Adversarial Coordination and Public Information Design:

Additional Material

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Abstract

This document contains additional results for the manuscript "Adversarial Coordination and Public Information Design." All numbered items (i.e., sections, subsections, lemmas, conditions, propositions, and equations) in this document contain the prefix "AM". Any numbered reference without the prefix "AM" refers to an item in the main text. Please refer to the main text for notation and definitions.

Section AM1 extends the result in Theorem 1* in the main text (about the optimality of perfectly coordinating the market response) to a class of economies in which (a) agents' prior beliefs need not be consistent with a common prior, nor be generated by signals drawn independently across agents, conditionally on θ, (b) the number of agents is arbitrary (in particular, finitely many agents), (c) payoffs can be heterogenous across agents, (d) agents have a level-K degree of sophistication, (e) the policy maker may possess imperfect information about the payoff state and/or the agents' beliefs, (f) the policy maker may disclose different information to different agents.

Section AM2 discusses the benefits to discriminatory disclosures, when the latter are feasible.

Section AM1: Generalization of Perfect Coordination Property

Consider the following amendment of the model in Section 2 in the main text.

Agents and exogenous information. Let N denote the set of agents; N is assumed to be measurable and can be finite or infinite. For each $i \in N$, let X_i denote a measurable set and define $\mathcal{X} \equiv \prod_{i \in N} X_i$. The set $\mathcal{X}$ is endowed with the product topology. For each $i \in N$, let $\Lambda_i : X_i \to \Delta(\Theta \times \mathcal{X})$ be a measurable function (with respect to the Borel sigma-algebra associated with X_i). The profile $\mathbf{x} = (x_i)_{i \in N} \in \mathcal{X}$ indexes the hierarchy of the agents' exogenous beliefs about θ and the beliefs of other agents.

The state of Nature in this environment is denoted by $\omega = (\theta, \mathbf{x}) \in \Omega \equiv \Theta \times \mathcal{X}$ and comprises the realization of the payoff fundamental θ and the exogenous profile of the agents' beliefs $\mathbf{x}$. Note that no restriction on the agents' belief profile $\mathbf{x}$ is imposed. In particular, the agents' beliefs need not be consistent with a common prior, nor be generated by signals drawn independently conditionally on θ.

Payoffs. Each agent's expected payoff differential between investing and not investing is given by

$$u_i(\theta, A) = \begin{cases} g_i(\theta, A) & \text{if } r = 1 \\ b_i(\theta, A) & \text{if } r = 0, \end{cases}$$

$i \in N$, where A denotes the aggregate investment (in case of finitely many agents, A coincides with the number of agents investing). The functions g_i and b_i are continuously differentiable and satisfy the same monotonicity assumptions as in the main text. In other words, for any $i \in N$, any (θ, A): (a) $\frac{\partial}{\partial \theta} g_i(\theta, A), \frac{\partial}{\partial \theta} b_i(\theta, A) \geq 0$, (b) $\frac{\partial}{\partial A} g_i(\theta, A), \frac{\partial}{\partial A} b_i(\theta, A) \geq 0$; and (c) $g_i(\theta, A) > 0 > b_i(\theta, A)$. Default occurs if and only if $R(\theta, A) \leq 0$, where R is increasing in (θ, A).

For simplicity, and to better highlight the novel effects, we abstract from the possibility that the regime outcome (i.e., default), as well as the agents' payoffs, may depend on variables z only imperfectly correlated with θ. As explained in Section 4 in the main text, the possibility of increasing the agents' expected payoffs while coordinating them on the same course of action extends to economies in which the regime outcome is a stochastic function of (θ, A). The optimality of policies satisfying the perfect coordination property also extends to these more general economies provided the planner's payoff satisfies Condition PC in Section 4 in the main text.

Disclosure Policies. Let $\mathcal{S}$ be a Polish space defining the set of possible disclosures to the agents. Let $m : N \to \mathcal{S}$ denote a *message function,* specifying, for each individual $i \in N$, the endogenous signal $m_i \in \mathcal{S}$ disclosed to the individual. Let $M(\mathcal{S})$ denote the set of all possible message functions with codomain $\mathcal{S}$. Let $\mathcal{P}$ be a partition of Ω and $h(\omega)$ the information set (equivalently, the cell) in $\mathcal{P}$ containing the state $\omega \in \Omega$. A *disclosure policy* $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ consists of a set $\mathcal{S}$ along with a mapping $\pi : \Omega \to \Delta(M(\mathcal{S}))$ measurable with respect to the σ-algebra defined by the partition

$\mathcal{P}$.[1] For each ω, $\pi(\omega)$ denotes the lottery whose realization yields the message function used by the policy maker to communicate with the agents. The case in which the partition $\mathcal{P}$ coincides with Ω corresponds to the case in which the policy maker is able to distinguish any two states in Ω (in this case the σ-algebra associated with Ω is the Borel σ-algebra).

Solution Concept. Agents have a level-K degree of sophistication. The policy maker adopts a conservative approach and evaluates the performance of any given policy on the basis of the "worst outcome" consistent with the agents playing (interim correlated) level-K rationalizable strategies. That is, for any given selected policy Γ, the policy maker expects the market to play according to the "most aggressive level-K rationalizable profile" defined as follows:

Definition AM1-1. Given any policy Γ, any $K \in \mathbb{N} \cup \{\infty\}$, the most aggressive level-K rationalizable profile (MARP-K) associated with Γ is the strategy profile $a^{\Gamma}_{(K)} \equiv (a^{\Gamma}_{(K),i})_{i\in[0,1]}$ that minimizes the policy maker's ex-ante expected payoff, among all profiles surviving K rounds of *iterated deletion of interim strictly dominated strategies.*

Hereafter we use IDISDS to refer to the process of iterated deletion of interim strictly dominated strategies.

Definition AM1-2. A policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ satisfies the **perfect-coordination property (PCP)** if, for any $\omega \in \Omega$, any message function $m \in supp[\pi(\omega)]$, any $i, j \in N$, $a^{\Gamma}_{(K),i}(x_i, m_i) = a^{\Gamma}_{(K),j}(x_j, m_j)$.

Fix an arbitrary policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$. For any $\omega \in \Omega$, any message function $m \in supp[\pi(\omega)]$, let $r(\omega, m; a^{\Gamma}_{(K)}) \in \{0, 1\}$ denote the regime outcome that prevails at ω when the distribution of endogenous signals is m, and agents play according to the strategy profile $a^{\Gamma}_{(K)}$.

Definition AM1-3. The disclosure policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ is **regular** if for any $\omega', \omega'' \in \Omega$ for which $h(\omega') = h(\omega'')$ and any $m \in supp\ [\pi(\omega')] = supp\ [\pi(\omega'')]$, $r(\omega', m; a^{\Gamma}_{(K)}) = r(\omega'', m; a^{\Gamma}_{(K)})$.

