(In)efficiency in Information Acquisition and Aggregation through Prices[*]

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Abstract

We study the acquisition of private information and its subsequent usage in financial markets. We show how inefficiencies in the collection of information relate to those in trading. As the cost of information declines, traders over-invest in information acquisition and trade too much on their private information. Generically, no policy exists based on the price of the financial asset and the individual trade volume inducing efficiency in both information acquisition and trading. Such an impossibility result turns into a possibility one when information acquisition is verifiable or when taxes can be made contingent on the aggregate volume of trade. When only ad-valorem taxes are available, they should not be used.

Keywords: information acquisition, aggregation through prices, learning and pecuniary externalities, team efficiency, optimal policy

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1 Introduction

Improvements in information technology are reducing the cost of acquiring and processing information. However, there are concerns about the social value of this bounty, in particular when it comes to the information acquired and used in financial trading. Policy proposals range from putting "sand in the wheels" of financial markets as a way of limiting speculative trading (the Tobin tax, which is an *ad-valorem* tax on transactions, is a prominent example) to subsidizing information acquisition because of its potential value as a public good. The merits of these proposals, however, are unclear.

In this paper, we develop a tractable framework to study both the positive and normative issues of interest. The model is designed to perform welfare analysis in economies with endogenous private information in which traders compete in demand schedules (generalized limit orders) and information is partially aggregated through prices.[1] In particular, we characterize the sources of inefficiency in the collection of private information prior to trading and relate them to possible inefficiencies in the limit orders that traders submit given their available information.

Our model is a linear-quadratic-Gaussian market microstructure in which a unit-mass continuum of traders compete by submitting a collection of generalized limit orders (equivalently, a demand schedule). The traders face uncertainty about the asset's value and the price-elastic supply of the asset. The asset supply may come from liquidity traders in a stock market, the central bank in a liquidity auction, the Treasury in a bond auction, or the regulator in the market for pollution permits. Before submitting their demand schedules, each trader collects a private signal about the asset's value whose noise is endogenous and correlated across the traders. Such a correlation may originate, for example, in the traders paying attention to common sources of information, with source-specific noise. Importantly, such a correlation, in addition to being realistic, has major implications for the (in)efficiency of the equilibrium acquisition and usage of information, as we discuss further on.

Our first main result is that, except in very special cases, absent policy interventions, the market does not use the information it collects efficiently. The inefficiency originates in the interaction between two externalities. First, traders do not account for the fact that a collective change in demand schedules may induce a change in the information contained in the equilibrium price, which in turn affects other agents' ability to align their trades with the asset's value (a familiar learning externality, which has been extensively investigated in the literature). Second, and more interesting, traders do not account for how the noise in their information, by shaping the schedules they submit, affects the equilibrium market-clearing price and, because the demands by the other traders are price elastic, the asset holdings and ultimately the consumption by other traders (a novel pecuniary externality that originates in the combination of noise in information with the use of limit orders). We isolate the pecuniary externality by considering a fictitious environment where traders are naive,

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[1]Competition in demand schedules is common in financial markets where limit orders are prevalent (e.g., in NYSE, more than 50% of trades are through limit orders, whereas market orders represent about 32% of total trading volume; see Li, Ye, and Zheng (2023)). Other markets where limit orders are pervasive include central bank liquidity auctions, Treasury auctions, and pollution rights markets.

in that they do not learn from prices, but are endowed with an exogenous public signal whose precision is the same as the one contained in the equilibrium market-clearing price. In such a fictitious environment, traders fail to account for the fact that variations in their schedules driven by noise in their private information trigger changes in the market clearing price and hence in the consumption by other agents. Because these variations are not justified by fundamentals, they are welfare reducing. Importantly, the inefficiency does not originate in the change in the agents' expenditures for given asset holdings but in the fact that other traders' asset holdings change due to the dependence of the limit orders on the equilibrium market-clearing price. Such a pecuniary externality is different from other pecuniary externalities considered in the literature originating in missing markets, collateral constraints, or other financial frictions. It vanishes when the noise in the agents' information vanishes or when the traders are restricted to submitting market orders (as we explain below, forcing the traders to submit market orders, while it eliminates both the learning and the pecuniary externalities, it need not however increase welfare because it makes the asset holdings less responsive to fundamental shocks). The pecuniary externality is also different from the familiar learning externality mentioned above, which has been extensively examined in the literature. In particular, whereas the learning externality makes traders under-react to private information, the pecuniary externality makes them over-react.

The knife-edge case in which the two externalities cancel each other out obtains when the equilibrium demand schedules are perfectly inelastic. When the equilibrium schedules are downward sloping, the pecuniary externality dominates and the equilibrium trades feature excessive sensitivity to private information. When, instead, the equilibrium schedules are upward sloping, the learning externality dominates and the sensitivity of the equilibrium limit orders to the traders' private information is inefficiently low. Interestingly, as the precision of the traders' private information grows (for example, because of reductions in the cost of acquiring and processing information due to technological progress), the pecuniary externality gains weight in relation to the learning externality.[2]

We show that, no matter whether traders over- or under-respond to their private information, the aforementioned inefficiencies in the equilibrium usage of information (equivalently, in trading) can be corrected using a non-linear tax-subsidy contingent on the equilibrium price of the asset and the individual volume of trade. More precisely, a linear-quadratic tax on the volume of trade paired with an ad-valorem tax on the dollar amount paid induces traders to submit efficient limit orders.

