“Soft” Affirmative Action and Minority Recruitment

Daniel Fershtman†    Alessandro Pavan‡

April 2020

Abstract

We study search, evaluation, and selection of candidates of unknown quality for a position. We examine the effects of “soft” affirmative action policies increasing the relative percentage of minority candidates in the candidate pool. We show that, while meant to encourage minority hiring, such policies may backfire if the evaluation of minority candidates is noisier than that of non-minorities. This may occur even if minorities are at least as qualified and as valuable as non-minorities. The results provide a possible explanation for why certain soft affirmative action policies have proved counterproductive, even in the absence of (implicit) biases.

*We thank Mariacristina De Nardi, Matthew Notowidigdo, Wojciech Olszewski, Nicola Persico, Analia Schlosser, and Marciano Siniscalchi for useful comments and suggestions.

†Eitan Berglas School of Economics, Tel-Aviv University. Email: danielfer@tauex.tau.ac.il
‡Department of Economics, Northwestern University. Email: alepavan@northwestern.edu
1 Introduction

In 2003, the National Football League established the “Rooney Rule”, a policy requiring teams to interview minority candidates for head coaching vacancies.\(^1\) This policy, versions of which have been applied across various industries\(^2\), is an example of “soft” affirmative action (SAA), a term typically referring to policies designed to change the composition of the candidate pool, rather than the criteria used during the hiring process. Contrary to “hard” affirmative action policies requiring direct consideration of minority status as a part of the hiring decision (e.g., employment quotas)\(^3\), such policies often involve taking steps to increase the share of minority candidates considered for a position, but subsequently treating candidates impartially; they are also far less controversial (Schuck, 2002).

SAA policies are also common in academic faculty recruitment. For example, Columbia University’s “Guide to Best Practices in Faculty Search and Hiring” offers a checklist of best practices in search and evaluation in tenure and tenure-track recruitment, which includes: creating a search plan including broader outreach; signaling a special interest in candidates contributing to the department’s diversity priorities; reaching out to colleagues to identify high-potential female and underrepresented minority candidates and encouraging them to apply to the position; requiring diversity statements; requiring an explanation (to the Dean) for being unable to find a sufficient number of competitive candidates from underrepresented groups, and detailing what steps were taken to identify such candidates.\(^4\)

This paper studies the effectiveness of SAA policies on minority recruitment. Indeed, while some SAA policies have proven successful (Heilman, 1980), others, such as the Rooney Rule, have been deemed ineffective, or even counterproductive. One possible explanation is the presence of implicit biases. Our analysis provides an alternative explanation based on the well-established observation that the evaluation of minority candidates is often noisier than that of non-minorities. We show that this property alone may be responsible for the negative effects of SAA on minority recruitment. We show that, while meant to increase the likelihood of hiring a minority candidate, such policies may in fact lead to the opposite outcome, even if minorities are preferred to non-minorities and are, on average, at least as qualified.


\(^2\)Facebook recently adopted a similar policy (https://money.cnn.com/2018/05/31/technology/facebook-board-diversity/index.html). In 2014, Senate Resolution (511) was introduced (though ultimately not enacted) to encourage companies to voluntarily establish policies to identify and interview qualified minority candidates for managerial openings at the director level or above.

\(^3\)Title VII of the Civil Rights Act of 1964 prohibits firms from establishing “hard” affirmative-action policies that require direct consideration of minority status during the hiring process.

\(^4\)See https://provost.columbia.edu/sites/default/files/content/BestPracticesFacultySearchHiring.pdf.
We propose a model of search, evaluation, and selection of candidates for a position. Consider, for example, the procedure for hiring a new university faculty member. The evaluation of candidates is sequential and involves a sequence of signals about the candidates’ qualification, including recommendation letters, interviews, seminars, and the evaluation of previous research and teaching output. The hiring process is governed by university recruitment standards, which may (but need not) include “hard” affirmative-action policies. We consider two types of SAA interventions: (1) promoting the expansion of the candidate pool in addition to evaluating those already in it, and (2) changing search practices in order to boost the number of minority candidates in the pool, at the expense of non-minorities. We allow for the possibility that the university may have different standards for accepting minority candidates and may place higher value on hiring a minority candidate. The evaluation procedure is carried out by a recruiting committee, which decides on the sequence of candidates’ evaluations as well as when (if at all) to search for additional candidates during the process.

We first consider the effects of SAA policies when hiring decisions are conducted under a myopic rule (i.e., when the sequence of candidates to evaluate and the decision to search for additional ones are based on myopic values attached to these actions). We show that SAA policies may reduce the probability of selecting a minority candidate, even if (i) hiring a minority candidate is at least as valuable as hiring a non-minority one, conditional on the candidate being qualified, (ii) minority candidates are ex-ante at least as likely to be qualified as non-minorities, and (iii) the probability that search delivers a minority candidate is at least as high as the probability it brings a non-minority one.

We then show the same conclusions apply under the optimal forward-looking rule. The mechanism underlying our results is the following. Search for additional candidates is an alternative to the evaluation of existing ones. Since SAA policies alter the desirability of expanding the candidate pool, they affect the relative desirability of evaluating candidates already in it. Since the evaluation of minority candidates is noisier, this may shift the evaluation of existing candidates in favor of non-minorities. Importantly, such policies may backfire both when they increase the relative attractiveness of search, and when they decrease it.

Consider first policies increasing the relative attractiveness of search (e.g., through the reduction of its costs). By making search more attractive, such policies may induce the committee to substitute the further evaluation of candidates whose early evaluation yielded negative results (e.g., unconvincing interviews, low test scores, etc.) with search for new candidates. Because the evaluation of minority candidates is noisier, the chances that negative early evaluations are due to noise rather than lack of qualification are greater for
minority candidates, who therefore suffer from the interruption of the evaluation process more than non-minorities. Our results show that this mechanism may be strong enough to undermine the ultimate chances that a minority candidate is chosen.

Next, consider policies reducing the attractiveness of search relative to a more thorough evaluation of candidates already in the pool. An example of such policies is the introduction of explicit requirements to shift search from fields/schools/areas populated primarily by non-minorities towards those populated by minorities. Because the candidates brought in by search are expected to be more difficult to evaluate, such policies ultimately reduce the attractiveness of search relative to a lengthier evaluation of candidates already in the pool. We show that the reduction in the attractiveness of search in turn may lead to a relatively more thorough evaluation of non-minority candidates compared to minority ones. Consequently, when the above effects are strong enough, despite raising the percentage of minority candidates in the pool, such policies may reduce the overall probability of hiring a minority candidate.

