Engagement Design

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Motivation

- Engaging agents
  - central to many economic problems

    - advertisement

    - matching

    - experimentation

    - gambling

    ...

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- Intermediaries
  - **benefit from longer engagement**
  - **control**
    - **quality of available alternatives**
    - **sequence**

This Paper

- Novel design problem
  - stochastic content of Pandora boxes
  - **sequence**
- Optimal design
  - **binary prize distributions** (over largest/lowest prize)
  - **homogenization** of boxes
- No costs in providing boxes
  - all boxes at outset
- Time cost in providing boxes
  - sequential provision optimal

Literature

- **Bayesian experimentation (multi-arm bandits)**
  - ...
  - Gittins (1974)
  - Weitzman (1979)
  - ...
- **Endogenous set of boxes**
  - Fershtman and Pavan (2025)
- **Engagement**
  - ...
  - Hebert and Zhong (2025)

Plan

1. Model
2. Searcher's optimal policy
3. Box design
4. Sequential vs simultaneous box provision
5. Conclusions

Model

Model

- **Players**
  - Designer (principal/intermediary/platform)
  - Searcher (agent/decision maker)
- Discrete time
- Designer
  - chooses each box's content (i.e., prize distribution F_j)
  - order boxes supplied
- Each box $j = 1, ..., m$
  - costs c to inspect/open
  - prize v_j drawn from $[0, \bar{v}]$ according to F_j (independently across boxes)
- Sequential provision
  - requested additional box delivered beginning of next period
  - DM opens boxes in any order of his choice
- Zero outside options
- Discount factors:
  - δ: agent
  - ρ: principal

Model

- **Searcher's problem**
  - $t = 0$, searcher
    - requests first box
    - opts out (outside option)
  - $t = 1$ (conditional on having requested box at $t = 0$), searcher
    - opens received box
    - requests new box
    - opts out
  - $t \geq 2$, searcher
    - walks away with one of opened boxes (payoff $\delta^t v_j$)
    - opens a box j received in previous periods (cost $\delta^t c$, $c \in (0, \bar{v})$) and learns box's content, v_j
    - requests new box (requested box available next period)

Model

- **Designer's problem**
  - sequence $\mathbf{F} = (F_1, ..., F_m)$
  - gross payoff of 1 for each opened box
  - box opened after t periods: discounted payoff ρ^t
  - if searcher selects box with prize v_j after t periods, designer incurs cost $\rho^t a v_j$, with $a \in [0, \delta/(c\rho)]$

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Searcher Optimal Policy

Searcher optimal policy (Fersthman and Pavan (2025))

Lemma

There exist

(a) **inspection indices** I_j *(fn* F_j*)*

(b) **expansion indices** E_j *(fn of* F_l*,* $l \geq j$*)*

s.t. searcher's optimal strategy is **generalized index policy.**

Searcher optimal policy (Fersthman and Pavan (2025))

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s.t. searcher's optimal strategy is **generalized index policy.**

Given $l \in (1, m)$ *boxes, searcher*

- *asks for box* $l + 1$ *if*
  - $E_{l+1} \geq I_j$ *for all boxes* $j \leq l$ *not opened yet*
  - $E_{l+1} \geq (1 - \delta)v_j$ *for any opened box* $j \leq l$
- *opens box* $j \leq l$ *if*
  - $I_j \geq E_{l+1}$
  - $I_j \geq I_s$ *for any box* $s \leq l$ *not opened yet*
  - $I_j \geq (1 - \delta)v_i$ *of every opened box* $i \leq l$
- *chooses opened box* $j \leq l$ *(terminating the search) if*
  - $(1 - \delta)v_j \geq E_{l+1}$
  - $(1 - \delta)v_j \geq I_i$ *for every unopened box* $i \leq l$
  - $v_j \geq v_s$ *for any other opened box* $s \leq l$*.*