A disclosure policy is thus regular if the default outcome induced by MARP-K compatible with Γ is measurable with respect to the policy maker's information (as captured by the partition $\mathcal{P}$).[2] With an abuse of notation, when we find it convenient to highlight the measurability restriction implied by the regularity of the policy, we will denote by $r(h(\omega), m; a^{\Gamma}_{(K)}) \in \{0, 1\}$ the regime outcome that prevails *at any state in* $h(\omega)$ under the message function m. Observe that, when the policy maker can perfectly distinguish between any two states, then any policy is regular.

[1] That is, by the collection of $\mathcal{P}$-saturated sets. Let $\mathcal{B}$ be the standard Borel σ-algebra associated with the primitive set Ω. A set $A \in \mathcal{B}$ is $\mathcal{P}$-saturated if $\omega \in A$ implies $h(\omega) \subseteq A$. Thus $A = \cup_{\omega\in A} h(\omega)$.

[2] Note that regularity is violated in the two-state-two-receiver model in Alonso and Zachariadis (2023)

Theorem AM1-1. *For any regular policy Γ, there exists another regular policy Γ^* satisfying the perfect-coordination property (PCP) and such that, for any ω, the probability of default under Γ^* is the same as under Γ.*

Proof of Theorem AM1-1. Let $\mathcal{A}^\Gamma \equiv \{(a_i(\cdot) : X_i \times \mathcal{S} \to [0,1])_{i\in N}\}$ denote the entire set of strategy profiles in the continuation game among the agents that starts with the policy maker announcing the policy Γ. For any $n \in \mathbb{N}$, let $T^\Gamma_{(n)}$ denote the set of strategies surviving n rounds of IDISDS under the original policy Γ, with $T^\Gamma_{(0)} = \mathcal{A}^\Gamma$. Denote by $\bar{a}^\Gamma_{(n)} \equiv (\bar{a}^\Gamma_{(n),i}(\cdot))_{i\in[0,1]} \in T^\Gamma_{(n)}$ the profile in $T^\Gamma_{(n)}$ that minimizes the policy maker's ex-ante payoff. Such a profile also minimizes the policy maker's interim payoff, as it will become clear from the arguments below. Hereafter, we refer to the profile $\bar{a}^\Gamma_{(n)}$ as the most aggressive profile surviving n rounds of IDISDS. The profiles $(\bar{a}^\Gamma_{(n)})_{n\in\mathbb{N}}$ can be constructed inductively as follows. The profile $\bar{a}^\Gamma_{(0)} \equiv (\bar{a}^\Gamma_{(0),i}(\cdot))_{i\in[0,1]})$ prescribes that all agents refrain from investing irrespective of their exogenous and endogenous signals; that is, each $\bar{a}^\Gamma_{(0),i}(\cdot)$ is such that $\bar{a}^\Gamma_{(0),i}(x_i, s) = 0$, for all $(x_i, s) \in X_i \times \mathcal{S}$.[3] Given any strategy profile $a \in \mathcal{A}^\Gamma$, any $i \in N$, let $U^\Gamma_i(x_i, m_i; a)$ denote the payoff that agent i with exogenous signal x_i and endogenous signal m_i obtains from investing, when all other agents follow the behavior specified by the strategy profile a. For any $n \geq 1$, the most aggressive strategy profile surviving n rounds of IDISDS is the one specifying, for each agent i, each $(x_i, m_i) \in X_i \times \mathcal{S}$, $\bar{a}^\Gamma_{(n),i}(x_i, m_i) = 1$ if $U^\Gamma_i(x_i, m_i; \bar{a}^\Gamma_{(n-1)}) > 0$ and $\bar{a}^\Gamma_{(n),i}(x_i, m_i) = 0$ if $U^\Gamma_i(x_i, m_i; \bar{a}^\Gamma_{(n-1)}) \leq 0$. The most aggressive level-K rationalizable strategy profile (MARP-K) consistent with the policy Γ is thus the profile $\bar{a}_{(K)}{}^\Gamma = (\bar{a}^\Gamma_{(K),i}(\cdot))_{i\in N}) \in T^\Gamma_K$. The case of fully rational agents in the main text corresponds to the limit in which $K \to \infty$. To be consistent with the notation in the main text, we denote MARP consistent with Γ by dropping the subscript K and denoting such profile by $\bar{a}^\Gamma \equiv ((\bar{a}^\Gamma_i(\cdot))_{i\in N}$, with $\bar{a}^\Gamma_i(\cdot) \equiv \lim_{\mathrm{K}\to\infty} \bar{a}^\Gamma_{(K),i}(\cdot)$, all $i \in N$.

Now, consider the policy $\Gamma^+ = (\mathcal{S}^+, \mathcal{P}, \pi^+)$, $\mathcal{S}^+ = \mathcal{S} \times \{0, 1\}$, obtained from the original policy Γ by replacing each message function $m : N \to \mathcal{S}$ in the support of each $\pi(\omega)$ with the message function $m^+ : N \to \mathcal{S}^+$ that discloses to each agent $i \in N$ the same message m_i disclosed by the original policy m, along with the regime outcome $r(\omega, m; \bar{a}^\Gamma_{(K)})$ that would have prevailed at (ω, m) under Γ when all agents play according to the most aggressive level-K rationalizable strategy profile $\bar{a}^\Gamma_{(K)}$ consistent with the original policy Γ. That is, for each $\omega \in \Omega$, each $m \in supp[\pi(\omega)]$, the policy Γ^+ selects the message function m^+ obtained from the original message function m by adding to its codomain the regime outcome $r(\omega, m; \bar{a}^\Gamma_{(K)})$ that would have prevailed at (ω, m) under MARP-K $\bar{a}^\Gamma_{(K)}$, with the same probability that Γ would have selected the original message function m. Hereafter, we denote by $m^+_i = (m_i, r(\omega, m; \bar{a}^\Gamma_{(K)}))$ the message sent to agent i under the new policy Γ^+ when the exogenous state is ω and the message function selected under the original policy Γ is m. Note that the assumption that Γ is regular implies that Γ^+ is measurable with respect to the σ-algebra generated by $\mathcal{P}$ and hence also regular.

[3] Note that, to ease the notation, we let each individual strategy prescribe an action for all $(x_i, m_i) \in \mathbb{R} \times \mathcal{S}$, including those that may be inconsistent with the policy Γ.