Our results are driven by the assumption that the evaluation of minority candidates is noisier than that of non-minority ones. This assumption is a central feature in the literature on statistical discrimination building on Phelps’ (1972) seminal contribution (e.g., Aigner and Cain, 1977, Borjas and Goldberg, 1979, Lundberg and Straz, 1983, Cornell and Welch, 1996, and more recently Chambers and Echenique, 2018, and Bardhi et al., 2020). The literature has offered various explanations for the relative difficulties in the evaluation of minority candidates. For instance, recruiting committees may have more limited experience evaluating minorities. Aigner and Cain (1977) suggest that majority-group candidates face “a more homogeneous set of environmental determinants of quality,” resulting in a lower variance in qualification, and hence less noisy evaluation. It may be easier to assess a candidate’s history if it followed a well-known path, and it may be easier to interpret a candidate’s references when they come from familiar letter writers. Furthermore, certain tests used in recruitment were initially designed with a specific group in mind; for example, it has long been argued that the SAT is more informative about the abilities of White students than African-American students (Linn, 1973, Fleming and Garcia, 1998).

The difficulty in evaluating minorities may also originate in differences in “background”

---

5In contrast to taste-based theories of discrimination (Becker, 1957), statistical discrimination theories explain group inequality without assuming prejudice or preference bias.

6An important exception is Arrow’s (1973) theory of statistical discrimination, which differs from Phelps’ and relies on coordination failures rather than differences in the evaluation of particular groups. See Fang and Moro (2011) for an overview of the statistical discrimination literature.
between minority candidates and recruiters.\textsuperscript{7} Such differences may impede the assessment of intangible qualities such as character, charisma, responsiveness, initiative, or focus, which are informal, but nevertheless potentially relevant for the position (see, e.g., Arrow, 1972, and Cornell and Welch, 1996). Relatedly, minority candidates may use “languages” (broadly defined) less familiar to the recruiting committee. Lang (1986) proposes a “language theory” of discrimination, whereby noisier evaluation of minorities is the result of differences in “language” impeding communication. Recent research also suggests that women tend to use more tentative language (qualifiers/disclaimers/hedges/intensifiers) in job interviews than men, which may be misinterpreted as lack of assertiveness (Leaper and Robnett, 2011). Similarly, output differences due to maternity/family duties may be acknowledged but difficult to quantify. Further, recruiters often (implicitly) use recent hires with a similar background as a benchmark for evaluating new candidates. If mostly white men were hired in the recent past, such benchmarking may provide recruiters with less effective tools for evaluating minority candidates through similarity comparisons.

Our results suggest that, without taking steps to improve the evaluation of minority candidates, attempts to shift the composition of the candidate pool in favor of minorities may simply amount to “checking a box,” or even prove detrimental. Such steps may include creating a sufficiently diverse recruiting committee and ensuring evaluation is based on a predetermined set of objective criteria with predetermined weights. In our model, if differences in the evaluation of candidates were entirely eliminated – as in the case of “blind” auditions – SAA would be guaranteed to increase the probability of hiring a minority candidate. If differences in candidates’ evaluation cannot be sufficiently reduced, “hard” affirmative action may be necessary to foster minority recruitment.

The literature on statistical discrimination has also studied the unintended effects of affirmative action, such as its negative effects on incentives to invest in human capital, productivity stereotyping, wage differentials, and occupational social status (see, e.g., Fryer and Loury, 2005).\textsuperscript{8} In an influential article, Coate and Loury (1993) show, for example, that a “patronizing equilibrium” may arise in which minorities’ incentives to invest in skills may be reduced when affirmative action is in place. This paper complements this literature by illustrating the possible negative effects of SAA directly on the dimension it is intended for – the enhancement of minority recruitment.

\textsuperscript{7} “Background” here may include race, sex, language, sexual preference, ethnic background, religious beliefs, and nationality.

\textsuperscript{8} The literature on the economics of affirmative action includes Welch (1976), Lundberg and Straz (1983), Chung (2000), Moro and Norman (2003), and Fryer and Loury (2013), and is too broad to be succinctly discussed here. For evidence on the effectiveness of affirmative action, see Holzer and Neumark (2000).
2 Model

The recruitment problem described below applies to a variety of environments. For concreteness, we focus on a university committee recruiting faculty, or students, for an available position. The recruitment of a candidate is subject to university approval standards. There are two categories of candidates, $A$ and $B$ (race, gender, field of study). Candidates’ qualifications $\theta \in \{L, H\}$ are unknown and independent ex-ante. Let $p^j_0 = \Pr(\theta^j = H)$ denote the prior that a category-$j$ candidate is qualified. The value of hiring a qualified category-$j$ candidate is $v^j > 0$, whereas the value of hiring any non-qualified candidate is zero. That is, the school wishes to hire only qualified candidates, but candidates of one category may be preferred.

Candidates can be recruited only if they are in the school’s candidate pool. For simplicity, we assume that at the outset there are only two candidates in the pool, one from each category. At each period $t = 0, 1, \ldots$, the committee either chooses a candidate to evaluate among those in the pool, or decides to expand the pool by searching for additional candidates. The evaluation of a candidate generates new information, formally captured by a signal about the candidate’s qualification. Again, for simplicity, we assume signals are binary. The evaluation of a qualified candidate from category $j$ yields a signal realization $s = 1$ with probability $q^j_H \equiv \Pr(s = 1|\theta^j = H)$, whereas the evaluation of a non-qualified category-$j$ candidate yields a signal realization $s = 0$ with probability $q^j_L \equiv \Pr(s = 0|\theta^j = L)$, with $q^j_H \geq 1 - q^j_L$. Conditional on a candidate’s type, the signals are iid draws from the above Bernoulli distribution.

Given a history $\sigma = (s_1, s_2, \ldots)$ of signal realizations, the posterior probability that a category-$j$ candidate is qualified will be denoted by $p^j(\sigma)$. The null history is denoted by $\sigma = \emptyset$.

Each time the committee searches for new candidates, it identifies a category-$j$ candidate with probability $\mu^j$, and no candidate with probability $1 - \mu^A - \mu^B$. For simplicity, we assume that beyond the opportunity cost of not being able to evaluate one of the candidates in the pool while searching, there is no direct cost for search. Likewise, the only cost of evaluating a candidate already in the pool is the cost of postponing search and the evaluation of other candidates.

