

# Human Capital Formation with Endogenous Credit Constraints\*

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## Abstract

This paper considers a human capital investment framework in which borrowing limits are endogenously derived from repayment incentives and, therefore, linked to future earnings prospects. Because human capital investments and credit limits are jointly determined, optimal investments differ considerably from those in standard models with exogenous constraints. Repayment incentives induce a positive relationship between credit limits and the ability, initial wealth, and human capital investments of individuals.

In our framework, investment responses to government policy are substantially different from those of a standard exogenous constraint model. Government policies affect incentives to default and, hence, optimal limits on private borrowing. Subsidies for investment in human capital should be accompanied by increases in borrowing limits, since borrowers are able to commit to repaying higher debts. Thus, education subsidies and student loans should be viewed as complements. Also, schooling subsidies, which relax credit limits, increase the dispersion of investment across ability types when constraints are endogenous, but not when constraints are exogenous. Contrary to conventional wisdom, public spending on education can crowd-in private spending, especially for the poor. Furthermore, when constraints are endogenous, higher labor income tax rates restrict borrowing and reduce investment in human capital (even when investments are tax deductible). We calibrate our model and show that the differences in predictions with standard models are quantitatively important.

**JEL Codes:** D31, E21, H52, I22, J24, J31.

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# 1 Introduction

It has long been argued that individuals from poor families are unable to borrow the resources needed to invest efficiently in their human capital (Becker [7]). The source of the problem is that human capital cannot serve as collateral for financial debts. Economists studying human capital investment decisions have incorporated this limitation by assuming that individuals face constraints on the amount they can borrow for school. While there has been intense debate about the importance of credit constraints for human capital investment (see Carniero and Heckman [12]), little attention has been paid to the nature of those constraints. Instead, economists have typically assumed that borrowing limits are exogenously given, independent of individual characteristics and decisions, and insensitive to government policies.

The *ad hoc* nature of exogenous credit constraints in economic models of human capital formation is dissatisfying given the potential importance of those limits in determining investment decisions and responses to government policies. There is little reason to think that all individuals face the same credit limit, that credit limits remain constant over the life cycle, or that credit limits are unaffected by government policies. To the contrary, recent empirical evidence suggests that credit limits vary across individuals with different levels of human capital (Keane and Wolpin [25]) and that borrowing limits change in response to changes in government bankruptcy policies (Gropp, Scholz, and White [20] and Monge, et al. [32]). When we allow credit limits to depend on the future earnings prospects of borrowers (as implied by optimal contracting between lenders and borrowers), we draw very different conclusions about cross-sectional differences in investment and about responses to government policies than when constraints are assumed to be fixed and the same for everyone. It is not enough, therefore, to ask how important credit constraints are in the market for human capital – it is also necessary to ask what determines credit limits, how they vary across individuals, and how they respond to government policy. In order to fully analyze the impacts of education subsidies or income taxes on human capital investment decisions, it is necessary to first develop a model that addresses these questions.

With this objective, we develop a life cycle framework of human capital formation in which credit constraints arise endogenously from individuals' incentives to default on their loan obligations. The model builds on earlier work by Kehoe and Levine [26], and Kocherlakota [28]. In equilibrium, creditors will only lend as much as they expect a borrower to repay.<sup>1</sup> Because lenders are limited in

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<sup>1</sup>Alvarez and Jermann [2] study the equilibrium valuation of financial assets. Other authors have used similar models to study the importance of durable goods (Krueger and Fernandez [19]) and pensions (Andolfatto and Gervais [3] and Lambertini [30]) in determining life cycle consumption decisions. Azariadys and Lambertini [6] study equilibrium aspects of an overlapping generations model with endogenous debt constraints and lifecycle consumption. Also within endogenous constraint frameworks, Krueger and Perri [29] study the effect of progressive taxation on insurance markets, and Attanassio and Ríos-Rull [5] explore the impact of outside transfers on insurance for villages. Chatterjee, et al., [15] examine default patterns in the market for unsecured consumer credit. None of the papers in

the punishments they can impose on borrowers that default, they will restrict the amount of credit they extend.<sup>2</sup> Individuals of heterogeneous abilities or those making different schooling choices will face different borrowing limits that depend on their incentives to default. Constraints will also vary over the life cycle as default incentives change.<sup>3</sup>

In our model, a link between borrowing constraints and future earnings prospects arises naturally from incentives to default and the penalties imposed on those who default. Credit laws typically create stronger incentives for individuals with higher earnings potential to repay their debt, because penalties for default disproportionately affect them. As a result, they can obtain relatively high amounts of credit. Those with low earnings potential have little incentive to repay loans and, consequently, face more stringent constraints. Human capital theory indicates that the more able an individual and the more he invests in his skills, the more earnings potential he will possess. In an efficient credit market, he should, therefore, be allowed to borrow more. Such an implication is consistent with recent empirical work by Keane and Wolpin. [25]

The link between human capital investment and credit availability creates additional incentives to invest that are absent in exogenous constraint models. This produces very different cross-sectional patterns in human capital investment. When constraints are endogenous, investment will be more skewed toward more able and wealthier individuals than when constraints are exogenous (given any distribution of initial endowments).

To the extent that government policies alter incentives to default, they will affect endogenously determined borrowing limits. Ignoring this effect can be misleading. For example, college subsidies are often discussed as a substitute for student loans. However, we find that with efficient credit markets and endogenously determined borrowing limits, lending should increase in response increased subsidies to schooling. This is because the subsidies encourage additional investments in human capital, which reduces future incentives to default. Thus, subsidy and loan policies can be better seen as complements rather than substitutes. This complementarity further implies that human capital investment responses to education subsidies are substantially larger when constraints are endogenously determined rather than held fixed. Furthermore, education subsidies expand differences in investment across ability types when constraints are endogenous but not when constraints are exogenous. In contrast to previous analyses of taxes and human capital, we find that increases in labor income taxes restrict endogenous credit limits and, therefore, reduce investment in human capital even when investments are tax deductible.

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this literature discuss human capital accumulation or education policy.

<sup>2</sup>Fay, Hurst, and White [18] empirically show that households are more likely to declare bankruptcy when the financial benefits of doing so are greater. Gropp, Scholz, and White [20] estimate that households living in states with larger bankruptcy asset exemptions are more likely to have their loan applications rejected and to be discouraged from borrowing. Monge, et al. [32] and the other studies in Pagano [34] examine how incentives to default translate into borrowing constraints in different countries.

<sup>3</sup>See Jappelli [24] and Fay, Hurst, and White [18] for empirical evidence on these relationships.

Credit constraints depend on interest rates and wage rates as highlighted by Kiyotaki and Moore [27]. To the extent that government policies are effective in changing investment decisions or wealth accumulation, they may have general equilibrium effects on credit constraints. Our model incorporates these effects, allowing borrowing constraints to adjust with changes in interest rates and wage rates caused by policy intervention.

The rest of this paper proceeds as follows. We first compare the standard treatment of human capital investment and exogenous borrowing constraints, which implicitly assumes that constraints are independent of individual abilities, choices, and government policy, with a model in which borrowing limits are assumed to be a function of future earnings. Modifying the nature of credit constraints in this way substantially alters the way we think about investment decisions. Section 3 then develops a model of human capital formation based on individual incentives to default on their loans. This model implies that individuals should be allowed to borrow a fraction of their future after-tax earnings, justifying our earlier treatment of variable constraints. Section 4 uses data from the U.S. to parameterize the model, and Section 5 quantitatively examines the importance of endogenously determined borrowing constraints. In particular, we study how the nature of credit constraints affects investment decisions across individuals and investment responses to government policy by contrasting the implications of our model with those of an exogenous constraint model. We also discuss the importance of different punishments levied on those who default. Section 6 concludes with a summary and discussion of avenues for future research.

## 2 The Nature of Borrowing Constraints

It is convenient to begin with a very simple model of human capital formation. Consider two-period lived individuals who must choose their investment in human capital during the first period and consumption in both periods. Agents are endowed with initial assets  $a$ . In the second period, they receive wage income  $w$  which is increasing in investment,  $y$ , and ability,  $e$ . In particular,  $w(e, y) = ey^\alpha$ . Investments may be subsidized at a rate  $\nu$ , and earnings may be taxed at rate  $\tau$ . In this economy, individuals choose investment,  $y$ , and borrowing,  $d$ , to

$$\max_{y,d} \{u(a + d - y(1 - \nu)) + \beta u((1 - \tau)w(e, y) - Rd)\},$$

where  $u(\cdot)$  is an (increasing, concave) utility function and  $\beta > 0$  is a time discount factor. The (gross) interest rate  $R \geq 1$  is given.

Without constraints on borrowing, individuals simply maximize lifetime earnings (net of investment costs) and borrow (or save) to optimally smooth consumption. Investment in human capital is, therefore, given by

$$y^* = \left( \frac{\alpha e(1 - \tau)}{R(1 - \nu)} \right)^{1/(1-\alpha)}.$$

Clearly, investment is increasing (and convex) in ability,  $e$ , and independent of initial assets,  $a$ . These two conditions hold as long as agents are unconstrained.

Since human capital cannot serve as collateral for financial debts, Becker [7] noted that individuals may be constrained in their ability to borrow for schooling. The literature on schooling has taken this into account by assuming that individuals face a limit,  $\bar{d}$ , on the amount they can borrow for school (e.g. Aiyagari, Greenwood, and Seshadri [1], Caucutt and Kumar [14], Hanushek, Yilmaz, and Leung, [21], and Keane and Wolpin [25]). Alternatively, some authors have modelled credit constraints by assuming that interest rates increase in the amount borrowed (e.g. Becker [7], Cameron and Taber [10], Card [11]). Both approaches typically assume that credit limits (or interest rates) are independent of individual characteristics and decisions, and both lead to similar conclusions about human capital investment behavior.<sup>4</sup>

Taking the first approach, we assume that borrowing constraints imply the following restriction on first-period debt:

$$d \leq \bar{d},$$

where the borrowing constraint  $\bar{d}$  is independent of individual characteristics and choices and government policy. In this sense, credit constraints are exogenously determined. This problem generates the following first order condition for human capital investment among constrained borrowers:

$$(1 - \nu)u'[a + \bar{d} - y(1 - \nu)] = \beta(1 - \tau)e\alpha y^{\alpha-1}u'[(1 - \tau)ey^\alpha - R\bar{d}].$$

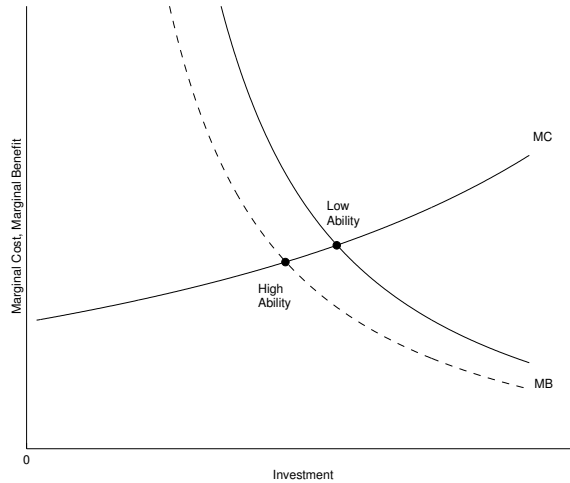
The left hand side represents the marginal cost of investment while the right hand side represents the marginal benefits. Both are shown in Figure 1 for individuals of two different ability levels. As in Becker [7], the marginal benefit curve (demand curve) is downward sloping due to diminishing marginal returns to investment and decreasing marginal utility of future consumption; the marginal cost curve (supply curve) is upward sloping whenever the borrowing constraint binds. When individuals are borrowing constrained, each extra dollar of investment must come out of current consumption, which becomes more and more costly in utility terms.