Let $\mathcal{A}^{\Gamma^+} \equiv \{(a_i(\cdot) : X_i \times \mathcal{S} \times \{0,1\}\} \to [0,1])_{i \in N}\}$ denote the set of strategy profiles in the continuation game among the agents that starts with the policy maker announcing the new policy Γ^+. For any $n \in \mathbb{N}$, let $T^{\Gamma^+}_{(n)} \subset \mathcal{A}^{\Gamma^+}$ denote the set of strategies surviving n rounds of IDISDS under the new policy Γ^+, with $T^{\Gamma^+}_{(0)} = \mathcal{A}^{\Gamma^+}$. Denote by $\bar{a}^{\Gamma^+}_{(n)} \equiv (\bar{a}^{\Gamma^+}_{(n),i}(\cdot))_{i \in N} \in T^{\Gamma^+}_{(n)}$ the profile in $T^{\Gamma^+}_{(n)}$ that minimizes the policy maker's ex-ante payoff, and observe that $\bar{a}^{\Gamma^+}_{(0)} \equiv (\bar{a}^{\Gamma^+}_{(0),i}(\cdot))_{i \in N}$ prescribes that all agents refrain from investing, irrespective of their exogenous and endogenous signals.

Step 1. First, we prove that, for any $i \in N$,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \forall a \in \mathcal{A}^{\Gamma}\}$$

$$\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 1); a) > 0 \, \forall a \in \mathcal{A}^{\Gamma^+}\}.$$

That is, any agent who finds it dominant to invest under Γ after receiving information (x_i, m_i) also finds it dominant to invest under Γ^+ after receiving information $(x_i, (m_i, 1))$. To see this, first use the fact that the game is supermodular to observe that, given any policy Γ,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \forall a \in \mathcal{A}^{\Gamma}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) > 0\}.$$

Likewise,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 0); a) > 0 \ \forall a \in \mathcal{A}^{\Gamma^+}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 0); \bar{a}^{\Gamma^+}_{(0)}) > 0\}.$$

Next, observe that, because under both $\bar{a}^{\Gamma}_{(0)}$ and $\bar{a}^{\Gamma^+}_{(0)}$ all agents refrain from investing, regardless of their exogenous and endogenous information, under both $\bar{a}^{\Gamma}_{(0)}$ and $\bar{a}^{\Gamma^+}_{(0)}$, default occurs if, and only if, $\theta \leq \bar{\theta}$ (with $\bar{\theta}$ defined by $R(\bar{\theta}, 0) = 0$). Then, note that, under Γ^+, for any $i \in N$, any $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\partial \Lambda_i^{\Gamma^+}(\omega, m | x_i, (m_i, 1)) = \frac{\mathbf{1}_{\{r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}}}{\Lambda_i^{\Gamma}(1 | x_i, m_i)} \partial \Lambda^{\Gamma}(\omega, m | x_i, m_i), \tag{AM1}$$

where

$$\Lambda_i^{\Gamma}(1 | x_i, m_i) \equiv \int_{\{(\omega, m) : r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}} \mathrm{d}\Lambda^{\Gamma}(\omega, m | x_i, m_i)$$

is the total probability that, under the policy Γ, agent i with information (x_i, m_i) assigns to the event $\{(\omega, m) \in \Omega \times M(\mathcal{S}) : r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}$. Under Bayesian learning, the agents' beliefs under the new policy policy Γ^+ thus correspond to "truncations" of their beliefs under the original policy Γ. In turn, this property of Bayesian updating implies that, for any $(x_i, m_i) \in X_i \times \mathcal{S}$ such that

$$U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) = \int_{(\omega, m)} \left(b_i(\theta, 1)\mathbf{1}\{\theta \leq \bar{\theta}\} + g_i(\theta, 1)\mathbf{1}\{\theta > \bar{\theta}\}\right) \mathrm{d}\Lambda_i^{\Gamma}(\omega, m | x_i, m_i) > 0,$$

it must be that

$$
\begin{aligned}
U_i^{\Gamma^+}(x_i,(m_i,1);\bar{a}_{(0)}^{\Gamma^+}) &= \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}\int_{(\omega,m)}\left(b_i(\theta,1)\mathbf{1}\{\theta\le\bar{\theta}\}+g_i(\theta,1)\mathbf{1}\{\theta>\bar{\theta}\}\right)\times \\
&\quad \times\mathbf{1}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=1\}}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i) \\
&> \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}\int_{(\omega,m)}\left(b_i(\theta,1)\mathbf{1}\{\theta\le\bar{\theta}\}+g_i(\theta,1)\mathbf{1}\{\theta>\bar{\theta}\}\right)\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i) \\
&= \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}U_i^{\Gamma}(x_i,m_i;\bar{a}_{(0)}^{\Gamma}) \\
&> 0,
\end{aligned}
$$

where the first equality follows from the truncation property of Bayesian updating, the first inequality from the fact that, for all $(\omega,m)\in\Omega\times M(\mathcal{S})$ such that $r(\omega,m;\bar{a}_{(K)}^{\Gamma})=0$, $\theta\le\bar{\theta}$, and hence $r(\omega,m;\bar{a}_{(0)}^{\Gamma})=0$, implying that

$$b_i(\theta,1)\mathbf{1}\{\theta\le\bar{\theta}\}+g_i(\theta,1)\mathbf{1}\{\theta>\bar{\theta}\}=b_i(\theta,1)<0,$$

the second equality from the definition of $U_i^{\Gamma}(x_i,m_i;\bar{a}_{(0)}^{\Gamma})$, and the second inequality from the fact that $U_i^{\Gamma}(x_i,m_i;\bar{a}_{(0)}^{\Gamma})>0$.

The above result implies that any an agent who, under Γ, finds it dominant to invest after receiving information (x_i,m_i) also finds it dominant to invest under Γ^+ after receiving information $(x_i,(m_i,1))$, as claimed.

Step 2. We now show that a property analogous to the one established in Step 1 applies to any other round of the IDISDS procedure. The result is established by induction. Take any round $n\in\{1,2,...,K\}$ and assume that, for any $0\le k\le n-1$, any $i\in[0,1]$,

$$
\begin{aligned}
&\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma}(x_i,m_i;a)>0\ \ \forall a\in T_{(k-1)}^{\Gamma}\} \\
&\subseteq\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma^+}(x_i,(m_i,1);a)>0,\ \ \forall a\in T_{(k-1)}^{\Gamma^+}\}.
\end{aligned}
\tag{AM2}
$$

Recall that this means that any agent who, under Γ, finds it optimal to invest when his opponents play *any* strategy surviving k rounds of IDISDS under Γ continues to find it optimal to invest when expecting his opponents to play *any* strategy surviving k rounds of IDISDS under Γ^+. Below we show that that the same property extends to strategies surviving n rounds of IDISDS. That is,

$$
\begin{aligned}
&\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma}(x_i,m_i;a)>0\ \ \forall a\in T_{(n-1)}^{\Gamma}\} \\
&\subseteq\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma^+}(x_i,(m_i,1);a)>0,\ \ \forall a\in T_{(n-1)}^{\Gamma^+}\}.
\end{aligned}
\tag{AM3}
$$

To see this, use again the fact that the game is supermodular to observe that

$$\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma}(x_i,m_i;a)>0\ \ \forall a\in T_{(n-1)}^{\Gamma}\}=\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma}(x_i,m_i;\bar{a}_{(n-1)}^{\Gamma})>0\}\tag{AM4}$$

and, likewise,

$$\begin{aligned}&\{(x_i,m_i)\in X_i{\times}\mathcal{S} : U_i^{\Gamma^+}(x_i,(m_i,1);a)>0,\quad \forall a\in T^{\Gamma+}_{(n-1)}\}\\ &\qquad = \{(x_i,m_i)\in X_i{\times}\mathcal{S} : U_i^{\Gamma^+}(x_i,m_i;\bar{a}^{\Gamma+}_{(n-1)})>0\},\end{aligned} \tag{AM5}$$

where recall that $\bar{a}^{\Gamma}_{(n-1)}$ (alternatively, $\bar{a}^{\Gamma+}_{(n-1)}$) is the most aggressive profile surviving $n-1<K$ rounds of IDSIDS under Γ (alternatively, Γ^+).