The entire recruiting process ends when the committee finds a candidate whose (category-adjusted) expected quality is large enough. Formally, assume that for any $j$ there exists a threshold $\bar{P}^j \in (0, 1]$ such that each candidate from category $j$ is given the slot at history $\sigma$. The results extend to more general search technologies, in particular, the possibility that each search yields a random number of candidates from each category. We assume a single candidate is discovered at each search to ease the exposition.
\( \sigma \) if and only if \( p^j(\sigma) \geq \bar{P}^j \). The threshold \( \bar{P}^j \) can be thought of as reflecting standards imposed by the university, which may, but need not, coincide with the preferences of the recruiting committee. To avoid trivialities, assume \( p^j_0 < \bar{P}^j \), which means each candidate must be evaluated at least once to be recruited.

A recruitment rule specifies in each period either a candidate to evaluate among those in the pool, or search for a new candidate. A recruitment rule is optimal if it maximizes the expected discounted payoff \( \mathbb{E} \left( \delta^T \bar{v}_T \right) \) over all feasible recruitment rules, where \( \delta \in (0,1) \) is the discount factor, \( T \) the (stochastic) time at which a selection is made, and \( \bar{v}_T \) the value of the selected candidate.\(^{10}\)

As anticipated in the Introduction, the key assumption behind our results is that the evaluation of minority candidates is noisier than that of non-minority ones. Formally, \( A \) is the minority category if and only if the evaluation of \( B \)-candidates is Blackwell more informative than that of \( A \)-candidates. Under the technology described above, this amounts to \( q^B_k \geq q^A_k \), \( k = H, L \), with at least one strict inequality.

### 3 Myopic Recruitment Rule

We start by assuming the recruiting process is conducted under a simple, myopic rule. Given a history of signal realizations \( \sigma \), denote by \( \lambda^j(\sigma) \) the probability that a candidate from category \( j \) with history \( \sigma \) is recruited after a single additional evaluation (that is, \( \lambda^j(\sigma) = \operatorname{Pr}(s : p^j(\sigma,s) \geq \bar{P}^j | j, \sigma) \)). The myopic value the committee attaches to an additional evaluation of a category-\( j \) candidate with history \( \sigma \) is equal to \( u^j(\sigma) = \lambda^j(\sigma)v^j \).

Likewise, the myopic value it attaches to an additional search is equal to the expected value \( u^S = \delta \left( \mu^A u^A(\emptyset) + \mu^B u^B(\emptyset) \right) \) of bringing a “blank-slate” candidate to the pool. Under a myopic rule, in each period, the committee selects the alternative (evaluation of a candidate from the pool, or search) with the highest myopic value. To avoid trivialities, assume \( u^A(\emptyset), u^B(\emptyset) > u^S \), so that upon being added to the pool each candidate has a greater myopic value than search.

Given \( v \equiv (v^A, v^B) \), \( p_0 \equiv (p^A_0, p^B_0) \), \( \mu \equiv (\mu^A, \mu^B) \), \( q \equiv (q^A_L, q^A_H, q^B_L, q^B_H) \), and \( \bar{P} \equiv (\bar{P}^A, \bar{P}^B) \), denote by \( \gamma^j(v, p_0, \mu, q, \bar{P}) \) the ex-ante probability a category-\( j \) candidate is selected under a myopic rule.

Our first result pertains to SAA policies aimed at expanding the size of the candidate pool (e.g., instructions to the committee to include areas typically ignored by the school in its searches). Formally, such policies can be captured by an improvement in search

\(^{10}\) If the selected candidate is from category \( j \) and the history of signals is \( \sigma \), the expected value of the candidate is \( p^j(\sigma)v^j \).
technology, i.e., an increase in $\mu^A$ and $\mu^B$ (possibly asymmetric across categories). To ease the exposition, and without any important implication for the results, we assume that, prior to the introduction of the policy, $\mu = (0, 0)$.

**Proposition 1.** Suppose $A$ is the minority category and that recruitment is conducted under a myopic rule. There exist $(v, p_0, \mu, q, \bar{P})$ such that $A$-candidates are (a) more valuable to the school ($v^A > v^B$), (b) more likely to be qualified ($p^A_0 > p^B_0$), (c) more likely to be identified through search ($\mu^A > \mu^B > 0$), (d) have a lower acceptance threshold ($P^A < P^B$), and yet policies promoting the expansion of the candidate pool may reduce the ex-ante probability that $A$-candidates are selected: $\gamma^A(v, p_0, \mu, q, \bar{P}) < \gamma^A(v, p_0, \bar{0}, q, \bar{P})$.

Under the assumptions in the proposition, an improvement in the search technology naturally increases the expected number of minority candidates considered for the position, as well as their share relative to non-minority ones. However, the increase in $\mu^A$ and $\mu^B$ – by making search more attractive – might come at the expense of lengthier evaluations of candidates whose early evaluations yielded negative results. Because minority candidates are more difficult to evaluate than non-minority ones, they are the ones who are more likely to suffer from the truncation in the evaluation process. The latter effect, when strong enough, may imply a reduction in the ex-ante probability of selecting a minority candidate.

Note that the probability of selecting $A$-candidates is reduced not only relative to the probability of selecting $B$-candidates, but overall; that is, promoting expansion of the candidate pool may reduce the ex-ante probability of recruiting $A$-candidates, despite increasing the overall probability of filling the slot.

Next, consider SAA policies aimed at increasing the probability of finding $A$-candidates at the expense of $B$-candidates (formally captured by an increase in $\mu^A$ and an equal reduction in $\mu^B$).

**Proposition 2.** Suppose $A$ is the minority category and that recruitment is conducted under a myopic rule. There exist $(v, p_0, \mu, q, \bar{P})$, with $v^A > v^B$, $p^A_0 > p^B_0$ and $P^A < P^B$ such that SAA policies aimed at increasing the likelihood of finding $A$-candidates at the expense of $B$-candidates reduce the ex-ante probability of selecting an $A$-candidate: $\gamma^A(v, p_0, (\mu^A, \mu^B), q, \bar{P}) > \gamma^A(v, p_0, (\mu^A + \zeta, \mu^B - \zeta), q, \bar{P})$, for $\zeta > 0$.

Under such SAA policies, $A$-candidates are more likely to be included in the candidate pool at the expense of $B$-candidates, increasing the probability of recruiting $A$-candidates. On the other hand, because $A$-candidates are more difficult to evaluate than $B$-candidates, such policies, contrary to those examined above, reduce the overall attractiveness of search
relative to a lengthier evaluation of existing candidates. Because $B$-candidates are the easiest to evaluate, the committee may substitute search primarily with the evaluation of the $B$-candidates. The proof in the Appendix shows that the latter effect may be strong enough to trigger a reduction in the ex-ante probability that minority candidates are selected.