Inspection and expansion indexes

- Box j's **inspection index**:

$$I_j = \sup_{\tau} \frac{\mathbb{E}^{\tau}\left[\sum_{s=0}^{\tau-1} \delta^s U_s\right]}{\mathbb{E}^{\tau}\left[\sum_{s=0}^{\tau-1} \delta^s\right]} = \frac{-c + \delta \int_{\frac{I_j}{1-\delta}}^{\overline{v}} v dF_j(v)}{1 + \frac{\delta}{1-\delta}\left(1 - F_j\left(\frac{I_j}{1-\delta}\right)\right)}$$

  where τ is stopping time

- Box j's **expansion index**:

$$E_j = \sup_{\tau,\chi} \frac{\mathbb{E}^{\chi}\left[\sum_{s=0}^{\tau-1} \delta^s U_s\right]}{\mathbb{E}^{\chi}\left[\sum_{s=0}^{\tau-1} \delta^s\right]},$$

  where τ is stopping time and χ is generic policy specifying, for each period,
    - whether to open one of **new boxes $l \geq j$ received after j-th expansion**
    - request additional box
    - select one of **boxes received after j-th expansion** opened already
- U_s: generic flow payoff

Expansion index: recursive structure

- Expansion index's **recursive structure**

$$E_j = \frac{\mathbb{E}^{\chi^*}\left[\sum_{s=0}^{\tau^*-1} \delta^s U_s\right]}{\mathbb{E}^{\chi^*}\left[\sum_{s=0}^{\tau^*-1} \delta^s\right]}$$

where

- U_s: generic flow payoff (cost or reward)
- χ^* : above index policy
- τ^*: first time at which, under χ^*, following are all weakly below E_j:

  (a) **next expansion index** E_l

  (b) **inspection indices** for boxes **received after** j**'s expansion**

  (c) **values** $(1-\delta)v_l$ **for boxes received and opened after** j**'s expansion**

Plan

Optimal Box Design

Binarizations

Definition

Given any $\mathbf{F} = (F_1, ..., F_m)$ s.t. $I_j \geq 0$ all j, **binarization** of $\mathbf{F}$ is design $\mathbf{F}^B = (F_1^B, ..., F_m^B)$ s.t. for any j:

1. F_j^B has support $\{0, \overline{v}\}$
2. $I_j^B = I_j$ (index preservation)

Binarizations

Proposition

For any $\mathbf{F}$*, designer's expected payoff under* $\mathbf{F}^B$ *at least as high as under* $\mathbf{F}$*.*

Binarizations: Proof Sketch

Lemma

Given any $\mathbf{F} = (F_1, ..., F_m)$ *s.t.* $I_j \geq 0$ *all* j*, binarization of* $\mathbf{F}$ *exists and s.t., for any* j*,* $p_j^B = \Pr(\tilde{v}_j = \overline{v})$ *given by*

$$p_j^B = \frac{I + c}{\delta[\overline{v} - I/(1-\delta)]}$$

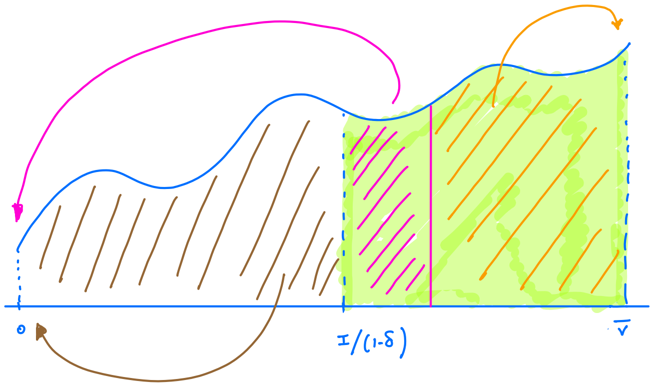
Binarizations: Proof Sketch

Lemma

Binarization $\mathbf{F}^B$ *s.t., for all* j, $p_j^B \leq 1 - F_j\left(\frac{I_j}{1-\delta}\right)$.