Our second main result is that inducing the traders to trade efficiently does not guarantee that they collect private information efficiently before submitting the orders. In particular, traders over-invest in information acquisition when the efficient schedules are downward sloping, and under-invest in information acquisition when they are upward sloping. In other words, when, in the laissez-faire equilibrium, the pecuniary externality prevails over the learning externality so that the traders over-

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[2]Provided that the noise in the traders' information is not too large, when the precision of the traders' information is relatively low, the learning externality dominates and the demand schedules are upward sloping, whereas the opposite is true (i.e., the pecuniary externality dominates and the demand schedules are downward sloping) for high levels of precision.

respond to their private information, forcing the traders to trade efficiently induces them to over-invest in the acquisition of private information. When, instead, the learning externality prevails over the pecuniary externality so that traders under-respond to private information, forcing the traders to trade efficiently induces them to under-invest in the acquisition of private information. The inefficiencies in the collection of information thus parallel those in the usage of information. Importantly, to uncover these results, one needs to allow the noise in the traders' private information to be correlated among the traders. If such a noise were independent, under the efficient demand schedules, the only effect of a variation in the precision of the traders' private information on welfare would be through the change in the dispersion of individual trades around the average trade. However, under the efficient demand schedules, the private and the social value of reducing such a dispersion coincide, so efficiency in the usage of information implies efficiency in the acquisition of information. It is common practice in the literature to assume that the noise in the agents' private signals is independent across agents. This assumption is made for tractability. In reality, the information the agents receive typically comes from common sources that are subject to noise at the source level. The attention agents allocate to such sources thus affects their exposure to both idiosyncratic and correlated noise. When this is the case, efficiency in trading does not guarantee efficiency in information acquisition.

We also show that, if traders could be trusted to submit the efficient demand schedules (an unrealistic hypothetical), then an *ad-valorem* tax on the dollar amount paid would induce the efficient collection of private information.

Next, we show that, absent any policy intervention, as information technology makes the collection of information cheaper, the economy eventually enters into a regime of over-investment in information acquisition and excessive trading on private information.[3] This is accompanied with inefficiently high price volatility, market depth, and price informativeness. In other words, the secular trend of improvement in information technology may have the undesirable effect of enticing over-investment in information acquisition and over-reaction to it in the trading of financial assets.[4] It follows that the interventions that are often advocated in the policy debate, which call for putting "sand in the wheels" of financial markets and subsidizing information collection, are potentially welfare reducing.

Motivated by the results described above, we then turn to the question of what policy interventions induce efficiency in both the acquisition and usage of information. We show that, generically, there exist no taxes/subsides contingent on the price of the asset and on the volume of individual trades that can induce efficiency in both the acquisition and the usage of information. This impossibility result, however, can be overturned by conditioning the tax/subsidy on the expenditure on

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[3]This result is fundamentally different from the over-investment in information acquisition that arises in economies in which information undermines insurance (see Hirshleifer (1971) for a seminal contribution). It is also different from the over-investment in information acquisition that arises in economies in which adverse selection precludes trade (see, e.g., Dang et al. (2017)).

[4]See, for example, Nordhaus (2015) on the sharp decline in the cost of computation (and therefore of information processing). See also Gao and Huang (2020) and Goldstein, Yang, and Zuo (2020) for the effects of the dissemination of corporate disclosures over the internet on the production of information by corporate outsiders.

information acquisition, when the latter is verifiable, or, when the latter is not verifiable, on the aggregate volume of trade. In the former case, efficiency in trading can be induced with standard taxes that depend only on the price paid and on the individual volume of trade, whereas efficiency in information acquisition can be induced through a separate tax/subsidy that depends on the amount of information purchased. In the latter case (i.e., when information acquisition is not verifiable), conditioning the marginal tax rate on the aggregate volume of trade provides the planner with more instruments to correct inefficiencies in the trades. Furthermore, by creating uncertainty over the marginal tax rate, such a contingency affects the agents' value for information and hence permits the planner to realign such a value to its social counterpart without undermining the efficiency of the limit orders submitted in equilibrium for given private information.

Finally, we show that if the government is restricted to using simple *ad-valorem* taxes (that is, taxes that are linear in individual expenditures on asset purchases), it should not use them. The result is striking given that ad-valorem taxes are typically proposed by advocates of policy interventions in financial markets.[5] The reason is that such taxes fail to change the sensitivity of the equilibrium trades to private information and/or the value the traders assign to collecting private information. They only change the sensitivity of the equilibrium schedules to the price. However, in the laissez-faire equilibrium, for given precision of private information and sensitivity of the schedules to the private signals, the sensitivity of the equilibrium schedules to the price is welfare-maximizing. As a result, these taxes bring the equilibrium farther away from the efficient allocation and hence are welfare decreasing. For the policy maker to improve over the laissez-faire equilibrium it is essential to use more sophisticated policies that are non-linear in individual expenditures on asset purchases, and with a marginal tax rate that depends on the aggregate volume of trade so as to induce efficiency not only in the submission of the limit orders but also in the collection of information.

Related literature The paper is related to several strands of the literature. The first one is the literature investigating the sources of inefficiency in the equilibrium usage of information. See, among others, Palfrey (1985), Vives (1988), Angeletos and Pavan (2007), Amador and Weill (2010), and Vives (2017). Among these works, the closest are Amador and Weill (2010) and Vives (2017). Both papers study inefficiency in information aggregation when traders submit demand schedules.[6] In these papers, the agents' private information is exogenous. In Amador and Weill (2010), traders always under-respond to their private information, whereas this is not the case in Vives (2017). None of these papers studies the sources of inefficiency in the collection of private information, relate them to the interaction between pecuniary and learning externalities, and investigate policy corrections,

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[5]Several EU countries use ad-valorem taxes for shares transactions (Belgium, France, Hungary, Italy, Portugal, and Spain) with the promise of switching to the common EU FTT once introduced. In Asia, ad valorem taxes are used by India, Malaysia, Pakistan, Thailand, Hong Kong, and Singapore. China has a stamp duty and so does Brazil (see Dowd (2020)).

