foreign firms with and without spillovers in models where the formation of skills is endogenous.
4. Endogenous Growth and Spillovers

Contrary to the assumption of the previous model, productive know-how is costly to accumulate. What we can do is the result of experimentation or training. Real resources such as materials and time are needed in the process of forming skills. Even if productive ideas are available, they require properly skilled individuals to make them productive.

In this section I will follow Stokey (1991) in assuming that skills are costly to accumulate and in that the level of skills of older managers determine the cost for younger potential managers to acquire their own skills. As in Stokey, there is an externality since managers are not compensated from their contribution of the set of ideas surrounding the youth. I will explore economies in which this externality can emanate from both, national and foreign managers.

4.1. Endogenous Accumulation of Skills

Consider now that to be able to master \( z'_0 \) units of productive know-how at maturity, a young individual must incur a cost in the current period of

\[
Z_g \phi \left( \frac{z'_0}{Z_g} \right)
\]

units of consumption goods. Here \( Z_g \) is the productive knowledge in the environment where the young agent live. The function \( \phi (\cdot) \) has the standard properties of an adjustment cost function: twice continuously differentiable, strictly increasing, strictly convex. I assume that \( \phi (0) = 0 \).

For the rest of the paper I will use the functional form

\[
\phi (x) = v_0 \frac{(x)^{1+v}}{1+v},
\]

where both \( v_0 > 0 \) and \( v > 0 \). Yet, I keep the use of the short-hand \( \phi (\cdot) \) to condense some of the formulas.

In a perfect-foresight equilibrium, each potential manager can foresee the payoffs \( \pi (z', w') \) (given by [2.2]) attainable with skills \( z' \) with wages \( w' \). Therefore, surrounded by ideas \( Z_g \), the optimal accumulation of a young agent that next period will become and active manager solves

\[
\max_{\{z'\}} \left\{ \beta \pi (z', w') - Z_g \phi \left( \frac{z'}{Z_g} \right) \right\}.
\]
This optimization consist of the difference of two convex functions. In all what follows, I impose the following restriction on parameters:

**Condition 2:** \( v > \frac{\alpha}{1-\alpha}. \)

Under this condition, the cost \( \phi(\cdot) \) is “more convex” than the rents \( \pi(\cdot, w') \) as functions of \( z' \). Therefore, the first order conditions are necessary and sufficient. Then, if condition 2 holds, the optimal level of skills for a future active manager is given by the first order condition

\[
\beta \alpha^{\frac{\alpha}{1-\alpha}} \left[ \frac{z'}{w'} \right]^{\frac{\alpha}{1-\alpha}} = v_0 \left[ \frac{z'}{Z_g} \right]^v.
\]

Otherwise, the optimal acquisition of skills is \( z' = 0 \).

Depending on whether we assume a closed or an open economy, and, in the latter, whether there are spillovers, we obtain different market clearing conditions for the values of \( w' \) and \( Z_g \). To complete the characterization of the optimal accumulation of skills we have to verify that the potential manager is not better off being a worker in both periods – and not investing in skills at all.

### 4.2. A Closed Economy

Assume that everyone in the cohort of mature potential managers command a level of skills \( Z_n \). Since the economy is closed, \( Z_g = Z_n \). In this case, the market clearing wages and rents are given by (2.3) and (2.4) respectively. Using these expression in (4.3) I obtain that \( G \), the gross growth in the level of skills is:

\[
G \equiv \frac{Z'_n}{Z_n} = \left[ \frac{\beta \theta^\alpha}{v_0} \right]^\frac{1}{\beta}.
\]

Hereafter I impose the following restriction on parameters:

**Condition 3:** \( \beta \theta^\alpha > v_0. \)

Under Condition 3, \( G > 1 \), and the economy exhibits constant and sustained growth. Notice that the economy does not exhibit transition dynamics. After one period, any pre-existing heterogeneity across managers disappears and the economy locates itself the unique BGP.

However, for these expressions to define an equilibrium, I also need to verify that occupation choices are optimal. Under Condition 1, the potential managers that are old prefer to be active managers. What is left is to verify that the young crop of potential managers will find it optimal to invest in skills and become active.
managers, obtaining a net present value of income of \( Z_n \left[ \alpha \eta^{\alpha - 1} - \phi(G) + \beta (1 - \alpha) \eta^{\alpha} G \right] \) over remaining a worker in both periods and obtaining \( Z_n \left[ \alpha \eta^{\alpha - 1} + \beta \eta^{\alpha - 1} G \right] \).

I impose now the following restriction on parameters:

**Condition 4:** \( v / (1 + v) > \alpha (1 + 1/\eta) \).

Under this condition, the following result can be directly verified:

**Lemma 4.1.** If Condition 4 holds then all potential managers invest in skills according to (4.4) and become active managers in the second period of their life.

The accumulation of skills requires real resources. In the closed economy, gross domestic \( Y_n \) and national \( Y_n \) output levels are equal and given by expression (3.2). Aggregate consumption is by \( C_n = Y_n - Z_n \omega \phi(G) \) as the aggregate cost of skill accumulation is subtracted.

### 4.3. A Small Open Economy

The intratemporal equilibrium conditions are the same as in the exogenous growth model. The free entry of foreign firms pins the wage of local young workers to the international level \( w_f \), each foreign managers hires \( n_f = \eta \) young workers. As before, if \( Z_n/Z_f < R_S \), then, national old potential managers become workers of multinational firms and \( m = 1 \). Otherwise, both national and multinational firms coexist and \( m = 1 - [Z_n/Z_f]^{1/\alpha} \).

#### 4.3.1. Isolated skill formation

Foreseeing that the cost of labor next period next period will be \( w'_n = \alpha \eta^{\alpha - 1} G Z_f \) and being surrounded by ideas \( Z_g = Z_n \), the solution for the optimal accumulation of skills (4.3) implies that the skills –relative to foreign country–of the next cohort of national managers would be

\[
\frac{Z'_n}{Z'_f} = \left( \frac{Z_n}{Z_f} \right)^\mu,
\]

where \( G \) is the BGP growth rate for a closed economy (4.4), and I have defined \( \mu \equiv v (1 - \alpha) / [v (1 - \alpha) - \alpha] \). Because of condition 1, it is the case that \( \mu > 1 \) and since \( (Z_n/Z_f) < 1 \), then \( \left( Z'_n/Z'_f \right) < (Z_n/Z_f) \). Since the home country’s managers are surrounded by an inferior set of ideas than the foreigners, their
optimal response to the presence of foreign competition in the future is to reduce their relative productivity.