Now let $A(\omega,m;a)$ denote the aggregate investment that, under Γ, prevails at (ω,m), when agents play according to $a\in\mathcal{A}^{\Gamma}$. Then take any $i\in N$ and any $(x_i,m_i)\in X_i\times\mathcal{S}$ such that

$$U_i^{\Gamma}(x_i,m_i;\bar{a}^{\Gamma}_{(n-1)})=\int_{(\omega,m)}u_i(\theta,A(\omega,m;\bar{a}^{\Gamma}_{(n-1)}))\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i)>0.$$

Because $\bar{a}^{\Gamma}_{(n-1)}$ is more aggressive than $\bar{a}^{\Gamma}_{(K)}$, in the sense that, for any $i\in N$, any $(x_i,m_i)\in X_i{\times}\mathcal{S}$, $\bar{a}^{\Gamma}_{(n-1),i}(x_i,m_i)\leq\bar{a}^{\Gamma}_{K,i}(x_i,m_i)$, then for all (ω,m),

$$r(\omega,m;\bar{a}^{\Gamma}_{(K)})=0\Rightarrow r(\omega,m;\bar{a}^{\Gamma}_{(n-1)})=0.$$

This implies that

$$\begin{array}{rl}\int_{(\omega,m)}u_i(\theta,A(\omega,m;\bar{a}^{\Gamma}_{(n-1)}))\mathbf{1}\{r(\omega,m;\bar{a}^{\Gamma}_{(K)})=0\}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i) & = \\ \int_{(\omega,m)}b_i(\theta,A(\omega,m;\bar{a}^{\Gamma}_{(n-1)}))\mathbf{1}\{r(\omega,m;\bar{a}^{\Gamma}_{(K)})=0\}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i) & < \quad 0.\end{array} \tag{AM6}$$

This observation, together with the truncation property in (AM1), implies that, for any $i\in N$, any $(x_i,m_i)\in X_i\times\mathcal{S}$ such that $U_i^{\Gamma}(x_i,m_i;\bar{a}^{\Gamma}_{(n-1)})>0$,

$$\begin{array}{rcl}U_i^{\Gamma^+}(x_i,(m_i,1);\bar{a}^{\Gamma}_{(n-1)}) & = & \int_{(\omega,m)}u_i(\theta,A(\omega,m;\bar{a}^{\Gamma}_{(n-1)}))\mathrm{d}\Lambda_i^{\Gamma^+}(\omega,m|x_i,m_i)\\ & = & \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}\int_{(\omega,m)}u(\theta,A(\omega,m;\bar{a}^{\Gamma}_{(n-1)}))\mathbf{1}\{r(\omega,m;\bar{a}^{\Gamma}_{(K)})=1\}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i)\\ & > & \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}\int_{(\omega,m)}u(\theta,A(\omega,m;\bar{a}^{\Gamma}_{(n-1)}))\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i)\\ & = & \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}U_i^{\Gamma}((x_i,m_i);\bar{a}^{\Gamma}_{(n-1)})\\ & > & 0,\end{array} \tag{AM7}$$

where the first and third equalities are by definition, the second equality follows from (AM1), the first inequality follows from (AM6), and the last inequality from the fact that $U_i^{\Gamma}(x_i,m_i;\bar{a}^{\Gamma}_{(n-1)})>0$, by assumption.

Next, note that $\bar{a}^{\Gamma}_{(n-1)}$ and $\bar{a}^{\Gamma+}_{(n-1)}$ are such that, for all $i\in N$, all $(x_i,m_i)\in X_i{\times}\mathcal{S}$, $\bar{a}^{\Gamma}_{(n-1),i}(x_i,m_i)$, $\bar{a}^{\Gamma+}_{(n-1),i}(x_i,(m_i,0))\in\{0,1\}$ and

$$\{(x_i,m_i)\in X_i\times\mathcal{S}:\bar{a}^{\Gamma}_{(n-1),i}(x_i,m_i)=1\}=\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma}(x_i,m_i;\bar{a}^{\Gamma}_{(n-2)})>0\}$$

and, likewise,

$$\{(x_i,m_i)\in X_i\times\mathcal{S}:\bar{a}^{\Gamma+}_{(n-1),i}(x_i,(m_i,1))=1\}=\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma^+}(x_i,(m_i,1);\bar{a}^{\Gamma+}_{(n-2)})>0\}.$$

Together properties (AM2), (AM4) and (AM5) imply that $\bar{a}^{\Gamma}_{(n-1)}$ and $\bar{a}^{\Gamma+}_{(n-1)}$ are such that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma}_{(n-1),i}(x_i, m_i) = 1 \Rightarrow \bar{a}^{\Gamma+}_{(n-1),i}(x_i, (m_i, 1)) = 1. \tag{AM8}$$

Condition (AM8), along with the fact that the game is supermodular, implies that

$$U_i^{\Gamma+}(x_i, (m_i, 1); \bar{a}^{\Gamma}_{(n-1)}) > 0 \Rightarrow U_i^{\Gamma+}(x_i, (m_i, 1); \bar{a}^{\Gamma+}_{(n-1)}) > 0. \tag{AM9}$$

Together (AM7) and (AM9) imply the property in (AM3).

