Identification of the oligopoly solution concept in a differentiated-products industry

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Abstract

A central part of the “new empirical industrial organization” has been the study of market power in homogenous-product industries. This paper discusses extension of these methods to differentiated-products industries. The requirements for identification of conjectural variation parameters are shown to be hard to satisfy in practice. An alternative menu-approach is discussed. © 1998 Elsevier Science S.A.

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1. Introduction

Much of the work in study of concentrated homogenous-product industries has attempted to identify the degree of market power (see Bresnahan, 1989). A central part of these studies is the estimation of conjectural variation (CV) parameters.¹ The main limitation of extending these studies to differentiated-products industries has been the difficulty of estimating demand in such cases (for a discussion of the problems in estimation and solutions see Nevo, 1997). However, with recent improvement in the methods of estimating demand for a large number of closely related products the time has come to ask if CV parameters can be identified in these industries? And if using CV parameters is the best way to test different solution concepts?

This note discusses both these questions. I show that, in principal, CV parameters are identified (just like they are in the homogenous-product case). However, in practice the exclusion restrictions required for identification will be hard to satisfy. Furthermore, an alternative “menu” approach is discussed and shown to be easier to identify.

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¹By a conjectural variation parameter I mean a continuous parameter that measures the degree of market power in an industry. This parameter can be interpreted as measuring, in some quantiative sense, how close the equilibrium outcome is to theoretical predictions, or as measuring the conjectures firms have regarding the competition. See Bresnahan (1989) for a discussion of the various alternatives. Here I abstract from interpretation and focus on identification issues.
2. The model

Assume we observe $J$ differentiated products, with aggregate demand for each product given by

$$Q_j = D(p_1, \ldots, p_J, Y, \alpha), \quad j = 1, \ldots, J,$$

where $Q_j$ is quantity demanded of variety $j$, $p_1, \ldots, p_J$ are the prices of all products, $Y$ is a vector of exogenous variables, and $\alpha$ is a vector of parameters to be estimated. Suppose there are $F$ firms, each of which produces some subset, $\mathcal{F}_j$, of the $J$ different brands. The profits of firm $f$ are

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j(W_f, \beta))Q_j - C_f,$$

where $mc(\cdot)$ denotes the marginal cost, which is a function of exogenous variables, $W$, and parameters to be estimated, $\beta$. Fixed costs are denoted by $C_f$. This specification assumes that marginal costs are constant. In a general setup the marginal costs are allowed to be a function of the quantities produced. The implications of this more general setup are discussed below.

Assuming (1) the existence of a pure-strategy Bertrand–Nash equilibrium in prices; and (2) that the prices that support it are strictly positive; the price $p_j$ of any product $j$ produced by firm $f$ must satisfy the first order condition

$$Q(p) + \sum_{r \in \mathcal{F}_j} (p_r - mc_r) \frac{\partial Q_r(p)}{\partial p_j} = 0.$$

These set of $J$ equations imply price–costs margins for each good. The markups can be solved for explicitly by defining $S_{jr} = - \frac{\partial Q_r}{\partial p_j}$, $j = 1, \ldots, J$,

$$\Theta_{jr} = \begin{cases} 1, & \text{if } \exists f: \{r, j\} \subseteq \mathcal{F}_f; \\ 0, & \text{otherwise} \end{cases}$$

and $\Omega_{jr} = \Theta_{jr}S_{jr}$. In vector notation the first order conditions become

$$Q(p) - \Omega(p - mc) = 0.$$

This implies a pricing equation

$$p = mc + \Omega^{-1}Q(p).$$

Different models of competition can be nested within this framework by setting various elements of the matrix $\Theta$ to one. For example, a single-product Nash Bertrand model corresponds to an $\Theta$ equal to the identify matrix, a multiproduct Nash Bertrand model corresponds to blocks of ones in $\Theta$, and a fully collusive model corresponds to a matrix of ones.

The econometric specification is completed by adding to both the demand and pricing equations econometric error terms, denoted $\varepsilon$ and $\eta$, respectively. The parameters of the model, $\alpha$ and $\beta$, are identified because both demand, specified in equation (1), and the supply relation, specified in equation (6), have excluded exogenous variables. The parameters can be estimated using simulations equations methods for different models of conduct, which correspond to different $\Theta$ matrices.
Furthermore, one can test between different models of conduct by comparing the “fit” of the various models. Formally, the test is performed by constructing tests of nonnested models as in Bresnahan (1987) or Gasami et al. (1992). In this approach the choice of the conduct model is from a finite set that is defined by the researcher according to her understanding and intuition. I refer to this approach as the menu approach.

