Problem set #1
Econ 308, Money and Banking
Solutions
Professor Monge. Winter 2002
TA: A Delavande

Exercise 1
1. (a) Present Value of your portfolio
\( r = 4.5\% \)
- Present value of balance: $4000
- Present value of the coupon bond:
\[
PV = \frac{500}{(1+r)^1} + \frac{500}{(1+r)^2} + \frac{500}{(1+r)^3} + \frac{500}{(1+r)^4} + \frac{500}{(1+r)^5} + \frac{500}{(1+r)^6} + \frac{500}{(1+r)^7} + \frac{500}{(1+r)^8} + \frac{500}{(1+r)^9} + \frac{10000}{(1+r)^{10}} = 10396
\]
- Present value of the T-bill
\[
PV = \frac{1500}{1+r} = 1435.4
\]
- Present Value of the coupon bond without exercising the option:
\[
PV = \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{2000}{(1+r)^2} = 2018
\]

When you compute the PV at the end of the first period, you find the same 2018 (Interest rate is supposed to be fixed). The price will be 1900, so you want to exercise the option.

So finally:
\[
PV = 2018 + \frac{-1900 + 2018}{1+r} = 2128.3
\]

- The present value of your debt is:
\[
PV = -\frac{2000}{1+r} - \frac{3000}{(1+r)^2} = -4661.1
\]

So the present value of your portfolio is:
\[
PV = 4000 + 10396 + 1435.4 + 2128.3 - 4661.1 = 13299.
\]

(b) Rate of return of your portfolio
\[
RET_t = \frac{C_t + P_{t+1} - P_t}{P_t}
\]

We just computed \( P_t = 13299 \).
\( C_1 \) represents the flows we get during the first period\(^1\):

\(^1\)Assuming that your checking account doesn’t pay interest rate.
\[ C_1 = 500 + 1500 + 100 - 1900 - 2000 = -1800 \]

Now, we need to compute \( P_2 \).

\[
P_2 = 4000 + \frac{500}{1 + r} + \frac{500}{(1 + r)^2} + \frac{500}{(1 + r)^3} + \frac{500}{(1 + r)^4} + \frac{500}{(1 + r)^5} + \frac{500}{(1 + r)^6} + \frac{500}{(1 + r)^7} + \frac{500}{(1 + r)^8} + \frac{500}{(1 + r)^9} + \frac{1000}{(1 + r)^9} + \frac{100}{(1 + r)^9} + \frac{2000}{(1 + r)^9} + \frac{100}{(1 + r)^9} + \frac{2000}{(1 + r)^9} \]

= 15521

So

\[
RET_1 = \frac{15521 - 13299 - 1800}{13299} = 3.17\% 
\]

Let’s compute \( RET_1 \).

\[ C_2 = 500 + 100 + 2000 + 100 - 3000 = -300 \]

\[
P_3 = 4000 + \frac{500}{1 + r} + \frac{500}{(1 + r)^2} + \frac{500}{(1 + r)^3} + \frac{500}{(1 + r)^4} + \frac{500}{(1 + r)^5} + \frac{500}{(1 + r)^6} + \frac{500}{(1 + r)^7} + \frac{10000}{(1 + r)^7} + \frac{10000}{(1 + r)^8} + \frac{10000}{(1 + r)^9} + \frac{2000}{(1 + r)^8} + \frac{2000}{(1 + r)^9} + \frac{10000}{(1 + r)^9} + \frac{10000}{(1 + r)^9} + \frac{2000}{(1 + r)^9} + \frac{2000}{(1 + r)^9} \]

= 16339.

\[
RET_2 = \frac{16339 - 15521 - 300}{15521} \approx 3.33\% 
\]

Let’s compute \( RET_3 \).

\[ C_3 = 500 + 100 + 2000 = 2600.0 \]

\[
P_3 = 4000 + \frac{10000}{(1 + r)^7} + \frac{10000}{(1 + r)^8} + \frac{10000}{(1 + r)^9} + \frac{2000}{(1 + r)^8} + \frac{2000}{(1 + r)^9} + \frac{10000}{(1 + r)^9} + \frac{10000}{(1 + r)^9} + \frac{2000}{(1 + r)^9} + \frac{2000}{(1 + r)^9} \]

= 16304.

\[
RET_3 = \frac{16304 - 16339 + 2600}{16339} \approx 15.69\% 
\]

and so on...