The young potential managers opts for the previous accumulation of skills only if it he becomes an active manager. This is, if

\[ w_f - Z_n \phi \left( \frac{Z'_n}{Z_n} \right) + \beta \pi \left( Z'_n, w'_f \right) > w_f + \beta w'_f. \]  \hspace{1cm} (4.6)

Define the function \( \Xi : [0, 1] \to R \) as

\[ \Xi (R) \equiv (1 - \alpha) (R) \frac{\mu}{1 + v} - \frac{1}{R^\mu (1 + v) - 1}. \]

Clearly, \( \Xi (0) = 0 \), and, under Condition 4, \( \Xi (1) > \alpha / \eta \). With this function, I can characterize the optimal occupation choice of young potential managers.

**Lemma 4.2.** The inequality in (4.6) holds if and only if \( \Xi \left( \frac{Z'_n}{Z'_f} \right) > \alpha / \eta \). Moreover, if Condition 4 holds, there exists a unique threshold \( R_N \), such that \( \Xi (R_N) = \alpha / \eta \), and for \( \Xi (R_0) < \alpha / \eta < \Xi (R_1) \) for all \( R_0 < R_N < R_1 \).

I omit a formal proof since the first part only entails the use (3.4), (3.3) and (4.5) in (4.6) and simplification; the second results does not depend on the monotonicity of \( \Xi (\cdot) \), i.e. the relative size \( \mu / (1 - \alpha) \) vs. \( \mu (1 + v) - 1 \), and follows solely from Condition 4, the fact that \( \Xi (0) = 0 \), and the continuity \( \Xi \).

Recall that if national potential managers become active, they would choose to have relatively less skills than the current generation, i.e. \( (Z'_n/Z'_f) < (Z_n/Z_f) \). Thus, if the current crop of domestic managers become workers for multinational firms, then the young cohort of potential managers will also choose to be workers. Therefore, the transition function \( \Gamma : [0, 1] \to [0, 1] \) of the relative skills of home managers is the following:

\[ \left( \frac{Z'_n}{Z'_f} \right) = \Gamma \left( \frac{Z'_n}{Z'_f} \right) = \left\{ \begin{array}{ll} \left( \frac{Z'_n}{Z'_f} \right)^{\mu} & \text{if } \left( \frac{Z'_n}{Z'_f} \right) > R_N \\ 0 & \text{otherwise.} \end{array} \right. \]

With this transition function, it is also immediate to verify the following result:

**Proposition 4.3.** For any initial condition \( R_N < (Z'_n/Z'_f) < 1 \), and let \( T \left( Z'_n/Z'_f \right) = \min \left\{ s \in \mathbb{N} : s \geq \frac{\ln (R_N)}{\mu \ln (Z'_n/Z'_f)} \right\} \). Then, for any \( t \geq T \left( Z'_n/Z'_f \right) \), \( Z'_n/Z'_f = 0 \) and \( m^t = 1 \). The sequence \( \left\{ \frac{Z'_n}{Z'_f} : t \geq 0 \right\} \) is strictly decreasing and \( \left\{ m^t : t \geq 0 \right\} \) is strictly increasing for \( t < T \left( Z'_n/Z'_f \right) \).
Regardless of how close the initial productivity of home country is with respect to the foreign country, as long as it is below, opening to foreign competition without somehow conveying their superior ability to endogenously produce skills will eventually lead the country to a “colonial” limiting point in which \( Y_g = 2\eta^{\alpha-1}Z_f \) because the output is entirely generated by multinational firms and national households consume the returns to their labor \( C_n = \alpha Y_g \). The result not only implies that this is the only limiting point of the economy but also that it will be reached in finite time.

### 4.3.2. Spillovers in skill formation

In this subsection I explore the behavior of the small open economy when the presence of foreign firms enhances the set of ideas to which national young agents are exposed in their formative years. I will show that the presence of these knowledge spillovers will revert the negative results of the previous model.

Following the same steps as before but using \( Z_g = Z_f^mZ_n^{1-m} \) in (4.3) the relative skills of that active national managers would accumulate —relative to their foreign counterpart— are given by:

\[
\left( \frac{Z_n'}{Z_f'} \right) = \left( \frac{Z_n}{Z_f} \right)^{\mu(1-m)},
\]

where \( \mu \) is as defined above. Notice that now the presence of foreign firms \( (m > 0) \) helps closing the gap between national managers with the foreign managers. Indeed, the closer is \( m \) to 1, the closer the ratio \( [Z_n' / Z_f'] \) gets to one. However, the value of \( m \) is an equilibrium outcome that depend on \( Z_n \) and \( Z_f \). Define \( \Gamma_0 : [0, 1] \rightarrow [0, 1] \) as the transition function of the skills conditional on potential managers becoming active managers (i.e. ignoring occupation choice). Using (3.5), \( \Gamma_0 (\cdot) \) is given by

\[
\left( \frac{Z_n'}{Z_f'} \right) = \Gamma_0 \left( \frac{Z_n}{Z_f} \right) = \left( \frac{Z_n}{Z_f} \right)^{\mu[Z_n/Z_f]^{1/m}}.
\]

[Insert Here Figure 1: \( \Gamma_0 \)]

Notice that as displayed in Figure 1, the function \( \Gamma_0 \) is non-monotone, \( \Gamma_0 (x) < \)
1 for any \( x \in (0, 1) \) and has two fixed points. The first one is:

\[
\frac{Z_n}{Z_f} = 1.
\]

Here, the country is at par with the rest of the world, \( m = 1 \) and the country is neither subject nor in need of spillovers from the rest of the world. The second fixed point is

\[
\frac{Z_n}{Z_f} = R_L \equiv \left[ \frac{1}{\mu} \right]^{1-\alpha} < 1,
\]

where the superindex \( L \) indicates that this is a “laggard” BGP, the country never fully catches up with the rest of the world, and a fraction \( m^L = \frac{\alpha}{(1-\alpha)v} \in (0,1) \) of the labor force works for multinational firms.

