# Knowing your Lemon before you Dump It

# Supplementary Material

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**Abstract**

This document contains additional material mentioned in the paper. All sections, conditions, and results specific to this document have the suffix "S" to avoid confusion with the corresponding parts in the main text. Section S.1 contains examples of generalized lemons and anti-lemons games. Section S.2 discusses the connection to other covert investment games.

## S.1 Examples of Generalized Lemons and Anti-Lemons Problems

In this section, we show how a number of games of interest fit into the general model of Section 2 in the paper. In all these examples, Assumption 1 (monotonicity) holds. Examples (a)-(e) are settings in which Assumption 2 in the paper (lemons) holds, whereas Examples (f)-(h) are settings in which Assumption 2' in the paper (anti-lemons) holds.

(a) *Jumpstarting frozen markets.* The suboptimal volume of trade in Akerlof's model motivates policy interventions (see also the analysis in Section 6 in the paper). Consider a government that, in the context of the Akerlof example in the main text, maximizes total welfare. The government can neither coerce the agents to trade nor prevent the existence of a free private market. However, it can influence the market outcome by purchasing some of the assets. The shadow cost of public funds used for such purchases is $1 + \lambda$, with $\lambda > 0$. The government, just like private buyers, does not know $\omega$ and values the assets at $\omega + \Delta$. Philippon-Skreta (2012) and Tirole (2012) show that, when the sellers' information is exogenous, the optimal policy has a simple form: The government purchases the lowest-quality assets, the market the intermediate-quality ones, and the best-quality

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assets are kept by the sellers. Furthermore, the optimum can be implemented by the government setting a price $r$ so as to maximize

$$\max_r \left\{ G(r;\rho^\dagger) \left[ \Delta - \lambda \left( r - M^-(r;\rho^\dagger) - \Delta \right) \right] \right\}, \tag{S.1}$$

where $\rho^\dagger$ is the sellers' cognition.[1] The first-term in the square bracket is the gain from trade, whereas the second term in the square brackets represents the deadweight loss on the deficit, reflecting the fact that, when the competitive private market breaks even, it is *as if* the government purchases all tendered assets itself.[2] The first-order condition for the optimal choice of $r$ is[3]

$$\frac{g(r;\rho^\dagger)}{G(r;\rho^\dagger)} = \frac{\lambda}{(1+\lambda)\Delta}. \tag{S.2}$$

Under the same differentiability and convexity assumptions as in the Akerlof example in the main text, we then have that the sellers' acquisition of information continues to be given by $-\int_r^{+\infty} G_\rho(m;\rho)dm = C'(\rho)$. The only difference is that $r$ is now chosen by the government instead of being determined by the market.

(b) *Partnerships.* Player $L$ has a project. She can associate player $F$ to it or do it alone. Bringing player $F$ on board creates synergies (lowers the cost of implementation), but forces $L$ to share the gains, which she does not want to do if the project is a good one. Player $L$'s payoff is $\omega - d_L$ if she does it alone and $r\omega - c_L$ if it is a joint project, where $r$ is the value share left to $L$ by (competitive) player $F$ and $c_L$ is player $L$'s reduced cost of project implementation. Let $c_F$ denote player $F$'s cost, with $c_L + c_F < d_L$. Given her posterior mean $m$, player $L$ chooses $a = 1$ if and only if $\delta_L(r,m) = d_L - c_L - (1-r)m \geq 0$. Provided that $m > 0$, Assumption 1 is then satisfied. Finally, $r = r(\rho^\dagger)$ solves

$$(1-r)M^-\left(\frac{d_L - c_L}{1-r};\rho^\dagger\right) = c_F. \tag{S.3}$$

Provided that (S.3) has one and only one solution, then Assumption 2 is also satisfied.[4]

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[1] To see this, let $m^g$ denote the critical threshold below which a seller of an asset of quality $m$ sells to the government. The social cost of the government's program is then equal to $\lambda \left\{ r - \mathbb{E}_{G(\cdot;\rho^\dagger)}[m|m \leq m^g] - \Delta \right\} G(m^g;\rho^\dagger)$, which accounts for the fact that the government can resell the assets at price $\mathbb{E}_{G(\cdot;\rho^\dagger)}[m|m \leq m^g] + \Delta$, and the proceeding from the sale can be used to reduce the distortions associated with future needs to collect money from taxpayers. The non-arbitrage condition between the government's program and the private market, along with the fact that buyers are competitive, then implies that $r = \mathbb{E}_{G(\cdot;\rho^\dagger)}[m|m \in [m^g, r]] + \Delta$. Combining the two conditions above and using the law of iterated expectations, we have that