Step 3. Equipped with the results in steps 1 and 2 above, we now prove that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma+}_{(K),i}(x_i, (m_i, 1)) = 1.$$

This follows directly from the fact that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma}_{(K),i}(x_i, m_i) = 1 \Rightarrow \bar{a}^{\Gamma+}_{(K),i}(x_i, (m_i, 1)) = 1, \tag{AM10}$$

which, in turn implies that, for any (ω, m),

$$r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1 \Rightarrow r(\omega, m; \bar{a}^{\Gamma+}_{(K)}) = 1.$$

Under Γ^+, the announcement that $r = 1$ thus reveals to the agents that (ω, m) is such that $r(\omega, m; \bar{a}^{\Gamma+}_{(K)}) = 1$. Because the payoff from investing is strictly positive when the bank avoids default, any agent i receiving a signal $(m_i, 1)$ thus necessarily invests. Under MARP-K consistent with the new policy Γ^+ thus all agents invest, regardless of their exogenous and endogenous private signals, when the policy publicly announces $r = 1$. That they all refrain from investing, irrespective of (x_i, m_i), when the policy announces $r = 0$ follows from the fact that $r = 0$ makes it common certainty among the agents that (ω, m) is such that $r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 0$ and hence that $\theta \leq \bar{\theta}$. But then, irrespective of (x_i, m_i), any agent $i \in N$ receiving exogenous information x_i and endogenous information $m_i^+ = (m_i, 0)$ finds it optimal to refrain from investing when expecting all other agents to abstain from investing no matter their exogenous and endogenous information. This implies that under MARP-K consistent with the new policy Γ^+, all agents refrain from investing when hearing that $r = 0$.

We conclude that the new policy Γ^+ satisfies the perfect coordination property and is such that, for any $(\theta, \mathbf{x}) \in \Theta \times \mathcal{X}$, the probability of default under Γ^+ is the same as under Γ. Q.E.D.

Section AM2: Discriminatory Disclosures

In this section, we consider an extension in which the policy maker can disclose different information to different market participants. The purpose of the section is to illustrate the possible benefits of discriminatory disclosures, when the latter are feasible. To maintain the analysis as simple as possible, we assume that the environment satisfies the conditions in Theorem 3*, implying that, if the policy maker were to restrict attention to non-discriminatory policies, the optimal policy would be a simple monotone pass/fail test failing with certainty all institutions with fundamentals below a cut-off θ^* and passing with certainty all the others.

We start by explaining that the benefits of discriminatory disclosures stem from the possibility of increasing the uncertainty each agent faces about the beliefs that rationalize other agents' behavior. We then consider a parametric setting in which the policy maker can engineer any public disclosure of her choice, but is constrained to use Gaussian signals when communicating privately with the agents. The advantage of such a parametric approach is that the combination of the exogenous and the endogenous private information can be conveniently summarized in a uni-dimensional sufficient statistics. This in turn permits us to relate the benefits of discriminatory disclosures to the type of securities issued by the banks (more generally, to the sensitivity of the agents' payoffs to the underlying fundamentals).[4]

In this section, to simplify the exposition, we assume away the shocks z imperfectly correlated with θ.

Subsection AM2.1: Benefits of Discriminatory Disclosures

Perhaps surprisingly, the reason why discriminatory disclosures may improve upon non-discriminatory ones has little to do with the possibility of tailoring the information disclosed to the agents to their prior beliefs. Discriminatory disclosures may outperform non-discriminatory ones because, by enhancing the dispersion of posterior beliefs, they make it harder for the agents to refrain from investing, thus permitting the policy maker to save a larger set of institutions.

To illustrate the point in the simplest possible way, consider an economy in which the agents' prior beliefs are homogenous (formally, this amounts to assuming the exogenous private signals x are completely uninformative). Next let $u(\theta, A)$ denote the payoff from investing when the fundamentals are θ and the aggregate investment is A. Notice that, for any $\hat{\theta}$ such that

$$\int u(\theta, 0)\mathrm{d}F(\theta|\theta > \hat{\theta}) > 0,$$

the most aggressive rationalizable strategy profile following the public announcement that $\theta > \hat{\theta}$ is such that every agent invests.[5] Under the assumptions of Theorem 3* in the main text, the optimal

[4] See also Li, Song and Zhao (2023) and Morris, Oyama and Takahashi (2024) for the characterization of the optimal *discriminatory* policy when agents do not possess any exogenous private information.

[5] The notation $F(\theta|\theta > \hat{\theta})$ stands for the common posterior obtained from the prior F by conditioning on the event that $\theta > \hat{\theta}$.

non-discriminatory policy is then a threshold rule with cut-off equal to[6]

$$\hat{\theta}^* = \inf\{\hat{\theta} \in \mathbb{R} \; s.t. \int u(\theta, 0) \mathrm{d}F(\theta|\theta > \hat{\theta}) > 0\}. \tag{AM11}$$

Suppose now the policy maker, instead of announcing whether θ is above or below some threshold $\hat{\theta}$, sends to each individual a private signal of the form $m_i = \theta + \sigma\xi_i$, where $\sigma \in \mathbb{R}_+$ is a scalar, and where the idiosyncratic terms ξ_i are drawn from a smooth distribution over the entire real line (e.g., a standard Normal distribution), independently across agents, and independently from θ. From standard results in the global games literature, we know that, as the private messages become highly precise (formally, as $\sigma \to 0^+$), in the absence of any public disclosure, under the most aggressive rationalizable profile, each agent invests if, and only if, his endogenous private signal falls above the threshold $\theta^{MS} \in (\underline{\theta}, \bar{\theta})$ implicitly defined by the unique solution to

$$\int_0^1 u(\theta^{MS}, A) \mathrm{d}A = 0. \tag{AM12}$$

As explained in the main text, the threshold θ^{MS} corresponds to the value of the fundamentals at which an agent who knows θ and holds *Laplacian* beliefs with respect to the aggregate investment[7] is indifferent between investing and not investing. Importantly, θ^{MS} *is independent of the initial common prior* and of the distribution of the noise terms ξ in the agents' signals. The above result thus implies that, with discriminatory disclosures, the policy maker can always guarantee that default never occurs for any $\theta > \theta^{MS}$. We then have the following result:[8]

Proposition AM2-1. *Assume the agents possess no exogenous private information about the underlying fundamentals. Let $\hat{\theta}^*$ be the threshold in (AM11) and θ^{MS} be the threshold in (AM12). Whenever $\theta^{MS} < \hat{\theta}^*$, discriminatory disclosures strictly improve upon non-discriminatory ones.*

The result follows directly from the arguments preceding the proposition. Because $\hat{\theta}^*$ can be arbitrarily close to $\overline{\theta}$ for particular prior distributions, and because θ^{MS} is invariant in the prior distribution from which θ is drawn, the result in Proposition AM2-1 is relevant in many cases of interest.

As anticipated above, the reason why discriminatory disclosures can improve upon non-discriminatory ones is that they permit the policy maker to enhance the dispersion of the agents higher-order beliefs. A higher dispersion in turn makes it more difficult for the agents to play adversarially to the policy maker (i.e, to refrain from investing). Formally speaking, when beliefs are sufficiently dispersed, an

[6]Here we follow the same abuse as in the main text and refer to the optimal non-discriminatory policy as the monotone policy whose threshold is given by $\hat{\theta}^*$.

[7]This means that the agent believes that the aggregate investment is uniformly distributed over $[0, 1]$.