In contrast to Becker's discussion, it is quite possible that higher ability individuals have a lower demand curve for investment and, hence, lower optimal investment. This will be the case whenever the *substitution effect* of ability is dominated by the *income* or *wealth effect*. The substitution effect encourages investment by increasing the return per unit of investment. A higher ability makes current consumption more expensive relative to future consumption, causing individuals to substitute current with future consumption by increasing investment. On the other hand, income effects discourage investment among higher ability individuals when consumption is a normal good.

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<sup>4</sup>Keane and Wolpin [25] are a notable exception in that they allow credit constraints to vary by age and human capital levels.

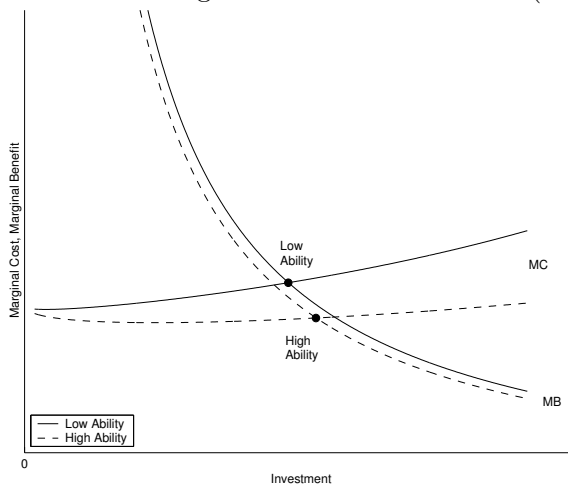
Figure 1: Marginal Benefits and Marginal Costs of Investment (Exogenous Constraints)



Higher ability implies a higher net present value of wealth for any level of investment. As a result, more able individuals want to increase both current and future consumption. Holding investment constant would increase future consumption in response to an increase in ability, but constrained agents can only increase current consumption by reducing investment. The net effect of ability on investment, therefore, depends on the balance of offsetting income and substitution effects. Notice, there would be no income effects with linear utility, in which case investment would unambiguously increase with ability among constrained borrowers (i.e. the case discussed by Becker [7]). More generally, if the intertemporal elasticity of substitution for consumption (IES) is less than one, as most empirical studies indicate (see Browning, Hansen, and Heckman [8]), then investment should be *decreasing* in ability – the case shown in Figure 1. This result is clearly counterfactual (e.g. Cameron and Heckman [9]).

This standard model hinges on the assumption that borrowing limits are uniform across agents and invariant to their ability and human capital investment choices. However, one might reasonably expect that more able individuals or those investing more in human capital can be counted on to repay their loans more reliably. Recognizing this, creditors would be willing to extend them more credit. In this case, human capital investment and credit limits are jointly determined. Incorporating this feature will substantially change the analysis. In the following section, we derive these *endogenous* credit limits from individual repayment incentives and optimal contracting between borrowers and lenders. For now, simply consider an economy in which individuals are able to borrow up to a fraction,  $\kappa \in (0, 1)$ , of their future after-tax earnings potential, so credit limits are an increasing function of ability and investments:  $\bar{d}(e, y) = \kappa(1 - \tau)w(e, y)$ . To keep the analysis simple, assume that ability is perfectly known by individuals and creditors, so there is no uncertainty or adverse selection. With these endogenous and variable credit limits, the first order

Figure 2: Marginal Benefits and Marginal Costs of Investment (Endogenous Constraints)



condition for constrained borrowers becomes

$$\begin{aligned} & ((1 - \nu) - \kappa e \alpha y^{\alpha-1}) u' [a + \kappa e y^\alpha - y(1 - \nu)] \\ & = \beta(1 - \tau)(1 - \kappa R) e \alpha y^{\alpha-1} u' [(1 - \tau)(1 - \kappa R) e y^\alpha]. \end{aligned}$$

The implied marginal cost and benefit curves for investment are shown in Figure 2. As in the standard exogenous constraint framework, the marginal benefit of investment is decreasing while the marginal cost is generally increasing. Marginal cost curves are flatter in this environment since higher investment expands borrowing opportunities. Expanded borrowing limits encourage investment among constrained borrowers. This effect is important when comparing the investment choices across ability types. Even in the case when income effects dominate substitution effects (IES is less than one), investment in human capital can be increasing in ability because more able individuals face a lower and flatter marginal cost curve. More able individuals can borrow more for any given level of investment, which reduces the cost of investment in terms of foregone first period consumption. Overall, investment is more likely to be increasing in ability when constraints are endogenous rather than exogenous. In fact, as long as out-of-pocket investment costs are greater than initial assets,  $(1 - \nu)y > a$ , it can be shown that the endogenous constraint model implies that investment will be increasing in ability.

The interest in exploring models with endogenous credit constraints does not end here. Government policies that affect the cost or the return of human capital can alter the incentives of individuals, changing their access to credit. Whether these effects exacerbate or reduce the direct effect of a policy depends on whether the constraints are relaxed or tightened. For example, it is quite possible in the endogenous constraint case that public schooling “crowds in” and not “out” private resources invested in human capital. This effect, which is incompatible with exogenous

credit constraints, can arise because public schooling expands credit limits. Interestingly, we find that the “crowding-in” of public schooling is more likely to take place for low income individuals. To examine this and other issues that might affect education decisions (e.g. income taxes and school subsidies), we construct a simple general equilibrium model in which the variable constraint function assumed in this section is endogenously derived from repayment incentives.

### 3 Default Incentives and Endogenous Borrowing Constraints

This section develops a model of human capital investment and endogenous borrowing constraints based on individual incentives to default. When lenders are limited in their ability to punish borrowers who default, we show that they will restrict individual borrowing to a fraction of future earnings. The model, therefore, provides a theoretical foundation for the simple variable borrowing constraint assumed in the previous section.

To determine the importance of different punishments faced by defaulting borrowers and to study how constraints may change over the life cycle, we consider an overlapping generations environment in which each generation lives for four periods. (Each generation contains a continuum of individuals with a mass of unity.) In each period  $t = 0, 1, 2, \dots$ , the economy is populated by students, young and old workers, and retirees. In the second period of work, all individuals have a single child at which time they give them bequests,  $b$ .<sup>5</sup> Within each cohort, there is two-dimensional intra-generational heterogeneity. Each generation is composed of agents initially endowed with different levels of physical assets,  $a$  (given to them by their parents as bequests), and different learning abilities,  $e$  (inherited from their parents following a Markov process described in detail below). The support of these characteristics is  $A \times E$ , and the probability measure  $\lambda$  describes the endogenous distribution of agents in this support.

Preferences at birth are given by

$$U_0 = u(c_0) + \beta u(c_1) + \beta^2 [u(c_2) + \rho v(b)] + \beta^3 u(c_3),$$

where  $\beta \in (0, 1)$  is the discount factor and  $c_j$  represents consumption at age  $j$ .<sup>6</sup> The function  $v(\cdot)$  reflects the utility of transferring resources to one’s own child. For analytical tractability, we shall assume that  $v(\cdot) = u(\cdot)$ . The degree of “altruism” will be determined by the parameter  $\rho \geq 0$ . Preferences from the point of view of a young worker, an old worker, and a retiree are given, respectively, by  $U_1 = u(c_1) + \beta [u(c_2) + \rho v(b)] + \beta^2 u(c_3)$ ,  $U_2 = u(c_2) + \rho v(b) + \beta^2 u(c_3)$ , and  $U_3 = u(c_3)$ . We assume that the period utility function is given by  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ , where  $\sigma > 0$ .

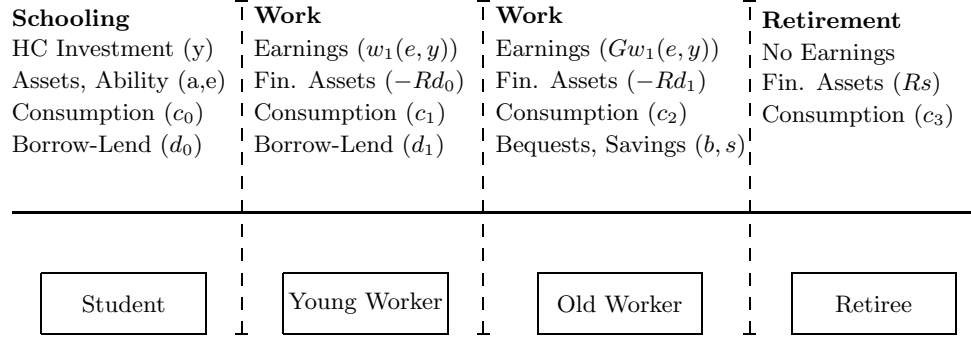
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<sup>5</sup>More generally, children may be born at any earlier period, but the bulk of expenses paid by parents are assumed to be paid in the second period of adulthood. Without changing the results, one could instead assume that individuals transfer bequests in the final period of life.

<sup>6</sup>We omit calendar time indices, since we focus on time invariant (stationary) equilibria.

The typical life cycle of all agents is described in Diagram 1.

**Diagram 1: Stages in the Life-Cycle of Agents**



Individuals in this economy must optimally convert physical capital into human capital, use their human capital to produce output and earn an income, transfer resources across periods to smooth consumption, and provide bequests to their offspring. In the first period, students use their initial endowment of assets,  $a$ , to finance consumption,  $c$ , and investment in human capital,  $y$ . Human capital investment is measured in units of the only consumption/capital good and captures not only time in school but also its quality and intensity. They may also borrow or lend. Positive  $d_0$  indicates the net debt of students. Young and old workers use their income from work to finance current and future consumption as well as bequests to their children given in the second period of adulthood. Their wages are proportional to their human capital,  $h$ , acquired from previous investment and work experience. Let  $d_1$  denote the debt ( $d_1 > 0$ ) or savings ( $d_1 < 0$ ) of young workers. Finally, let  $s$  denote the savings (always positive in equilibrium) of older workers. Retirees no longer work and consume their savings.

We assume that ability and investments are complementary in the production of human capital. Specifically, the human capital of a young worker with ability  $e$  who invested  $y$  units during his youth is given by

$$h_1 = ey^\alpha,$$

where  $\alpha \in (0, 1)$  is the investment-elasticity of the human capital production function.

Individuals accumulate work experience that augments their human capital. As a result, the human capital of old workers is given by

$$h_2 = Gh_1 = Ge y^\alpha, \quad G > 1.$$

The earnings of young and old workers are given by

$$w_1(e, y) = wh_1 = wey^\alpha \quad \text{and} \quad w_2(e, y) = wh_2 = wGe y^\alpha,$$

respectively, where  $w$  denotes the equilibrium wage per unit of human capital.

When making decisions, individuals have perfect knowledge of their ability,  $e$ , how their earnings depend on  $e$  and  $y$ , the equilibrium price for labor,  $w$ , and the gross interest rate,  $R$ . We also assume that creditors have this information.

The main insight of the endogenous constraint model is that institutional frameworks protect, albeit imperfectly, the rights of creditors. Institutions often allow creditors to discipline defaulting borrowers by impeding future borrowing, destroying their credit rating, garnishing a fraction of their earnings, and seizing part of their owned assets (currently and in a pre-specified future). We attempt to capture these forces in an admittedly stylized way.

