Matching Auctions

Daniel Fershtman Alessandro Pavan

MATCH-UP 2017

Motivation

- Mediated matching central to "**sharing economy**"

- Most matching markets intrinsically **dynamic — re-matching**

  - shocks to profitability of existing matching allocations

  - gradual resolution of uncertainty about attractiveness

  - preference for variety

- Re-matching, while pervasive, largely ignored by matching theory

This paper

- Dynamic matching
  - mediated (many-to-many) interactions
  - evolving private information
  - payments
  - capacity constraints
- Applications
  - scientific outsourcing (Science Exchange)
  - lobbying
  - sponsored search
  - internet display advertising
  - lending (Prospect, LendingClub)
  - B2B
  - health-care (MEDIGO)
  - organized events (meetings.com)
- Matching auctions
- Profit vs welfare maximization

Plan

- **Model**
- Matching auctions
- Profit maximization
- Distortions
- Conclusions

Model

- Profit-maximizing platform mediates interactions between 2 sides, A, B

Model

- Profit-maximizing platform mediates interactions between 2 sides, A, B
- Agents: $N_A = \{1, ..., n_A\}$ and $N_B = \{1, ..., n_B\}$, $n_A, n_B \in \mathbb{N}$

Model

- Profit-maximizing platform mediates interactions between 2 sides, A, B
- Agents: $N_A = \{1, ..., n_A\}$ and $N_B = \{1, ..., n_B\}$, $n_A, n_B \in \mathbb{N}$
- Period-t match between agents $(i, j) \in N_A \times N_B$ yields gross payoffs

$$v_{ijt}^A = \theta_i^A \cdot \varepsilon_{ijt}^A \quad \text{and} \quad v_{ijt}^B = \theta_j^B \cdot \varepsilon_{ijt}^B$$

θ_i^k: "vertical" type

ε_{ijt}^k: "horizontal" type (time-varying match-specific)

Model

- Profit-maximizing platform mediates interactions between 2 sides, A, B
- Agents: $N_A = \{1, ..., n_A\}$ and $N_B = \{1, ..., n_B\}$, $n_A, n_B \in \mathbb{N}$
- Period-t match between agents $(i, j) \in N_A \times N_B$ yields gross payoffs

$$v_{ijt}^A = \theta_i^A \cdot \varepsilon_{ijt}^A \quad \text{and} \quad v_{ijt}^B = \theta_j^B \cdot \varepsilon_{ijt}^B$$

  θ_i^k: "vertical" type

  ε_{ijt}^k: "horizontal" type (time-varying match-specific)

- Agent i's period-t (flow) type ($i \in N_A$):

$$v_{it}^A = (v_{i1t}^A, v_{i2t}^A, ..., v_{in_B t}^A)$$

Model

- Profit-maximizing platform mediates interactions between 2 sides, A, B
- Agents: $N_A = \{1, ..., n_A\}$ and $N_B = \{1, ..., n_B\}$, $n_A, n_B \in \mathbb{N}$
- Period-t match between agents $(i, j) \in N_A \times N_B$ yields gross payoffs

$$v_{ijt}^A = \theta_i^A \cdot \varepsilon_{ijt}^A \quad \text{and} \quad v_{ijt}^B = \theta_j^B \cdot \varepsilon_{ijt}^B$$

  θ_i^k: "vertical" type

  ε_{ijt}^k: "horizontal" type (time-varying match-specific)

- Agent i's period-t (flow) type ($i \in N_A$):

$$v_{it}^A = (v_{i1t}^A, v_{i2t}^A, ..., v_{in_Bt}^A)$$

- Agent i's payoff ($i \in N_A$):

$$U_i^A = \sum_{t=0}^{\infty} \delta^t \sum_{j \in N_B} v_{ijt}^A \cdot x_{ijt} - \sum_{t=0}^{\infty} \delta^t p_{it}^A$$

  with $x_{ijt} = 1$ if (i, j)-match active, $x_{ijt} = 0$ otherwise.

Model

- In each period $t \geq 1$, platform can match up to M pairs of agents

  - space, time, services constraint

Model

- In each period $t \geq 1$, platform can match up to M pairs of agents

  - space, time, services constraint

- Many-to-many matching

Model

- In each period $t \geq 1$, platform can match up to M pairs of agents

  - space, time, services constraint

- Many-to-many matching

- Platform's profit:

$$\sum_{t=0}^{\infty} \delta^t \left(\sum_{i \in N_A} p_{it}^A + \sum_{j \in N_B} p_{jt}^B - \sum_{i \in N_A} \sum_{j \in N_B} c_{ijt}(x^{t-1}) \cdot x_{ijt} \right)$$

Model

- Each θ_l^k drawn independently from (abs cont.) F_l^k over $\Theta_l^k = [\underline{\theta}_l^k, \bar{\theta}_l^k]$

Model

- Each θ_l^k drawn independently from (abs cont.) F_l^k over $\Theta_l^k = [\underline{\theta}_l^k, \bar{\theta}_l^k]$