$$M^-(r;\rho^\dagger) = \frac{G(m^g;\rho^\dagger)}{G(r;\rho^\dagger)}\mathbb{E}_{G(\cdot;\rho^\dagger)}[m|m \leq m^g] + \frac{G(r;\rho^\dagger) - G(m^g;\rho^\dagger)}{G(r;\rho^\dagger)}\mathbb{E}_{G(\cdot;\rho^\dagger)}[m|m \in [m^g, r]]$$

which gives the formula in (S.1).

[2] It is important, though, that the government does not buy all these assets and lets the market rebound. Otherwise, the market would nonetheless rebound, and the sellers' anticipation of this rebound would force the planner to buy assets at an even higher price: See the papers mentioned above for details.

[3] The condition uses the fact that $\partial M^-(r;\rho^\dagger)/\partial m^* = g(r;\rho^\dagger)[r - M^-(r;\rho^\dagger)]/G(r;\rho^\dagger)$.

[4] Note that for (S.3) to admit one and only one solution it must be that $M^-\left(d_L - c_L;\rho^\dagger\right) \geq c_F$. When this condition holds, (S.3) admits at least one solution. Such a solution is unique if, and only if, in addition to the

(c) *Disclosure of Hard Information.* In another important variant of Akerlof's game, the seller wants to sell for sure (she has no value for the good, say), and either has no information about $\omega$ (with probability $1-\rho$) or knows $\omega$ (with probability $\rho$), as in Dye's (1985) model. So $G(m;\rho)=\rho G(m)$ for $m<\omega_0$ and $G(m;\rho)=\rho G(m)+1-\rho$ for $m\geq\omega_0$, where $\omega_0$ is the prior mean of $G$. In contrast with Akerlof's soft-information lemons game, the seller's decision is not whether to put the good in the market (a foregone conclusion), but whether to reveal the state of Nature when knowing it. A well-established literature, surveyed by Milgrom (2008), has studied such an incentive to disclose. A natural extension of the disclosure model consists in thinking of $\rho$ (the precision of information) as endogenous.

In order to apply the general results, we must define the actions and the corresponding $\delta_L$ function. Let $a=1$ stand for the decision of non disclosing and $a=0$ for the decision of disclosing the state of Nature. The rationale for this choice is that player $F$'s beliefs about player $L$'s cognition matter only if there is no disclosure. As in the Akerlof example in the main text and in Example (a) above, let $\omega+\Delta$ denote the buyer's utility. Then let $r$ denote the price offered by the buyer in the absence of disclosure. The seller thus obtains $m+\Delta$ if she discloses, and $r$ if she does not disclose, so that $\delta_L(r,m)=r-(m+\Delta)$, so that Assumption 1 (monotonicity) in the main text is satisfied.

To compute $r(\rho^\dagger)$, note that, when cognition is exogenously fixed at $\rho^\dagger$, the seller discloses if and only if she is informed and $m>m^*(r)=r-\Delta$. Hence $r(\rho^\dagger)$ solves

$$r=\frac{(1-\rho^\dagger)\omega_0+\rho^\dagger\int_{-\infty}^{r-\Delta}mdG(m)}{1-\rho^\dagger+\rho^\dagger G(r-\Delta)}+\Delta. \tag{S.4}$$

Our results below apply to this environment as well. However, in this setting, the expected value of $m$ conditional on player $L$ engaging (i.e., not disclosing) does not coincide with $M^-\left(m^*(r(\rho^\dagger));\,\rho^\dagger\right)$. This is because player $L$, when not receiving any information, has no choice but to engage, irrespective of whether her posterior expected value of $\omega$ (which is equal to $\omega_0$) is below or above $m^*$.[5] The analog of Assumption 2 (lemons) in the main text in this setting is that the sign of $dr(\rho^\dagger)/d\rho^\dagger$ coincides with the sign of $\partial\hat{M}\left(r(\rho^\dagger);\,\rho^\dagger\right)/\partial\rho^\dagger$ where, for any $(r,\rho^\dagger)$

$$\hat{M}\left(r;\,\rho^\dagger\right)\equiv\frac{(1-\rho^\dagger)\omega_0+\rho^\dagger\int_{-\infty}^{r-\Delta}mdG(m)}{1-\rho^\dagger+\rho^\dagger G(r-\Delta)}$$

denotes the expected value of $m$ conditional on $L$ engaging optimally against a reaction of $r$ by $F$, under cognition $\rho^\dagger$. It is easy to see that this is the case whenever the solution to (S.4) is unique, which is always the case when $r-\hat{M}\left(r;\,\rho^\dagger\right)$ is increasing in $r$.

