Exercice 1

1. In equilibrium,

\[ M_s = M_d \]
\[ M_s = P \times Y \times \exp(a_o - a_1 r) \]

Taking the log, we get:

\[ \ln M_s = \ln P + \ln Y + a_o - a_1 r \]

\[ \Leftrightarrow \]

\[ r^* = \frac{\ln P + \ln Y + a_o - \ln M_s}{a_1} \]

This gives us:

\[ \frac{\partial r^*}{\partial P} \geq 0 \]
\[ \frac{\partial r^*}{\partial Y} \geq 0 \]

An increase in P and Y increases the demand for money, which increases interest rate.

2. The market clearing condition is

\[ M_s = M_d \]
\[ M_s = P \times Y \times \exp(a_o - a_1 r_0) \]

3. The M we observe is the equilibrium outcome. So we have for all t:

\[ \ln M_t - \ln P_t - \ln Y_t = a_o - a_1 r_t + \varepsilon_t \]

So we can use Least Square to estimate \( a_o \) and \( a_1 \). \( \varepsilon_t \) is the error term.

This is exactly equivalent to estimate

\[ y_t = x_t' \beta + \varepsilon_t \]

with \( y_t = \ln M_t - \ln P_t - \ln Y_t \)
and \( x_t' = [1 \ r_t] \) and \( \beta = \begin{bmatrix} a_o \\ -a_1 \end{bmatrix} \).

An estimate \( b \) of \( \beta \) is \( b = (XX')^{-1}X' y \).
**Exercice 2**

1. **Question 5**
   Nobody will choose gold alone (lower expected return, higher uncertainty than the stock market). You may want to hold some as hedge.

2. **Question 6**

   \[
   \text{cov}(r_{\text{best}}, b_{\text{cane}}) = \sum_s P(s) [r_{\text{best}}(s) - E(r_{\text{best}})] [r_{\text{cane}}(s) - E(r_{\text{cane}})]
   \]
   \[
   = 0.5(25 - 10.5)(7 - 6) + 0.3(10 - 10.5)(-5 - 6) + 0.2(-25 - 10.5)(20 - 6)
   \]
   \[
   = -90.5
   \]

   We need \(\sigma_{\text{cane}}^2\)

   \[
   \sigma_{\text{cane}}^2 = 0.5(7 - 6)^2 + 0.3(-5 - 6)^2 + 0.2(20 - 6)^2
   \]
   \[
   = 76
   \]

   \[
   \rho(\text{best-cane}) = \frac{\text{cov}(r_{\text{best}}, b_{\text{cane}})}{\sigma_{\text{cane}} \sigma_{\text{best}}} = \frac{-90.5}{\sqrt{76} \times 18.9} = -0.549
   \]

   It is still a good hedging, but not as good as before.

- First method to get \(\sigma_p\)

Assuming equal weight of each stock, we get:

\[
E(p/s = \text{bullish}) = 0.5 \times 25 + 0.5 \times 7 = 16\%
\]
\[
E(p/s = \text{bearish}) = 0.5 \times 10 + 0.5 \times (-5) = 2.5\%
\]
\[
E(p/s = \text{crisis}) = 0.5 \times (-25) + 0.5 \times 20 = -2.5\%
\]

This gives us the following expected return for the portfolio:

\[
E(p) = 0.5 \times 16 + 0.3 \times 2.5 - 0.2 \times 2.5
\]
\[
= 8.25
\]

Then we get:

\[
\sigma_p^2 = 0.5(16 - 8.25)^2 + 0.3(2.5 - 8.25)^2 + 0.2(-2.5 - 8.25)^2
\]
\[
= 63.063
\]
- Using rule 5 to get $\sigma_p$
Then we get:

$$\sigma_p^2 = 0.5^2 \times 18.9^2 + 0.5^2 \times 76 + 2 \times 0.5 \times 0.5 \times (-90.5)$$
$$= : 63.063$$

The 2 methods are consistent!