- Period-t horizontal type ε_{ijt}^k drawn from cdf $G_{ijt}^k(\varepsilon_{ijt}^k \mid \varepsilon_{ijt-1}^k, x^{t-1})$

Model

- Each θ_l^k drawn independently from (abs cont.) F_l^k over $\Theta_l^k = [\underline{\theta}_l^k, \bar{\theta}_l^k]$
- Period-t horizontal type ε_{ijt}^k drawn from cdf $G_{ijt}^k(\varepsilon_{ijt}^k \mid \varepsilon_{ijt-1}^k, x^{t-1})$
- Agents observe θ_i^k prior to joining, but learn (ε_{ijt}^k) over time

Model

- **Exogenous processes**:
  - G_{ijt}^{k} and c_{ijt} independent of x^{t-1}

Model

- **Exogenous processes**:
  - G^k_{ijt} and c_{ijt} independent of x^{t-1}
- **Endogenous processes:**
  - when $x_{ijt-1} = 0$, $\varepsilon^k_{ijt} = \varepsilon^k_{ijt-1}$ a.s.
  - when $x_{ijt-1} = 1$, dependence of G_{ijt} on x^{t-1} through $\sum_{s=1}^{t-1} x_{ijs}$
  - $c_{ijt}(x^{t-1})$ depends on x^{t-1} through $\sum_{s=1}^{t-1} x_{ijs}$
  - example 1: experimentation in Gaussian world ($\varepsilon^k_{ijt} = \mathbb{E}[\omega^k_{ij} | (z^k_{ijs})_s]$)
  - example 2: preference for variety

Model

- **Exogenous processes**:
  - G^k_{ijt} and c_{ijt} independent of x^{t-1}
- **Endogenous processes:**
  - when $x_{ijt-1} = 0$, $\varepsilon^k_{ijt} = \varepsilon^k_{ijt-1}$ a.s.
  - when $x_{ijt-1} = 1$, dependence of G_{ijt} on x^{t-1} through $\sum_{s=1}^{t-1} x_{ijs}$
  - $c_{ijt}(x^{t-1})$ depends on x^{t-1} through $\sum_{s=1}^{t-1} x_{ijs}$
  - example 1: experimentation in Gaussian world ($\varepsilon^k_{ijt} = \mathbb{E}[\omega^k_{ij} | (z^k_{ijs})_s]$)
  - example 2: preference for variety
- ε drawn independently across agents and from θ (given x)

Plan

- Model
- **Matching auctions**
- Profit maximization
- Distortions
- Conclusions

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta^k_{l0} \in \Theta^k_l$ at price $p^k_l(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\rightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$
  - each match $(i, j) \in N_A \times N_B$ assigned **score** $S_{ijt} \in \mathbb{R}$

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$
  - each match $(i,j) \in N_A \times N_B$ assigned **score** $S_{ijt} \in \mathbb{R}$
  - matches with highest (nonnegative) score implemented (up to capacity)

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$
  - each match $(i, j) \in N_A \times N_B$ assigned **score** $S_{ijt} \in \mathbb{R}$
  - matches with highest (nonnegative) score implemented (up to capacity)
  - unmatched agents pay nothing

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$
  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions
- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$
  - each match $(i, j) \in N_A \times N_B$ assigned **score** $S_{ijt} \in \mathbb{R}$
  - matches with highest (nonnegative) score implemented (up to capacity)
  - unmatched agents pay nothing
  - matched agents pay $p_{lt}^k(\theta_0, \theta_t, b_t, x^{t-1})$

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$
  - each match $(i, j) \in N_A \times N_B$ assigned **score** $S_{ijt} \in \mathbb{R}$
  - matches with highest (nonnegative) score implemented (up to capacity)
  - unmatched agents pay nothing
  - matched agents pay $p_{lt}^k(\theta_0, \theta_t, b_t, x^{t-1})$

- Each **bilateral score** S_{ijt}

  - depends on information about (i, j) only

  - independent of past bids

Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_{l0}^k \in \Theta_l^k$ at price $p_l^k(\theta_0)$

  - higher status $\longrightarrow$ more favorable treatment in subsequent auctions

- At any $t \geq 1$:
  - agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$
  - agents *bid* $b_{lt}^k \equiv (b_{ljt}^k)_{j \in N_{-k}}$, one for each partner from side $-k$
  - each match $(i, j) \in N_A \times N_B$ assigned **score** $S_{ijt} \in \mathbb{R}$
  - matches with highest (nonnegative) score implemented (up to capacity)
  - unmatched agents pay nothing
  - matched agents pay $p_{lt}^k(\theta_0, \theta_t, b_t, x^{t-1})$

- Each **bilateral score** S_{ijt}

  - depends on information about (i, j) only

  - independent of past bids

- Full transparency - bids, payments, membership, matches all public.