To determine the dynamics of the skills in the country, the occupation choices of both, young and old potential managers must be consider. As before, even if they already had invested in skills \( Z_n \), old potential managers would rather become workers for a multinational firm if their relative productivity is low enough: \( Z_n/Z_f < R_S \). In this case, \( m = 1 \), and \( Z_g = Z_n \) since only foreign knowledge surrounds the young cohort. In this case we have the extreme opposite implication with respect to the model without spillovers. Now, the cost of accumulating for the national young cohort is the same as for foreign youth. Therefore, they both would choose the same level of skills, \( Z'_n = Z'_f \) and, in the next period, the country fully catches up.

On the other hand, before investing in skills, each agent compares the alternative not investing in skills and being a worker in both periods versus the alternative of optimally investing in skills and being a manager next period. Following the same steps as in the previous section, I define the function \( \Phi (\cdot) \) to characterize the occupation/investment decision of young potential managers. Let

\[
\Phi (x) \equiv (1 - \alpha) \Gamma_0(x) \frac{1}{1+v} - \frac{1}{1+v} \Gamma_1(x) \frac{1+\mu}{\mu} (x)^{-1(1+v)}.
\]

Similarly as with a Lemma in the previous section, it is easy to verify that young potential managers will invest and become active managers if and only if \( \Phi (x) > \alpha/\eta \). However, contrarily to the function \( \Xi (\cdot) \) of the model with no spillovers, even if Condition 4, the presence of spillovers induce non-monotonicities that can lead \( \Phi (\cdot) \) to cross multiple times \( \alpha/\eta \) in the interval \([0,1]\). Therefore, the transition function \( \Gamma (\cdot) \) has to be defined directly using the function \( \Phi (\cdot) \):
\[
\frac{Z_n'}{Z_f'} = \Gamma \left( \frac{Z_n}{Z_f} \right) = \begin{cases} 
1 & \text{if } Z_n/Z_f < R_S, \\
0 & R_S < Z_n/Z_f, \text{ and } \Phi \left( \frac{Z_n}{Z_f} \right) < \alpha/\eta \\
\Gamma_0 \left( \frac{Z_n}{Z_f} \right) & \text{otherwise.}
\end{cases}
\] (4.8)

In the first branch, as explained above, only foreign knowledge is active in the country and the national young potential managers become active and the economy converges in one period. The second branch indicates the possibility that old potential managers become active but young ones do not invest. In this case, the activation of the lower skills of national managers blocks the entry of foreign knowledge and this happens to the extreme of making it pointless for the youth to invest in skills to compete with the next period entry of foreign managers. But, since in the next period \(m' = 1\), then \(Z'_n = Z'_f\), and the economy will converge in two periods. In the last case, both cohorts are active.

Notice that the system exhibits predatory-prey dynamics: as foreign firms enter, local managers accelerate their skills formation and reduce their distance with the rest of the world. This is reinforced by the fact that as they increase their skills, next period the mass of foreign firms diminish, reducing the effective competition for workers. In turns, the stock of productive ideas that young agents are being exposed to is also diminished, the country increases its lag with respect of the world, implying a higher presence of foreign firms in the subsequent period and so on.

**Proposition 4.4.** The following results hold: (a) If \(\Phi (R_L) < \alpha/\eta \) or \(R_L < R_S\), then \(Z_n/Z_f = 1\) is the unique resting point of \(\Gamma\) and is globally stable. (b) If instead \(\Phi (R_L) > \alpha/\eta \) and \(R_L > R_S\), then both \(Z_n/Z_f = 1\) and \(Z_n/Z_f = R_L\) are resting points and \(R_L\) is locally stable. Moreover, if for all \(x \in (R_S, 1), \Gamma_0 (x) > R_S\) and \(\Phi (x) > \alpha/\eta\), then,

\[
\lim_{t \to \infty} \left( \frac{Z_n}{Z_f} \right) = \begin{cases} 
R_L & \text{if } \left( \frac{Z_n^0}{Z_f^0} \right) > R_S \\
1 & \text{otherwise.}
\end{cases}
\]

**Proof.** See appendix. ■

Figure 2 displays several cases of the transition function. In all the panels, the dash line is the function \(\Gamma_0\) with the thick line is for the function \(\Gamma\). In all cases, if \(\left( \frac{Z_n^0}{Z_f^0} \right) < R_S\), then the country converges in levels after one period in which
all output was generated by multinational firms. On the other hand, for some initial conditions, the $R_L$ steady state is the limiting point in the cases in which $\Gamma$ crosses the 45° line at $R_L$ and the convergence can be cyclical.

[Insert Figure 2: different cases for $\Gamma_0, \Phi, \Gamma$]

In sum, even if we do not get the disastrous results of the model with competition without spillovers, the competition effect is still present and either completely dominates in one period and the country converges in levels, or remains operative forever blocking the level convergence of the country. The expectation that by opening to foreign firms the national productivity will increase may lead to disappointment since, foreseeing future competition, national firms may opt to reduce, not increase their efforts to increase productivity.\footnote{\textsuperscript{15}Aitken and Harrison (1999) discuss a similar “competition” effect driven by strategic interaction in the context of a static partial equilibrium model.}

\section*{5. Endogenous Growth with Firms Producing Skills}

In this section I consider an environment in which the formation of skills takes place inside the firm. As in Boyd-Prescott (1987a,b), Chari-Hopenhayn (1991) and Jovanovic-Nyarko (1995), the formation of skills of a young worker depends on the set of skills and the actions of the manager for whom he works.\footnote{\textsuperscript{16}See also the extensions of the Chari-Hopenhayn model by Agarwal, R. et al (2004) and Filson and Franco (2006).} Since managers must be engaged in training the workers, a well functioning market exists. Since the costs and returns of the investment in skills are fully foreseen, the equilibrium skill formation is socially efficient.\footnote{\textsuperscript{17}In Chari-Hopenhayn (1991) skill formation is exogenous. In Jovanovic-Nyarko (1995) adverse selection precludes the efficiency of the equilibrium.} These transactions, of course, can take place across national borders without the presence of externalities.

