1 Hedging Interest Rate Risk One more time

Your portfolio is as follows: The total liabilities are $42,500 with a duration $DUR_L = 3.7$. Your assets total 50,200 and have a duration of $DUR_A = 2$. What is the duration gap in your portfolio? Would you be better off if the interest rate declines? Explain the intuition.

2 Default Risk and Interest Rates

Imagine a world with risk neutral investors, i.e. their decisions are entirely determined by the expected returns of different assets. Imagine also that there are two investment possibilities. The first one is to invest in bonds issued by a very reliable borrower, e.g. the U.S. Federal Government. The probability of default is negligible, and the interest rate paid is equal to $i_s$. Alternatively, investors can lend to a not so reliable borrower. To simplify, assume that with probability $p$ this borrower will default. In that case he will not pay interests at all and will only repay a fraction $f < 1$ of the principal. With probability $(1-p)$ the borrower will not default and will repay the entire principal, plus the interest rate $i_r$ on it.

1. What is the interest rate that the risky borrower needs to pay to obtain resources? Explain what is the effect on $i_r$ of changes in $i_s, p, f$.

2. Assume that $i_s = 5\%$. Compare the interest paid by two firms. The first is characterized by $p = 0.1, f = 0.95$. The second $p = 0.2$ and $f = 0.7$.

3 Pure Expectations Theory of The Term Structure of the Interest Rates in a Very Simple Economy

Imagine a simple world in which the short (one period) interest rates can only take two values: low $i_l = 2.5\%$ or high $i_h = 6.5\%$. If, in the current period interest rates are high, in the next period they will be high with 65% probability and low with 35% probability. Conversely, if current interest rates are low, they will be low next period with probability 85% and high with probability 15%. Assume that the assumptions on the pure expectations theory are valid. Thus, the yield-to-maturity of security of different terms must equate the expected return of any roll-over scheme.

1. Assume that the current interest rate is high. $(i = i_h)$ What is the (average) expected interest rate for the next period? Under PET, which is forward rate for next period? What should be the yield to maturity of a two period bond?
2. Answer the previous questions but assume now that the current interest rate is low \((i = i_l\%)\).

3. For each case \(i = i_{low}\) and \(i = i_{high}\), What should be the forward rate for the short interest rate in two periods? What should be the yield to maturity of a three period bond?

4 The Yield Curve

1. Historical records show that interest rates are high in the beginning of a recession and they decline as the recession advances. Conversely, the interest rates typically increase during periods of fast growth. According to the pure expectations theory, what should be the typical shape of the yield curve during a recession and during a boom.

2. Many different sources, e.g. newspapers, internet sites, etc, provide information on the behavior of financial markets. Interest rates, including the term structure of the interest rates are closely watched. Choose one of these sources (WSJ, FT, CNNFN, etc) and attach the graph of the yield curve for the US as of Thursday 31, 2002. Does the observed curve suggest that interest rates will decline? Discuss.