PROBLEM SET #4
Econ 308, Money and Banking

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1 Ex-ante vs. Ex-post Real Interest Rates (25%)  
When buying or selling a security, agents are fully informed on the nominal interest rate of that security. This is not the case for the nominal interest rate, as the actual inflation rate for the period is not known. Once we learn what was the effective inflation rate, one can compute what is the effective or ex-post real interest rate. The expected or ex-ante real interest rate is the average real return that agents would get given the probabilities of the inflation.

1. Imagine that the nominal interest rate is $i_n = 10\%$. Inflation for the year can take three different values $\pi_L = 1\%, \pi_M = 4\%$ and $\pi_H = 8\%$, with probabilities $p_L = 30\%, p_M = 50\%$ and $p_H = 20\%$, respectively.

   (a) What is the expected inflation rate $\pi^e$? (HINT: $\pi^e = \sum_{j=1}^{N} p_j \pi_j$)

   (b) Calculate what is the effective real interest rate $i_{\text{real}}$ for each of the possible results $j = L, M, H$

   (c) What is the expected real interest rate $i^e$? (HINT: $i^e_{\text{real}} = \sum_{j=1}^{N} p_j i_{\text{real}}(j)$)

   (d) It is common to see the use of the approximation $i_{\text{real}} = i_n - \pi^e$. Under which circumstances this formula will give a good approximation?

2. In many countries, governments issue debt instruments that have fixed real interest rates. This is, nominal returns are adjusted according to the actual inflation rate to deliver the pre-set real interest rate. One could use the bond prices (or their yields to maturity) to obtain estimates of the market’s expectations about inflation. Imagine that in a given country the government issue real bonds with $i_{\text{real}} = 5\%$. Nominal bonds pay $i_N = 12\%$. What is the implicit expected inflation for the year? Assume that agents are risk neutral and that in fact they are indifferent between the two bonds.

2 Liquidity Premium Theory of The Term Structure of the Interest Rates in a Very Simple Economy, (25%)  
Imagine a simple world in which the short (one period) interest rates can only take two values: low $i_t = 2\%$ or high $i_h = 6\%$. If, in the current period short term interest rates are high, in the next period they will be high with 65\% probability and low with 35\% probability. Conversely, if current interest rates are low, they will be low next period with probability 80\% and high with probability 20\%. Instead of assuming the the pure expectations theory, in this economy, forward rates incorporate a liquidity premium of $l_p = 1.5\%$. This is the forward rates from any period $t$ to $t+1$ are given by, $f_{t, t+1} = E[i_{t+1}] + l_p$, i.e. equal to the expected short term interest rate at time $t+1$ plus the liquidity premium.
1. Assume that the current interest rate is high. \((i = i_h)\) What is the (average) expected interest rate for the next period? Under LP theory, which is forward rate for next period? What should be the yield to maturity of a two period bond?

2. Answer the previous questions but assume now that the current interest rate is low \((i = i_l\%)\).

3. For each case \(i = i_{ow}\) and \(i = i_{high}\), What should be the forward rate for the short interest rate in two periods? What should be the yield to maturity of a three period bond?

4. To what extent can be said that the LP theory in this example is consistent with empirical regularities of the yield curve?

3 The Yield Curve (15%)

Many different sources, e.g. newspapers, internet sites, etc., provide information on the behavior of financial markets. Interest rates, including the term structure of the interest rates are closely watched. Choose one of these sources (WSJ, FT, CNNFN, etc) and attach the graph of the yield curve for the US as of this week. According to the PET, does the observed curve suggest that interest rates will decline? What about under the LP theory and empirical regularities?

4 Equilibrium Interest Rates (35%)

Imagine a simple economy with two different types of agents: households and entrepreneurs. There are, many agents of each type, so we can safely assume that they take the interest rate \(r\) as given. For this problem we will assume that there is equal number of agents of each type and to simplify the algebra we can safely assume that \(N_h = N_e = 1\). Households own a fraction \(\phi \in (0, 1)\) of the equity of the firms managed by entrepreneurs. Denote by \(\pi(z, r)\) the (optimized) profits of a firm with productivity \(z\) when the equilibrium interest rate is \(r\). (We will solve for this function later.) Thus, at the end of the period, each household receives a fraction \(\phi \pi(z, r)\) of the profits of the firm while entrepreneurs receive the remaining \((1 - \phi)\pi(z, r)\).

In this problem you should think of entrepreneurs as not saving at all and simply consuming their income of the period.

Households value consumption in the beginning of the period \(c_0\) and also consumption at the end of the period. Their utility function is given by

\[
\sqrt{c_0} + \beta \sqrt{c_1}
\]

where \(\beta \in (0, 1)\) is a discount factor. Households have an initial wealth \(a > 0\), which can be used for current consumption or for savings \(s\). The gross returns of the savings, \(s(1 + r)\), and the dividends \(\phi \pi(z, r)\) determine their consumption at the end of the period, i.e. \(c_1 = s(1 + r) + \phi \pi(z, r)\).

On the other hand, entrepreneurs need to finance the capital \(k\) used in production. Thus, they have to repay \(k(1 + r)\). The production function is given by \(zk^\alpha\) where \(z\) is the level of productivity and \(\alpha \in (0, 1)\) is a parameter of the production function. Assume that entrepreneurs only value consumption at the end of the period.

1. Argue that regardless of the value of \(\phi\) entrepreneurs would want to maximize the profits of the firms. Taking \(r\) as given, write down the optimization problem for entrepreneurs. Write down the expression indicating the demand for funds (supply of bonds) of the entrepreneurs for each interest rate. Find out the formula for \(\pi(z, r)\).

2. Taking \(r\) as given, write down the optimization problem for households. Write down the expression indicating the supply of funds (demand for bonds) of the households for each interest rate.

3. What would be the effect of an increase in the productivity \(z\) in the supply and demand schedules for funds.
4. Write down the formula for the equilibrium interest rate and the total capital used by firms.

5. What is the relationship between the equilibrium interest rate and: (a) Wealth of households \(a\); (b) Productivity of firms \(z\), (c) Distribution of the ownership of the firms \(\phi\). Explain the intuition.