[6]See also Kyle (1989), Vives (2011), and Rostek and Weretka (2012) for related models of strategic competition in schedules.

which are the key contributions of our paper.

The second strand is the literature on information acquisition in markets. See Diamond and Verrecchia (1981) and Verrecchia (1982) for earlier contributions. More recently, Peress (2010) examines the trade-off between risk sharing and information production, whereas Manzano and Vives (2011) study information acquisition in markets with correlated noise, while Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) study information acquisition in markets with multiple risky assets. Dávila and Parlatore (2021) study the effect of trading costs on information aggregation and acquisition. Mondria, Vives, and Yang (2021) study a model where traders have to exert effort (pay attention) to reduce noise in the interpretation of the information content of the price. None of these papers studies inefficiency in information acquisition and how it relates to inefficiency in trading.[7] Vives (1988) shows that, in a Cournot economy in which a continuum of privately-informed traders with conditionally independent private signals submit market orders, both the decentralized acquisition of information and the equilibrium trades are efficient. In the present paper, we show that the same result extends to economies in which the information collected in equilibrium is subject to correlated noise, provided that the traders are restricted to submitting market orders instead of richer supply/demand functions. When traders submit market orders, neither the pecuniary externality nor the learning externality of conditioning on prices are present. As a result, the planner cannot improve on the laissez-faire equilibrium if it is equally restricted to using only market orders. However, welfare is typically higher under limit orders even if trading and information acquisition are inefficient. This is because limit orders permit the traders to use the market-clearing price to adjust their asset holdings based on the information held by other traders (see also the discussion in Appendix 2 at the end of the document).

Efficiency in the usage of information implies efficiency in information acquisition in the macro business-cycle economies considered in Angeletos, Iovino, and La'O (2020). In these economies, prices imperfectly aggregate information, as in our paper, but agents have access to complete markets that permit them to fully insure against idiosyncratic consumption risk. In contrast, in our economy, markets are incomplete, in the sense that traders consume the returns to their own investments; in such economies, policies that correct inefficiencies in the usage of information need not induce efficiency in the collection of information. Colombo, Femminis, and Pavan (2025) consider an economy in which production is affected by investment spillovers. They show, among other things, that simple state-invariant subsidies to technology adoption induce efficiency in production when information is exogenous but not when it is endogenous. In the latter case, more sophisticated Pigouvian-like taxes where the marginal rates depend on aggregate output and on the aggregate investment in the new technology are necessary to induce efficiency in both the usage and the acqui-

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[7]See also the literature on the Grossman-Stiglitz paradox, namely on the lack of incentives to acquire information when prices are fully revealing (see Grossman and Stiglitz (1980), and Vives (2014) for a potential resolution of the paradox). Related is also the literature on strategic complementarity/substitutability in information acquisition (see, among others, Ganguli and Yang (2009), Hellwig and Veldkamp (2009), Manzano and Vives (2011), Myatt and Wallace (2012), and Pavan and Tirole (2022)).

sition of information. That paper, however, abstracts from information aggregation through prices, which is the focus of the present paper. Colombo, Femminis, and Pavan (2014) show that efficiency in actions need not imply efficiency in information acquisition when individual payoffs depend on the dispersion of individual actions around the average action. In the present paper, we show that, even in the absence of such externalities, efficiency in usage does not imply efficiency in information acquisition when agents compete in schedules. The relation between efficiency in information usage and in information acquisition is also investigated in Angeletos and Sastry (2023) and Hébert and La'O (2023). The first paper considers economies in which markets are complete, whereas the second one considers an abstract linear-quadratic game as in Angeletos and Pavan (2007). In both papers, like in ours, agents learn from the behavior of others. However, neither of these two papers analyzes the welfare implications of such a learning originating from the agents submitting price-contingent schedules. There are no pecuniary externalities in either of these papers, whereas such externalities play a key role in our analysis. As explained above, these externalities naturally arise when agents submit demand schedules. Importantly, neither Angeletos and Sastry (2023) nor Hébert and La'O (2023) identify whether agents over- or under-invest in information acquisition, whether they over- or under-respond to their private information and to the market-clearing price, and which policies can correct inefficiency in information acquisition and trading, which is the focus of our analysis.

The third strand is the recent literature on the impact of technological progress on the collection of information and its usage in financial markets. Farboodi, Matray, and Veldkamp (2018) show that the growth of big data, combined with the size distribution of firms, can lead to a decline in price informativeness for smaller firms. Peress (2005) shows that a declining cost of information collection is outweighed by a parallel decline in the cost of entry to financial markets and the interaction between the two can explain several empirical anomalies. Malikov (2019) shows that falling information costs can actually contribute to a rise in passive investment by reducing the cost of, and therefore the returns to, stock picking. Several papers (see, among others, Azarmsa (2019), Mihet (2018), and Kacperczyk, Nosal, and Stevens (2019)) show that technological progress that facilitates the collection of information can lead to increasing levels of inequality. Unlike most of the work in this literature, we focus on the normative implications of technological improvements in the collection of information.