3. Identification of CV parameters

An alternative, to the method described in the previous section, is to replace the matrix $\Theta$ defined by equation (4), which is derived directly from different economic models, with a matrix of parameters. These parameters are equivalent to the conjectural variations (CV) parameters used in much of the literature that measures market power in homogenous goods industries. The reader is referred to Bresnahan (1989) for a discussion of the interpretation of these parameters. Here I ask under what conditions are these parameters identified jointly with the cost and demand parameters?

3.1. A simple example

In order to examine the question I study the simplest model possible. Assume there are only two single product firms, and that both demand and marginal cost are given by linear functions. The demand function for brand $j$ is

$$Q_j = \alpha_{j0} + \alpha_{j1}p_1 + \alpha_{j2}p_2 + Y'\alpha_j + \epsilon_j, \quad j = 1, 2,$$

and the marginal cost function is

$$mc_j = \beta_{j0} + W'\beta_{j1}, \quad j = 1, 2.$$

Note that $Y$ and $W$ may include more than a single variable and these variables are assumed to be different between cost and demand equations. I assume that the dimension of $W$ is such that the parameters of the demand function are identified.\(^2\) Substituting the linear demand and cost specifications into equation (6), and allowing for CV parameters rather than the structure of equation (4), the supply relation becomes

$$p_j = \beta_{j0} + W_j'\beta_{j1} - A \left( \theta_{ii} \frac{\partial Q_i}{\partial p_i} Q_j - \theta_{ij} \frac{\partial Q_j}{\partial p_i} Q_i \right) + \eta$$

$$= \beta_{j0} + W_j'\beta_{j1} - A (\theta_{i0}\alpha_i Q_j - \theta_{i0}\alpha_i Q_i) + \eta, \quad j = 1, 2, \quad i = 3 - j = 2, 1,$$

where

$$A = \left( \begin{array}{cc} \frac{\partial Q_2}{\partial p_2} & \frac{\partial Q_1}{\partial p_1} \\ \frac{\partial Q_1}{\partial p_1} & \frac{\partial Q_2}{\partial p_2} \end{array} \right)^{-1} = \left( \begin{array}{cc} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{array} \right)^{-1}.$$

\(^2\)In practice identifying parameters of the demand for differentiated products is a difficult problem, potentially harder than the one discussed here. For a discussion of the problem and possible solutions see Nevo (1997) and references therein.
Given the assumptions made the demand parameters are identified. Therefore, a necessary condition for the parameters \((\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22})\) to be identified is that the dimension of the exogenous variables in the demand equation, \(Y\), is at least two. In the case where we have \(J\) products, the dimension of \(Y\) has to be at least \(J\). Thus, in principal the CV parameters are identified.

In practice, however, it is hard, if not impossible, to find such a large number of exogenous variables that influence demand but are uncorrelated with the shock in the pricing equation. Various marketing variables might be included in the demand equation; however, it is unlikely that these are uncorrelated with the pricing decision. Seasonal parameters that influence demand are good candidates; however, they are unlikely to be of large enough dimension.

In summary, even for this relatively simple example there is little hope of identifying all the CV parameters in practice.

3.2. The general case

The problems presented in the previous section continue to hold if we examine a more general demand function. Furthermore, if marginal costs are not constant and vary with quantity, then additional terms enter the pricing equation. This augments the problem. Now, we not only require more instrumental variables, but also for each quantity variable that enters demand\(^1\) we require an instrumental variable that rotates, not just shifts, the demand curve. The argument is essentially the same as that given in Bresnahan (1982) and Lau (1982).

4. Discussion

Adding CV parameters to the model has some appeal because it allows the researcher to obtain a quantitative estimate of how “close” the observed data is to some theoretical model of conduct. However, the above discussion shows that in practice the exclusion restrictions required for identification will be hard to meet, even if only several CV parameters are estimated. Beyond suggesting a research strategy, this result suggests that the identifying assumptions made by those studies that claim to identify CV parameters in differentiated-products industries should be carefully examined.

The alternative menu approach, presented in Section 2, does not require additional instrumental variables, beyond those required to estimate the demand and supply parameters, yet at the same time allows to test between different models of conduct. This approach has the additional advantage of directly linking estimation and theory.

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\(^1\)One could make the argument that all quantities produced by a multiproduct firm, as well as quantities produced by other firms, influence marginal cost.
References