To save on notation, in this section I will assume that $\omega = \eta = 1$, i.e. all young agents are potential managers. Therefore, I need to impose that $\alpha < 1/2$ for condition 1 to hold. This simplifying assumption shortens the mathematical expressions and allows a more direct comparison with the original work of Boyd and Prescott.
5.1. Technology and the problem of the Firm

As before, a firm is a team of a manager and \( n \) workers.\(^{18}\) In addition of consumption goods \( y \), firms produce skills \( z' \) that young workers can use in the next period when they are managers. The technology frontier of \((y, z')\) is as follows. A team of \( n \) workers led by a manager with skills \( z \) can produce \( y \) units of consumption goods and \( z' \) units of skills for each one of the young worker according to:

\[
y = z \left[ n^\alpha - n \phi \left( \frac{z'}{z} \right) \right]. \tag{5.1}
\]

where \( \phi (\cdot) \) is the same increasing, convex and twice differentiable function above.

According to their with skills \( z \), each manager hires workers in competitive labor markets. Managers with different set of skills may opt to offer different prospects of training for young workers. However, in equilibrium, young workers must be indifferent to work for either one. Denote by \( W_t \) the market clearing level of utility of a young agent at time \( t \). Then, the same value \( W_t \) must be obtain by a young agent that works and is trained by a high skilled manager as one that works and is trained by a low skilled manager.

In this environment, training is at the interior of the firm. To provide \( z' \) units of skills to one worker, a manager with skills level \( z \) incurs a cost \( z \phi \left( \frac{z'}{z} \right) \). Here \( \phi (\cdot) \) is as defined above, implying that total and marginal costs of training are increasing in \( z' \) and decreasing \( z \). The benefit of training, which is foreseen by both manager and worker, is given by \( \beta \pi_{t+1} (z') \), the discounted value of the rents for the would be manager. Internalizing the costs and benefits of training, the effective net cost of each worker for the manager is \( [W_t + z \phi \left( \frac{z'}{z} \right) - \beta \pi_{t+1} (z')] \). A manager maximizes his \( \pi_t (z) \) by choosing optimally the number of workers \( n \) and that \( z' \), the training provided to each one:

\[
\pi_t (z) = \max_{(n,z')} \left\{ z \left[ n^\alpha - n \phi \left( \frac{z'}{z} \right) \right] - n \left[ W_t - \beta \pi_{t+1} (z') \right] \right\}. \tag{5.2}
\]

The first result is that, in the margin, the costs and benefit of skill transmission are fully internalized. Assume the following condition:

\(^{18}\)In the previous two models, since learning is outside the firm, we could interpret \( n \) to be units of labor services. In this model \( n \) needs to be, literally, the number of workers. Integer problems do not arise since we can assume that, need it be, a young agent can work for two or more identical managers (managers with the same \( z \)). Likewise, a manager can hire
**Condition 5:** The function $\pi_{t+1}(\cdot)$ is increasing and satisfies the conditions that $\pi_{t+1}(0) = 0$, and $\lim_{x \to +\infty} \left[ \frac{\pi_{t+1}(x)}{\phi(x)} \right] = 0$.

This condition is a generalization of condition 1 and simply implies that the solution of (5.2) is bounded.

**Proposition 5.1.** Assume that Condition 4 holds. Then, (a) the optimal transfer $z_t^*(z)$ of skills is independently of the number of workers in the firm $n_t^*(z)$; and (b) the returns to the manager are increasing in $z$ and given by

$$
\pi_t(z) = (1 - \alpha) z [n_t^*(z)]^\alpha,
$$

his marginal contribution to the production of goods (gross of training costs). (c) The function $n_t^*(z)$ is strictly increasing in $z$.

**Proof.** See appendix.

The intuition for this result is straightforward. Managers optimize how they provide $W_t$, either in terms of consumption goods or in terms of training. The optimal provision of skills requires that the marginal return of those skills equalize the marginal cost for the firm of providing them.

The profits $\pi_t(z)$ are increase more than proportional with $z$ since $n^*(z)$ is also strictly increasing function of $z$. In turn, there are two forces that imply, that $n^*(z)$ is increasing. First, as in the Lucas’ span of control models of the previous sections, the marginal product of labor –in producing goods– is increasing in $z$. Second, a higher $z$ reduces the cost of any training $z'$, reducing the effective cost of labor for the entrepreneur.

A direct implication of the previous proposition is the following:

**Corollary 5.2.** The optimal $z_t^*(z)$ is strictly increasing in $n_{t+1}^*(\cdot)$, the number of young workers that the current trainee will control in the next period.

Obviously, if the current trainee is expecting to be a worker and have no one under his control, the returns of investing in skills are zero. In general equilibrium, the level of skills and the number of workers for each managers depend on the skills of ancestors and of the size of the progeny.
5.1.1. A simplified example.

Before examining the dynamics for closed and open economies, it is convenient to examine a simplified example. Assume an economy with three periods: \( t, t + 1 \) and \( t + 2 \). Assume that the market prices of young workers for periods \( t \) and \( t + 1 \) are, respectively, \( W_1 \) and \( W_2 \). Therefore, for any \( z'' \), \( \pi_{t+2}(z'') = 0 \) and the optimal formation for skills at time \( t + 1 \) is equal to zero. Therefore, using (2.2), the rents for managers in \( t + 1 \) are

\[
\pi_{t+1}(z') = \theta \left[ z' \right]^{1/(1-\alpha)} \left[ W_2 \right]^{-\alpha}.
\]

Skills will be valuable in \( t + 1 \). Using this function \( \pi_{t+1}(z') \) in (5.2) the optimal transfer of skills \( z'^*_t(z) \) is given by

\[
z'^*_t(z) = (z)^\mu \left[ \frac{\beta \theta}{(1-\alpha) v_0} \right]^{\frac{\alpha}{1-\alpha}} \left[ W_2 \right]^{-\mu},
\]

which more than proportionally with \( z \) because \( \mu > 1 \), and goes down with \( W_2 \). For a manager with \( z \) skills, define the effective cost \( \hat{W}(z) \equiv W_1 - \left\{ \beta \pi_{t+1}([z'^*_t(z)] - z\phi (\frac{z'^*_t(z)}{z}) \right\} \). After some algebra it can be shown that it is a strictly decreasing function of \( z \):

\[
\hat{W}(z) = W_1 - z \zeta \left[ \frac{z}{W_2} \right]^{\frac{\alpha}{1-\alpha}} \left[ \frac{\alpha + \beta}{\alpha (1-\alpha)} \right] \frac{1}{1-\alpha}
\]

where \( \zeta \) is a positive constant.\(^{19}\) The number of workers for a manager with skills \( z \) at time \( t \) is \( n_t^* (z) = \left[ \alpha z / (\hat{W}(z)) \right]^{1/(1-\alpha)} \), and his rents are \( \pi_t (z) = \theta z^{1/(1-\alpha)} \left[ \hat{W}(z) \right]^{-\alpha} \). Therefore, for the same \( W_1 \) both values are higher and more elastic to variations in \( z \) than in the standard Lucas’ span of control model where training takes place outside the firm.