condition above, $c_F-\frac{\partial}{\partial m^*}M^-\left(\frac{d_L-c_L}{1-r};\,\rho^\dagger\right)(d_L-c_L)>0$ for any $r$ that solves (S.3). The last property always holds when $G(m;\rho^\dagger)/g(m;\rho^\dagger)$ is increasing in $m$ and $c_F>d_L-c_L$ for, in this case, $\frac{\partial}{\partial m^*}M^-\left(\frac{d_L-c_L}{1-r};\,\rho^\dagger\right)\in(0,1)$.

[5] Furthermore, it is easy to see that any solution to (S.4) is such that $\omega_0>r-\Delta=m^*(r)$.

(d) *(Interdependent herding) entry games.* Firm $L$ decides whether to enter a market. Firm $F$ then decides whether to follow suit. Firm $F$ uses the information revealed by firm $L$'s decision, but, in contrast with most herding models, payoffs are interdependent and so externalities are not purely informational. Suppose for instance that $L$ and $F$ are rivals, with per-customer profit $\pi^m$ under monopoly and $\pi^d < \pi^m$ under duopoly.[6] The state of Nature $\omega$ here represents information correlated with the two firms' entry costs. Specifically, assume that firm $L$'s entry cost is $\omega$ whereas firm $F$'s entry cost is $\omega + \varepsilon$, where $\varepsilon$ is drawn from $\mathbb{R}$, according to the distribution $H(\varepsilon)$ with density $h(\varepsilon)$, independently from $\omega$. Importantly, the realization of $\varepsilon$ is unknown to firm $L$ when it decides whether to enter. Let $r$ denote the probability of non-entry by firm $F$ and let $m$ denote firm $L$'s posterior expected value of $\omega$. We then have that

$$\delta_L(r, m) \equiv \left[ r\pi^m + (1 - r)\pi^d \right] - m,$$

implying that $m^*(r) = r\big(\pi^m - \pi^d\big) + \pi^d$. Assumption 1 is thus satisfied. In this application, $r(\rho^\dagger)$ is then the solution to $r = 1 - H(\pi^d - M^-\big(m^*(r);\, \rho^\dagger\big))$. Assumption 2 is satisfied whenever the solution to this equation is unique, which is the case if the density $h$ of $H$ satisfies $h(\pi^d - M^-\big(m^*(r);\, \rho^\dagger\big)) < 1$, the distribution $G(m; \rho^\dagger)$ of $m$ is such that $G(m; \rho^\dagger)/g(m; \rho^\dagger)$ is increasing in $m$, which implies that $\partial M^-\big(m^*(r);\, \rho^\dagger\big)/\partial m^* < 1$, and $\pi^m - \pi^d \in (0, 1)$.

(e) *Marriage.* Consider the following variant of Spier (1992)'s model, augmented with cognition. Players $L$ and $F$ decide whether to get married. Getting married has value $v_L$ and $v_F$ for $L$ and $F$, respectively, provided that all goes well, which has probability $\omega$ distributed on $[0, 1]$. With probability $1 - \omega$, instead, things go wrong in which case the players divorce, obtaining utility $v_i - \mathscr{L}_i$, $i = L, F$. The divorce can, however, be made less painful (raising the utility to $v_i - \ell_i$, with $0 < \ell_i < \mathscr{L}_i$, $i = L, F$) through a covenant spelling out the outcome in case of divorce. Adding the covenant costs a fixed amount $c_i < \mathscr{L}_i - \ell_i$ to player $i = L, F$, implying that it is efficient to add the covenant if the parties want to marry but expect to divorce with a sufficiently high probability. The value of $v_L$ is large enough that player $L$ wants to marry regardless of whether the covenant is introduced ($v_L \geq \mathscr{L}_L$). In contrast, player $F$'s value $v_F$ is distributed on $[(1 - \omega_0)\mathscr{L}_F, +\infty)$ according to the c.d.f. $H$ and is $F$'s private information. Player $L$ may acquire information about $\omega$ and then chooses between a contract with ($a = 1$) and without ($a = 0$) covenant.[7] Player $F$ then decides whether to accept to marry. Because $v_F - (1 - \omega_0)\mathscr{L}_F \geq 0$, in the absence of any information and any covenant, player $F$ always accepts to marry, no matter the realization of $v_F$. Let $r$ denote the probability that player $F$ accepts to marry when the proposed contract includes the covenant (i.e., when player $L$ engages). This game also satisfies Assumptions 1 and 2. To see

[6] One can also perform the analysis for complementors, with $\pi^d > \pi^m$.