4 Optimal (forward-looking) Rule

We now show that the effects identified above are not a mere consequence of the committee following a myopic rule – they may emerge also under an optimal rule.

4.1 Preliminaries

Consider first an environment in which the candidate pool is exogenous and constant over time (this amounts to $\mu^A = \mu^B = 0$). As we show in the Online Appendix, the optimal rule is an index rule (Gittins and Jones, 1974). Each candidate is assigned an index $V^j$ that depends only on the candidate’s category, $j$, and the candidate’s history of signal realizations, $\sigma$. The indices take the following form (see the proof of Lemma 1 in the Online Appendix)

$$V^j(\sigma) = \sup_{\tau^j > 0} \frac{\mathbb{E} \left[ \delta^{\phi^j} \left( 1 - \delta^{\tau^j - \phi^j} \right) 1_{\{\phi^j < \tau^j\}} \tilde{v}^j | j, \sigma \right]}{1 - \mathbb{E} \left[ \delta^{\tau^j} | j, \sigma \right]}$$

(1)

where $\tau^j$ is a (stochastic) stopping-time, $\phi^j$ is the (stochastic) time at which the candidate’s (posterior) probability of success $p^j(\sigma)$ exceeds the acceptance threshold $\bar{P}^j$ for the first time, and $\tilde{v}^j \in \{0, v^j\}$ denotes the candidate’s value to the organization. The index (1) is the maximal average expected discounted payoff (averaging over discounted time) from the sequential evaluation of the candidate, ignoring the existence of the other candidates.

**Lemma 1.** Assume the candidate pool is fixed. In each period, the optimal recruitment rule evaluates the candidate with the highest index, as defined in (1).

Next, consider an environment where the candidate pool can be expanded through search. Let

$$V^S = \sup_{\tau^A, \tau^B > 0} \frac{\delta \sum_{j \in \{A, B\}} \mu^j \mathbb{E} \left[ \delta^{\phi^j} \left( 1 - \delta^{\tau^j - \phi^j} \right) 1_{\{\phi^j < \tau^j\}} \tilde{v}^j | j, \emptyset \right]}{1 - \sum_{j \in \{A, B\}} \mu^j \mathbb{E} \left[ \delta^{\tau^j} | j, \emptyset \right]}$$

(2)
be the index of search, where $\tau^j$, $\phi^j$ and $\tilde{v}^j$ are as defined above. The index maximizes the average expected discounted payoff from the sequential evaluation of the first candidate that arrives as the result of search.

**Lemma 2.** Suppose the candidate pool can be expanded through search. The optimal recruitment rule consists in evaluating in each period one of the candidates in the pool with the highest index, as defined in (1), provided this index is greater than $V^S$, and searching for new candidates otherwise.

The fact that the optimal rule takes an index form follows from arguments similar to those in the literature on branching bandits (e.g., Weiss, 1988, Weber, 1992). In Fershtman and Pavan (2019), we show that, under appropriate conditions, the problem of searching for “arms” is a special case of the branching problem. In that paper we also provide a novel proof for the optimality of an index rule, and derive a recursive characterization of the index for search which we use here to arrive at (2). In the Online Appendix of this paper, we show that the optimality of an index rule carries over to the recruitment problem under consideration here, despite the irreversibility of the recruitment decisions.\(^{11}\)

Under the optimal rule, the value the committee assigns to the additional evaluation of any candidate in the pool takes into account the effects of the results of the additional evaluation on the desirability of all further evaluations. Likewise, the value the committee assigns to search accounts for the fact that the new candidates may be evaluated multiple times. Note that $V^S$ is directly linked to the indices of the candidates expected to be identified by search. Specifically, the optimal stopping-time $\tau$ in (2) is the first time at which the index of any candidate that arrives as a result of search drops below $V^S$.

### 4.2 Soft affirmative action

The following result shows that the negative effects of improvements in search technology on the recruitment of minorities may occur also under optimal (forward-looking) policies. Let $\Gamma^j(v, p_0, \mu, q, \overline{P})$ denote the ex-ante probability of selecting a category-$j$ candidate under the optimal rule.

**Proposition 3.** Suppose $A$ is the minority category and that recruitment is conducted under an optimal rule. There exist $(v, p_0, \mu, q, \overline{P})$ with $v^A > v^B$, $p_0^A > p_0^B$, $\overline{P}^A < \overline{P}^B$ and $\mu^A > \mu^B > 0$ such that SAA policies promoting the expansion of the candidate pool may reduce the ex-ante probability that $A$-candidates are selected: $\Gamma^A(v, p_0, \mu, q, \overline{P}) < \Gamma^A(v, p_0, \tilde{0}, q, \overline{P})$.

\(^{11}\)In general, index policies are not guaranteed to be optimal in the presence of irreversible decisions.
The intuition for the result is similar to the one for the myopic rule. Qualified minority candidates whose initial evaluation yields negative results due to noise may not have the opportunity to prove themselves when search is an attractive alternative to further evaluation. Although improvements in the search technology may increase the presence of minority candidates in the pool, they may reduce the ex-ante probability that the position is given to a minority candidate. Importantly, this may happen despite the fact that improvements in the search technology may bring more minority candidates to the pool than non-minority ones.

Similarly, policies that increase $\mu^A$ at the expense of $\mu^B$ may also backfire by reducing the ex-ante probability that $A$-candidates are selected.

**Proposition 4.** Suppose $A$ is the minority category and that recruitment is conducted under an optimal rule. There exist $(v, p_0, \mu, q, \overline{P})$, with $v^A > v^B$, $p^A_0 > p^B_0$, and $\overline{P}^A < \overline{P}^B$, such that SAA policies increasing the likelihood that search brings $A$-candidates at the expense of $B$-candidates reduce the ex-ante probability of selecting an $A$-candidate: $\Gamma^A(v, p_0, (\mu^A, \mu^B), q, \overline{U}) > \Gamma^A(v, p_0, (\mu^A + \zeta, \mu^B - \zeta), q, \overline{U})$, for $\zeta > 0$.