- Idea:
  - only realizations above index matter in index computation
  - by shifting mass to $\overline{v}$ binarizations reduce prob v exceeds $I/(1-\delta)$

Binarizations: Proof Sketch



Binarizations: Proof Sketch

Lemma

Binarization $\mathbf{F}^B$ *s.t., for all* j*, all* $a, b \in [0, I_j/(1-\delta)]$*, with* $b \geq a$*,* $F_j^B(a) \geq F_j(b)$.

- Stopping always occurs when $v_j \geq I_j/(1-\delta)$
- Binarizations increase prob. prize below any cut-off below $I/(1-\delta)$
  - reduce stopping prob.

Binarizations: Proof Sketch

Lemma

Binarization $\mathbf{F}^B$ *s.t., for all* j, $E_j^B \leq E_j$.

- (Difficult step): induction, together w. each F_j^B assigning zero prob to $(0, l_j]$

Binarizations: Proof Sketch

Above properties imply following:

1. Prob. searcher stops after opening a box smaller under $\mathbf{F}^B$ (searcher always stops when $(1-\delta)v_j \geq I_j$)

2. Under $\mathbf{F}^B$, searcher may swap expansion with inspection
   $\Rightarrow$ boxes with lower I_j opened earlier
   $\Rightarrow$ lower probability of stopping after j opened
   $\Rightarrow$ less delay in opening (higher expected NPV from box opening)

3. Expected discounted cost to designer smaller under $\mathbf{F}^B$
   - same box: $\mathbb{E}[v_j|F_j^B] \leq \mathbb{E}[v_j|F_j]$
   - different boxes:s-th box opened under $\mathbf{F}^B$ has smaller index than s-th box opened under $\mathbf{F}$ (advantageous swapping)

Box Homogenization

Let $\mathbf{F}^*$ be design s.t., for all boxes,

- F_j^* binary with support $\{0, \overline{v}\}$
- $p_j^* = c/(\delta\overline{v})$

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Theorem

Design $\mathbf{F}^*$ *optimal.*

Box Homogenization

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- F_j^* binary with support $\{0, \overline{v}\}$
- $p_j^* = c/(\delta\overline{v})$

Theorem

Design $\mathbf{F}^*$ optimal.

- No benefit to box differentiation

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Sequential vs Simultaneous Provision

Sequential vs Simultaneous Provision

Corollary

Suppose no (time) cost in supplying boxes. Then simultaneous provision optimal

Sequential vs Simultaneous Provision

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Suppose no (time) cost in supplying boxes. Then simultaneous provision optimal

- Same boxes under simultaneous and sequential provision
- However, under simultaneous provision
  - smaller delay in openings
  - **larger discounted payoff (strictly $\rho < 1$)**

Sequential vs Simultaneous Provision

Corollary

Suppose it takes designer one period to provide each box. Designer's payoff **higher under sequential provision** *(strictly if $\rho < 1$).*

Sequential vs Simultaneous Provision

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- Sequential provision economizes on delivery (time) cost in case of early stopping

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Conclusions

Conclusions

- Sequential engagement design
- Promise of attractive future options:
  - discourages stopping
  - may induce searcher to postpone exploration of available options
- Optimal design
  - two effects perfectly balanced
  - **equalization of all options**
- No cost to provide alternatives
  - **simultaneous dispatch**
- When it takes time to provide alternatives
  - **sequential dispatch**
- Under both simultaneous and sequential provision
  - **binary distributions**
  - equalization of all inspection and expansion indices

Additional Work

- Joint box and info design (about box content)
  - more constrained problem (Bayesian plausibility)
  - however, same results as above
  - full disclosure optimal
- Physical costs of expanding box pool
  - more complex problem
  - binarizations may bring expansion indices below lowest prize value
  - conjecture: depending on discount factor
    - either same policy as above
    - or increasing sequence of inspection indices with $I_j = E_{j+1}$

Most Important Slide

THANK YOU