[7] As anticipated above, in this application, both actions are adverse-selection sensitive, but $a = 0$ is less so than $a = 1$.

this, first note that[8]

$$\delta_L(r,m) = r[v_L - (1-m)\ell_L - c_L] + (1-r)\cdot 0 - [v_L - (1-m)\mathscr{L}_L].$$

Hence, $\delta_L(r,m)$ satisfies Assumption 1. Next, note that, in this example, $m^*(r)$ is given by

$$m^*(r) = \max\left\{\frac{r(v_L - \ell_L - c_L) - (v_L - \mathscr{L}_L)}{\mathscr{L}_L - r\ell_L}; 0\right\}.$$

Hence $r(\rho^\dagger)$ is given by the solution to $r = 1 - H(\ell_F + c_F - M^-(m^*(r);\rho^\dagger)\ell_F)$. Provided that the above equation admits a unique solution (which is the case when $r + H(\ell_F + c_F - M^-(m^*(r);\rho^\dagger)\ell_F)$ is increasing in $r$) Assumption 2 holds in this example too.

(f) *Start-up followed by liquidation.* An entrepreneur (player $L$) must decide whether to start a new business. Starting the project costs the entrepreneur $c_L > 0$ and generates cash flows equal to $1-\omega$. Before being able to collect the project's cash flows, the entrepreneur may need to liquidate the project (for example, because of a preference shock that makes consumption at the time the projects payoffs off no longer valuable to $L$, as in Diamond and Dybvig (1983)). Early liquidation occurs with probability $p$ and results in the entrepreneur collecting a price $r$ for the assets from a pool of risk-neutral competitive investors (player $F$). The entrepreneur's value from starting the project (i.e., the engagement decision in this application) is equal to $\delta_L = (1-p)(1-m) + pr - c_L$. The entrepreneur thus starts the project if and only if $m < m^*(r) = (1 - p + pr - c_L)/(1-p)$. Assuming, for simplicity, that the value of the project in the investors' hands is also equal to $1-m$, we then have that $r = 1 - M^-(m^*(r);\rho)$. Hence Assumptions 1 and 2' are satisfied.

(g) *Warfare.* Country $L$ is a potential invader and must decide whether to engage in a fight ($a = 1$) or abstain from doing so ($a = 0$). The state of nature $\omega$ represents the probability that country $F$ wins in case of a fight. Let $r$ denote the probability that country $F$ surrenders without fighting back. The payoff that $L$ obtains in case of victory is 1, whereas the cost of a defeat is $c_L$, implying that $\delta_L(r,m) = r + (1-r)(1 - m - mc_L)$. Hence Assumption 1 holds. Furthermore, in this game, $L$ engages if and only if $m \le m^*(r)$ where

$$m^*(r) = \frac{1}{(1-r)(1+c_L)}.$$

Similarly, letting country $F$'s payoff from victory be equal to 1 and its loss in case of defeat be equal to $c_F$, we have that country $F$ concedes if and only if

$$M^-(m^*(r(\rho^\dagger));\rho^\dagger) - \Big(1 - M^-(m^*(r(\rho^\dagger));\rho^\dagger)\Big)\, c_F \le 0.$$

Assuming that $c_F$ is drawn from some cumulative distribution $H$, we then have that $r(\rho^\dagger)$ is given

[8]Observe that the absence of a covenant is "good news" about $m$.

by the solution to

$$r(\rho^{\dagger}) = 1 - H\left(\frac{M^{-}(m^{*}(r(\rho^{\dagger}));\rho^{\dagger})}{1 - M^{-}(m^{*}(r(\rho^{\dagger}));\rho^{\dagger})}\right). \tag{S.5}$$

Hence, this is an anti-lemon problem, in that the decision by player $L$ to engage carries information that the state is one in which, if player $F$ were to fight back, he would likely lose, thus making $F$ play in a friendlier way towards player $L$. Whenever equation (S.5) admits a unique solution, Assumption A2' then holds: an increase in cognition by player $L$, when it leads to a reduction in $M^{-}(m^{*}(r(\rho^{\dagger}));\rho^{\dagger})$, induces player $F$ to respond with an action that is friendlier to $L$ (i.e., he surrounds more often).