For tractability, assume that defaulting agents are excluded from further borrowing and any savings earn a lower rate of return  $R_d = \phi R$ ,  $\phi \in (0, 1)$ . The latter punishment captures a number of effects. For example, if creditors can seize all savings deposited in formal markets, individuals will be forced to save in informal sectors or use a ‘backyard’ technology offering a lower rate of return. Alternatively, creditors may be able to seize a fraction of all physical assets saved by defaulting agents as reflected in Chapter 7 bankruptcy filings. We also assume that individuals must forfeit a fraction  $\gamma \in (0, 1)$  of their earnings if they choose to default. Wage earnings can be garnished up to 10-15% for those who default on federal student loans. Tax refunds can also be seized.<sup>7</sup> More generally, individuals with positive income but zero assets are typically required to repay some of their loan, where that repayment is likely to depend on their level of income. And, because defaulting borrowers receive a bad credit rating, they face difficulties in borrowing to purchase a home and may be forced to rent instead. Given the sizeable tax breaks provided for home mortgages, this implies that lucrative tax breaks may be foregone by those who default. Poor credit ratings may also make renting more costly for those who default, as property owners may be reluctant to rent to them. To the extent that these costs are positively related to earnings, they are reflected in  $\gamma$ . Overall, these punishment parameters  $(\phi, \gamma)$  should be viewed as approximations to a more complex system that relates punishments to the income and savings of individuals choosing to default. We assume that all of these punishments apply for only one period.

Clearly, the relevance of these three different punishments varies across the life cycle of agents. Being shut out of the lending market may be extremely costly for young workers who wish to borrow against higher future earnings. Yet, it has little impact on those about to retire. On the contrary, older workers earn higher incomes and hold more assets. For them, the ability of creditors to seize their assets or income if they default can be quite costly. These costs are likely to be smaller for

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<sup>7</sup>Student loans receive special treatment in bankruptcy proceedings. In general, they cannot be discharged under Chapter 7 bankruptcy and they must be paid in full under Chapter 13 bankruptcy re-payment plans except under extreme circumstances. Simple failure to make student loan payments, or default, can result in wage garnishments, seizure of tax refunds, and a poor credit rating. See Lochner and Monge [31] for a detailed description of student loan default and the ensuing punishments.

the young worker just out of school.

Creditors foresee the repayment incentives of agents. Therefore, they will only lend up to the maximum amount a borrower will willingly repay. Credit limits should incorporate all the (observable) information of borrowers, which (in this framework) includes their wealth, age, ability, and any human capital investments. As a result, agents with different characteristics will face different credit limits. Furthermore, individuals can affect the amount of credit they receive by choosing to invest more or less in their human capital. Borrowing limits should, therefore, be viewed as functions of age, ability, and investments rather than fixed constants. Because of this, any change in the environment that alters the costs or returns of physical and human capital investment will induce a change in the credit limits faced by everyone.

Given preferences, demographics, technologies and the institutional environment  $(\gamma, \phi)$ , it is possible that unrestricted consumption and investment decisions are not incentive compatible for some agents. Credit constraints bind when the unrestricted plan entails levels of debt that are so high that the agent is better off defaulting and enduring the associated punishment. Rational creditors with full information will restrict the amount of credit so that the agent never chooses to default. As a result, the expected discounted utility of repaying one's debt will always be equal to or better than the one attainable by defaulting. With perfect information and without uncertainty, there will be no default in equilibrium.

Let  $V_1^d$  and  $V_2^d$  be the value functions obtainable by defaulting for young and old workers, and  $V_0$ ,  $V_1$ , and  $V_2$  be the values attainable in equilibrium for students and young and old workers. The agent's problem is given by

$$\begin{aligned} V_0(a, e) &= \max_{\{y, d_0: V_1(w_1(e, y), d_0) \geq V_1^d(w_1(e, y))\}} \{u(a + d_0 - y) + \beta V_1(w_1(e, y), d_0)\}, \\ V_1(w_1, d_0) &= \max_{\{d_1: V_2(w_2(e, y), d_1) \geq V_2^d(w_2(e, y))\}} \{u(w_1 + d_1 - R d_0) + \beta V_2(G w_1, d_1)\}, \end{aligned} \quad (1)$$

$$V_2(w_2, d_1) = \max_{\{s, b\}} \{u(w_2 - b - R d_1 - s) + \rho v(b) + \beta u(R s)\},$$

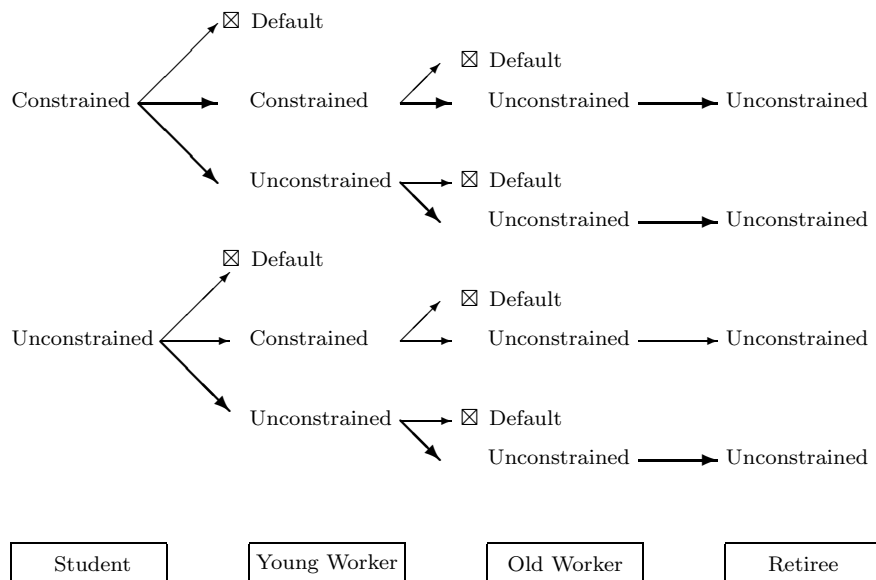
where

$$V_1^d(w_1) = \max_{\{d_1: d_1 \leq 0\}} \{u(w_1(1 - \gamma) + d_1) + \beta V_2(G w_1, R d_1)\}, \quad \text{and} \quad (2)$$

$$V_2^d(w_2) = \max_{\{s, b\}} \{u(w_2(1 - \gamma) - b - s) + \rho v(b) + \beta u(R d_1 s)\}.$$

Diagram 2 displays the possible life cycle investment/consumption plans. The  $\boxtimes$  s indicate that the market will block any investment plan that triggers default.

Diagram 2: Transitions in the Life-Cycle of the Agents



Depending on initial wealth and ability, credit limits may bind for students. In subsequent periods, the decisions made in previous periods will determine whether constraints bind. Young workers with positive debt would typically want to roll-over some of their debt to smooth consumption across their adult life. However, if the debt becomes too high, the amount needed to borrow for the unrestricted consumption plan could trigger default in the next period. Finally, the presence of retirement causes older workers to save (rather than borrow). Therefore, credit constraints are not relevant at that stage. In summary, loans made to students and young workers need to be compatible with individual incentives to repay during the two periods of work. Constraint levels may change between the schooling and work stages due to changes in incentives to default.

The thick lines in Diagram 2 indicate the only three possible paths in equilibrium. Individuals that are unconstrained as students will remain unconstrained throughout the rest of their lives.<sup>8</sup> Intuitively, if a student knew he would be constrained the next period, he would borrow and consume less while young. He would continue to reduce his current consumption until either the future constraint was no longer binding or until his current borrowing became constrained. On the other hand, individuals that are constrained as students may or may not remain constrained in the first period of work.

For any given pair  $(w, R)$  fixed, the model can be solved almost entirely in closed form. Regions

<sup>8</sup>To see why, assume that to the contrary, an agent is unconstrained when a student but that he is constrained later in life. Because he is not constrained during as a student, his investment  $y$  must equal the unconstrained optimum. Thus, the binding constraints only distort consumption. As an old worker, the agent cannot be credit constrained as he wants to save. The only possibility is that he is constrained as a young worker (i.e.  $u'(c_1) > \beta R u'(c_2)$ ). Since he is unconstrained during as a student,  $u'(c_0) = \beta R u'(c_1)$ . Combining these equations produces  $u'(c_0) > (\beta R)^2 u'(c_2)$ , which is a contradiction implying that the agent was constrained during the first two periods of life.

of  $(w, R)$  in which individuals default will never be reached in equilibrium. Still, to solve the model and obtain an expression for the credit constraints, it is necessary to obtain the value functions and the value of default for all nodes of the tree in Diagram 2.

Constraints will be non-binding for old workers as individuals save for retirement.<sup>9</sup> The value function for an old worker is

$$V_2(w_2, d_1) = \begin{cases} \Theta_\rho(R_d) \frac{[(1-\gamma)w_2]^{1-\sigma}}{1-\sigma} & \text{if } d_1 > \mu w_2 \text{ (the agent defaults)} \\ \Theta_\rho(R) \frac{[w_2 - R d_1]^{1-\sigma}}{1-\sigma} & \text{if } d_1 \leq \mu w_2 \text{ (the agent re-pays),} \end{cases}$$

where  $\Theta_\rho(R)$  is a strictly positive, strictly increasing function of  $R$ :

$$\Theta_\rho(R) = \begin{cases} (\rho + \Theta_0(R)) \left[ \frac{(\Theta_0(R)/\rho)^{-1/\sigma}}{1 + (\Theta_0(R)/\rho)^{-1/\sigma}} \right]^{1-\sigma} & \text{if } \rho > 0 \\ \Theta_0(R) & \text{if } \rho = 0 \end{cases}$$

and the function  $\Theta_0(R)$ , which would apply in the absence of altruism ( $\rho = 0$ ), is given by

$$\Theta_0(R) \equiv \left( \frac{(\beta R)^{-1/\sigma} R}{1 + (\beta R)^{-1/\sigma} R} \right)^{1-\sigma} + \beta \left( \frac{R}{1 + (\beta R)^{-1/\sigma} R} \right)^{1-\sigma}.$$

This function is obtained from calculating the optimal retirement savings and bequest decisions of old workers. If an agent carried over a large debt relative to his wage earnings, it could be optimal to default on it. In such a case, there are two costs of not repaying the debt. He loses a fraction of his earnings and earns a lower rate of return on his savings. At this age, the restriction on future borrowing is irrelevant. The function  $\mu$  that defines the default region is given by

$$\mu(R) \equiv \frac{1}{R} \left( 1 - (1 - \gamma) \left[ \frac{\Theta_\rho(R_d)}{\Theta_\rho(R)} \right]^{\frac{1}{1-\sigma}} \right).$$

The level of debt that triggers default is a constant fraction of current earning levels. This fraction is given by  $\mu = \mu(R)$ , which is a strictly decreasing function of the gross interest rate  $R$ . This fraction is also increasing in the punishments creditors can impose on those who default (i.e.  $\mu$  is increasing in  $\gamma$  and decreasing in  $\phi$ ). Because creditors foresee optimal default decisions, the function  $\mu$  also defines the credit constraints, i.e. the limits of the net debt that young workers can carry over to the next period. With perfect information, lenders will never allow the debt of young workers to exceed  $\mu w_2$  in this environment. While  $\mu$  only depends on interest rates, punishment parameters  $\gamma$  and  $\phi$ , and preferences, borrowing limits depend on investments and ability through future earnings,  $w_2$ . To the extent that earnings are increasing in  $y$  and  $e$ , so will the amount of borrowing that is allowed.