A fourth strand is the literature building on Tobin (1978)'s proposal to put sand in the wheels on foreign exchange transactions as a way to curb volatility and speculation. Similar interventions have been advocated for financial markets (see, e.g., Stiglitz (1989) and Summers and Summers (1989)). High volumes of speculation (particularly in the short term) and/or "noise trading" are typically assumed to be detrimental to welfare. However, some theoretical work shows that a tax on financial transactions may increase price volatility and lower market depth and welfare (see, among others, Kupiec (1996), Sørensen (2017), and Song and Zhang (2005)). Subrahmanyam (1998) and Dow and Rahi (2000) show that a quadratic transaction tax may have ambiguous effects on speculators' profits and on the welfare of other traders. Umlauf (1993), Colliard and Hoffmann (2017), and Deng,

Liu, and Wei (2018) document a negative impact of transaction taxes on trading volume and an ambiguous impact of the same taxes on price volatility and market depth. Using transaction data from the Italian Stock Exchange, Cipriani, Guarino, and Uthemann (2022) estimate a model with price elastic informed and noise traders to assess the effects of a transaction tax on informed and noise traders. They find that the tax reduces trading activity and price volatility, but also reduces price informativeness for most stocks. Our paper contributes to this literature by showing that simple ad-valorem taxes are welfare reducing and that efficiency in both the usage and the acquisition of information requires conditioning tax bills on the expenditure on information acquisition (when the latter is verifiable) or on the aggregate volume of trade (when information acquisition is not verifiable, for example because it originates in attention).

Finally, in this paper, we assume that higher investments in information acquisition can reduce the agents' exposure to correlated noise in information. Recent work by Woodford (2012a), Woodford (2012b), and Nimark and Sundaresan (2019) shows that rational inattention can also explain the agents' exposure to correlated noise, and that the equilibrium of a rationally-inattentive economy shares several features with those of an economy in which the agents' use of information is "biased" in the sense of prospect theory. Particularly related in this respect is Frydman and Jin (2022), which shows how rational inattention can lead to endogenous bias in valuation, and that the noise in perception is closely linked to the bias in perception. Our paper shares with this literature the property that investments in information acquisition also affect the agents' exposure to correlated noise, something that, from the perspective of an outside observer, may look like a bias in decision making.

Organization. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 compares the equilibrium to the efficient use of information, identifies the sources of inefficiency in trading, and shows how certain taxes/subsidies may restore efficiency. Section 4 identifies inefficiencies in information acquisition and discusses policy corrections. Section 5 concludes. All proofs are in Appendix 1 at the end of the document (the proofs are self-contained but expanded derivations can be found in the Supplementary Material). Appendix 2 contains results for the case in which traders are restricted to submitting market orders.

2   Model

In this section, we describe the environment, as well as the traders' problem of choosing the private information to acquire prior to trading and a demand schedule.

2.1   Environment

The market is populated by a continuum of traders, indexed by $i \in [0, 1]$, trading a homogenous and perfectly divisible asset. Let x_i denote the quantity of the asset demanded by trader i and $\tilde{x} = \int x_i di$

the traders' aggregate demand. Each trader i's payoff from purchasing x_i units of the asset at price p is given by $\pi_i \equiv (\theta - p) x_i - \lambda x_i^2/2$, where λ is a positive scalar, and where $\theta \sim N(0, \sigma_\theta^2)$. The variable θ proxies for the traders' gross common value from purchasing the asset, whereas the term $\lambda x_i^2/2$ is a quadratic trading or adjustment cost whose role is to induce imperfectly elastic demands.[8]

Traders face an exogenous inverse asset supply $p = \alpha - u + \beta \tilde{x}$, where α and β are positive scalars, and where $u \sim N(0, \sigma_u^2)$ is an aggregate shock.[9] Such a supply may originate from the combination of various operations of liquidity traders such as pension or index funds trading the asset as part of broader market portfolios, along with the operations of large liquidity suppliers such as central banks trading the asset as part of their liquidity programs. The planner believes the costs of such a supply to be equal to $(\alpha - u) \tilde{x} + \beta \tilde{x}^2/2$. This specification permits us to equivalently interpret the supply of the asset as coming from a "representative supplier" with payoff $p\tilde{x} - (\alpha - u) \tilde{x} - \beta \tilde{x}^2/2$. In this case, the term $\alpha - u$ proxies for the opportunity cost for the representative supplier of unloading the asset, and $\beta \tilde{x}^2/2$ for a quadratic trading or adjustment cost.[10] Both the traders and the planner treat such a supply as exogenous to their own operations. Importantly, the planner includes the utility of the representative supplier into the welfare function (equivalently, accounts for the social costs of unloading the asset) when computing the efficient allocations.

To simplify the derivation of the equilibrium formulas, we assume that the variables θ and u are independently distributed. The results, however, extend to the case where they are imperfectly correlated. For notational purposes, given any Gaussian random variable h with variance σ_h^2, we denote by $\tau_h \equiv 1/\sigma_h^2$ the variable's precision.

The traders do not know θ. They privately collect information about θ prior to submitting their demand schedules, but account for the fact that the equilibrium market-clearing price imperfectly aggregates the traders' dispersed information about θ.

Formally, each trader observes a signal $s_i \equiv \theta + \epsilon_i$, where $\epsilon_i \equiv f(y_i)(\eta + e_i)$ is a combination of idiosyncratic and correlated noise. Precisely, the noise variable $\eta \sim N(0, \sigma_\eta^2)$ is perfectly correlated among the traders whereas the variables $e_i \sim N(0, \sigma_e^2)$ are i.i.d. among the traders. The variables $(\theta, u, \eta, (e_i)_{i \in [0,1]})$ are jointly independent. The exposure of trader i to the noise variables (η, e_i) is a decreasing function f of $y_i \in \mathbb{R}_+$. Depending on the context, y_i can be interpreted either as the amount of information acquired by the trader, or the attention the latter allocates to exogenous sources of information. The cost of y_i is given by a differentiable function $\mathcal{C}(y_i)$, with $\mathcal{C}'(y_i), \mathcal{C}''(y_i) > 0$ for all $y_i \geq 0$. All the main results hold for general cost functions. This is because inefficiencies in

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[8]See also Vives (2011) and Rostek and Weretka (2012) for examples of models with a quadratic adjustment cost.