(h) *Leadership.* Like in Hermalin (1998)'s theory of leadership, consider a setting in which a leader has information about the profitability of a project and benefits from binging on board a partner. Contrary to Hermalin (1998), however, assume that the leader's information is endogenous. Specifically, suppose that player $L$'s gain from starting the project is $\delta_L(r, m) = 1 - m + r - c_L$, where $1 - m$ is the probability that the project succeeds, $r$ is the probability that player $F$ joins the venture, and $c_L$ is $L$'s cost of initiating the project. Hence Assumption 1 holds. Player $F$, after observing $L$'s decision to initiate the project, decides whether to join. If he does, his payoff is equal to $1 - m + 1 - c_F$, whereas, if he does not, it is equal to zero. Again, this is an anti-lemon problem, in that the decision by $L$ to engage (here to start a project) is good news for player $F$, instead of bad news. Assuming that $c_F$ is drawn from some cumulative distribution $H$, we then have that the probability that $F$ joins is given by the solution to

$$r(\rho^{\dagger}) = H\left(2 - M^{-}\left(1 + r(\rho^{\dagger}) - c_L;\rho^{\dagger}\right)\right). \tag{S.6}$$

Hence, Assumption 2' holds whenever equation (S.6) admits a unique solution.

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## S.2 Relation to Other Covert Investment Games

The paper's emphasis is on information acquisition, a choice motivated both by the applications and by the fact that cognitive investments are the ultimate covert investments. But the results may also be useful for other covert investments: capacity acquisition, learning by doing, arms buildup, and so on.

Suppose that there are two players, playing a "second-stage" normal-form game with actions $a_L$, $a_F \in \mathbb{R}$. One of the players, here player $L$, makes a "first-stage" investment $\rho \in \mathbb{R}$ at an increasing investment cost $C(\rho)$.[9] Payoffs are $\phi_L(a_L, a_F) - \psi(a_L, \rho) - C(\rho)$ for player $L$ and $\phi_F(a_L, a_F)$ for

[9] The analysis can be extended to the case where both players make period-1 investments. The insights are not fundamentally different from those discussed here.

player $F$, where all functions are $C^2$ and satisfy $\partial^2\psi/\partial a_L \partial\rho < 0$ and

$$\frac{\partial^2 \phi_i}{\partial a_i \partial a_j} \begin{cases} > 0 & (SC) \\ \text{or} & \\ < 0 & (SS) \end{cases}$$

for $i, j = L, F$, $j \neq i$.

That is, the investment $\rho$ lowers player $L$'s marginal cost of action $a_L$, and the strategic interaction between the two players involves either strategic complementarity ($SC$) or strategic substitutability ($SS$). For example, $a_i$ may stand for firm $i$'s output, $\rho$ an investment that lowers the marginal cost of production, and the two firms' output choices may be either strategic complements or substitutes.

Assume, for simplicity, that, if player $L$'s investment was common knowledge, the normal-form game in $(a_1, a_2)$ would have a unique and stable equilibrium. In such a game, player $F$'s equilibrium action $a_F(\rho^\dagger)$ as a function of player $L$'s anticipated investment $\rho^\dagger$, is increasing in $\rho^\dagger$ under SC and decreasing in $\rho^\dagger$ under $SS$.

Consistently with the analysis above, suppose that player $L$'s actual investment $\rho$ is not observed by player $F$ (so de facto the game is a simultaneous-move game in actions $(\rho, a_L)$, for player $L$, and $a_F$, for player $F$). One can then define player $L$'s optimal action when she deviates from her equilibrium investment. The above assumptions imply that player $L$'s optimal action $a_L(\rho, \rho^\dagger)$ when player $F$ expects $\rho^\dagger$ and player $L$'s actual investment is $\rho$ is non-decreasing in $\rho^\dagger$ under either $SC$ or $SS$.

This environment is similar to that considered in the industrial organization literature on the taxonomy of business strategies[10], except for one important twist. The investment choice $\rho$ is not observed by player $F$ and so has no commitment effect; rather, what matters for the outcome of the normal-form game is the anticipation $\rho^\dagger$ by firm $F$ of firm $L$'s choice as well as the actual choice $\rho$ by firm $L$ (of course, in a pure-strategy equilibrium, $\rho^\dagger = \rho$).