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<sup>9</sup>Here, we ignore the case of a very generous pay-as-you-go pension system, in which poor agents could wish to borrow even as older workers.

We can also obtain the amount of bequest transferred by a parent. If  $W_2(e, a)$  denotes the net wealth of an old worker who started life with  $(e, a)$ , then his child will start life with an initial value of wealth given by  $b(e, a) = W_2(e, a)/(1 + (\Theta_0(R)/\rho)^{1/\sigma})$ .

With these results, the value function for a young worker can be determined analytically. Assume that the ratio  $G/(1 - \gamma)$  is high enough with respect to  $\beta\phi R$  so that defaulting young workers will not want to save – they will simply consume their current earnings, net of the seized portion.<sup>10</sup> In addition to determining whether or not an individual defaults on his current debts, it is also necessary to determine whether he is able to accumulate further debts as freely as optimization requires. Borrowing may be restricted from future default incentives, as indicated by  $\mu$ .

Depending on the “state”  $(w_1, d_0)$  for a young worker, he will either default on earlier loans (in which case he cannot borrow again), face a constraint on additional borrowing, or make unconstrained choices. The levels of debt that define each of these regions can be expressed as a fraction of current earnings, so

$$V_1(w_1, d_0) = \begin{cases} \delta(R) \frac{[w_1(1+G/R)-Rd_0]^{1-\sigma}}{1-\sigma} & \text{if } d_0 \leq \kappa_0 w_1 \text{ (the agent is unconstrained)} \\ \frac{[w_1(1+\mu G)-Rd_0]^{1-\sigma}}{1-\sigma} + \beta\Theta_\rho(R) \frac{[G(1-\mu)w_1]^{1-\sigma}}{1-\sigma} & \text{if } \kappa_0 w_1 < d_0 \leq \kappa_1 w_1 \text{ (the agent is constrained)} \\ \frac{w_1^{1-\sigma}}{1-\sigma} [(1-\gamma)^{1-\sigma} + \beta\Theta_\rho(R)G^{1-\sigma}] & \text{if } d_0 > \kappa_1 w_1 \text{ (the agent defaults)} \end{cases}$$

where<sup>11</sup>

$$\kappa_0(\mu, R) = \frac{\mu G(1 + (\beta\Theta_\rho(R)R)^{-1/\sigma} R) + 1 - (\beta\Theta_\rho(R)R)^{-1/\sigma} G}{R}$$

$$\kappa_1(\mu, R) = \frac{1 + \mu G - [(1-\gamma)^{1-\sigma} + \beta\Theta_\rho(R)G^{1-\sigma}[1 - (1-\mu)^{1-\sigma}]]^{\frac{1}{1-\sigma}}}{R}.$$

In the first region for  $d_0$ , debt carried over from youth is relatively low ( $d_0 \leq \kappa_0 w_1$ ) such that re-payment is preferred to default. Furthermore, the agent is not constrained from borrowing additional funds during his first period of work. From that date onwards, he is able to finance the unrestricted consumption profile. At the other extreme, if debt left over from youth is high enough ( $d_0 > \kappa_1 w_1$ ), the young worker is better off defaulting and enduring punishment from lenders. In equilibrium, lenders will recognize this and restrict debts to be no greater than  $\kappa_1 w_1$  for students. For mid-level initial debts ( $d_0 \in (\kappa_0 w_1, \kappa_1 w_1]$ ), young workers do not owe enough to make default worthwhile, however, they are constrained from carrying a debt of more than  $\mu w_2$  into the next

<sup>10</sup>Considering the possibility that defaulting agents want to save does not add much complexity but also does not change the main characteristics and behavior of the credit constraints.

<sup>11</sup>The function  $\delta(R) = \frac{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}} (1 + \rho^{\frac{2\sigma-1}{\sigma}}) \beta^2 (\beta R)^{\frac{2(1-\sigma)}{\sigma}}}{[1 + (\beta R)^{(1/\sigma)} (1 + \rho^{-1/\sigma})/R + (\beta R)^{(2/\sigma)}/R^2]^{1-\sigma}}$  derives directly from unrestricted optimization. Then, the unrestricted borrowing function,  $d_1(d_0, w_1)$ , can be calculated.  $\kappa_0$  is obtained by solving the equality  $\mu G w_1 = d_1(d_0, w_1)$  (i.e. the maximum level of debt  $d_0$  that is consistent with the credit constraint not binding in that period). To obtain the value of  $\kappa_1$ , equate the value of default with the value of being credit constrained:  $\frac{[w_1(1+\mu G)-Rd_0]^{1-\sigma}}{1-\sigma} + \beta\Theta_\rho(R) \frac{[G(1-\mu)w_1]^{1-\sigma}}{1-\sigma} = \frac{w_1^{1-\sigma}}{1-\sigma} [(1-\gamma)^{1-\sigma} + \beta\Theta_\rho(R)G^{1-\sigma}]$ . This is the relevant equality, because  $\delta(R) \frac{[w_1(1+G/R)-Rd_0]^{1-\sigma}}{1-\sigma} \geq \frac{[w_1(1+\mu G)-Rd_0]^{1-\sigma}}{1-\sigma} + \beta\Theta_\rho(R) \frac{[G(1-\mu)w_1]^{1-\sigma}}{1-\sigma}$  for all  $(w_1, d_0)$ .

period because of future incentives to default. In this case, the end of period net debt for a young worker will be the maximum allowed ( $d_1 = \mu w_2$ ). In equilibrium, student debts are limited to  $d_0 \leq \kappa_1 w_1$ , while debts for young workers are constrained to  $d_1 \leq \mu w_2$ . The factor  $\kappa_0$  simply characterizes the level of student debt above which individuals want to borrow more in the first period of work than will be allowed by lenders.

The rate of growth in earnings,  $G$ , plays an important role in determining whether young workers will be constrained from borrowing. Given  $(w_1, d_0)$ , individuals that experience substantial wage growth will want to borrow a lot as students and young workers to smooth consumption. On the other hand, those with high wage growth will face a greater penalty from default – the inability to borrow again is costly since they can no longer smooth consumption and wage garnishments will be greater. These individuals are, therefore, allowed to borrow more than those with less wage growth. Whether or not they are more likely to face binding constraints depends on the balance of these two forces – their greater demand for credit and their greater incentives to re-pay their loans.

Credit constraints will only bind in the first two periods of life, since agents want to *save* and not borrow in the period prior to retirement. Thus, there are only three possibilities for the constraints: (i) the credit constraints bind for the first two periods, (ii) they bind for only the first period, or (iii) they never bind. However, due to the uniformity of preferences, only one of the first two possibilities can arise in any given equilibrium – it is not possible that, in the same stationary equilibrium, some agents are constrained for only one period, while others are constrained for two.

There are two types of equilibria in which at least some individuals are constrained. A Type I equilibrium consists of a group of agents for whom the credit constraints bind in the first two periods and another group (which may not exist) that is always unconstrained. In this equilibrium,  $\kappa_0 < \kappa_1$  and students who are constrained will continue to be constrained during their first period of work. The borrowing constraints are defined by  $d_0 \leq \kappa_1 w_1$  for students and by  $d_1 \leq \mu w_2$  for young workers. A Type II equilibrium consists of one group that is only constrained as students and another group (which may not exist) that is always unconstrained. In this equilibrium, some students are constrained to borrow no more than  $\kappa_1 w_1$ , while all workers are able to borrow freely. Individuals that were constrained as students are unconstrained in later periods either because the borrowing constraints they faced initially were very stringent or because constraints during adulthood are quite loose.

The conditions determining which of these two possible types of equilibria arise depends on the interest rate (as we illustrate below). That is, whether or not the inequality  $\kappa_0 < \kappa_1$  holds depends on  $R$ . Both  $\kappa_1$  and  $\mu$  are decreasing with respect to the interest rate. A higher interest rate increases the amount the agent has to repay and, therefore, less debt is needed to trigger default. The factor  $\kappa_0$  may be increasing or decreasing in  $R$ . It captures two effects. On the one hand, the lower the

interest rate, the more interested a young worker will be in borrowing. Thus, the punishment of not being allowed to borrow will be more costly. This increases the maximum debt required for the constraint  $\mu$  to bind. On the other hand, the lower the interest rate, the lower the effective liability of the agent, which makes it more attractive to re-pay any debts. Numerical simulations can produce cases in which either effect dominates. If the punishment of not being able to borrow is so costly that individuals are willing to relinquish resources for the opportunity to increase their consumption smoothing, then  $\kappa_0$  will be negative and a Type I equilibrium is guaranteed.

## General Equilibrium

We now make explicit the set  $E$  of different ability types as well as the intergenerational transmission of ability.

**A. 1.** *The support of abilities is given by a finite set  $E \subset \mathbb{R}_+$ .*

Let  $\mathcal{E}$  be the power set of  $E$ . The transmission of abilities from parents to children is given by the transition function  $P : E \times \mathcal{E} \rightarrow [0, 1]$ .

A stationary economy is one in which a time-invariant probability space describes the population at each point in time. Given the structure of the model, the economy is stationary if and only if there is a time invariant distribution for  $(a, e)$  among youth. Since bequests are non-negative, the set of possible initial asset levels is contained in  $A \subset \mathbb{R}_+$ . Let  $\mathcal{B}_+$  denote the Borel sets  $\mathbb{R}_+$  and  $\mathcal{A}$  denote the Borel sets on  $A$ . Finally, let  $\mathcal{F}$  be the product space  $\mathcal{E} \times \mathcal{A}$ . For a measure  $\lambda$  defined over the space  $(E \times A, \mathcal{F})$  to be invariant, it is required that, given the bequest function  $b(e, a)$ ,  $\lambda$  “reproduces” itself over time. That is, for any set  $B \in \mathcal{E} \times \mathcal{B}_+$ ,

$$\lambda(B) = \int_{E \times A} \chi_{\{(e', b(e, a)) \in B\}}(e, a) P(e', e) \lambda(de \times da). \quad (\text{Agg1})$$

where  $\chi$  is the indicator function.

Given  $\lambda$ , aggregate human capital  $H$  is given by

$$H = (1 + G) \int_{E \times A} ey(a, e)^\alpha \lambda(da \times de). \quad (\text{Agg2})$$

The net savings of all agents determines the aggregate physical capital stock,  $K$ :

$$K = \int_{E \times A} [s(a, e) - d_0(a, e) - d_1(a, e)] \lambda(da \times de). \quad (\text{Agg3})$$

Aggregate output,  $Q$ , is produced with physical and human capital according to the production function  $Q = F(K, H)$ , where  $F(K, H)$  is increasing and concave in both  $K$  and  $H$ .

Regardless of the contracting environment, aggregate human and physical capital determine the market-clearing wage and interest rate. Market clearing prices must be

$$w = \frac{\partial F(K, H)}{\partial H}, \quad R = 1 - \delta + \frac{\partial F(K, H)}{\partial K}. \quad (\text{MC})$$

where  $\delta \in (0, 1)$  is the depreciation rate of physical capital.