The mechanism behind the result in Proposition 4 is similar to the one under the myopic rule. Such policies reduce the overall attractiveness of search relative to a more careful evaluation of the candidates already in the pool. Because, among those candidates already in the pool, $A$-candidates are the most difficult to evaluate, the committee may substitute search with the evaluation of the $B$-candidates at the expense of the $A$-candidates. When this effect is strong enough, such policies may have the unintended effect of reducing the ex-ante probability of selecting a minority candidate.

## 5 Discussion

The Rooney rule was adopted in 2003. Yet in 2020, there are only three African-American head coaches, the same number as in 2003, prompting criticism viewing the rule as merely a means of “checking a box”.\(^{12}\)

Why are certain SAA policies unsuccessful? This paper suggests a possible explanation based on differences in the effectiveness of evaluating minority and non-minority candidates, which does not presume any bias in decision making. SAA policies do increase the percentage of minority candidates considered for a position, but also alter the desirability

---

\(^{12}\)See the *Washington Post*, Jan 5, 2020: “The dearth of black coaches in the NFL is a problem that somehow still hasn’t been fixed”.

11
of searching for more candidates relative to evaluating those already in the pool. The purpose of this article is to show that this effect may be strong enough to lead to a reduction in the overall probability that a minority candidate is selected. Unaccompanied by steps to reduce difficulties in the evaluation of minority candidates, SAA is thus not guaranteed to deliver its desired effects; it may indeed amount to “checking a box,” or in some cases even prove counterproductive.

References


A Proofs

Proof of Proposition 1. Suppose \( q_H^A \in (0,1) \), and \( q_L^A = q_H^B = q_L^B = 1 \). That is, the evaluation of \( B \)-candidates is perfectly revealing, whereas the evaluation of \( A \)-candidates takes the typical “no-news-is-bad-news” form. Under this technology, \( \lambda^B(\emptyset) = p_0^B \), \( \lambda^A(\emptyset) = q_H^A p_0^A \), and \( \lambda^A(0) = q_H^A p^A(0) \), with

\[
p^A(0) = \frac{(1 - q_H^A)p_0^A}{(1 - q_H^A)p_0^A + 1 - p_0^A}.
\]

Status-quo technology (\( \mu^A = \mu^B = 0 \)). The \( A \)-candidate is evaluated first if

\[
u^A(\emptyset) = p_0^A q_H^A v^A > p_0^B v^B = u^B(\emptyset).
\]

Assume (3) holds. Then \( \gamma^A(v,p_0,\vec{0},q,P) = p_0^A (1 - p_0^B + p_0^B q_H^A) \).

Improved search technology (\( \mu^A, \mu^B > 0 \) with \( \mu^B = 1 - \mu^A \)). Condition (3), along with the assumption that \( u^B(\emptyset), u^A(\emptyset) > u^S \), implies that

\[
\frac{p_0^A q_H^A v^A}{u^A(\emptyset)} > \frac{p_0^B v^B}{u^B(\emptyset)} > \delta \left( \mu^A p_0^A q_H^A v^A + \mu^B p_0^B v^B \right) \frac{1}{u^S}.
\]

Now suppose that

\[
\frac{(1 - q_H^A)p_0^A q_H^A v^A}{u^A(0)} < \delta \left( \mu^A p_0^A q_H^A v^A + \mu^B p_0^B v^B \right).
\]

Condition (5) implies a single negative evaluation of an \( A \)-candidate suffices to trigger search. Denote by \( \gamma^A_S \) the probability of selecting an \( A \)-candidate after search is carried out. Under a myopic rule, when search is carried out, any existing candidate is never evaluated again. Therefore,

\[
\gamma^A_S = \mu^A \left( \lambda^A(\emptyset) + (1 - \lambda^A(\emptyset)) \gamma^A_S \right) + \mu^B (1 - p_0^B) \gamma^A_S.
\]

Rearranging and using \( \mu^A + \mu^B = 1 \), we have that

\[
\gamma^A_S = \frac{\mu^A \lambda^A(\emptyset)}{\mu^A \lambda^A(\emptyset) + \mu^B p_0^B}.
\]
Hence,
\[\gamma^A(v, p_0, \mu, q) = \lambda^A(0) + (1 - \lambda^A(0))(1 - p^B_0)\gamma^A_S = p^A_0 q^A_H \left( 1 + \frac{\mu^A(1 - \lambda^A(0))(1 - p^B_0)}{\mu^A \lambda^A(0) + \mu^B p^B_0} \right).\]

Given (3)-(5), \(\gamma^A(v, p_0, \mu, q, \overline{P}) < \gamma^A(v, p_0, 0, q, \overline{P})\) if
\[p^A_0 (1 - p^B_0 + p^B_0 q^A_H) > p^A_0 q^A_H \left( 1 + \frac{\mu^A(1 - \lambda^A(0))(1 - p^B_0)}{\mu^A \lambda^A(0) + \mu^B p^B_0} \right),\]

or, equivalently,
\[\mu^B p^B_0 (1 - q^A_H) > \mu^A(1 - p^A_0) q^A_H.\]  

(6)

The result in the proposition follows by observing that there exists a non-empty open set of parameter values satisfying both the restrictions in the proposition and Conditions (3)-(6) (the following is an example: \(\delta = 0.9, p^A_0 = 0.8, p^B_0 = 0.7, v^A = 1.5, v^B = 1, q^A_H = 0.6, \mu^A = 2/3\)).

**Proof of Proposition 2.** Let \(q^A_L = q^B_L = 1\) and \(1 > q^B_H > q^A_H > 0\). Then \(\lambda^j(0) = p^A_0 q^A_H, p^j(0) = (1 - q^A_H)p^A_0 + q^A_H/\left(1 - q^A_H p^A_0\right)\), and \(\lambda^j(0) = q^A_H p^A(0)\). Also assume \(\mu^B = 1 - \mu^A\).

**Status-quo technology (\(\zeta = 0\))** Suppose that

\[
\frac{\lambda^B(0) v^B}{u^B(0)} > \frac{\lambda^A(0) v^A}{u^A(0)} > \delta \left( \frac{\mu^A \lambda^A(0) v^A + \mu^B \lambda^B(0) v^B}{u^A(0)} \right) > \frac{(1 - q^A_H) \lambda^A(0) v^A}{1 - q^A_H p^A_0}, \frac{(1 - q^B_H) \lambda^B(0) v^B}{1 - q^B_H p^B_0}.\]  

(7)

Condition (7) implies that evaluating any blank-slate candidate is preferred to search, whereas search is preferred to evaluating any candidate whose first evaluation yielded a negative result.