[8]The proposition shows that the condition $\theta^{MS} < \hat{\theta}^*$ is sufficient for discriminatory policies to strictly improve upon non-discriminatory ones. When, instead, $\theta^{MS} \geq \hat{\theta}^*$, whether or not the optimal policy is discriminatory depends on the prior F and on the sensitivity of the agents' payoffs to θ. See Li, Song and Zhao (2023) and Morris, Oyama and Takahashi (2024) for a characterization of the optimal discriminatory policy when payoffs are constant in θ.

agent receiving a private signal indicating that the bank may collapse under a sufficiently large attack (i.e, in case few agents invest) may nonetheless invest if he expects many other agents to have received extreme signals indicating that the fundamentals are strong enough for the bank not to collapse, no matter the aggregate investment. In this case, investing may become *iteratively dominant* for this individual. The optimality of discriminatory policies thus follows from a "divide-and-conquer" logic reminiscent of the one in the vertical contracting literature (see, e.g., Segal (2003) and the references therein). Importantly, when discriminatory policies outperform non-discriminatory ones, this is not because they mis-coordinate the response by the market (recall that, by virtue of Theorem AM1-1 in the present document, the optimal policy always satisfies the perfect-coordination property, irrespectively of whether or not it is discriminatory), but because, by enhancing the heterogeneity in structural uncertainty, they make it difficult for market participants to coordinate on an adversarial course of action when the planner recommends that they invest.

Subsection AM2.2: Payoff Sensitivity and the Optimality of Discriminatory Policies

We conclude by showing how the optimality of discriminatory policies may depend to the sensitivity of the agents' payoffs to the underlying fundamentals and relate such sensitivity to the type of securities issued by the banks under scrutiny. To gain on tractability, we consider an environment in which the prior distribution F from which θ is drawn is an improper uniform distribution over the entire real line and where the agents' exogenous private signals are given by $x_i = \theta + \sigma_\eta \eta_i$, with $\eta_i \sim \mathcal{N}(0, 1)$.[9] Furthermore, to facilitate the aggregation of the agents' exogenous and endogenous signals into a uni-dimensional statistics, we restrict attention to the following parametric structure. The policy maker can engineer any public disclosure of her choice but is constrained to sending signals of the Gaussian form $\widetilde{m}_i = \theta + \sigma_\xi \xi_i$, with $\xi_i \sim \mathcal{N}(0, 1)$, when communicating privately with the agents. The restriction to Gaussian private applies only to the information the policy maker discloses *privately* to the agents, over and above the information conveyed by the public test. In each state θ, the endogenous information $m_i = (\widetilde{s}, \widetilde{m}_i)$ disclosed to each agent i thus comprises a public signal $\widetilde{s}$, along with a private signal $\widetilde{m}_i$. Under such a structure, the quality of the private signals is then conveniently parametrized by the variance $\sigma_\xi^2 > 0$ of the endogenous noise terms.

We also assume the agents' payoff from investing depends on the aggregate investment A only through the effects of the latter on the default outcome. In other words, we assume that there exist strictly increasing functions $\bar{g}(\theta)$ and $\bar{b}(\theta)$ such that the payoff of each agent investing is equal to $\bar{g}(\theta)$ in case the bank does not default and equal to $\bar{b}(\theta)$, in case the bank defaults. The payoff from not investing is equal to zero. Finally, we assume that the function R determining the default outcome

[9] The assumption that F is an improper uniform distribution is standard in the global-game literature. It simplifies the formulas below, without any serious effect on the results. Also observe that the entire hierarchy of the agents' beliefs is well defined, despite the prior being improper.

takes the same linear form $R(\theta, A) = \theta - 1 + A$ as in the baseline model.[10]

Then observe that the information contained in each pair $(x_i, \widetilde{m}_i)$ of exogenous and endogenous private signals is the same as the information contained in the sufficient statistics

$$z_i \equiv \frac{\sigma_\xi^2 x_i + \sigma_\eta^2 \tilde{m}_i}{\sigma_\eta^2 + \sigma_\xi^2},$$

which, given θ, is normally distributed with mean θ and variance $\sigma_z^2 \equiv \left(\sigma_\eta^2 \sigma_\xi^2\right) / \left(\sigma_\eta^2 + \sigma_\xi^2\right)$. Hence, the policy maker's choice of the discriminatory component of her disclosure policy can be conveniently reduced to the choice of the standard deviation σ_z of the above sufficient statistics, with $\sigma_z \in (0, \sigma_\eta]$.

Arguments analogous to those establishing Theorem 2* in main document then imply that, for any realization $\widetilde{s}$ of the endogenous public signal, the most aggressive rationalizable strategy profile a^Γ is characterized by a unique cut-off $\bar{z}(\widetilde{s})$ (whose value depends on the distribution from which the public signal is drawn) such that, for all $i \in [0,1]$, $a_i^\Gamma(x_i, (\widetilde{s}, \tilde{m}_i)) = \mathbf{1}\{z_i > \bar{z}(\tilde{s})\}$. Moreover, arguments similar to those establishing Theorem 2* in the main text imply that, for any given choice of σ_z^2, the optimal public announcement is binary with $\tilde{s} \in \{0, 1\}$ — that is, the public test has a pass/fail structure. Finally, from Theorem AM1-1 in the present document, the optimal policy has the perfect-coordination property which means that, given σ_z^2, $\bar{z}(0) = +\infty$, and $\bar{z}(1) = -\infty$. That is, all agents invest when $\tilde{s} = 1$, and they all refrain from investing when $\widetilde{s} = 0$.

Next, let Φ denotes the cdf of the standard Normal distribution, and, for any $\theta \in [0, 1]$, define

$$z_{\sigma_z}^*(\theta) \equiv \theta + \sigma_z \Phi^{-1}(\theta),$$

to be the private statistics threshold such that, when all agents refrain from investing when $z_i < z_{\sigma_z}^*(\theta)$ and invest when $z_i > z_{\sigma_z}^*(\theta)$, default occurs when the fundamentals fall below θ and does not occur when they are above θ.[11]

For any $(\theta_0, \hat{\theta}, \sigma_z)$, let $\psi(\theta_0, \hat{\theta}, \sigma_z)$ denote the payoff from investing of an agent with private statistic $z_{\sigma_z}^*(\theta_0)$, when regime change occurs for all $\theta \leq \theta_0 \in [0, 1]$, public information reveals that $\theta \geq \hat{\theta}$, and the total precision of private information is σ_z^{-2}. Then let

$$\theta_{\sigma_z}^{inf} \equiv \inf \left\{ \hat{\theta} : \ \psi(\theta_0, \hat{\theta}, \sigma_z) > 0 \ \text{ all } \ \theta_0 \in [0, 1] \right\}.$$

Note that, for any $\hat{\theta} > \theta_{\sigma_z}^{inf}$, under the most aggressive rationalizable strategy profile, all agents invest after the public signal reveals that $\theta \geq \hat{\theta}$. Hereafter, we assume that all agents invest also when public disclosures reveal that $\theta \geq \theta_{\sigma_z}^{inf}$. This simplifies the exposition below by permitting us to talk about the "optimal policy." As discussed in the main body, the latter does not formally exist when agents are expected to play according to the most aggressive rationalizable profile. However,

[10] The results below extend to more general payoff functions, as long as the agents' exogenous signals x are sufficiently precise.