All the above leads to the following standard definition:

**Definition 1. Stationary Equilibrium** *Given the demographics  $\{E \times A, \mathcal{F}, P\}$ , preferences  $\{u(\cdot), v(\cdot), \beta, \rho\}$ , technologies  $\{ey^\alpha, F(\cdot, \cdot), \delta\}$ , and the credit market contracting environment  $(\gamma, \phi)$ , a stationary equilibrium is a price pair  $\{w, R\}$ , an invariant distribution  $\lambda$ , aggregate capital stocks  $\{H, K\}$  and individual allocations  $\{y(a, e), d_0(a, e), d_1(a, e), s(a, e)\}$  such that*

1. *Given  $\{w, R\}$ , the allocation  $\{y(a, e), d_0(a, e), d_1(a, e), b(a, e), s(a, e)\}$  solves the individual problem for all  $(a, e) \in A \times E$ ;*
2. *The aggregate stocks are consistent with individual decisions (equations Agg1, Agg2 and Agg3); and,*
3. *Prices clear the markets (equations MC).*

Because of their importance in the actual U.S. economy, we include government policies like public schooling and education subsidies in our calibration of the model. Their inclusion affects the decisions and constraints of all agents. It also requires the inclusion of a government budget constraint in the definition of a stationary equilibrium. These changes are straightforward and discussed further in the next section of the paper.

## 4 Parameterizing the Model

To quantitatively assess the effect of government policies and relevance of endogenously determined borrowing constraints, we need to specify empirically grounded parameter values for preferences, intergenerational transmission of ability, technologies, and credit institutions. Our main objective here is to better understand the quantitative importance of the endogenous nature of borrowing constraints. We parameterize the model using data from the U.S. economy. In particular, we use data on schooling, ability, and wages in the National Longitudinal Survey of Youth (NLSY) and schooling costs from the Digest of Education Statistics [33]. We also generate life cycle consumption patterns that are consistent with the patterns estimated in the literature (e.g. Carroll and Summers [13] and Attanasio, et al. [4]).

In our set up, a period is interpreted as approximately 15 years (where people are born around age 10). Using a yearly discount factor of  $1/1.06$ , the discount factor in the model is  $\beta = 1.06^{-15} = 0.4173$ . A CES production function is assumed where

$$F(K, H) = A [\xi K^{-r} + (1 - \xi)H^{-r}]^{-1/r}.$$

Our base case assumes complementarity between physical and human capital with  $r = 0.2$ .<sup>12</sup> An annual depreciation rate of 8% is assumed for physical capital, so  $\delta = 1 - 0.92$ <sup>15</sup>. All other parameters of the model are determined using the data and procedures described in the following subsections.

#### 4.1 Determining Parameters for Wage Growth and Human Capital Production

We empirically estimate the parameters  $\alpha$ ,  $G$ , and the distribution of  $e$  using wage and earnings data for men from the National Longitudinal Survey of Youth (NLSY) and average direct schooling expenditures per pupil from the Digest of Education Statistics. [33] The NLSY is particularly useful for our purposes, since it contains often used measures of cognitive ability in the Armed Forces Qualifying Test (AFQT), and it follows the same individuals over the early part (first 17 years) of their careers. To aid in estimation of heterogeneous learning abilities, we categorize individuals in the NLSY into five ability groups (corresponding to each of the five quintiles in the national distribution) according to their scores on the AFQT.

A two-step procedure is used to estimate the parameters of interest. First, total costs for each year of schooling are computed from direct expenditures and foregone earnings. Working men in the NLSY are used to compute foregone earnings. In calculating direct expenditures, we use average expenditures per pupil from 1980-89 for primary and secondary schooling, and for universities and colleges, since those years correspond to the years most men in our NLSY sample made their marginal schooling decisions. Next, we use a standard log wage regression and NLSY data on male earnings to estimate the human capital parameters  $\alpha$  and  $G$  as well as the distribution of ability. These parameter estimates are summarized in Table 1. See Appendix A for details of the two-step estimation procedure.

Table 1: Estimated Human Capital Parameter Values

Parameter	Estimate
$\alpha$	0.6146
$G$	2.1275
$e_1$	1.0000
$e_2$	1.2184
$e_3$	1.3054
$e_4$	1.3159
$e_5$	1.3916

Estimates for  $e_j$  provide a measure of the quantitative effect of ability on earnings. Someone from the highest ability group will earn 39% more than someone from the lowest ability group

<sup>12</sup>We have also explored a version with  $r = -0.2$  in which human and physical capital are substitutes in production. Tables analogous to those presented below show very little substantive difference.

conditional on the same human capital investment.

## 4.2 Intergenerational Transmission of Ability

The estimates of  $e_j$  indicate the earnings capacity for agents in different quintiles of cognitive ability. By construction, each of these ability types has a 20% mass, which we take to be the stationary distribution for ability. The intergenerational transmission of ability is assumed to be a five-point Markov chain with transmission probabilities defined by the matrix

$$P = \begin{pmatrix} 1 - (5/2)\pi & \pi & \pi/2 & \pi/2 & \pi/2 \\ \pi & 1 - 3\pi & \pi & \pi/2 & \pi/2 \\ \pi/2 & \pi & 1 - 3\pi & \pi & \pi/2 \\ \pi/2 & \pi/2 & \pi & 1 - 3\pi & \pi \\ \pi/2 & \pi/2 & \pi/2 & \pi & 1 - (5/2)\pi \end{pmatrix}.$$

We choose  $\pi = 0.1763$  to generate an intergenerational correlation of ability equal to 0.5. See Daniels, et al., [16] for a recent discussion of these estimates. The results we focus on are quite robust to changes in the specification for  $P$  or the values of  $\pi$  that we use.

## 4.3 Calibrating Altruism, Credit Market Punishments, and Aggregate Technology Parameters

We choose the parameters  $\rho$ ,  $\gamma$ ,  $\phi$ ,  $A$ , and  $\xi$ , such that the stationary equilibrium in the economy mimics main features of the U.S. economy. Features we replicate include an equilibrium real interest rate of  $R = 1.04^{15}$ , average wage earnings of young workers,  $E(w_1)$ , average expenditures on education,  $E(y)$ , and average consumption growth over the life cycle ( $E(c_1)/E(c_0)$ ,  $E(c_2)/E(c_1)$ , and  $E(c_3)/E(c_2)$ ). Mean wages are calculated using the log wage regression estimates from the NLSY data described above to remove any influences of differential cohort size. We compute a mean wage income for the first 15 years of work to be \$247,145. Using the empirical distribution for schooling by ability and the expenditures reported in Table A-1, we compute the mean investment in the NLSY to be approximately \$92,673. In our calibration, we divide these amounts by 1,000 to normalize units in thousands of dollars. We replicate average consumption growth rates of  $E(c_1)/E(c_0) = 2.5$ ,  $E(c_2)/E(c_1) = 1.15$ , and  $E(c_3)/E(c_2) = 0.8$ , which roughly correspond to the patterns reported in Carroll and Summers [13] and Attanasio, et al. [4].<sup>13</sup>

Given the considerable public involvement in the U.S. education system, we introduce a subsidy schedule for human capital investment in our calibration. Investments are assumed to be subsidized fully through  $y_0 = 50$  (roughly grade 10), after which they are subsidized at the rate  $\nu = 0.5$  (reflecting the fact that foregone earnings are a sizeable fraction of schooling costs once individuals are old enough to work but that the government also significantly subsidizes the direct costs of

<sup>13</sup>These (and other) studies report estimated life cycle profiles for consumption that differ considerably. The consumption growth factors we use lie within the range of estimates produced by these studies.

high school and college). Flat wage taxes are levied to balance the federal budget (i.e. to pay for the subsidies to investment). See the following section for a more complete discussion of how this policy environment affects the constraint functions.

The calibrated parameters are shown in Table 2 along with the data target values.<sup>14</sup> The intertemporal elasticity of substitution,  $\sigma$ , can be directly determined from  $\beta$ , the desired interest rate, and the ratio of expected consumption  $E(c_3)/E(c_2)$ . There is no simple mapping between any of the other parameters and the data targets within each column, as the parameter values needed can only be obtained by solving for the stationary equilibrium repeatedly until all five of those target values are matched.

Table 2: Calibrated Parameter Values and Target Data (Base Case)

Parameter	Value	Target Data
$\sigma$	1.2804	$\sigma = \log(\beta R) / \log(E(c_3)/E(c_2)), E(c_3)/E(c_2) = 0.8$
$\rho$	0.2286	$E(y) = 93$
$A$	13.593	$E(w_1) = 247$
$\xi$	0.0664	$R = 1.04^{15} = 1.80$
$\gamma$	0.1285	$E(c_1)/E(c_0) = 2.5$
$\phi$	0.9834	$E(c_2)/E(c_1) = 1.15$

The calibration implies that individuals who default face costs equal to 13% of their earnings and 2% of their savings. These cost factors correspond well to official policies toward individuals who default on federal student loans, which specify that wage earnings can be garnished up to 10-15%. The role of assets in determining punishments for those defaulting on student loans is less clear from a legal standpoint, reflecting the fact that most recent graduates have little or no assets to seize; however, the equilibrium is quite insensitive to changes in  $\phi$ .

#### 4.4 The Base Case Economy

This section describes important characteristics of the calibrated base economy. Equilibrium interest rates and skill prices are presented in Table 3 along with aggregate human and physical capital levels, the wage tax rate, and the average educational subsidy amount. A wage tax rate of 9.23% is needed to pay for education subsidies, which average \$71 per person.

Because ability is correlated across generations, more able individuals tend to receive larger bequests from their parents, who were also likely to have been more able. While most lower ability individuals begin with assets of less than \$70, most high ability individuals begin with assets above that amount. The distribution of initial assets is summarized in column 1 of Table 4. The difference in average initial assets for the most able and least able is about \$17.

<sup>14</sup>We verify that the steady state is unique within a region around these parameter values.

Table 3: Base Economy

Variable	Value
$w$	12.2102
$R$	1.8009
$\tau$	0.0923
$H$	63
$K$	40
$E(\textit{subsidy})$	71

Table 4: Base Case Initial Assets, Investment, Earnings, Debt, and Consumption

Statistic	$a$	$y$	$w_1$	$d_0$	$d_1$	$c_0$	$c_1$	$c_2$
$E(\cdot)$	74	93	247	30	35	83	206	235
$E(\cdot e_1)$	64	77	176	21	25	71	147	168
$E(\cdot e_2)$	71	89	235	28	33	80	196	224
$E(\cdot e_3)$	76	96	263	32	37	85	219	251
$E(\cdot e_4)$	78	98	269	32	38	87	224	256
$E(\cdot e_5)$	81	103	293	35	42	90	244	279
$Var(\cdot)$	162	153	1,796	26	36	114	1,247	1,628
$E(Var(\cdot e))$	125	74	198	3	4	73	138	180

Figure 3 shows how human capital investments increase with ability and initial assets. For any given level of initial assets, the least able individuals invest about \$5 less than the second lowest ability group and about \$10 less than the most able. The affects of ability on investment appear to be fairly constant across initial asset levels. For every \$1 increase in initial assets, investment tends to increase by nearly \$.80 for all ability groups.

The overall distribution of investment in the economy depends on the joint distribution of initial assets and ability. The positive correlation between ability and initial assets results in much higher investment among the more able, as seen in Figure 4, which graphs the conditional distribution of investment for each ability type. As Table 4 shows, average investment differences by ability (column 2) are substantial. This is also clearly reflected in Figure 4, which shows that the distributions of investment for the lowest and highest ability groups barely overlap. Much of the investment difference is due to the correlation between ability and initial assets. Recall that Figure 3 implies that, conditional on initial assets, the difference in investment between the least and most able should be about \$10. This reflects the direct effect of ability through it's effect on the marginal returns to investment (and the ability to borrow as we discuss more below). The \$26 average difference in investment between the most and least able individuals reported in Table 4 reflects this direct relationship as well as the fact that more able individuals also tend to have higher initial assets.