**Affirmative action in search (\(\zeta > 0\)).** Assume

\[
\frac{(1 - q^A_H) \lambda^B(0) v^B}{1 - q^B_H p^B_0} > \delta \left( (\mu^A + \zeta) \lambda^A(0) v^A + (\mu^B - \zeta) \lambda^B(0) v^B \right) > \frac{(1 - q^A_H) \lambda^A(0) v^A}{1 - q^A_H p^A_0}.\]  

(8)

Note that (8) implies that, after a single negative evaluation, search is preferred to a second evaluation if the candidate is a B-candidate, whereas the opposite holds for A-candidates.

**Comparison.** Denote by \(\gamma^A_S(z), z \in \{0, \zeta\}\), the probability of selecting an A-candidate after search is launched, with \(z = 0\) in case of the default technology, and \(z = \zeta\) under SAA.
Under the myopic rule, once search is carried out, any candidate already in the pool is never evaluated again. Therefore,

\[ \gamma_A^S(0) = \mu_A (\lambda_A(\emptyset) + (1 - \lambda_A(\emptyset)) \gamma_A^S(0)) + \mu_B (1 - \lambda_B(\emptyset)) \gamma_A^S(0), \]

which implies

\[ \gamma_A^S(0) = \frac{\mu_A \lambda_A(\emptyset)}{\mu_A \lambda_A(\emptyset) + \mu_B \lambda_B(\emptyset)}. \]

It follows that

\[ \gamma_A^s(v, p_0, (\mu_A, \mu_B), q, \overline{P}) = (1 - \lambda_B(\emptyset))(\lambda_A(\emptyset) + (1 - \lambda_A(\emptyset)) \gamma_A^S(0)) \]

\[ = (1 - \lambda_B(\emptyset)) \lambda_A(\emptyset) \left( \frac{\mu_A + \mu_B \lambda_B(\emptyset)}{\mu_A \lambda_A(\emptyset) + \mu_B \lambda_B(\emptyset)} \right). \]

Similarly, Conditions (7)-(8) imply that

\[ \gamma_B^A(\zeta) < (\mu_A + \zeta)(\lambda_A(\emptyset) + (1 - \lambda_A(\emptyset)) \gamma_B^A(\zeta)) + (\mu_B - \zeta)(1 - \lambda_B(\emptyset))(1 - \lambda_B(0)) \gamma_B^A(\zeta), \]

where the inequality follows from the fact that a $B$-candidate may potentially be evaluated more than twice before search is launched. Rewriting the above inequality, we have that

\[ \gamma_B^A(\zeta) < \frac{(\mu_A + \zeta) \lambda_A(\emptyset)}{(\mu_A + \zeta) \lambda_A(\emptyset) + (\mu_B - \zeta)(\lambda_B(\emptyset) + \lambda_B(0)(1 - \lambda_B(0)))}. \]

Therefore,

\[ \gamma_A^s(v, p_0, (\mu_A + \zeta, \mu_B - \zeta), q, \overline{P}) < \gamma_A^s(v, p_0, (\mu_A, \mu_B), q, \overline{P}) \text{ if } \]

\[ \frac{\mu_A}{\mu_A \lambda_A(\emptyset) + \mu_B \lambda_B(\emptyset)} > \frac{(1 - \lambda_B(0))(\mu_A + \zeta)}{(\mu_A + \zeta) \lambda_A(\emptyset) + (\mu_B - \zeta)(\lambda_B(\emptyset) + \lambda_B(0)(1 - \lambda_B(0)))}. \]

The result in the proposition follows by observing that there exists a non-empty and open set of parameter values satisfying both the restrictions in the proposition and Con-
Proof of Proposition 3. Consider the same evaluation technology as in Proposition 1: \( q_A^H \in (0,1) \), and \( q_A^L = q_B^H = q_B^L = 1 \). The index of a blank-slate \( j \)-candidate is then equal to

\[
V_j^j(\emptyset) = \frac{\lambda_j(\emptyset) v^j}{1 - \delta(1 - \lambda_j(\emptyset))},
\]

as the optimal \( \tau^j \) in the definition of the index specifies stopping immediately (\( \tau^j = 1 \)) following a negative signal \( s = 0 \), and never stopping (\( \tau^j = \infty \)) following a positive signal \( s = 1 \). Recall that, under the assumed evaluation technology, \( \lambda_B(\emptyset) = p_B^0 \) and \( \lambda_A(\emptyset) = p_A^0 q_A^H \). Furthermore, \( V^B(0) = 0 \) and \( V^j(1) = v^j \). It is also easy to see that, analogously to (10),

\[
V^A(0) = \frac{\lambda^A(0) v^j}{1 - \delta(1 - \lambda^A(0))}.
\]

Status-quo technology (\( \mu^A = \mu^B = 0 \)). In this case, \( V^S = 0 \). The \( B \)-candidate is then evaluated first if

\[
V^B(\emptyset) = \frac{p_B^0 v^B}{1 - \delta(1 - p_B^0)} > \frac{p_A^0 q_A^H v^A}{1 - \delta(1 - p_A^0 q_A^H)} = V^A(\emptyset).
\]

Assuming (11) holds, \( \Gamma^A(v,p_0,\vec{0},q,P) = (1 - p_B^0)^A_0 \).

Improved search technology (\( \mu^A, \mu^B > 0 \), with \( \mu^B = 1 - \mu^A \)). From Lemma 2, the optimal stopping-time \( \tau \) in (2) is the first time at which the index of the candidate discovered through search drops below \( V^S \). Now suppose

\[
V^A(\emptyset), V^B(\emptyset) > \frac{\delta (\mu^A p_A^0 q_A^H v^A + \mu^B p_B^0 v^B)}{1 - \delta^2 + \delta^2 (\mu^A p_A^0 q_A^H + \mu^B p_B^0)} > V^A(0).
\]

We argue that the index of search is then equal to

\[
V^S = \frac{\delta (\mu^A p_A^0 q_A^H v^A + \mu^B p_B^0 v^B)}{1 - \delta^2 + \delta^2 (\mu^A p_A^0 q_A^H + \mu^B p_B^0)}.
\]

To see this, recall that the optimal stopping-times \( \tau^j \) in (2) are the first times at which the index of the arriving candidate drops weakly below \( V^S \). Now suppose this time coincides with the first time at which an evaluation yields a negative signal realization \( s = 0 \), and
else is equal to $\tau^j = +\infty$. Using these observations, (13) follows from the definition of the search index. Condition (12), along with the fact that $V^B(0) = 0$, guarantee the optimal stopping-times $\tau^j$ in (2) are indeed the ones assumed above.