5.2. A Closed Economy

Consider now an economy in which all managers have a level of know-how \( z \). Obviously, this means that \( Z_n^* = z \). Since the production of goods has decreasing

\(^{19}\)The formula is \( \zeta \equiv \left[ \frac{1}{(1-v)\theta (1-\alpha)} \right]^{1/(1-\alpha)} \left[ \frac{(1-\alpha - \alpha)}{(1-\alpha)} \right] \frac{1}{(1-\alpha)} \left[ \beta \theta \right]^{( \frac{1}{1-\alpha})(1-\alpha)\theta (1-\alpha)} \).
returns to $n$ and the cost of producing skills is linear in $n$, managers with the same skills hire the same number of workers. Since in the first period all managers have the same skills, then each one will hire $n_n = 1$ workers.

As shown in the previous proposition, the optimal level of skills transferred are independent of the number of workers for a manager and solve

$$\max_{(z')} \left[ \beta \pi_{t+1}(z') - z \phi \left( \frac{z'}{z} \right) \right]. \tag{5.3}$$

Therefore, all firms will transfer (and all young will receive) the same training $z'$, implying that, along the equilibrium path, within a period, all managers will be identical ($z = Z_n$) and each one will hire $n_n = 1$ workers. Under such circumstances, the rents of all managers are

$$\pi_n = (1 - \alpha) Z_n.$$

Because of this, the solution $z'^\ast(z)$ to (5.3) would be proportional $z$, implying that for a constant $G$, the law of motion of the aggregate (average) level of skills follows

$$Z'_n = G Z_n.$$

Since the costs and benefits of skills are internalized, then the market clearing value of $W$ would satisfy

$$W = z \left[ \alpha + \beta (1 - \alpha) G - \phi(G) \right].$$

Therefore, to characterize a balanced-growth path we only need to determine the value of $G$. Unless $z' = 0$ or $z' = +\infty$, and even if the problem 5.2 is not globally concave, the global optimal skill formation must locally satisfy a the necessary first order condition

$$\beta \pi'_{t+1}(z') = \phi' \left( \frac{z'}{z} \right). \tag{5.4}$$

and an envelope condition:

$$\pi'_n(z) = n^\alpha - n \phi \left( \frac{z'}{z} \right) + \left( \frac{z'}{z} \right) \phi' \left( \frac{z'}{z} \right). \tag{5.5}$$

Impose $n = 1$, use the functional form for $\phi(\cdot)$ and let $G = \left( \frac{z'}{z} \right)$, implying the equation
\[ v_0 G^\nu = \beta \left[ 1 + \frac{v v_0 G^{1+v}}{1+v} \right]. \] (5.6)

The left hand side of this equation is the marginal cost while the right hand side is the marginal benefit. A root \( G \) of this expression is only relevant if satisfies three conditions: (a) it is a “maximization”, i.e. the marginal benefit crosses the marginal cost from above; (b) net output is positive:

\[ 1 - \phi (G) > 0, \]

and (c) agents prefer to be workers during their youth and managers during their prime age instead of workers in both periods. For this to be the case in a balance growth path, we need that

\[ \beta (1 - 2\alpha) G > \phi (G), \]

i.e. the (discounted) difference in the income of managers vs. workers compensate for the training costs.

The existence and uniqueness of a BGP requires restrictions on the parameters so that there is no explosive or implosive growth.

**Condition 6:** The parameters \((v_0, v, \beta)\) satisfy the inequalities

\[ v_0 (1 + v) / [1 + v (1 + v_0)] < \beta < (v_0 / [1 + v])^{\frac{1}{1+v}}. \]

**Proposition 5.3. (Existence and Uniqueness of a BGP.)** Assume that Condition 4 holds. Then there exist a unique \( G \in (1, \beta^{-1}) \) that solves (5.6), satisfies the “maximization” condition for skills, old agents prefer to be managers and net output \( 1 - \phi (G) \) is positive.

[Insert Figure 3: a double panel. a) Parameter restrictions; b) RHS and LHS]

Figure 3 graphs the RHS and the LHS of (5.6) for a set of parameter that satisfy condition 5. At \( G = 0 \), the RHS is positive and LHS is equal to 0. Since the RHS is more convex than the LHS, if the curves cross, they cross twice. The second crossing is not relevant because it defines a local minimum. Therefore, the optimal \( G \) is either the first crossing or a point to the right of \( G \). A degenerate solution \( G \rightarrow \infty \) is ruled out because we require \( 1 - \phi (G) \) to be positive. Condition
implies that the first crossing is above 1 (and hence, the economy grows) but below \( \beta^{-1} \) (hence, the value of the firm is bounded from above) and that the second crossing implies \( 1 - \phi(G) \) to be negative. Therefore, under condition 5, the first crossing is the global optimum.

I now verify that the equilibrium allocation is efficient. Assume that a social planner starts with a cohort of old agents, all with the same expertise \( Z_n \). The planner must decide how many young workers to assign to each manager and how much skills \( Z'_n \) to invest in each of the young workers. Because of decreasing returns in production, each manager will be allocated the same number of young workers, and the aggregate production of goods is \( Z_n \). The aggregate cost of skill formation is \( Z_n \phi(Z'_n/Z_n) \). Therefore, the resources available for consumption in the period are \( Z_n [1 - \phi(Z'_n/Z_n)] \) and the value function \( S(Z_n) \) for the planner is defined by the Bellman Equation (BE)

\[
S(Z_n) = \max_{\{Z'_n\}} \{Z_n [1 - \phi(Z'_n/Z_n)] + \beta S(Z'_n)\}.
\]

This problem can be equivalently written as the optimal choice of the rate of growth \( G = Z'_n/Z_n \). Therefore, the problem can be written as

\[
S(Z_n) = \max_{G} \{Z_n [1 - \phi(G)] + \beta S(GZ_n)\}.
\]