Figure 3: Human Capital Investment as a Function of Ability and Initial Assets

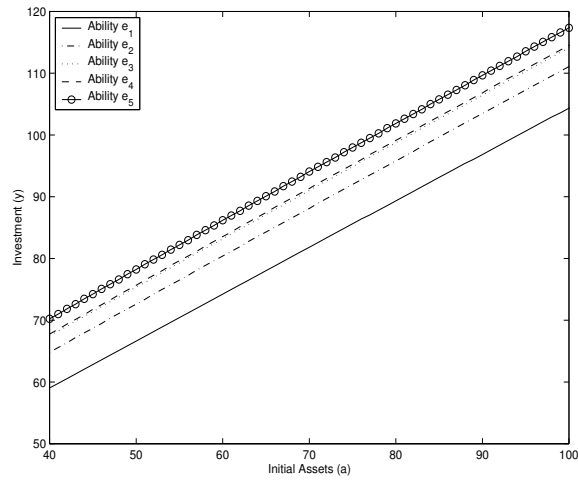


Figure 4: Distribution of Human Capital Investment by Ability

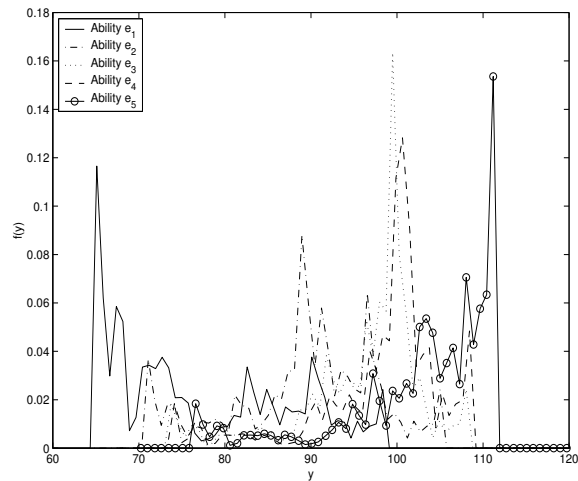


Figure 5: Debt for Students ( $d_0$ ) as a Function of Ability and Initial Assets

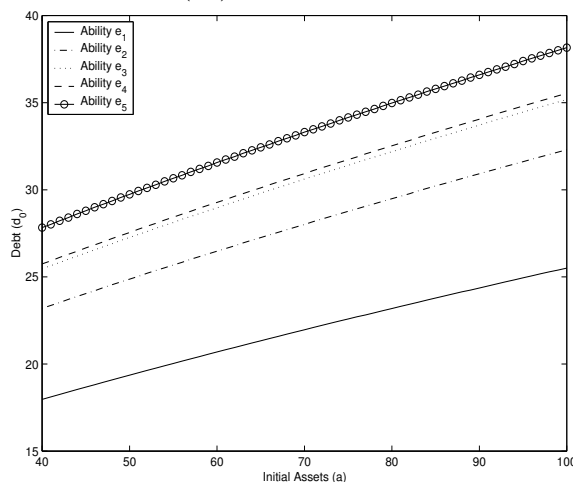
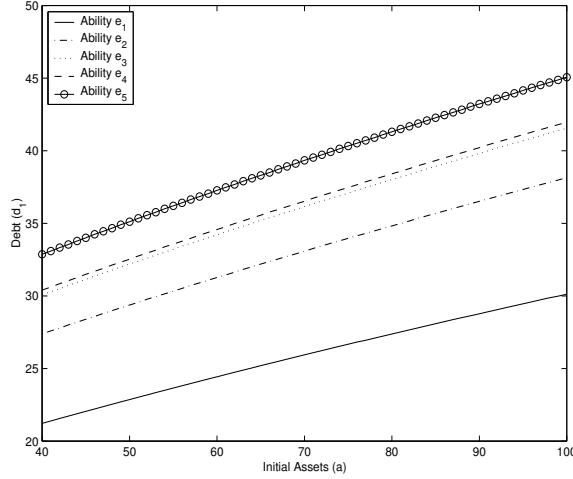


Table 4 reports a number of other interesting statistics for the economy. For example, column 3 reports average earnings in the population, average earnings conditional on ability, the variance in earnings in the population, and the average conditional variance (i.e. the average of each variance conditional on ability) for individuals during the first period of work. Differences in earnings across ability types reflect both differences in investment and differences in the return to investment. Thus, more able individuals earn substantially more than their less able counterparts.

Columns 4 and 5 of Table 4 report the same statistics on debt for students and young workers. In equilibrium, all students and young workers are constrained, so the entries in the table reflect the amount of borrowing allowed. On average, students are able to borrow up to \$30 and young workers \$35, but there is considerable variation related to initial asset levels and ability as shown in Figures 5 and 6. Allowable debt increases in initial assets (by about \$1 for every \$6 increase in assets), because individuals with higher initial assets will invest more in their human capital. This is also true for ability; however, allowable debt would also increase with ability if investment were held constant since more able individuals will earn higher returns on their investments. Holding initial assets constant, the most able can borrow about \$10-15 more than the least able during their youth and first period of work – these differences are sizeable given the relatively low levels of average debt. Thus, more able individuals not only have an advantage in that they can produce more human capital for any given investment, but they also tend to begin life with higher initial assets and greater access to credit.

The maximum allowable debt depends on future earnings power relative to the factors  $\kappa_1$  and  $\mu$ , which are reported in Table 5 along with  $\kappa_0$ . Notice that  $\kappa_0 < 0$ , which implies that the economy is in a Type I equilibrium – constraints bind during youth and the first period of work. Creditors will allow debt during youth to reach 13.2% of (after-tax) earnings next period, and they will allow

Figure 6: Debt for Young Workers ( $d_1$ ) as a Function of Ability and Initial Assets



young workers to borrow up to 7.3% of their (after-tax) earnings the following period.

Table 5: Base Economy

Variable	Value
$\kappa_0$	-0.0860
$\kappa_1$	0.1323
$\mu$	0.0734

As discussed earlier, these constraint factors depend on the equilibrium interest rate. Figure 7 shows this relationship for  $R$  corresponding to annual interest rates ranging from 2-6%. Since  $\frac{d_1}{d_0} = \left(\frac{\mu}{\kappa_1}\right)G$  and  $\frac{\kappa_1}{\mu} < G$  over the entire range of  $R$ , the figure suggests that individuals will generally be allowed to borrow more during the first period of work than during their youth. The fraction of future earnings that youth can borrow tends to be more responsive to changes in interest rates, so we might expect greater general equilibrium effects on borrowing at this time than during the first period of work.

Finally, we examine consumption in the economy. The final three columns of Table 4 report summary statistics on consumption for the first three periods (note that  $c_3 = 0.8c_2$  for everyone). Consumption increases substantially (by a factor of 2.5, on average) from youth to the first period of work. This is by design, since we calibrate the economy to produce this increase. As shown in the table, the increase is substantially greater (in percentage terms as well as levels) for the more able types. This is largely because more able individuals earn a higher return on their investments and are, therefore, willing to give up more consumption in the short run to finance those investments. Figure 8 graphs some representative consumption profiles for individuals with identical initial assets ( $a = 75$ ) but who differ in ability. While consumption varies very little by ability for students with

Figure 7: Dependence of Solvency Constraints on the Interest Rate

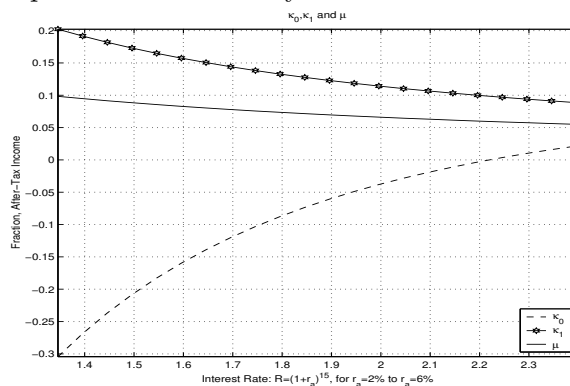
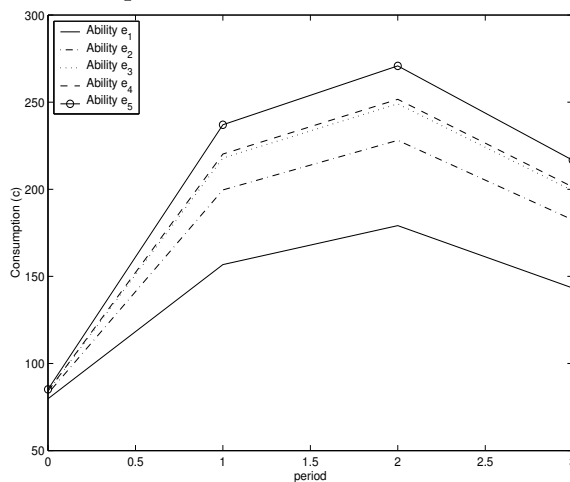


Figure 8: Lifecycle Consumption Profiles for Individuals with Initial Assets of \$75



the same level of initial assets, there are sizeable differences in consumption among workers and retirees.

#### 4.5 Quantitatively Comparing Exogenous and Endogenous Borrowing Constraints

The analysis in Section 2 suggested that investment behavior will depend on the assumptions made about the nature of credit constraints. We now ask whether the endogenous nature of credit constraints plays an important role in determining allocations and investment decisions or whether quantitatively similar results would obtain in a model with exogenously imposed constraints. Table 6 compares the role of ability and initial assets in our model of endogenous constraints with a model that assumes invariant exogenous borrowing constraints. To focus only on the role of the constraints, all parameters, including  $\tau$ ,  $R$ , and  $w$ , are held fixed at their base case levels. The first column reports levels of debt, optimal investment,  $dy/da$ , and  $dy/de$  for someone with the median

ability ( $e = 1.3054$ ) and average amount of initial assets ( $a = 74.30$ ) in the base case endogenous constraint economy. Column two reports the corresponding values for an exogenous constraint model with  $d_0$  and  $d_1$  set to the same levels of debt as in the endogenous constraint model. Thus, the only difference between the two columns is the endogeneity of the credit constraints.<sup>15</sup> Notice that investment is nearly 25% higher in the endogenous constraint model. Treating debt constraints as exogenous would cause one to overstate the degree of under-investment in human capital caused by credit market imperfections. Because individuals who invest more are able to borrow more in an efficient contracting economy, the endogeneity of constraints generates an additional incentive to invest that is not present when constraints are exogenously determined.

Table 6: Comparison of Endogenous and Exogenous Constraint Models

	Endogenous Constraints	Exogenous Constraints
$d_0$	31.30	31.30
$d_1$	36.96	36.96
$y$	94.34	75.73
$dy/da$	0.78	0.58
$dy/de$	34.41	-14.63

Notes: Based on an individual with  $a = 74.3$  and  $e = 1.3054$ . Levels of debt for the exogenous constraint economy are set to be the same as those determined in the endogenous credit market economy.

Investment also increases much more with initial assets (34% more) in the endogenous constraint model. Again, this difference reflects the fact that constrained individuals with higher initial assets will, all else equal, invest more in their human capital. This relaxes borrowing constraints in the endogenous constraint framework, which further increases investment.

While investment also increases with ability in the endogenous constraint framework, it is decreasing in ability when the constraints are exogenous. The exogenous constraint framework predicts that more able individuals will invest *less* in their human capital given any level of initial assets. As noted earlier, this is because a higher ability improves future earnings and consumption, which makes consumption while young relatively more valuable. When debt constraints are endogenous, this effect is more than offset by the increased capacity to borrow resulting from a greater future earnings capacity.