2.

- \( r = 7\% \)
- Present value of balance: $4000
- Present value of the coupon bond:
\[
PV = \frac{500}{1+r} + \frac{500}{(1+r)^2} + \frac{500}{(1+r)^3} + \frac{500}{(1+r)^4} + \frac{500}{(1+r)^5} + \frac{500}{(1+r)^6} + \frac{500}{(1+r)^7} + \frac{500}{(1+r)^8} + \frac{500}{(1+r)^9} + \frac{10000}{(1+r)^{10}} = 8595.3
\]
- Present value of the T-bill

\[
PV = \frac{1500}{1+r} = 1401.9
\]

- Present Value of the coupon bond without exercising the option:

\[
PV = \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{2000}{(1+r)^2} = 1927.7
\]

When you compute the PV at the end of the first period, you find the same 1927.7 (Interest rate is supposed to be fixed). The price is 1900, so you want to exercise the option. So finally:

\[
PV = 1927.7 + \frac{-1900 + 1927.7}{(1+r)} = 1953.6
\]

- The present value of your debt is:

\[
PV = -\frac{2000}{1+r} - \frac{3000}{(1+r)^2} = -4489.5
\]

So the present value of you portfolio is:

\[
PV = 4000 + 8595.3 + 1401.9 + 1953.6 - 4489.5 = 11461.
\]

Rate of return: same (just apply \(r=7\%\)).

(b) Approximation formula: \(dp/p == -DUR \times di/(1+i)\)

Let’s look at an increase in the interest rate from 4.5\% in period 1 to 7\% in period 2:

To compute the duration, we use the following formula:

\[
Dur = \frac{\sum tCP_t/(1+r)^t}{\sum CP_t/(1+r)^t}
\]

Going back to \(r=4.5\%\).

\[
C_1 = -1800
\]
\[
C_2 = -300
\]
\[
C_3 = 2600
\]
after that, for \(n=4\) to 10:

\[
C_n = 500
\]
so applying the formula:

\[
DUR = 7.75
\]

Note: you should use excel to do those computations.
Let’s see how the approximation formula performs:

\[ dp/p = -DUR \times di/(1 + i) = -7.75 \times (0.045 - 0.07)/(1 + 0.045) = .18541 \]

We know that \( dp/p = \frac{PV(r=4.5\%) - PV(r=7\%)}{PV(r=4.5\%)} = \frac{13299 - 11461}{13299} = .13821 \)
Exercice 2

If you accept the offer now, PV=500 000
If you accept next year:

\[
P V' = \frac{-10000}{(1+0.01)^1} + \frac{5000}{(1+0.01)^2} + \frac{20000}{(1+0.01)^3} + \frac{20000}{(1+0.01)^4} + \frac{20000}{(1+0.01)^5} + \frac{500000}{(1+0.01)^{12}}
\]

+ \frac{5000}{(1+0.01)^6} + \frac{5000}{(1+0.01)^7} + \frac{5000}{(1+0.01)^8} + \frac{5000}{(1+0.01)^9} + \frac{5000}{(1+0.01)^{10}} = 551610

So you should wait!

Exercice 3

(a) Current yield:

\[
i_c = \frac{C}{P_0} = \frac{10}{96} = 10.417\%
\]

(b) \( RET = \frac{C_0 + P_1 - P_0}{P_0} = \frac{10+97-96}{96} = 11.458\% \)

\[
RET = i_c + \frac{P_1 - P_0}{P_0}
\]

(c) \( RET = \frac{C_1 + P_2 - P_1}{P_1} = 0 \)

(d) The yield to maturity \( y \) has to satisfy the following formula in the first period:

\[
96 = \frac{10}{y+1} + \frac{10}{(y+1)^2} + \frac{10}{(y+1)^3} + \frac{100}{(y+1)^4}, \text{ Solution is } y = .11656
\]

The yield to maturity \( y \) has to satisfy the following formula in the second period:

\[
97 = \frac{10}{y+1} + \frac{10}{(y+1)^2} + \frac{100}{(y+1)^3}, \text{ Solution is } y = .1177
\]

The yield to maturity \( y \) has to satisfy the following formula in the third period:

\[
97 = \frac{10}{y+1} + \frac{100}{y+1}, \text{ Solution is } y = .13402
\]