Since the return function is homogeneous of degree one (HD1) in \( Z_n \) and the feasible set is clearly convex, then, the operator defined by this BE maps HD1 functions into itself. However, the operator is not a contraction unless we restrict the feasible set \( G \in [0, G_0] \) for any \( G_0 < \beta^{-1} \). Thus restricted, following Alvarez and Stokey (1998), it can be shown that the unique fixed point of this BE is of the form \( S(Z_m) = S_0 Z_m \) for a constant \( S_0 \) that satisfies

\[
S_0 = \max_{G \in [0, \beta_0^{-1}]} \left\{ \left[1 - v_0 \frac{(G^{1+v})}{1+v}\right] + \beta GS_0 \right\} \quad (5.7)
\]

The first order condition for an interior optimum is

\[
v_0 G^v = \beta S_0, \quad (5.8)
\]

\footnotetext{Additional restrictions on the parameters \((v_0, v, \alpha)\) must be imposed so that the social planner would not want to break the symmetry. See the appendix for the full argument and the sufficient conditions for symmetry.}
and is sufficient. Therefore, the constant $S_0$ must satisfy

$$S_0 = 1 - v_0 \left( \left( \frac{\beta S_0}{v_0} \right)^{\frac{1}{v}} \right)^{1+v} + \beta G S_0.$$ 

If instead, we use (5.8) and write the previous equation in terms of $G$ we obtain (5.6), the equilibrium growth rate. Therefore, under Condition 6, there is a unique fixed point and it coincides with the equilibrium one.

5.3. A Small Open Economy.

Now, consider the case of free entry of foreign managers in a small economy. Instead of spillovers, foreign managers can directly train the workers under their control. The technology frontier of $(y, z'; z, n)$ is the same for domestic and foreign managers. The difference between national and foreign managers is that the latter have more advanced skills $Z_f > Z_n$.

To rule out an arbitrage, foreign managers must be indifferent between operating inside the country or staying abroad. The absence of frictions implies that all the adjustment is in the extensive margin of the mass of foreign managers entering the country. Each one obtains a return $\pi_f = (1 - \alpha) Z_f$. The market clearing price of a local young agent is the same as in the rest of the world $W_n = W_f$. Foreign managers face the same problem and market conditions as in their home country. Therefore, they hire one young worker and transfer $GZ_f$ units of skills.

National managers face the same problem and market conditions but they have a lower level of skills $Z_n$. In any point in time, young workers have to be indifferent among the managers for which they can work. Managers provide the same compensation $W_n$ but with different combination of payments and skill transfers.

The equilibrium is simplified by the fact that in any point in time there can be at most three ‘types’ of managers. The first are the foreign managers. The second type are ‘new’ sector of managers who were directly trained by a foreign managers or can trace a foreign manager in his genealogical tree of training. All those ‘new’ managers are identical of each other because each cohort has the BGP $G$ times the skills of the previous one. Finally, there are the ‘deep-rooted’ managers, which are the progeny of managers who never worked for foreign managers.

To compute an equilibrium it is convenient to solve the social planner’s problem.
of the country. The planner has to decide how many workers to assign to each of the managers and how much to invest in the skills of each. The planner considers the output generated in each production unit, the cost of skill formation and the market compensation of foreign managers.

Denote by \( x \in [0, 1] \) the mass of deep-rooted and \( 1 - x \) the mass of new national managers. The ‘state’ for the recursive planner’s problem is \((x, Z_n, Z_f)\). He has to choose the future values \( x' \) and \( Z_n' \). With \( x \) production units and \((x'/x)\) workers allocated to each and skills \( Z_n' \) for the next cohort, aggregate output net of training costs are as follows:

The deep-rooted sector provides

\[
Z_n x \left[ (x'/x)^{\alpha} - (x'/x) \phi \left( Z_n'/Z_n \right) \right] = Z_n \left[ x^{1-\alpha} (x')^{\alpha} - x' \phi \left( Z_n'/Z_n \right) \right],
\]

the national new sector provides

\[
(1 - x) Z_f [1 - \phi (G)],
\]

and the foreign sector provides

\[
[(1 - x') - (1 - x)] Z_f [\alpha - \phi (G)] = [x - x'] Z_f [\alpha - \phi (G)].
\]

The \( G \) in these expressions is the efficient growth rate of the BGP. Since the social planner takes as given \( \pi_f \), the market price of a foreign manager, it would be optimal to assign one worker to each of the \((1 - x)\) new local managers and to invest in \( Z_f G \) units of skills in each of these workers. Any expansion or contraction of this sector will be done in the extensive margin – how many foreign managers to bring to the country. Each foreign manager is paired with one worker, therefore, the expansion of the new sector is equal to \((1 - x') - (1 - x) = x - x'\). From the output of the foreign sector, both, the cost of training and the repatriated foreign rents have to be subtracted. Adding the three net outputs and rearranging, aggregate national consumption is

\[
C_n^C (x, Z_n, Z_f, x', Z_n') \equiv Z_n \left[ x^{1-\alpha} (x')^{\alpha} - x' \phi \left( Z_n'/Z_n \right) \right] + Z_f \left[ [1 - \phi (G)] - x [1 - \alpha] - x' [\alpha - \phi (G)] \right].
\]

which clearly indicates the trade-off between allocating one worker to the deep-rooted sector versus the new sector.
The social planner has the option to liquidate the deep-rooted sector and allocate all the old agents to be workers in a multinational firm. For this, the country would bring extra foreign managers, pair each of them with an old worker. The foreign manager must be paid his market price \((1 - \alpha) Z_f\) and, since there is no training, the home country receives \(\alpha Z_f\). In this case, aggregate national consumption is:

\[
C_n^L (x, Z_f) \equiv Z_f \{[1 - \phi (G)] - x [1 - 2\alpha]\},
\]

which does not depend on \(Z_n\) since the deep-rooted sector is being shut down and the values \(x'\) and \(Z_n'\) are being set to zero.