One can also use these comparisons to decompose the effects of initial endowments on investment

<sup>15</sup>While not shown, the same patterns emerge for all other combinations of  $a$  and  $e$  in our model economy.

in the endogenous constraint model. For example, the endogenous constraint model predicts that a one dollar increase in initial assets raises investment in human capital by about 78 cents. From the exogenous constraint comparative statics, we observe that about 58 cents of that increase would occur if debt levels were held constant. Therefore, about 25% of the total increase in investment can be traced to the endogenous nature of the constraints.

The contrasts are stark and clearly argue for more carefully analyzing the sources of borrowing constraints. Not only do the two models predict very different levels of investment for any given amount of observable debt, but they also predict very different correlations between investment and individual ability and asset endowments. As we show below, policy responses to investment subsidies and wage taxes are also quite different in the two models.

## 5 Government Policies

In an efficient credit market, government policies will affect investment through three channels. First, they may directly alter the costs and benefits of investment. Second, they may change the limits on borrowing faced by individuals. Finally, general equilibrium changes in the price of human capital and interest rates can affect investment through changes in the costs and benefits of investment as well as the amount of lending creditors will provide. Because the main contribution of our model is the endogeneity of credit constraints, we pay particular attention to the effect of government policies on borrowing limits and how changes in those limits affect individual human capital investment decisions.

### 5.1 Labor Income Taxation

The effect of labor income taxation on human capital formation is the subject of an extensive literature, which generally assumes that perfect capital markets exist.<sup>16</sup> It is well known that if all the costs of human capital investment are in the form of foregone earnings and individuals can borrow and lend freely, then the partial equilibrium effect of a proportional tax on earnings is neutral. Tax impacts in general equilibrium models with perfect credit markets are also negligible. [17, 22, 35] This is because a proportional tax on earnings reduces both the costs and benefits of human capital at the same rate. More generally, the response of unconstrained agents to a tax on labor income depends on the fraction of investment that is tax deductible, which is generally assumed to be the fraction of total investment that is represented by foregone earnings. When direct costs of investment (e.g. tuition, books, etc.) exist and are not tax deductible, a tax on earnings will generally reduce incentives to invest.

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<sup>16</sup>See Davies and Whalley [17], Heckman, Lochner, and Taber [22], and Trostel [35] for a few recent studies.

In our model, all investment costs are from foregone consumption goods and assumed to be non-deductible, so an increase in labor taxes reduces the returns to investment without affecting the costs (ignoring general equilibrium effects). This suggests that labor income taxes should reduce investment and consumption for unconstrained agents. Now, consider the effects of a labor income tax on individuals who are borrowing constrained. For exogenous fixed constraints, the tax reduces future earnings and consumption, which increases the marginal value of future earnings. This effect counteracts any disincentive effects of an income tax and can even cause investment to be increasing in income tax rates. This assumes that constraint levels are unaffected by policy.

Our model of efficient lending suggests that endogenously determined constraints would become more stringent following an increase in income tax rates due to a reduction in future after-tax earnings (assuming individuals cannot default on their taxes, constraints on borrowing are given by  $\kappa_1(1-\tau)w_1$  and  $\mu(1-\tau)w_2$ ). The fact that the government captures part of the returns from human capital investment reduces individual incentives to repay financial liabilities, thereby restricting the amount of credit lenders will offer. While wage taxes will not change the constraint factors  $\mu$  and  $\kappa_1$  when prices are held constant, allowable debt is scaled by the factor  $1-\tau$  reflecting the fact that future net-of-tax earnings determine default incentives. This effect causes investment to decline relatively more in response to an income tax when constraints are endogenous rather than exogenously held fixed. Furthermore, declines in investment cause further restrictions on credit, which feeds back into investment decisions. Finally, general equilibrium effects may also affect credit limits. It should be noted that the reductions in credit and resulting declines in investment associated with a tax increase would occur even if all human capital investments were assumed to be tax deductible. This stands in sharp contrast to the standard result of complete neutrality.

The impacts of an increase in tax rates can easily be examined in our calibrated base economy. In this economy, a 1% increase in the wage tax rate produces a 0.5 percentage point decline in investment for the average individual (when  $w$  and  $R$  are held constant).<sup>17</sup> The effect of such a tax increase on investment would only be half that size if constraints were exogenous. Thus, the endogeneity of credit constraints accounts for one-half of the total effect of labor income taxes on investment.

In computing the equilibrium effects of a tax increase on investment, one must consider changes in the distribution of bequests and initial assets that might occur. Furthermore,  $R$  and  $w$  are likely to change in a general equilibrium environment. Following a 1% increase in labor income tax rates, we find that average investment in the economy would decline from \$92.7 to \$90.9 (accounting for changes in the distribution of initial assets). Further accounting for general equilibrium changes in  $R$  and  $w$  causes average investment to decline even more to \$90.3. Thus, our model of endogenous

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<sup>17</sup>We no longer balance the budget in this experiment, since we want to identify the impacts of a change in tax rates alone.

constraints predicts that average human capital investment would decline by more than 2.5% in response to a 1% increase in wage taxes.

## 5.2 Subsidies to Education

Now, assume that labor income taxes are used to finance a proportional subsidy,  $\nu$ , on the schooling expenses of young agents. Thus, the self-financing constraint requires that  $\nu \int_{A \times E} y(a, e) \lambda(da \times de) = \tau wH$ . Young agents deciding how much to invest in human capital must solve the problem

$$V_0(a, e) = \max_{\{(d_0, y): d_0 \leq \kappa_1(1-\tau)wey^\alpha\}} \{u(a + d_0 - y(1 - \nu)) + \beta V_1((1 - \tau)wey^\alpha, d_0)\}. \quad (3)$$

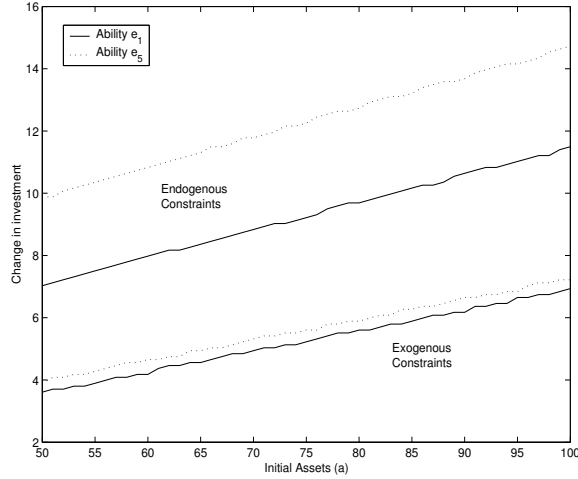
Abstracting from general equilibrium effects, the regime may increase or reduce the unconstrained investment depending on whether  $\nu \gtrless \tau$ . Among constrained youth, subsidies encourage investment, much like an increase in assets – they increase the amount of investment that can be supported for a given level of debt. A more unique result of this model concerns the effects of the policy on the constraints themselves. As above, labor taxes serve to reduce the amount of allowable debt, counteracting the increase in available funds from the subsidy. However, by encouraging human capital investments over consumption during youth, subsidies increase future earnings power producing an indirect effect on the amount of borrowing allowed. Thus, subsidies to investment will lead to greater increases in investment when constraints are endogenous rather than exogenously fixed. The final impact of the combined tax and subsidy policy on lending depends entirely upon whether after-tax earnings increase or decrease.

We numerically analyze the impacts of an increase in the rate of subsidy to schooling to show how our model of endogenous constraints responds. The base model is described in the calibration procedure and roughly corresponds to the current U.S. education system. Investments are assumed to be subsidized fully through  $y_0 = 50$  (i.e. the tenth grade) and at the rate  $\nu = 0.5$  above that level. Flat wage taxes are levied to balance the federal budget (i.e. to pay for the subsidies to investment).

We begin our analysis by studying the role of the endogenous nature of credit constraints in determining investment responses to an investment subsidy. Specifically, Figure 9 shows changes in human capital investment in response to an increase in the subsidy rate to 0.55, holding tax rates and prices constant. Responses are shown for both high and low ability individuals with a broad range of initial assets when credit constraints are assumed to be exogenous and when constraints are endogenously determined in an efficient contracting environment. The figure clearly shows that investment increases substantially more in response to investment subsidies when constraints are endogenously determined. More interestingly, investment responses are much larger for the most able students relative to less able students when constraints are endogenously determined. In

contrast, investment responses are quite similar across ability types when constraints are assumed to be exogenous. Overall, the endogeneity of constraints leads to greater responses in investment and to greater inequality in investment decisions across ability types when subsidies to investment are increased.

Figure 9: Changes in Investment in Response to a 5% Increase in Investment Subsidies



We now analyze the full impacts of an increase in the subsidy to investment in our model when taxes on labor income must be raised to pay for the subsidy increase. The base case equilibrium prices ( $w, R$ ), wage tax rate ( $\tau$ ), aggregate human and physical capital stocks ( $H, K$ ), credit constraint functions ( $\mu, \kappa_0, \kappa_1$ ), and average subsidy amount are shown in the first column of Table 7. The table also shows average investment amounts and earnings for individuals of different ability ( $e_j$ ) levels. Average debt, savings, and consumption are also shown at different ages.

Table 7 also reports the impacts of increasing the rate of subsidy,  $\nu$ , from 0.5 to 0.55 in both partial and general equilibrium. Column 1 reflects the base case equilibrium. Column 2 shows the partial equilibrium changes when the prices of human and physical capital are held constant (the tax rate is allowed to adjust to maintain a balanced budget). These results reflect the direct impacts of an increased subsidy on decisions and credit constraints (as well as the distribution of initial assets/bequests). The final column of the table reports the new general equilibrium values once  $w$  and  $R$  are allowed to change, showing the added impact of changing prices on decisions and constraints.

In response to the increased rate of subsidization, human capital investment increases by nearly 20% resulting in a 13% increase in aggregate human capital. Holding  $w$  and  $R$  (and, therefore,  $\kappa_1$  and  $\mu$ ) constant, the amount of debt held by students and young workers increases by about 10%. This is because individuals respond to the subsidy by increasing their investment in human capital, which increases their future earnings power (by about 12%) and relaxes limits placed on

Table 7: Increased Subsidy on Investment

Variable	$\nu = 0.55$		
	Base Case ( $\nu = 0.5$ )	Partial Equilibrium	General Equilibrium
$w$	12.2102	12.2102	12.2081
$R$	1.8009	1.8009	1.8045
$\tau$	0.0923	0.0967	0.0967
$\kappa_0$	-0.0860	-0.0860	-0.0850
$\kappa_1$	0.1323	0.1323	0.1319
$\mu$	0.0734	0.0734	0.0733
$H$	63	71	71
$K$	40	44	44
$E(\text{subsidy})$	71	83	83
$E(y)$	93	111	110
$E(y(e_1))$	77	91	91
$E(y(e_2))$	89	107	106
$E(y(e_3))$	96	115	115
$E(y(e_4))$	98	118	117
$E(y(e_5))$	103	124	123
$V(y)$	153	247	244
$E(V(y e))$	74	121	119
$E(w_1)$	247	276	275
$E(w_1(e_1))$	176	195	195
$E(w_1(e_2))$	235	262	261
$E(w_1(e_3))$	263	294	294
$E(w_1(e_4))$	269	301	300
$E(w_1(e_5))$	293	328	327
$V(w_1)$	1,796	2,357	2,341
$E(V(w_1 e))$	198	282	280
$E(d_0)$	30	33	33
$E(d_1)$	35	39	39
$E(c_0)$	83	88	88
$E(c_1)$	206	229	228
$E(c_2)$	235	262	261

their borrowing. The small increase in the tax rate needed to balance the budget is not enough to offset these forces. The relaxation of borrowing constraints also serves to increase investment in addition to early consumption.