2. Pb 1 from appendix A

$$ret = \frac{div + E(P_2) - P_1}{P_1}$$

$$S1 \; ret = \frac{-12}{12} = -1$$
$$S2 \; ret = \frac{0.25 + 2 - 12}{12} = -0.8125$$
$$S3 \; ret = \frac{0.40 + 14 - 12}{12} = 0.2$$
$$S4 \; ret = \frac{0.60 + 20 - 12}{12} = 0.716$$
$$S5 \; ret = \frac{0.85 + 30 - 12}{12} = 1.57$$

a.

$$E(ret) = -1 \times 0.1 - 0.8125 \times 0.2 + 0.2 \times 0.4 + 0.716 \times 0.25 + 1.57 \times 0.05$$
$$= 0.075$$

The median is the minimum t st $\text{Prob}(ret \leq t) \geq 0.5$.

So in our case, the median is 0.2

because $\text{Prob}(ret \leq -0.8125) = 0.3$ and $\text{Prob}(ret \leq 0.2) = 0.7$

The mode is the most frequent value occuring, i.e 0.2.

b.

$$\sigma_p^2 = 0.1(-1 - 0.075)^2 + 0.2(-0.8125 - 0.075)^2 + 0.4(0.2 - 0.075)^2 + 0.25(0.716 - 0.075)^2 + 0.05(1.57 - 0.075)^2 = .49382$$

so $\sigma_p = \sqrt{.49382} = .70272$

mean abs dev= $0.1 \times |{-1} - 0.075| + 0.2 \times |{-0.8125} - 0.075| + 0.4 \times |0.2 - 0.075|$
$$+ 0.25 \times |0.716 - 0.075| + 0.05 \times |1.57 - 0.075| = .57$$

c. The first moment is the mean.
The second moment is the variance.
\[ M_3 = 0.1(-1 - 0.075)^3 + 0.2(-0.8125 - 0.075)^3 + 0.4(0.2 - 0.075)^3 + 0.25(0.716 - 0.075)^3 + 0.05(1.57 - 0.075)^3 = -3.0346 \times 10^{-2} \]

This is a left-skewed (negative) distribution.

3. Pb 1 from appendix B

Expected utility without insurance:

\[
EU = 0.999 \ln(200000 + 50000(1 + 0.06)) + 0.001 \ln(50000(1 + 0.06)) = 12.44
\]

Suppose \( x \) is the maximum amount of insurance you are willing to pay.
You should be indifferent between insuring your house and paying \( x \) or not insuring.
so we have the following equation:

\[
\ln(200000 + 50000(1 + 0.06) - x) = 12.44
\]

\[
2.53 \times 10^5 - x = \exp(12.44)
\]

\[
x = 2.53 \times 10^5 - \exp(12.44)
\]

\[
x = 289.4
\]

**Exercise 3**

a. We have

\[ u(W) = \frac{W^\gamma}{\gamma} \]

Taking the first and second derivative:

\[
\begin{align*}
    u'(W) &= W^{\gamma - 1} \\
    u''(W) &= (\gamma - 1)W^{\gamma - 2}
\end{align*}
\]

So for \( \gamma > 1 \), \( u''(W) > 0 \) so \( u(.) \) is convex, and the agent is then risk-lover.
So for \( \gamma < 1 \), \( u''(W) < 0 \) so \( u(.) \) is concave, and the agent is then risk-averse.
So for \( \gamma = 1 \), \( u''(W) = 0 \) so \( u(.) \) is linear, and the agent is then risk-neutral.
It makes sense to have \( \gamma < 0 \). Your utility is negative, but still increasing in wealth.

2. \[
\begin{align*}
    u(CE) &= 0.15u(8) + 0.40u(5) + 0.45u(10) \\
    CE^\gamma &= 0.15 \times 8^\gamma + 0.4 \times 5^\gamma + 0.45 \times 10^\gamma \\
    CE &= (0.15 \times 8^\gamma + 0.4 \times 5^\gamma + 0.45 \times 10^\gamma)^{\frac{1}{\gamma}}
\end{align*}
\]
3. The smaller $\gamma$, the more risk-averse the agent is. (the CE is getting smaller as $\gamma$ decreases).

**Exercise 4**

If you don’t undertake the project, your expected utility is:

$$EU_{np} = u(W(1 + r)) - v(0)$$

If you undertake the project, your expected utility is:

$$EU_p = \max\{pu(y_s) + (1 - p)u(y_f) - v(1), qu(y_s) + (1 - q)u(y_f) - v(0)\}$$

(a) The agent will undertake the project if

$$EU_p \geq EU_{np}$$

(b) You will exert effort if

$$pu(y_s) + (1 - p)u(y_f) - v(1) \geq qu(y_s) + (1 - q)u(y_f) - v(0)$$