[9]As usual, the role of this shock is to prevent the price from being fully revealing of the information the traders collectively possess.

[10]Suppose there is a continuum of suppliers of measure 1, indexed by j, each with the same payoff $px_j + h_j - (\alpha - u) x_j - \beta x_j^2/2$ from selling the asset, with x_j denoting the amount of the asset sold by supplier j and h_j an idiosyncratic portfolio or utility shock, with $\int h_j dj = 0$. The aggregate payoff of the suppliers is equal to $p\tilde{x} - (\alpha - u) \tilde{x} - \beta \tilde{x}^2/2 - \beta var(x_j)/2$. Because all supplier sell the same amount of shares $x_j = \tilde{x} = (p - \alpha + u)/\beta$ in equilibrium, $var(x_j) = 0$, implying that the inverse asset supply is $p = \alpha - u + \beta \tilde{x}$ and the suppliers' aggregate payoff is $p\tilde{x} - (\alpha - u) \tilde{x} - \beta \tilde{x}^2/2$.

the acquisition of information originate in differences between the private and the social value of information which are invariant in $\mathcal{C}$. However, the existence of equilibria, both in the laissez-faire economy (Proposition 6) and under the optimal policies (Propositions 10 and 11) requires that individual payoffs be quasi-concave in y_i. As we show in the Appendix, this is always the case when $\mathcal{C}$ is sufficiently convex and $\mathcal{C}'(y)$ is small for small y. Specifically, there are constants $L, M \in \mathbb{R}_{++}$ (defined in the Appendix) such that equilibrium exists and is unique (within the family of equilibria in affine strategies) when $\mathcal{C}'(0) \leq L$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > M$. For example, when $\mathcal{C}(y) = By^2/2$, with $B \in \mathbb{R}_{++}$, these conditions are jointly satisfied for B large enough.

The idea behind the above information structure is that traders learn from a variety of information sources differing in their noises and in the extent to which such noises are correlated among the traders.[11] To maintain the analysis simple, we assume that the information received from such sources is summarized in a uni-dimensional statistics and that the investment y_i in information acquisition reduces the exposure to both idiosyncratic and correlated noise according to a function f taking the form $f(y) = y^{-1/2}$. Such an assumption allows us to express the precision

$$\tau_\epsilon(y) \equiv \frac{\tau_e \tau_\eta}{\tau_e + \tau_\eta} y \tag{1}$$

of the endogenous noise term $\epsilon \equiv f(y)(\eta + e)$ as a linear function of y. The analysis below is facilitated by the uni-dimensionality of the traders' investment in information acquisition. However, the key insights extend to richer specifications in which $y_i = (y_i^\eta, y_i^e)$, with y_i^η and y_i^e parametrizing the traders' exposure to common and idiosyncratic noise, respectively.

Timing. At $t = 0$, the traders simultaneously make their investments $(y_i)_{i \in [0,1]}$ in information acquisition. At $t = 1$, the traders observe their private signals $(s_i)_{i \in [0,1]}$. At $t = 2$, the traders simultaneously submit their demand schedules. At $t = 3$, the market clears, the equilibrium price is determined, the equilibrium trades are implemented, and payoffs are realized.

Remark. While our analysis is motivated by trading in financial markets, the model can also be applied to many other environments. For example, a regulator may be supplying pollution permits to firms that need them to produce. Assuming that each unit of output requires a permit and that each permit has a unit price of p, we then have that π_i is firm i's profits net of its expenditure on permits, $p\tilde{x}$ is the total revenue the regulator obtains from the sale of the permits, and $(\alpha - u)\tilde{x} + \beta\tilde{x}^2/2$ is the cost to the regulator of allocating $\tilde{x}$ permits, with such a cost accounting for the environmental impact of pollution. The uncertainty the firms face over the inverse supply function of pollution permits then reflects the uncertainty over the regulator's tolerance for pollution.

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[11]The correlation in the noise may in turn reflect error at the "source" level as, e.g., in Myatt and Wallace (2012).

2.2 Traders' problem

Given $I_i \equiv (y_i, s_i)$, trader i chooses a demand schedule that maximizes, for each price p, the trader's expected payoff

$$\mathbb{E}\left[(\theta - p)\, x_i - \lambda \frac{x_i^2}{2} | I_i, p\right]$$

taking into account how the price p co-moves with the traders' fundamental value θ, the supply shock u, and the common noise η in the traders' information. The expectation is over θ. The solution to this problem is the demand schedule given by

$$X(p; I_i) = \frac{1}{\lambda}(\mathbb{E}[\theta | I_i, p] - p), \tag{2}$$

where $\mathbb{E}[\theta | I_i, p]$ denotes the trader's expectation of θ given the trader's investment y_i in information, the realization s_i of the trader's private signal, and the price p.[12]

At $t = 0$, each trader $i \in [0, 1]$ then selects y_i to maximize the expected profit

$$\mathbb{E}\left[\left(\theta - p - \frac{\lambda}{2} X(p; I_i)\right) X(p; I_i)\right] - \mathcal{C}(y_i),$$

where the expectation is over (s_i, θ, p), given y_i. Following the pertinent literature, we focus on equilibria and on team-efficient allocations (defined below) in which the market-clearing price p is an affine function of all aggregate variables (θ, u, η), and where all agents acquire information of the same quality (equivalently, pay the same attention to all relevant sources), and follow the same rule to map their private information into the demand schedules.

