Utility
Certainty equivalent rate
Risk neutral
Risk lover
Mean-variance criterion
Indifference curve

Expected return
Variance
Standard deviation
Covariance
Correlation coefficient

Selected readings

A classic work on risk and risk aversion is:

Some good statistics texts with business applications are:

Problems

1. Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either $50,000 or $150,000 with equal probabilities of .5. The alternative risk-free investment in T-bills pays 5 percent per year.
   a. If you require a risk premium of 10 percent, how much will you be willing to pay for the portfolio?
   b. Suppose that the portfolio can be purchased for the amount you found in (a). What will be the expected rate of return on the portfolio?
   c. Now suppose that you require a risk premium of 15 percent. What is the price that you will be willing to pay?
   d. Comparing your answers to (a) and (c), what do you conclude about the relationship between the required risk premium on a portfolio and the price at which the portfolio will sell?

2. Consider a portfolio that offers an expected rate of return of 10 percent and a standard deviation of 15 percent. T-bills offer a risk-free 8 percent rate of return. What is the maximum level of risk aversion for which the risky portfolio is still preferred to bills?

3. Draw the indifference curve in the expected return—standard deviation plane corresponding to a utility level of 5 percent for an investor with a risk aversion coefficient of 3. Hint: choose several possible standard deviations, ranging from 5 percent to 25 percent and find the expected
rates of return providing a utility level of 5. Then plot the expected return-standard deviation points so derived.

4. Now draw the indifference curve corresponding to a utility level of 4 percent for an investor with risk aversion coefficient \( A = 4 \). Compare your answers to questions 3 and 4, what do you conclude?

5. Draw an indifference curve for a risk-neutral investor providing utility level 5 percent.

6. What must be true about the sign of the risk aversion coefficient, \( A \), for a risk lover? Draw the indifference curve for a utility level of 6 percent for a risk lover.

Consider historical data showing that the average annual rate of return on the S&P 500 portfolio over the past 60 years has averaged about 8.5 percent more than the Treasury bill return and that the S&P 500 standard deviation has been about 21 percent per year. Assume these values are representative of investors' expectations for future performance and that the current T-bill rate is 6 percent. Use these values to answer questions 7 to 9.

7. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights as follows:

<table>
<thead>
<tr>
<th>( W_{T-bill} )</th>
<th>( W_{S&amp;P} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
</tr>
</tbody>
</table>

8. Calculate the utility levels of each portfolio of question 7 for an investor with \( A = 3 \). What do you conclude?

9. Repeat question 8 for an investor with \( A = 5 \). What do you conclude?

Reconsider the Best and SugarKane stock market hedging example in the text, but assume for questions 10 to 13 that the probability distribution of the rate of return on SugarKane stock is as follows:

<table>
<thead>
<tr>
<th>Probability</th>
<th>Bullish Stock Market</th>
<th>Bearish Stock Market</th>
<th>Sugar Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of return</td>
<td>10%</td>
<td>3</td>
<td>-5%</td>
</tr>
</tbody>
</table>

10. If Humanex's portfolio is half Best stock and half SugarKane, what are its expected return and standard deviation? Calculate the standard deviation from the portfolio returns in each scenario.

11. What is the covariance between Best and SugarKane?

12. Calculate the portfolio standard deviation using rule 5 and show that the result is consistent with your answer to question 10.
Question 5B.1. Suppose the utility function is $U(W) = \sqrt{W}$.

a. What is the utility level at wealth levels $50,000$ and $150,000$?

b. What is expected utility if $p$ still equals $.5$?

c. What is the certainty equivalent of the risky prospect?

d. Does this utility function also display risk aversion?

e. Does this utility function display more or less risk aversion than the log utility function?

Does revealed behavior of investors demonstrate risk aversion? Look at the prices and past rates of return in financial markets, we can answer with a resounding “yes.” With remarkable consistency, riskier bonds are to lower prices than are safer ones with otherwise similar characteristics. Risky stocks also have provided higher average rates of return over long periods of time than less risky assets such as T-bills. For example, over the 1926 to period, the average rate of return on the S&P 500 portfolio exceeded T-bill return by about 8.5 percent per year.

It is abundantly clear from financial data that the average, or representative investor exhibits substantial risk aversion. For readers who recognize that financial assets are priced to compensate for risk by providing a risk premium and at the same time feel the urge for some gambling, we have a concrete recommendation: direct your gambling desire to investment in financial assets. As Von Neumann once said, “The stock market is a casino with the house in your favor.” A small risk-seeking investment may provide all the excitement you want with a positive expected return to boot!

Problems: Appendix B

1. Suppose that your wealth is $250,000. You buy a $200,000 home and invest the remainder in a risk-free asset paying an annual interest of 6 percent. There is a probability of 0.01 that your house will burn down and its value be reduced to zero. With a log utility of a year's worth of your wealth, how much would you be willing to pay for insurance on the beginning of the year? (Assume that, if the house does not burn down, its end-of-year value still will be $200,000.)

2. If the cost of insuring your house is $1 per $1,000 of value, what is the certainty equivalent of your end-of-year wealth if you insure your house at:

   a. $\frac{1}{2}$ its value
   
   b. Its full value
   
   c. $1\frac{1}{2}$ times its value.