Given Lemma 2 and the stationarity of the search index, if search is preferred to the evaluation of a candidate at some history, it continues to be preferred to the evaluation of that candidate in all future periods.

Now let $\Gamma^A_s$ denote the probability of selecting an $A$-candidate once search is launched. Given the above conditions,

$$\Gamma^A_s = \mu^A (\lambda^A(\emptyset) + (1 - \lambda^A(\emptyset)) \Gamma^A_s) + \mu^B (1 - p_0^B) \Gamma^A_s,$$

which yields

$$\Gamma^A_s = \frac{\mu^A \lambda^A(\emptyset)}{\mu^A \lambda^A(\emptyset) + \mu^B p_0^B}.$$

Therefore, the ex-ante probability of selecting an $A$-candidate is equal to

$$\Gamma^A(v, p_0, \mu, q, \mathbf{p}) = (1 - p_0^B) (\lambda^A(\emptyset) + (1 - \lambda^A(\emptyset)) \Gamma^A_s) = (1 - p_0^B) \lambda^A(\emptyset) \left( \frac{\mu^A + \mu^B p_0^B}{\mu^A \lambda^A(\emptyset) + \mu^B p_0^B} \right).$$

Under Conditions (11)-(12), $\Gamma^A(v, p_0, \mu, q, \mathbf{p}) < \Gamma^A(v, p_0, \tilde{q}, q, \mathbf{p})$ if

$$(1 - p_0^B) p_0^A > (1 - p_0^B) \lambda^A(\emptyset) \left( \frac{\mu^A + \mu^B p_0^B}{\mu^A \lambda^A(\emptyset) + \mu^B p_0^B} \right).$$

The latter condition is equivalent to Condition (6). The result in the proposition now follows by observing that there exists a non-empty open set of parameter values satisfying the restrictions in the proposition, Conditions (11)-(12), and Condition (6) (for example: $\delta = 0.9, p_0^A = 0.75, p_0^B = 0.7, v^A = 1.2, v^B = 1, q_H^A = 0.19$, and $\mu^A = 0.52$).

**Proof of Proposition 4.** Consider the same evaluation technology as in the proof of Proposition 2 (i.e., $q^A_L = q^B_L = 1$, and $1 > q_H^B > q_H^A > 0$) and assume $\mu^B = 1 - \mu^A$. The same arguments as in the proof of Proposition 3 imply that $V^j(\emptyset) = \lambda^j(\emptyset) v^j / (1 - \delta(1 - \lambda^j(\emptyset)))$ and $V^j(0) = \lambda^j(0) v^j / (1 - \delta(1 - \lambda^j(0)))$, with $\lambda^j(\emptyset) = p_0^j q_H^j$.
\[ \lambda^j(0) = p^j(0)q_H^j = \frac{(1-q_H^j)p^j_0q_H^j}{1-q_H^j p^j_0}. \]

Assume
\[ \frac{p_B^0 q_H^B v^B}{1-\delta(1-p_B^0 q_H^B)} > \frac{p_A^0 q_H^A v^A}{1-\delta(1-p_A^0 q_H^A)} \tag{14} \]
and observe that the latter condition implies \( V^B(\emptyset) > V^A(\emptyset) \), so that the \( B \)-candidate is evaluated first before search is launched.

*Status-quo technology* (\( \zeta = 0 \)). Assume
\[ V^A(\emptyset), V^B(\emptyset) > \frac{\delta (\mu^A p_A^0 q_H^A v^A + \mu^B p_B^0 q_H^B v^B)}{1-\delta^2 + \delta^2 (\mu^A p_A^0 q_H^A + \mu^B p_B^0 q_H^B)} > V^A(0), V^B(0), \tag{15} \]
and note that the above condition implies that the optimal stopping-time in the formula for \( V^S \) is the first time at which an evaluation yields a realization \( s = 0 \) which in turn implies that
\[ V^S = \frac{\delta (\mu^A p_A^0 q_H^A v^A + \mu^B p_B^0 q_H^B v^B)}{1-\delta^2 + \delta^2 (\mu^A p_A^0 q_H^A + \mu^B p_B^0 q_H^B)}. \tag{16} \]

*Affirmative action in search* (\( \zeta > 0 \)). Assume
\[ V^A(\emptyset), V^B(0) > \frac{\delta (\mu^A + \zeta) A(\emptyset) v^A + \delta (\mu^B - \zeta) B(\emptyset) (\lambda^B(\emptyset) + \delta (1-\lambda^B(\emptyset)) \lambda^B(0))}{1-\delta^2 ((\mu^A + \zeta) (1-\lambda^A(\emptyset)) + (\mu^B - \zeta) (1-\lambda^B(\emptyset)) (1-\lambda^B(0)) \delta)} > V^A(0), V^B(0,0), \tag{17} \]
where
\[ V^B(0,0) = \frac{\lambda^B(0,0) v^B}{1-\delta(1-\lambda^B(0,0))}, \]
with \( \lambda^B(0,0) = p_B^0(0,0)q_H^B \) and
\[ p_B^0(0,0) = \frac{(1-q_H^B)^B(0)}{1-q_H^B p_B^0(0)}. \]

Condition (17) implies that it takes one negative signal realization \( s = 0 \) for the index of an \( A \)-candidate to drop below
\[ V^S = \frac{\delta (\mu^A + \zeta) A(\emptyset) v^A + \delta (\mu^B - \zeta) B(\emptyset) (\lambda^B(\emptyset) + \delta (1-\lambda^B(\emptyset)) \lambda^B(0))}{1-\delta^2 ((\mu^A + \zeta) (1-\lambda^A(\emptyset)) + (\mu^B - \zeta) (1-\lambda^B(\emptyset)) (1-\lambda^B(0)) \delta)}, \tag{18} \]
and two negative signal realizations $s = 0$ for the index of a $B$-candidate to drop below the value of $V^S$ in (18), thus implying that the search index is indeed the value in (18).