3 Inefficiency in trading

In this section, we fix the traders' investments in information acquisition y_i and assume that $y_i = y$ for all i. We thus also fix the precision of the traders' private information τ_ϵ, as defined in (1). We first solve for the traders' equilibrium demand schedules and then compare them to their efficient counterparts. The analysis permits us to identify inefficiency in the usage of information (that is, in trading) and policies alleviating the inefficiency. Because y is held fixed, to ease the notation, we drop it from the arguments of many of the functions we introduce below when there is no risk of confusion.

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[12]Our linear-quadratic model is close to the standard CARA-Normal one, except for the fact that, in the latter, the denominator of the asset demand is the product of the traders' constant risk aversion coefficient and the conditional variance of the asset value.

3.1 Equilibrium usage of information

In any symmetric equilibrium in which the price is an affine function of (θ, u, η), each trader's demand schedule is an affine function of her private signal s_i and the price p. That is,

$$X(p; I_i) = a^* s_i + \hat{b}^* - \hat{c}^* p,$$

for some scalars $(a^*, \hat{b}^*, \hat{c}^*)$ that depend on the exogenous parameters of the model, as well as on the investment y in information.[13] Aggregating across traders, we then have that the aggregate demand is equal to $\int X(p; I_i) di = a^*(\theta + f(y)\eta) + \hat{b}^* - \hat{c}^* p$. As usual, the idiosyncratic errors in the traders' signals wash out in the aggregate demand.[14] However, the agents' information-acquisition activity (parametrized by y) impacts the aggregate demand through its effect on the traders' exposure to common noise η. This property has important implications for the positive and normative results we discuss below. Letting

$$z \equiv \theta + \underbrace{f(y)\eta - \frac{u}{\beta a^*}}_{\omega}, \tag{3}$$

we then have that the equilibrium price must satisfy

$$p = \frac{\alpha + \beta \hat{b}^*}{1 + \beta \hat{c}^*} + \frac{\beta a^*}{1 + \beta \hat{c}^*} z. \tag{4}$$

The information about θ contained in the market-clearing price is thus the same as the one contained in the endogenous public signal z whose noise $\omega \equiv f(y)\eta - u/\beta a^*$ is a combination of the common noise η in the traders' private information and the shock u to the supply of the asset. Given y and the sensitivity a^* of the traders' demand schedules to their private information s_i, we then have that the precision of the noise ω in the endogenous signal z contained in the price is equal to $\tau_\omega(a^*)$, with the function $\tau_\omega(a)$ given by

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 \tau_u y \tau_\eta}{\beta^2 a^2 \tau_u + y \tau_\eta}. \tag{5}$$

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[13] The reason why we denote the sensitivity $\hat{c}^*$ of the equilibrium demand schedules to the price and the constant term $\hat{b}^*$ in the equilibrium demand schedules with the $\wedge$ symbol is that, in the Appendix, we use the notation $x_i = a^* s_i + b^* + c^* z$ to denote the induced trades (the volume of the asset purchased/sold by each trader i) as a function of the trader's private information and the endogenous signal z contained in the equilibrium price. We do not use $\wedge$ for the sensitivity a^* of the equilibrium demand schedules to the traders' private information s_i because that sensitivity is the same no matter whether one looks at the submitted demand schedules or the induced trades.

[14] This is because we make the convention that the analog of the strong law of large numbers holds for a continuum of independent random variables with uniformly bounded variances. The last property holds as long as the y_i's have a common lower bound strictly larger than 0.

Let Λ be the function defined, for all τ_ω, by

$$\Lambda(\tau_\omega) \equiv \frac{y^2\tau_\eta^2(\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega\tau_\epsilon(\tau_\theta + 2y\tau_\eta)}{\tau_\epsilon y\tau_\eta(y\tau_\eta - \tau_\omega)}. \tag{6}$$

and, for any a, let $\hat{C}$ and $\hat{B}$ be the functions given by

$$\hat{C}(a) \equiv \frac{\beta(\lambda+\beta)a^2\tau_\theta\tau_u + y\tau_\eta\tau_\theta - \beta ay\tau_\eta\tau_u(1 - (\lambda+\beta)a)}{y\tau_\eta(\lambda\tau_\theta + \beta^2 a\tau_u)}, \tag{7}$$

and

$$\hat{B}(a) \equiv \frac{\alpha}{\beta+\lambda}\left(\lambda\hat{C}(a) - 1\right). \tag{8}$$

We then have the following result:

Proposition 1 (equilibrium trading). *Suppose $y_i = y$ for all i, with y exogenous. There exits a unique symmetric equilibrium. The sensitivity of the traders' equilibrium demand schedules to their private information, a^*, is given by the unique real root to the equation*

$$a^* = \frac{1}{\lambda\Lambda(\tau_\omega(a^*))}, \tag{9}$$

and is such that $0 < a^* < 1/\lambda$. Given a^*, the equilibrium values of the other two parameters $\hat{c}^*$ and $\hat{b}^*$ defining the equilibrium demand schedules are given by the functions (7) and (8).

Fixing the quality of the traders' private information y, the equilibrium demand schedules thus solve a familiar fixed-point problem in which the traders correctly account for the information contained in the market-clearing price, and the latter is consistent with the submitted demand schedules. As anticipated above, the novelty relative to previous work is the presence of common noise in the traders' information, η, which is present in both the aggregate demand schedule and the market-clearing price.