[11] Given that $R(\theta, A) = \theta - 1 + A$, $z_{\sigma_z}^*(\theta)$ is implicitly defined by the equation $\Phi\left(\frac{z^* - \theta}{\sigma_z}\right) = \theta$.

because the policy maker can always guarantee that, no matter the selection of the rationalizable strategy profile, each agent invests for any $\theta > \theta^{inf}_{\sigma_z}$, we find the abuse justified.

Proposition AM2-2. *Suppose the policy maker is constrained to using Gaussian signals when communicating privately with the agents. Let*

$$\sigma_z^* \equiv \arg\min_{\sigma_z \in (0,\sigma_\eta]} \theta^{inf}_{\sigma_z}.$$

The optimal disclosure policy has the following structure. The policy maker publicly announces whether $\theta < \theta^{inf}_{\sigma_z^*}$, or whether $\theta \geq \theta^{inf}_{\sigma_z^*}$. In addition, when $\theta \geq \theta^{inf}_{\sigma_z^*}$, the policy maker sends a Gaussian private signal to each agent of precision $\sigma_\xi^{-2} = [\sigma_\eta^2 - (\sigma_z^*)^2]/(\sigma_z^*)^2\sigma_\eta^2$.

The result follows from the arguments preceding the proposition – note that the precision of the endogenous private information σ_ξ^{-2} in the proposition is the one that, together with the precision of the exogenous signals σ_η^{-2} yields a total precision σ_z^{-2} for the sufficient statistics z_i that minimizes the threshold $\theta^{inf}_{\sigma_z}$ defining the default outcome.

Equipped with the result in Proposition AM2-2, we can then identify primitive conditions under which the optimal policy is non-discriminatory. By virtue of Proposition AM2-2, discriminatory disclosures strictly dominate non-discriminatory ones if, and only if, $\sigma_z^* < \sigma_\eta$ (equivalently, if, and only if, there exists $\sigma_z < \sigma_\eta$ such that $\theta^{inf}_{\sigma_z} < \theta^{inf}_{\sigma_\eta}$). For any precision σ_z^{-2} of the agents' private statistics, let $\theta^{\#}_{\sigma_z}$ denote the unique solution to the equation $\psi(\theta^{\#}_{\sigma_z}, \theta^{inf}_{\sigma_z}, \sigma_z) = 0$. Note that, under MARP, $\theta^{\#}_{\sigma_z}$ identifies the fundamental threshold below which regime change occurs when the total precision of the agents' private information is σ_z^{-2} , and the endogenous disclosure of public information reveals that $\theta \geq \theta^{inf}_{\sigma_z}$. Let[12]

$$D(\theta, \theta^{\#}_{\sigma_z}) \equiv \begin{cases} \bar{b}'(\theta) \text{ if } \theta < \theta^{\#}_{\sigma_z} \\ \bar{g}'(\theta) \text{ if } \theta \geq \theta^{\#}_{\sigma_z}. \end{cases}$$

Proposition AM2-3. *Suppose that, for any $\sigma_z \in (0, \sigma_\eta]$,*

$$\mathbb{E}[D(\theta, \theta^{\#}_{\sigma_z})(\theta - \theta^{\#}_{\sigma_z})|z^*(\theta^{\#}_{\sigma_z}), \theta \geq \theta^{inf}_{\sigma_z}; \sigma_z] \geq 0. \tag{AM13}$$

Then the optimal policy is non-discriminatory.

The formal proof is below. Here we first discuss the intuition behind the result and its implications. The condition in Proposition AM2-3 is a measure of the sensitivity of the marginal agent's net payoff from investing to the underlying fundamentals.[13] To see this, note that the condition is

[12]Here $\bar{b}'$ and $\bar{g}'$ denote the derivatives of the $\bar{b}$ and $\bar{g}$ functions, respectively.

[13]The marginal agent is the one with signal $z^*_{\sigma_z}(\theta^{\#}_{\sigma_z})$.

equivalent to[14]

$$\frac{\mathbb{E}[\bar{g}'(\theta)(\theta-\theta^{\#}_{\sigma_z})|z^*(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{\#}_{\sigma_z};\sigma_z]}{\mathbb{E}[\bar{g}(\theta)|z^*(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{\#}_{\sigma_z};\sigma_z]}\geq\frac{\mathbb{E}[\bar{b}'(\theta)(\theta^{\#}_{\sigma_z}-\theta)|z^*(\theta^{\#}_{\sigma_z}),\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z});\sigma_z]}{\mathbb{E}[|\bar{b}(\theta)||z^*(\theta^{\#}_{\sigma_z}),\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z});\sigma_z]}.$$

The left-hand side is the elasticity of the marginal agent's expected net payoff from investing with respect to the underlying fundamentals, in case of no default. The right-hand side is the corresponding elasticity in case of default.[15]

To gather some intuition, consider the case in which, when default occurs, the agents' payoff differential between investing and not investing is constant in the underlying fundamentals (i.e., $\bar{b}'(\theta)=0$ for all θ). In this case, the marginal agent faces only *upside risk*. Hence, when the quality of private information decreases (which amounts to a mean-preserving increase in risk), the agent's expected net payoff from investing increases. Starting from any policy that discloses private information to the agents (i.e., for which $\sigma_z<\sigma_\eta$), the policy maker can then do better by reducing the precision of the agents' private information. In this case, the optimal policy is non-discriminatory.

The value of Proposition AM2-3 is in indicating how the optimality of discriminatory disclosures relates to the sensitivity of the agents' payoffs to the underlying fundamentals. In turn, such sensitivity typically depends on the type of security issued by the banks. For example, the above condition is more likely to hold when investors are *equity holders*. In this case, when the bank defaults, their claims are junior (i.e., subordinated) with respect to those from other stake holders with higher seniority (e.g., bond holders). In case of default, the agents' payoff then amount to a liquidation value that is typically little sensitive to the exact amount of the bank's performing loans (the bank's fundamentals). On the contrary, when the bank does not default (i.e., when the government succeeds in persuading the bank's equity holders to stay put), the value of the equity-holders' claims reflect the bank's long-term profitability, which is sensitive to the amount of the bank's performing loans. The result in Proposition AM2-3 thus indicates that discriminatory disclosures are more likely to be beneficial when the banks are seeking external funding by issuing bonds than when they do so by issuing equity.