Let $T_L(\rho, \rho^\dagger) \equiv \max_{a_L} \left\{\phi_L(a_L, a_F(\rho^\dagger)) - \psi(a_L, \rho) - C(\rho)\right\}$ denote player $L$'s payoff when her actual investment is $\rho$ and player $F$ anticipates investment $\rho^\dagger$. The above assumptions imply that, whether $SC$ or $SS$ prevails, for all $(\rho, \rho^\dagger)$ and $(\hat\rho, \hat\rho^\dagger)$ with $\hat\rho \geq \rho$ and $\hat\rho^\dagger \geq \rho^\dagger$, the following "expectation conformity" condition is satisfied:

$$T_L(\hat\rho, \hat\rho^\dagger) - T_L(\rho, \hat\rho^\dagger) \geq T_L(\hat\rho, \rho^\dagger) - T_L(\rho, \rho^\dagger).$$

Consequently, let $\rho$ (alternatively, $\hat\rho$) denote player $L$'s optimal investment when player $F$ expects investment $\rho^\dagger$ (alternatively, $\hat\rho^\dagger$).[11] Expectation conformity implies that there is complementarity

[10] See, e.g., Bulow et al (1985), and Fudenberg and Tirole (1984).

[11] That is, given $\rho^\dagger$, $(a_L, \rho)$ is such that

$$(a_L, \rho) \in \arg\max_{\tilde\rho, \tilde a_L} \left\{\phi_L(\tilde a_L, a_F(\rho^\dagger)) - \psi_L(\tilde a_L, \tilde\rho) - C(\tilde\rho)\right\}$$

with $\{a_L(\rho^\dagger), a_F(\rho^\dagger)\}$ denoting the Nash equilibrium of the normal-form game under common knowledge that $L$

between investment and anticipation of investment: $(\hat{\rho}_1 - \rho_1)(\hat{\rho}_1^\dagger - \rho_1^\dagger) \geq 0$. This is so both when the stage-2 game involves strategic substitutes or strategic complements.

The intuition goes as follows: Suppose that firm $F$ expects $L$ to invest more and therefore to produce more output. It then raises its output under $SC$ and decreases it under $SS$. In either case, firm $L$ is induced to raise its output, vindicating a higher investment in the first place. It can also be checked that when there are two equilibria ($\rho = \rho^\dagger$ and $\hat{\rho} = \hat{\rho}^\dagger$), player $L$ is better off in the high-investment one, again regardless of the type of strategic interaction ($SC$ or $SS$).

Let us draw a formal analogy between the generalized lemons game of this paper and the covert investment game described above. The investment $\rho$ in the present paper is player $L$'s cognition, that is, her choice of information structure. To interpret the generalized lemons game as a covert investment game, it suffices to assume that, in the "stage-2" game, player $L$ wants to take an action equal to her investment. For example, one can think of $a_L$ as the information used by player $L$ in the stage-2 game. In this spirit, the assumption that $L$ maximizes her payoff by choosing $a_L = \rho$ simply reflects the idea that player $L$ makes full use of the acquired information (see also the discussion in the next section about how to interpret $\rho$ as capacity in entropy reduction, or the maximal slope of the induced stochastic choice rule). In this case, $\psi(a_L, \rho) = 0$ if $a_L = \rho$, whereas $\psi(a_L, \rho) = -\infty$ otherwise, which is a discontinuous version of the complementarity relationship $\partial^2\psi_L/\partial a_L \partial\rho < 0$ in the investment game. Letting $a_F = r$, we then have that

$$\phi_L(a_L, r) \equiv \int_{-\infty}^{m^*(r)} \delta_L(m, r) dG(m; a_L)$$

and so $\partial^2\phi_L(r(\rho^\dagger), a_L)/\partial a_L \partial r < 0$ whenever condition $\mathrm{B}(\rho, \rho^\dagger) > 0$ in Proposition 1 in the main text is satisfied, for $a_L = \rho$. Furthermore, Condition $\mathrm{A}(\rho^\dagger) < 0$ in Proposition 1 implies that $dr/da_L < 0$. Summarizing, when the two conditions of Proposition Proposition 1 are met, the lemons game can be seen as an investment game with strategic substitutes (SS). In contrast, many anti-lemon games are investment games with strategic complements.

invested $\rho^\dagger$ (i.e., under symmetric information).

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