As we show in the Appendix, $1/\Lambda(\tau_\omega(a))$ is the coefficient of the projection of θ on (s_i, p) accounting for the endogenous information z contained in the market-clearing price. Using the formulas for τ_ϵ and τ_ω, one can show that, for any a,

$$\Lambda(\tau_\omega(a)) = 1 + \frac{\beta^2 a^2\tau_u + \tau_\theta}{\tau_e y} + \frac{\beta^2 a^2\tau_u\tau_\theta}{\tau_e y^2\tau_\eta} + \frac{\tau_\theta}{y\tau_\eta}. \tag{10}$$

It is easy to see that, when $y \to \infty$, for any a, $\Lambda(\tau_\omega(a)) \to 1$, in which case $a^* \to 1/\lambda$, $\hat{c}^* \to 1/\lambda$, and $\hat{b}^* \to 0$. This should be expected. As the noise in the agents' signals s_i vanishes, the equilibrium schedules converge to the complete-information ones $(s_i - p)/\lambda$, for all p and s_i. Similarly, as $\tau_e \to \infty$, for any a, $\Lambda(\tau_\omega(a)) \to 1 + \tau_\theta/y\tau_\eta$, in which case $a^* \to y\tau_\eta/\lambda(\tau_\theta + y\tau_\eta)$, $\hat{c}^* \to 1/\lambda$ and $\hat{b}^* \to 0$. This also should be expected. When the idiosyncratic noise in the agents' signals vanishes, the price does not contain any information about θ that the agents do not know already through their signal s_i, in

which case

$$\mathbb{E}\left[\theta|I_i,p\right] = \frac{y\tau_\eta}{y\tau_\eta + \pi_\theta}s_i$$

where $y\tau_\eta$ is the precision of the signal s_i when $\tau_e \to \infty$. On the other hand, when either $y \to 0$, or $\tau_e \to 0$, or $\tau_\eta \to 0$, for any a, $\Lambda(\tau_\omega(a)) \to \infty$, implying that $a^* \to 0$, $\hat{c}^* \to 1/\lambda$, and $\hat{b}^* \to 0$. When the noise in the agents' signals diverges to infinity, the agents' stop responding to their signals and their schedules converge to $-p/\lambda$, that is, the agents' short-sell the asset when its price is positive.

3.2 Efficient usage of information

To isolate the inefficiencies in the equilibrium usage of information, we first identify the welfare losses (relative to the full-information benchmark) under any symmetric profile of affine demand schedules. We then characterize the demand schedules that minimize these losses (equivalently, that maximize ex-ante welfare) over the relevant class. The comparison between the equilibrium and the efficient schedules permits us to identify the inefficiency in equilibrium trading. By considering a fictitious environment in which traders do not learn from prices, we then identity the pecuniary externalities that, jointly with the familiar learning externalities that are present when agents learn from prices, are responsible for the inefficiency. Finally, we show how the interaction between the two externalities relates to the slope of the demand schedules. We discuss policies correcting the inefficiency in trading at the end of the section.

3.2.1 Welfare losses

Ex-post welfare is given by

$$W \equiv \int \left(\theta x_i - \frac{\lambda}{2}x_i^2\right) di - \left(\alpha - u + \beta\frac{\tilde{x}}{2}\right)\tilde{x}.$$

The integral term is the total payoff that the traders derive from purchasing the asset. The remaining term is the supply cost. The traders' payoffs are net of the expenses they incur to purchase the asset. These expenses do not appear in the welfare function because they are a zero-sum transfer between the traders and the relevant asset suppliers and all agents' payoffs are linear in consumption. Importantly, note that W accounts for the cost of unloading the asset (equivalently, for the utility $p\tilde{x} - (\alpha - u)\tilde{x} - \beta\tilde{x}^2/2$ of the representative supplier).

It is easy to see that the trades that maximize ex-post welfare are given by $x_i = x^o$ for all i, with

$$x^o \equiv (\theta + u - \alpha)/(\beta + \lambda). \tag{11}$$

When traders know θ, these trades coincide with those sustained in equilibrium, which is a manifes-

tation of the First Welfare Theorem.[15]

Next, let W^o denote welfare under the first-best allocation, and $WL \equiv \mathbb{E}[W^o] - \mathbb{E}[W]$ denote the ex-ante expected welfare losses that arise when the traders purchase the asset in a quantity different from the first-best level, due to asymmetric information. When each function $X(p; I_i)$ is affine in s_i and p, the welfare losses can be expressed as follows (the derivations are in the Appendix):

$$WL = \frac{1}{2}(\beta + \lambda)\mathbb{E}[(\tilde{x} - x^o)^2] + \frac{\lambda}{2}\mathbb{E}[(x_i - \tilde{x})^2]. \tag{12}$$

The term $\mathbb{E}[(\tilde{x} - x^o)^2]$ captures the losses due to the discrepancy between the aggregate level of trade $\tilde{x}$ and its first-best counterpart, x^o. The term $\mathbb{E}[(x_i - \tilde{x})^2]$, instead, captures the losses due to the dispersion of the individual trades around the average level.