The increase in human capital investment drives down the price of human capital in column 3, although the change is quite small. Interest rates also increase slightly. Because prices change very little in general equilibrium, changes in the constraint factors  $\kappa_1$  and  $\mu$  are negligible. As a result, changes in allowable debt are almost entirely due to changes in investment and not changes in prices or the amount of debt allowed for any given amount of future earnings. General equilibrium effects play little role in the measured responses to an increase in the rate of subsidy.

It is significant that these results imply that borrowing limits should be relaxed when education subsidies are increased. This is in direct contrast to the standard discussion of federal student loan and subsidy policy, which generally assumes there is a trade-off between the two.

### 5.3 Bankruptcy Policy

We have, thus far, taken punishment policies as given. However, one might ask how changes in the ability of creditors to discipline defaulting borrowers would affect credit limits and human capital investment. Such an analysis may be useful for comparing education decisions across countries with different lending environments or for thinking about reforming current U.S. bankruptcy code.

Given wage and interest rates, tougher punishments on default increase the feasible credit for each agent. Unconstrained agents will not change their decisions, while constrained agents, facing stricter punishments, can commit to increased borrowing and investment allowing greater human capital investment and smoother consumption profiles. Thus, all constrained individuals will be strictly better off.

Changes in punishments do not necessarily have the same effect on borrowing limits for students and young workers. Based on our calibration, an increase in  $\gamma$  would increase  $\kappa_1$  by nearly twice as much as  $\mu$ , so changes in the fraction of earnings that can be seized from defaulters have a much greater impact on the fraction of future earnings that can be borrowed by students. However, because earnings are so much larger for old workers than young workers, the change in actual borrowing limits is similar for students and young workers. In contrast, an increase in the amount of savings that can be seized from defaulters ( $1 - \phi$ ) has a slightly greater impact on  $\mu$  than on  $\kappa_1$ . Thus, the effect on actual borrowing limits is greater on young workers rather than students; although, both effects are quite small.

Overall, the amount of earnings that can be seized plays a much more important role in determining borrowing constraints in our calibrated economy. Completely eliminating the ability of creditors to seize saved assets from defaulters would not eliminate all borrowing, whereas eliminat-

ing the ability of creditors to seize earnings would.

## 6 Conclusions

This paper explores how the nature of credit constraints affects the way we think about human capital investment decisions and the response of those decisions to government policies. In many ways, assumptions about credit constraints have important implications for investment.

We move away from the standard *ad hoc* treatment of borrowing constraints and human capital by modelling the endogenous formation of credit constraints when individuals have incentives to default on their loans and creditors are limited in their capacity to punish those who default. In our framework, punishment strategies determine the personal cost of default for all borrowers. Because this cost is likely to vary across the population and over the life cycle, so will limits on borrowing. Those who face greater costs from a punishment policy will be given greater access to credit, since lenders understand that those borrowers are more likely to repay their debts in order to avoid punishment. We demonstrate that if lenders can seize a fraction of defaulting borrowers earnings and/or savings and can prevent them from borrowing in the future, then the amount of debt lenders will extend to borrowers depends on their future earnings capacity. More able individuals will be extended more credit, because potential punishments are more costly for them. Lenders will also extend greater credit to those who invest more in their human capital for the same reason. Furthermore, credit limits will vary over the life cycle, reflecting changes in the individual cost of default punishments. In our calibrated economy, allowable debt increases with age as both savings and earnings increase.

The nature of constraints affects the cross-sectional distribution of investment in human capital. When constraints are endogenously determined, as in our model, ability and human capital investment should be positively correlated in the population. This need not be true when constraints are assumed to be exogenous and invariant. More generally, our model of endogenous constraints implies that investment in human capital should be more skewed towards the most able and wealthiest individuals when compared with models assuming that constraints are exogenous.

In our model, borrowing constraints not only vary in the population, but they are also influenced by government policy and macroeconomic forces. Policies that increase the returns to investment (e.g. education subsidies) should increase the amount of credit lenders extend, since individuals will optimally choose to invest more in their human capital, increasing their future earnings capacity. Thus, more extensive loan programs should go hand-in-hand with increased subsidies for schooling. This sheds new light on the standard discussion that posits a tradeoff between lending and subsidies.

The endogenous response of constraints to education and tax policy leads to interesting predictions about how investment is affected. We find that investment in human capital responds much

more to education subsidies when constraints are endogenously determined rather than exogenously fixed. Furthermore, education subsidies enhance differences in investment across ability types when constraints are endogenous but not when they are exogenous. While standard models of human capital formation find that flat income taxes have no effect on human capital formation when investment costs are tax deductible (e.g. foregone earnings), our model of endogenous constraints predicts that lenders will restrict credit and investment will decline when taxes are raised.

The lessons learned from studying the endogenous formation of borrowing constraints in efficient credit markets can be useful for improving federal student loan programs. While the current government lending system has some features of an efficient credit market (e.g. the amount of loans offered is tied to the quantity and, indirectly, quality of education), it does not fully adjust credit limits to reflect differences in future earnings capacity. Whether this results in over- or under-investment in human capital is not immediately obvious. Lochner and Monge [31] find that limits may be too stringent on more able individuals choosing lucrative careers and too lax on less able students choosing careers with little financial return.

Future research should focus on better understanding the precise punishments creditors have at their disposal. It is also important to consider the role of uncertainty about the future and, perhaps, private information in determining the optimal structure for credit markets. In addition to adding realism, these extensions are likely to generate default in equilibrium, which can then be studied jointly with decisions to invest in human capital.

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## Appendix A: Estimating Parameters for Wage Growth and Human Capital Production

A two-step procedure is used to estimate the human capital production parameters of interest. First, total costs for each year of schooling are computed from direct expenditures and foregone earnings. Then, a log wage regression is used to estimate human capital parameters of the model.

In calculating direct expenditures, we use average expenditures per pupil from 1980-89 for primary and secondary schooling, and for universities and colleges, since those years correspond to the years most men in our NLSY sample made their marginal schooling decisions.<sup>18</sup> Table A-1 reports the present value of direct expenditures for each year of schooling.<sup>19</sup> In 1999 dollars, direct costs for 12 years of schooling amount to almost \$60,000, and expenditures for 16 years of schooling cost about \$100,000. While a substantial fraction of these direct costs are subsidized by the government, they still reflect direct inputs. We include them here because the objective at this moment is to estimate the human capital production function.

Working men in the NLSY are used to compute foregone earnings. We use a standard log earnings regression that controls for experience (age - education - 6), experience-squared, and indicators for each AFQT quintile to compute average annual earnings for men of all schooling and ability combinations. Foregone earnings are set to zero for those with 9 years of schooling under the assumption that individuals generally cannot work before age 14 in most states. For individuals with more than 9 years of schooling, foregone earnings are computed as the earnings for someone with 9 years of schooling (appropriately adjusted each year for experience that would have been acquired) had they chosen to work rather than attend school each year thereafter. Unlike direct expenditures, foregone earnings are specific to the ability group. Not surprisingly, more able individuals forego more earnings for each year of school they attend. The sum of foregone earnings and direct expenditures for each year of schooling are given by AFQT quintile in Table A-1. For someone in the highest ability quintile, total expenditures on schooling for a college graduate are approximately \$136,000.

In step two, we use wage data in the NLSY to estimate  $\alpha$ ,  $G$ , and ability levels  $e$  for each of the five types, recognizing that our model produces the following wage relationships:

$$\begin{aligned} \log(w_1) &= \log(w) + \log(e) + \alpha \log(y) \\ \log(w_2) &= \log(w) + \log(e) + \alpha \log(y) + \log(G). \end{aligned}$$

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<sup>18</sup>Direct expenditures for individuals attending school for twelve or fewer years are simply the present value of annual primary/secondary expenditures for the appropriate number of school years. For those attending 13 or more years of school, direct expenditures include 12 years of primary/secondary expenditures plus college tuition expenditures.

<sup>19</sup>Consistent with our equilibrium calibration, a 4% interest rate was assumed in calculating the present value of all costs. Costs are discounted back to the time of school entry.

Table A-1: Total Schooling Costs by Year of School and AFQT Quintile (1999 dollars)

Years of School	Direct Expenditures	Total Expenditures				
		Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
8	41,505	41,505	41,505	41,505	41,505	41,505
9	45,836	45,836	45,836	45,836	45,836	45,836
10	50,001	52,444	53,166	53,705	53,708	53,939
11	54,005	59,214	60,754	61,903	61,908	62,401
12	57,856	66,162	68,617	70,450	70,458	71,244
13	68,372	80,110	83,579	86,169	86,181	87,292
14	78,485	93,982	98,563	101,983	101,999	103,465
15	88,208	107,781	113,567	117,886	117,906	119,758
16	97,558	121,094	128,579	133,862	133,887	136,152
17	106,547	134,719	143,575	149,881	149,911	152,614
18	115,191	148,227	158,520	165,900	165,935	169,099
19	123,503	161,587	173,373	181,866	181,906	185,548
20	131,495	174,762	188,080	197,717	197,763	201,894

Notes:

- 1) Direct expenditures assume average expenditure per pupil in primary and secondary schooling through grade 12. Additional expenditures for higher grades are taken from average expenditures per student in all colleges and universities. [33]
- 2) Foregone earnings are calculated from a regression of log wage income on AFQT quintile, education indicators, experience and experience-squared. Foregone earnings are based on someone with 9 years of schooling plus the corresponding level of experience. Total costs equal direct expenditures plus foregone earnings. See text for details.
- 3) Cost measures are discounted at a 4% annual interest rate beginning with school entry.

This simple specification motivates the following log wage regression:

$$\log(\text{wage}) = a + \alpha \log(y) + \sum_{j=2}^5 \eta_j AFQT_j + \gamma_1 X + \gamma_2 X^2 + \varepsilon$$

where  $y$  is taken from total expenditures on schooling for the appropriate schooling-ability classification in Table A-1,  $AFQT_j$  represents an indicator for AFQT quintile  $j$ , and  $X$  represents work experience (defined as age - education - 6). This wage regression maintains the assumption that ability, the logarithm of education expenditures, and experience (incorporated in  $G$  in our simplified model) should enter a log wage equation linearly.<sup>20</sup>

The point estimate for  $\alpha$  is 0.6146. It directly corresponds to the parameter of interest – the elasticity of human capital with respect to investments – and is used in our simulations below. Since it is impossible to distinguish between  $\log(w)$  and  $\log(e_1)$ , one must normalize either the price of skills or the lowest ability level. Normalizing  $e_1 = 1$ , estimates for higher ability learning levels are given by  $e_j = \exp(\eta_j)$ .

Finally, the returns to experience given by  $\gamma_1$  and  $\gamma_2$  can be used to determine  $G$ . Because log earnings are quadratic in experience, any estimate of  $G$  will depend on how we define the two work periods of the model. Since we are interested in studying borrowing and savings behavior for individuals in school and recently out of school, we compute  $G$  using the ratio of wages for someone with 15 years of experience relative to zero years of experience yielding a value of 2.1275 for  $G$ .

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<sup>20</sup>See Heckman, Lochner, and Todd [23] for an empirical discussion of these assumptions over the second half of the 20th Century.