Condition (9) in the proof of Proposition 2 then implies

$$\Gamma^A (v, p_0, (\mu^A, \mu^B), q, \overline{p}) > \Gamma^A (v, p_0, (\mu^A + \zeta, \mu^B - \zeta), q, \overline{p}).$$

The result in the proposition then follows by observing that there exists a non-empty open set of parameter values satisfying both the restrictions in the proposition and Conditions (14), (15), (17), and (9) (e.g., $\delta = 0.9, p_0^A = 0.69, p_0^B = 0.68, \nu^A = 1.01, \nu^B = 1, q_H^A = 0.4, q_H^B = 0.8, \mu^A = 0.15, \text{ and } \zeta = 0.01$).
Online Appendix

B Omitted proofs

Proof of Lemma 1. In the absence of search, the recruitment problem can be mapped into a multi-armed bandit problem, with candidates corresponding to different “arms”. This problem, however, is not a standard one, because of the irreversibility of the recruiting decision. Notwithstanding the fact that many multi-arm bandit problems with irreversible choice fail to admit an index solution, the arguments below imply that an index rule is optimal in our model.

To see this, consider the following fictitious environment. Let the flow payoff from the pull of each arm be equal to zero at each pull for which \( p_j(\sigma) < \bar{P}_j \) (such pulls correspond to the evaluation of candidates which, given the university’s acceptance rules, are not acceptable yet). Once \( p_j(\sigma) \geq \bar{P}_j \), the pull of the arm generates a constant flow payoff equal to \( p_j(\sigma^{\phi_j})v_j(1-\delta) \) in each subsequent period, and the “state” of the arm does not change any more (here \( \sigma^{\phi_j} \) is the history of signal realizations at the first time at which the candidate’s probability of being qualified exceeds the acceptance threshold \( \bar{P}_j \)). Further assume that at any point in time the committee can pull any arm – even if for some candidate \( p_j(\sigma) \geq \bar{P}_j \) as the result of previous pulls. In other words, in contrast to the true model, the pull of an arm corresponding to a candidate for whom \( p_j(\sigma) \geq \bar{P}_j \) does not preclude the possibility of pulling other arms in subsequent periods. This environment satisfies all the conditions guaranteeing the optimality of an index rule in the classic multi-armed bandit problem. Therefore, the optimal rule in such fictitious environment selects in each period the arm with the highest Gittins index. The Gittins index of each candidate is equal to

\[
V_j(\sigma) = \sup_{\tau_j > 0} \frac{E \left[ \sum_{s=\phi_j}^{\tau_j-1} \delta^s p_j(\sigma^{\phi_j})(1-\delta) | j, \sigma \right]}{1 - E \left[ \delta^{\tau_j} | j, \sigma \right]}
= \sup_{\tau_j > 0} \frac{E \left[ \delta^{\phi_j} \left( 1 - \delta^{\tau_j - \phi_j} \right) 1_{\{\phi_j < \tau_j\}} \tilde{v}_j | j, \sigma \right]}{1 - E \left[ \delta^{\tau_j} | j, \sigma \right]}
\]

Next note that, once an arm is pulled for which \( p_j(\sigma) \geq \bar{P}_j \), its index \( V_j(\sigma) = p_j(\sigma^{\phi_j})v_j(1-\delta) \) remains the same at all subsequent periods, and the indices of the other arms remain the same. Hence, under the optimal rule in the fictitious environment, once a candidate is found qualified (i.e., \( p_j(\sigma) \geq \bar{P}_j \)), that candidate is selected in each subsequent period. Because the fictitious environment is a relaxation of the primitive one, it follows
that the optimal rule in the primitive environment coincides with the one in the fictitious environment.

Proof of Lemma 2. In the presence of search, the recruitment problem can be thought of as a generalization of the classic multi-armed bandit problem in which the decision maker can search for new arms, in addition to pulling one of the existing ones. As shown in Fershtman and Pavan (2019) (see also the literature on branching bandits), in the absence of irreversible decisions, the optimal rule for such problems continues to be an index rule, but with a special index for search. The latter index is the maximal expected average discounted payoff from searching or pulling any of the new arms that arrive as the result of search, where the maximization is over both a stopping time and a rule that chooses among search and the pull of the new arms that arrive as the result of search. Arguments similar to those in the proof of Lemma 1 imply that, notwithstanding the irreversibility of the recruiting decision, the optimal rule for the recruiting problem under consideration is an index rule. The results in Fershtman and Pavan (2019) also imply that the search index is equal to

\[ V^S = \sup_{\tau, \pi} \frac{\delta E^{\pi, \tau} \left[ \sum_{s=1}^{\tau-1} \delta_s r^{\pi}_s \right]}{1 - E^{\pi, \tau} \left[ \delta^\tau \right]}, \tag{19} \]

where \( \tau \) is a stopping time, \( \pi \) is a rule that chooses between evaluation of one of the new candidates brought in by search and further search, and \( r^{\pi}_s \) is the flow payoff under the rule \( \pi \), with the latter equal to zero when \( \pi \) selects search or one of the candidates for which \( p_j^i(\sigma) < \bar{P}_j \), and is equal to \( p_j^i(\sigma^{\phi_j}) v_j^i (1 - \delta) \) when \( \pi \) selects a candidate for whom, either at the present period or at a previous period, \( p_j^i(\sigma) \geq \bar{P}_j \) (recall that \( \sigma^{\phi_j} \) is the history of signal realizations at the first time at which the candidate’s probability of being qualified exceeds the acceptance threshold).

Importantly, note that the index of search is independent of any information pertaining to the candidates already in the candidate pool.

It can further be shown that the optimal rule \( \pi \) in (19) is in fact the same index rule that characterizes the solution to the entire problem, and that the optimal stopping time \( \tau \) in (19) is the first time at which the search index and the index of all newly arrived arms fall below the index of search when the latter was launched. In the context of the specific problem under examination here, because the search technology is stationary, \( \tau \) coincides with the first time at which the index of the first candidate identified by search
falls below the index of search. Given these features, (19) can be rewritten as

$$V^S = \sup_{\tau^A, \tau^B > 0} \frac{\delta \sum_{j \in \{A,B\}} \mu^j \mathbb{E} \left[ \sum_{s=\phi^j}^{\tau^j-1} \delta^s p^j (\sigma^{\phi^j}) n^j (1 - \delta) |j, \emptyset \right]}{1 - \sum_{j \in \{A,B\}} \mu^j \mathbb{E} \left[ \delta^{\tau^j} |j, \emptyset \right]}$$

$$= \sup_{\tau^A, \tau^B > 0} \frac{\delta \sum_{j \in \{A,B\}} \mu^j \mathbb{E} \left[ \delta^{\phi^j} (1 - \delta^{\tau^j-\phi^j}) 1_{\{\phi^j < \tau^j\}} n^j |j, \emptyset \right]}{1 - \sum_{j \in \{A,B\}} \mu^j \mathbb{E} \left[ \delta^{\tau^j} |j, \emptyset \right]}.$$