The value function \(V\) for the social planner’s problem is

\[
V (x, Z_n, Z_f) = \max \{V_C (x, Z_n, Z_f), V_L (x, Z_f)\},
\]

where \(V_C (x, Z_n, Z_f)\) is the value under maintaining some agents in the deep-rooted sector and \(V_L (x, Z_f)\) is if the deep-rooted sector is liquidated. These functions are given by:

\[
V_C (x, Z_n, Z_f) = \max_{\{x' \in [0,1], Z_n' \geq 0\}} \{C_C (x, Z_n, Z_f, x', Z_n') + \beta V (x', Z_n', G Z_f)\},
\]

and

\[
V_L (x, Z_f) = C_n^L (x, Z_f) + \beta S (G Z_f),
\]

where \(S (\cdot)\) is the value function of the closed economy.

Notice that \(C_n^L\) is \(HD1\) in \(Z_f\) and \(C_C\) is \(HD1\) in \((Z_f, Z_n)\). Defining \(r \equiv \frac{Z_n}{Z_f}\), and factoring out \(Z_f\), following Alvarez and Stokey (1998), I define the functions:

\[
c_C (x, r, x', r') \equiv r \left[ x^{1-\alpha} (x')^\alpha - x' \phi (Gr' / r) \right] + [1 - \phi (G)] - x [1 - \alpha] - x' [\alpha - \phi (G)],
\]

\[
c_L (x, x') \equiv [1 - \phi (G)] - x [1 - 2\alpha].
\]

The normalized BE satisfies:

\[
v (x, r) = \max \{v_C (x, r), v_L (x)\},
\]

where the normalized value of continuing is

\[
v_C (x, r) = \max_{\{x' \in [0,1], r' \geq 0\}} \{c_C (x, r, x', r') + \beta v (x', r')\},
\]

and the normalized value of liquidating is

\[
v_L (x) = c_L (x) + \beta S_0,
\]

where \(S_0\) is as defined above for the closed economy.
Lemma 5.4. If condition 6 holds, there exists a unique solution $v$ to the BE defined by (5.12), (5.13) and (5.14).

This result is immediate because under condition 5, $G\beta < 1$, and the BE is a contraction.

[Insert Figure 4: a triple panel. a) V; b) R; c) x]

For a developing country $r < 1$, and the solution of this problem is simple. It is immediate that $c_C(x, r, x', r')$ is strictly increasing in $r$. Therefore, (5.13) maps increasing functions $v(x', r')$ with respect to $r'$ into strictly increasing functions $v_C(x, r)$ w.r.t. $r$. Since $v_L(x)$ does not depend on $r$, for all $x$ there is an $r_L(x)$ so that for all $r \leq r_L(x)$, it is optimal liquidate the sector and allocate the labor units embedded in the old agents and put them to work for foreign or new sector firms. In this case, $v(x, r) = v_L(x)$. Therefore, given $x$, the value function $v(x, r)$ has initially a flat section and then a strictly increasing section.

With respect to $x$, notice that $c_L(x)$ is strictly decreasing in $x$ since $\alpha < 1/2$ implying that $v_L(x)$ is strictly decreasing in $x$. This is because, in a BGP, it would be more productive to have those $x$ agents instead of workers being managers with a ratio $r = 1$ of skills. Also, whenever $r < 1$, $c_C(x, r, x', r')$ is strictly decreasing with respect to $x$. Hence, (5.13) maps decreasing functions $v(x', r')$ with respect to $r'$ into strictly decreasing functions $v_C(x, r)$ w.r.t. $x$. Therefore, when $r < 1$, $v(x, r)$ strictly decreasing for all $0 < x < 1$. This is a result that we expected. The more mass we have in the laggard sector the lower the aggregate output.

[Insert Figure 5: opening up: fraction, relative productivity, aggregate output]

Figure 5 considers the evolution of an closed economy that opens up. In the initial stage $x = 1$ and $r < 1$. The optimal response is for the deep-rooted sector to shrink and fully disappear in finite time. Notice that over time the optimal response is also to reduce $r$. In the period before the planner sets $x = 0$, it is pointless to invest any resources in skills and then $r' = 0$. After that, foreign firms disappear from the country, the economy is fully renewed catching up developed countries.

It is important to notice that the transfer of skills from multinational firms materialize in a new sector of firms, not in the pre-existing sector of firms. This
is indeed, somewhat in line with empirical findings. The model implies that the presence of foreign firms should hurt the productivity of pre-existing firms – because of the competition effect and the absence of spillovers. However, the economy as a whole fully catches up. When facing competition, the optimal behavior of foreign firms is to use their ability to form national skills in order to hire local workers.


In this paper I used simple general equilibrium growth models to study the impact of multinational firms on the formation of skills and the long run behavior of output a small developing country. Within the context of a simple model environment, I examined three different growth models: (a) an exogenous growth model, (b) an endogenous growth model with an externality in the formation of skills and (c) an endogenous growth model in which skills are internally produced in the firm. The impact of multinational firms on the host country in models (a) and (b) is via spillovers. These two models are rather standard in the growth literature and the existence and measurement of spillovers have been the subject of a vast empirical literature. Spillovers are also the tenet underlying much debate and policy proposals and programs. I show that spillovers are not sufficient to propel the country to catch up with developed countries.

In model (c) there are no spillovers and any transfer of skills is the result of a market transaction. In a competitive environment, firms will optimally transfer a level of skills that gradually obliterate the existent sector and creates a new sector of domestic firms that are at par with the ones in developed countries. In finite time, the small country converges to the income level of developed countries. Spillovers are not necessary.

These results are very suggestive about the role of government policy. In model (a), a benevolent government would definitely want to subsidize foreign firms. In model (b) optimal policy can be quite kinky. For instance, if local skills are very low, a subsidy would be pointless since the country will converge next period. For higher initial level of skills, the government may want to subsidize foreign firms fully obliterating the local firms for one period and converge to the developed country level in the next. The competition effect in this model can also introduce interesting issues of time consistency.

In model (c) the equilibrium is efficient and the government must not subsidize.
However, if there are obstacles to the transmission of skills from foreign firms to local agents, a government may want to undo the obstacle by providing a subsidy. But it is necessary to go beyond these somewhat speculative arguments. Optimal government policy in models with obstacles to the flow of firms and/or the transmission of skills deserve a rigorous and comprehensive consideration.