Proof of Proposition AM2-3. We establish the result by showing that, when Condition (AM13) holds, for any fixed $\hat{\theta}$, the function $\Psi(\hat{\theta},\sigma_z)\equiv\min_{\theta_0\in[0,1]}\psi(\theta_0,\hat{\theta},\sigma_z)$ is increasing in σ_z. Moreover, in this case, the regime threshold in the absence of any public disclosure, $\theta^*_{\sigma_z}$, implicitly defined by $\psi(\theta^*_{\sigma_z},-\infty,\sigma_z)=0$, is decreasing in σ_z, with $\lim_{\sigma_z\to0+}\theta^*_{\sigma_z}=\theta^{MS}$.

To ease the notation, let $\sigma=\sigma_z$. By the envelope theorem, we have that $\frac{\partial}{\partial\sigma}\Psi(\hat{\theta},\sigma)=\frac{\partial}{\partial\sigma}\psi(\bar{\theta}_\sigma,\hat{\theta},\sigma)$,

[14] See also Iachan and Nenov (2015) for a similar condition in a related class of games of regime change.

[15] Observe that, for the marginal agent with signal $z^*_{\sigma_z}(\theta^{\#}_{\sigma_z})$,

$$\mathbb{P}[\theta\geq\theta^{\#}_{\sigma_z}|z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{inf}_{\sigma_z};\sigma_z]\mathbb{E}[\bar{g}(\theta)|z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{\#}_{\sigma_z};\sigma_z]=$$
$$\mathbb{P}[\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z})|z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{inf}_{\sigma_z};\sigma_z]\mathbb{E}[|\bar{b}(\theta)||z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z});\sigma_z].$$

with $\bar{\theta}_\sigma \in \arg\min_{\theta_0 \in [0,1]} \psi(\theta_0, \hat{\theta}, \sigma)$. Note that, for any $\theta_0 > \hat{\theta}$, any σ,

$$\frac{\partial}{\partial\sigma}\psi(\theta_0,\hat{\theta},\sigma) = \frac{\partial}{\partial\sigma}\int_{\hat{\theta}}^{\infty}\left(\bar{b}(\theta)\mathbf{1}_{\theta<\theta_0} + \bar{g}(\theta)\mathbf{1}_{\theta\geq\theta_0}\right)\frac{\phi\left(\frac{z^*_\sigma(\theta_0)-\theta}{\sigma}\right)}{\sigma\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}\mathrm{d}\theta$$

$$= \frac{\partial}{\partial\sigma}\frac{\int_{1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}^{1}\left(\bar{b}(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A<1-\theta_0}+\bar{g}(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A>1-\theta_0}\right)\mathrm{d}A}{\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

$$= \frac{\int_{1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}^{1}\left(\bar{b}'(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A<1-\theta_0}+\bar{g}'(z^*_\sigma(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A>1-\theta_0}\right)(\Phi^{-1}(\theta_0)-\Phi^{-1}(1-A))\mathrm{d}A}{\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

$$+\frac{(\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)(\theta_0-\hat{\theta})}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

where the second equality follows from the change of variables $A = 1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\theta}{\sigma}\right)$ along with the fact that, by definition, $1-\Phi\left(\frac{z^*_\sigma(\theta_0)-\theta_0}{\sigma}\right) = 1-\theta_0$, while the third equality from using $z^*_\sigma(\theta) = \theta+\sigma\Phi^{-1}(\theta)$. Lastly, by reverting the change of variables, and letting

$$D(\theta,\theta_0) \equiv \begin{cases}\bar{b}'(\theta) \text{ if } \theta < \theta_0\\ \bar{g}'(\theta) \text{ if } \theta \geq \theta_0,\end{cases}$$

we have that

$$\begin{aligned}\frac{\partial}{\partial\sigma}\psi(\theta_0,\hat{\theta},\sigma) &= \frac{\int_{\hat{\theta}}^{\infty} D(\theta,\theta_0)(\theta-\theta_0)\phi\left(\frac{z^*_\sigma(\theta_0)-\theta}{\sigma}\right)\mathrm{d}\theta + (\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)\left(\theta_0-\hat{\theta}\right)}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}\\ &= \sigma^{-1}\mathbb{E}[D(\theta,\theta_0)(\theta-\theta_0)|z^*_\sigma(\theta_0),\theta\geq\hat{\theta}] + \frac{(\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)\left(\theta_0-\hat{\theta}\right)}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta_0)-\hat{\theta}}{\sigma}\right)}.\end{aligned}$$

When evaluated at $\hat{\theta} = \theta^{inf}_\sigma$ and $\theta_0 = \theta^{\#}_\sigma$, because $\psi(\theta^{\#}_\sigma, \theta^{inf}_\sigma, \sigma) = 0$, we have that the above expression becomes

$$\frac{\partial}{\partial\sigma}\psi(\theta^{\#}_\sigma,\theta^{inf}_\sigma,\sigma) = \sigma^{-1}\mathbb{E}[D(\theta,\theta^{\#}_\sigma)(\theta-\theta^{\#}_\sigma)|z^*_\sigma(\theta^{\#}_\sigma),\theta\geq\theta^{inf}_\sigma] + \frac{|b(\theta^{inf}_\sigma)|\phi\left(\frac{z^*_\sigma(\theta^{\#}_\sigma)-\theta^{inf}_\sigma}{\sigma}\right)\left(\theta^{\#}_\sigma-\theta^{inf}_\sigma\right)}{\sigma^2\Phi\left(\frac{z^*_\sigma(\theta^{\#}_\sigma)-\theta^{inf}_\sigma}{\sigma}\right)}.$$

It is now easy to see that Condition (AM13) implies that $\frac{\partial}{\partial\sigma}\psi(\theta^{\#}_\sigma,\theta^{inf}_\sigma,\sigma) > 0$.

The above property implies that, fixing $\theta^{inf}_{\sigma_z}$, a marginal increase in the standard deviation of the agents' private information at σ_z increases $\Psi(\theta^{inf}_{\sigma_z},\sigma_z)$. Furthermore, because the threshold $\theta^{\#}_{\sigma_z}$

solves $\psi(\theta^{\#}_{\sigma_z}, \theta^{inf}_{\sigma_z}, \sigma_z) = 0$, we have that, by increasing σ_z while keeping the threshold $\theta^{inf}_{\sigma_z}$ fixed, the policy maker guarantees that, for any $\theta > \theta^{inf}_{\sigma_z}$, $\psi(\theta, \theta^{inf}_{\sigma_z}, \sigma_z) > 0$. Next, note that $\theta^{inf}_{\sigma_z}$ is decreasing in σ_z. This follows from the fact that, for any σ_z, any $\theta > \hat{\theta}$, $\psi(\theta, \hat{\theta}, \sigma_z)$ is strictly increasing in $\hat{\theta}$ (this last property in turn follows from Lemma 2 in Angeletos, Hellwig and Pavan (2007)). From the above results, we thus have that, starting from any discriminatory policy, a reduction in the precision of the agents' private information (i.e., a marginal increase in σ_z) lowers the fundamental threshold $\theta^{inf}_{\sigma_z}$ below which regime default occurs, thus improving the policy maker's payoff. Q.E.D.

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