1 Monetary Aggregates and Interest Rates

Imagine an economy in which the demand for money can be characterized by

\[ M^d = P Y \exp(a_0 - a_1 r) \]

where \( P \) is the price level, \( Y \) is GDP, \( r \) is the nominal interest rate and \( a_0, a_1 \) are positive constants.

1. Assume that the supply of money is \( M^s \); write the formula for the equilibrium interest rate. What is the effect of changes in \( P \) and \( Y \) on the equilibrium interest rate? Explain the intuition.

2. Imagine that instead of fixing the quantity of money, the Central Bank aims to fix the interest rate at a level \( r_0 \). Write down the formula for the quantity of money that clears the market.

3. (a little bit harder) Given your previous answers, imagine that an econometrician observes the realized time series on prices, output, interest rates and money \( (P_t, Y_t, r_t, M_t) \) for a number of periods in a particular economy. Can he infer (estimate) the parameters on the demand for money, i.e. \( a_0, a_1 \)? Explain.

2 Problems from Course Packet

Answer the following problems:

1. Question 5 (page 153) and 6 (page 156).
2. Problem 1 from Appendix A (page 167)
3. Problem 1 from Appendix B (page 171)

3 Certainty Equivalents

Imagine that your wealth is invested in a risky portfolio. With probability 15% your wealth at the end of the period will be 8, with probability 40% it will be 5 and with probability 45% it will be 10. Assume that the utility function is given by \( u(W) = W^\gamma \).

1. Characterize whether the agent is risk averse, risk neutral or risk lover according to whether \( \gamma > 1, \gamma = 1, \gamma < 1 \). Does it make economic sense to have \( \gamma < 0 \)? Explain.
<table>
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<th>( \gamma )</th>
<th>Certain Equivalent</th>
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<td>2.5</td>
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2. Complete the following Table with the certain equivalent of the portfolio above for each of the parameter values for \( \gamma \).

3. Explain the relationship between \( \gamma \) and the degree of risk aversion of the agent.

**Risk Taking with Exertion of Effort**

You are an entrepreneur with initial wealth \( A \). You can deposit all or any part of this wealth and obtain a gross interest rate of \((1 + r)\). You can also invest your wealth in a new venture in which the prospects of success depend on your effort. In this potential venture, you need to exert effort before realizing the outcome. There are two possibilities: if the venture is a success you will obtain a payoff \( y_s \); if the venture is a failure, you simply obtain \( y_f \). If you put effort \((e = 1)\), the probability of success is \( p \in (0, 1) \) but if you do not put any effort the probability of success is only \( q < p \).

You enjoy wealth but dislike effort. Indeed your preferences are given by

\[
E[u(\text{Wealth})] - v(\text{effort})
\]

where \( v(1) > v(0) \) (effort is costly) and \( u[\cdot] \) is an increasing function.

Please, indicate under which conditions: (a) the agent will undertake the project, and (b) will exert effort on it.