A. Proofs

Proof of Lemma XXX (Transition Function for Endogenous Growth with Spillovers)

First, notice that if the conditions for (a) hold, then necessarily the economy will reach the set \([0, R_S]\) in a finite number of steps and therefore converge to 1. On the other hand, if the conditions for (b) hold, given that the function \(\Gamma_0\) crosses the 45\(^0\) line from above and that it is continuous, there is an \(\epsilon > 0\) such that the ball \(B(R_L, \epsilon)\) is such that \(B(R_L, \epsilon) \subset (R_L, 1)\), \(\Gamma_0[B(R_L, \epsilon)] \subset B(R_L, \epsilon)\), and \(\Gamma'_0(x)\) all \(x \in B(R_L, \epsilon)\). Therefore, \(R_L\) is locally stable. Finally, if \(\Gamma_0(x) > R_S\) and \(\Phi(x) > \alpha\), then if the economy starts in a position where old agents become managers, it will always remain there, and in this case the limiting point is \(R_L\).

Characterization of the problem of managers.

Under Condition 3 the optimal provision of skills if finite and, given \(z\) and \(W_t\), the maximized value \(\pi_t(z)\) is bounded. However, under condition 3, the optimal \(z^{t*}_t(z)\) is finite. Notice that

\[
\pi_t(z) = \max_{(n,z')} \left\{ z \left[ n^\alpha - n \phi \left( \frac{z'}{z} \right) \right] - n \left[ W_t - \beta \pi_{t+1}(z') \right] \right\}
\]

\[
= \max_{(n,z')} \left\{ zn^\alpha - nW_t + n \left[ \beta \pi_{t+1}(z') - z \phi \left( \frac{z'}{z} \right) \right] \right\}
\]

\[
= \max_{(n)} \left\{ zn^\alpha - nW_t + n \max_{(z')} \left\{ \beta \pi_{t+1}(z') - z \phi \left( \frac{z'}{z} \right) \right\} \right\}
\]

which proves (a). The maximization \(\max_{(z')} \left[ \beta \pi_{t+1}(z') - z \phi \left( \frac{z'}{z} \right) \right]\) is not necessarily convex and the optimal is not necessarily interior, but under condition 3, the maximum is attained by a finite value of \(z^{t*}_t(z)\). If \(z^{t*}_t(z) = 0\), then the
maximization reduces to
\[ \pi_t(z) = \max_{(\alpha)} \{ z [n^\alpha] - n W_t \} \]  
\[ = (1 - \alpha) z [n_t^*(z)]^\alpha, \]  
(A.1)

where the second equality follows from the necessary and sufficient condition
\[ z\alpha [n_t^*(z)]^{\alpha-1} = W. \]  
If instead \( z_t^*(z) > 0 \), then
\[ \pi_t(z) = \max_{(n)} \left\{ z n^\alpha - n W_t + n \left\{ \beta \pi_{t+1}([z_t^*(z)] - z \phi \left( \frac{z_t^*(z)}{z} \right) \right\} \right\}, \]

which is a convex maximization. The necessary and sufficient first order condition for \( n_t^*(z) \)
\[ z\alpha [n_t^*(z)]^{\alpha-1} = W_t - \left\{ \beta \pi_{t+1}([z_t^*(z)] - z \phi \left( \frac{z_t^*(z)}{z} \right) \right\}. \]  
(A.3)

Multiplying both sides of this equation by \( n_t \), and the result in the maximand, we obtain the stated result in (b). To verify (c), notice that the envelope theorem implies that the derivative of the RHS of (A.3) while the LHS is strictly increasing in \( z \) for any given \( n_t^*(z) \). Therefore, \( n_t^*(z) \) has to increase with \( z \).

**Balanced Growth Path with Internal Diffusion.**

Both LHS and RHS expressions are continuous and strictly increasing as functions of \( G \). To save on notation, LHS(\( x \)) and RHS(\( x \)) indicate the respective expressions evaluated at \( x \). First, notice that \( LHS(0) = 0 \) and \( RHS(0) = \beta > 0 \). Also, for \( \beta \in (0, 1) \) and \( v, v_0 > 0 \), \( RHS(\cdot) \) has more curvature than \( LHS(\cdot) \) and RHS is eventually higher than the LHS. By direct derivation, both curves have the same slope at \( \beta^{-1} \). Then, if \( RHS(\beta^{-1}) > LHS(\beta^{-1}) \) then \( RHS(x) > LHS(x) \) for all \( x \geq 0 \) and no BGP exists. Instead, iff \( \beta < (v_0/ [1 + v])^{\frac{1}{1+\sigma}} \) then, \( RHS(\beta^{-1}) < LHS(\beta^{-1}) \). In such case, there are two values \( 0 < G_0 < \beta^{-1} < G_1 \) such that \( RHS(G_0) = LHS(G_0) \) and \( RHS(G_1) = LHS(G_1) \) and \( RHS(x) \leq LHS(x) \) \( \forall x \in (G_0, G_1) \). In addition iff \( v_0 (1 + v) / [1 + v (1 + v_0)] < \beta \), then \( LHS(1) < RHS(1) \) implying that \( 1 < G_0 < G_1 \). Interestingly iff \( \beta < (v_0/ [1 + v])^{\frac{1}{1+\sigma}} \) then \( LHS \left( (v_0/ [1 + v])^{\frac{1}{1+\sigma}} \right) > RHS \left( (v_0/ [1 + v])^{\frac{1}{1+\sigma}} \right) \) implying that necessarily \( G_1 > (1 + v )/v_0^{\frac{1}{1+\sigma}} \). This rules out \( G_1 \) because it implies \( \phi(G_1) > 1 \). Therefore, only \( G_0 \) satisfies both conditions and since \( G_0 \in (1, \beta^{-1}) \) the argument is complete.
References


Transition Function of Relative Productivities

$Z_d/Z_f$

Stable BGP

Unstable BGP
Transition Function: \( \left[ \frac{Z_d'}{Z_f'} \right] = \Gamma \left( \frac{Z_d}{Z_f} \right) \)

\[
\Gamma(\frac{Z_d}{Z_f}) = 0.60571 \\
\Phi(\frac{Z_d}{Z_f}) = 0.91446 \\
45^\circ \quad \alpha \\
\frac{Z_d}{Z_f} = \frac{Z_d'}{Z_f'} \\
R_1 = 0.60571 \\
R_L = 0.91446
Transition Function: \( \frac{Z_d'}{Z_f'} = \Gamma \left( \frac{Z_d}{Z_f} \right) \)
Transition Function: \( \begin{bmatrix} Z_d/Z_l \end{bmatrix} = \Gamma \left( Z_d/Z_l \right) \)