Myopic score

Definition

A **myopic score** (with weights β)

$$S^m_{ijt} \equiv \beta^A_i(\theta^A_{i0}) \cdot b^A_{ijt} + \beta^B_j(\theta^B_{j0}) \cdot b^B_{ijt} - c_{ijt}(x^{t-1}),$$

Index

Definition

Index score (with weights β)

$$S^I_{ijt} \equiv \sup_{\tau} \frac{\mathbb{E}^{\lambda_{ij}|\theta_0,\theta_t,b_t,x^{t-1}}\left[\sum_{s=t}^{\tau} \delta^{s-t} \cdot S^{m;\beta}_{ijs}\right]}{\mathbb{E}^{\lambda_{ij}|\theta_0,\theta_t,b_t,x^{t-1}}\left[\sum_{s=t}^{\tau} \delta^{s-t}\right]}$$

- τ: stopping time
- $\lambda_{ij}|\theta_0, \theta_t, b_t, x^{t-1}$: process over myopic scores under truthful bidding, when $\varepsilon^k_{ijt} = \frac{b^k_{ijt}}{\theta^k_{it}}$

Payments (PST + BV)

- Payments for $t \geq 1$ designed to make payoffs proportional to marginal contributions to weighted surplus

- Membership fees for $t = 0$ designed to induce agents to participate and select status designed for their vertical type.

Plan

- Model
- Matching auctions
- **Profit maximization**
- Distortions
- Conclusions

Profit maximization

- Consider the weights

$$\hat{\beta}_l^k(\theta_{l0}^k) \equiv 1 - \frac{1 - F_l^k(\theta_{l0}^k)}{f_l^k(\theta_{l0}^k)\theta_{l0}^k}$$

Theorem

(i) **Exogenous processes**: *suppose, under myopic scoring rule with weights $\hat{\beta}$, all agents, at $t = 0$, expect non-negative match quality. Then a myopic scoring rule with weights $\hat{\beta}$ is profit-maximizing.*

(ii) **Endogenous processes**: *suppose, under index scoring rule with weights $\hat{\beta}$, all agents, at $t = 0$, expect non-negative match quality. In addition, suppose that either (a) $M = 1$, or (b) $M \geq n_A \cdot n_B$, or (c) $1 < M < n_A \cdot n_B$ and environment is "separable." Then an index scoring rule with weights $\hat{\beta}$ is profit-maximizing.*

Plan

- Model
- Dynamic matching auctions
- Profit maximization
- **Distortions**
- Conclusions

Welfare maximization

- Efficient auctions have same structure as profit-maximizing auctions, but with

$$\beta_l^k(\theta_l^k) = 1$$

Distortions

Theorem

Assume horizontal types ε **non-negative**

(1) Suppose $M \geq n_A \cdot n_B$:

$$\chi_{ijt}^{P} = 1 \Rightarrow \chi_{ijt}^{W} = 1$$

(2) Exogenous processes with any M, or endogenous processes with $M = 1$:

$$\sum_{(i,j) \in N_A \times N_B} \chi_{ijt}^{W} \geq \sum_{(i,j) \in N_A \times N_B} \chi_{ijt}^{P}$$

(3) Endogenous processes with $1 < M < n_A \cdot n_B$: if matching stops at $T < \infty$ under profit maximization, then

$$\sum_{t=1}^{\infty} \sum_{(i,j) \in N_A \times N_B} \chi_{ijt}^{W} \geq \sum_{t=1}^{\infty} \sum_{(i,j) \in N_A \times N_B} \chi_{ijt}^{P}$$

(*) Above conclusions can be reversed with negative horizontal types (upward distortions)

Conclusions

- Mediated (dynamic) matching
  - agents learn about attractiveness of partners over time
  - shocks to profitability of matching allocations
  - preferences for variety
- Matching auctions
  - similar in spirit to GSPA for sponsored search BUT
  (i) value of experimentation
  (i) costs of info rents
- Matching distortions → regulation
- Ongoing/future work
  - alternative indexes based on empirical distributions
  - population dynamics
  - no payments

Thank You!

Separable environments

- Let

$$\underline{S}_{ijt} \equiv \inf_{s \leq t} \left\{ S^{I}_{ijs} \right\}$$

Definition (separability)

Environment is *separable* under index rule with weights β if, for any $t \geq 1$, any $(i,j),(i',j') \in N_A \times N_B$, any $(\theta_0, \theta_t, \varepsilon^t, x^{t-1})$,

$$\underline{S}_{ijt} > \underline{S}_{i'j't} \geq 0 \;\Rightarrow \underline{S}_{ijt} \cdot (1-\delta) \geq \underline{S}_{i'j't}$$

- Separability imposes restrictions only on "*downside risk*"

Example (bad news)

Separability holds if period-1 indexes sufficiently apart for all pairs for which $S^{I;\beta}_{ij1} \geq 0$, and, at all $t \geq 2$, either $S^{I;\beta}_{ijt} \geq S^{I;\beta}_{ijt-1}$, or $S^{I;\beta}_{ijt} < 0$.