3.2.2 Efficient demand schedules

Consistently with the rest of the literature (see, among others, Vives (1988), Angeletos and Pavan (2007), Amador and Weill (2012), and Vives (2017)), we define the efficient use of information as the demand schedule that minimizes the ex-ante welfare losses over the set of demand schedules that are affine in the private signals and the price. While the welfare definition accounts for the costs of supplying the asset, the optimization is over the traders' demand schedules, respecting the exogeneity of the supply of the asset. This definition permits us to isolate the inefficiencies in the traders' usage of information. Accordingly, we say that $(a^T, \hat{b}^T, \hat{c}^T)$ identifies the efficient use of information if, and only if, when all traders submit the demand schedules $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$, the welfare loses are as small as under any other affine schedule $x_i = a' s_i + \hat{b}' - \hat{c}' p$.[16]

Lemma 1 (**efficiency of demands for given sensitivity to private information**). *For any sensitivity a of the demand schedules to the traders' private information, the values of $\hat{c}$ and $\hat{b}$ in the demand schedules that minimize the welfare losses are given by the same functions (7) and (8) that define the equilibrium demand schedules.*

Inefficiencies in trading, if present, are thus entirely due to the sensitivity of the equilibrium demand schedules to private information. Given such a sensitivity, the response of the equilibrium schedules to the price and the unconditional level of trade are efficient. Using Lemma 1, we can express the welfare losses as a function $WL(a, \tau_\omega(a))$ of the sensitivity of the traders' schedules to their private information and the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price (the expression for $WL(a, \tau_\omega(a))$ is in the Appendix – proof of Proposition 2). The

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[15]Clearly, the theorem does not require that the traders know u. In fact, it does not even require that they know θ. It suffices that they have no way of learning about their payoffs beyond what they know prior to trading.

[16]Again, we use the symbol ˆ to distinguish the efficient demand schedules from the efficient trades.

efficient level of a, which we denote by a^T, is thus the value of a that minimizes $WL(a, \tau_\omega(a))$. Let

$$\Delta(a) \equiv -\frac{\beta^2 y^3 \tau_\eta^3 \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 \left(\beta^2 a^2 \tau_u + y\tau_\eta\right)^2 \left(\tau_\omega(a) + \tau_\theta\right) \left(y\tau_\eta - \tau_\omega(a)\right)},$$

and

$$\Xi(a) \equiv \frac{\tau_\eta \beta \left(\tau_\omega(a) + \tau_\theta\right)}{\lambda \tau_e \left(y\tau_\eta - \tau_\omega(a)\right)}.$$

We then have the following result:

Proposition 2 (**efficient trading**). *Suppose that $y_i = y$ for all i, with y exogenous. The planner's problem has a unique solution. The sensitivity a^T of the traders' demand schedules to their private information is implicitly given by the solution to*

$$a^T = \frac{1}{\lambda} \frac{1}{\Lambda(\tau_\omega(a^T)) + \Delta(a^T) + \Xi(a^T)} \tag{13}$$

and is such that $0 < a^T < 1/\lambda$. Given a^T, the other two parameters defining the efficient demand schedules, $\hat{c}^T$ and $\hat{b}^T$, are given by the same functions in (7) and (8) that describe the corresponding coefficients under the equilibrium usage of information.

When, for any a, $\hat{b}$ and $\hat{c}$ are set optimally, the welfare losses $WL(a, \tau_\omega(a))$ are a convex function of a reaching a minimum at $a = a^T$, with $0 < a^T < 1/\lambda$. Note that the equation (13) that determines the value of a^T differs from the one in (9) yielding the equilibrium value of a^* only by the two terms $\Delta(a)$ and $\Xi(a)$ in the denominator of the right-hand side of (13). The term $\Delta(a)$ is essentially a scaling of

$$\frac{\partial WL(a, \tau_\omega(a))}{\partial \tau_\omega(a)} \frac{\partial \tau_\omega(a)}{\partial a}.$$

Therefore, this term can be thought of as a proxy for the familiar learning externality originating in the fact that traders do not internalize that the sensitivity of their demand schedules to their private information determines the informativeness of the equilibrium price and hence the possibility for other traders to use the price as an endogenous signal for θ when choosing how many shares to purchase. Note that, after replacing the formula for $\tau_\omega(a)$ into the formula for $\Delta(a)$, we have that

$$\Delta(a) = -\frac{\beta^2 y \tau_\eta \tau_u \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 \left[\beta^2 a^2 y \tau_\eta \tau_u + \tau_\theta \left(\beta^2 a^2 \tau_u + y\tau_\eta\right)\right]}.$$

This term is always negative reflecting the under-response of the equilibrium demand schedules to private information. Essentially, traders do not consider that responding more to their private information leads to a more informative price and hence to more efficient trades. The social planner, instead, internalizes this effect and asks that the traders respond more to their private information.

Naturally this learning externality vanishes when $\tau_e \to \infty$, for, in this case, there is no idiosyn-

cratic noise in the agents' signals. Recall that, in this case, $a^* \to y\tau_\eta/\lambda(\tau_\theta + y\tau_\eta)$, and, $\Delta(a^*) = 0$ when $a^* = y\tau_\eta/\lambda(\tau_\theta + y\tau_\eta)$. Likewise, this externality vanishes when $y \to 0$, i.e., when $s_i = \theta$ almost surely. In this case, $a^* \to 1/\lambda$, and $\Delta(1/\lambda) \to 0$ as $y \to \infty$. In the same vain, the externality vanishes when $\tau_u \to 0$ for, in this case, the price becomes uninformative (because of the high volatility of the supply schedule), no matter the sensitivity of the agents' schedules to their private information.

The term $\Xi(a)$, instead, is a pecuniary externality. When the traders respond to their private signals, they do not internalize that variations in their demands due to noise in their signals impact other traders' asset holdings through the dependence of other traders' demands on the price. Being noise-driven, such variations are not justified in the planner's eyes. The planner thus asks that the traders respond less to their private signals to reduce the welfare losses of such noise-driven variations. Note that, after replacing the formula for $\tau_\omega(a)$ into the formula for $\Xi(a)$, we have that

$$\Xi(a) = \frac{\beta \left[\beta^2 a^2 \tau_u y \tau_\eta + \tau_\theta \left(\beta^2 a^2 \tau_u + y\tau_\eta \right) \right]}{\lambda \tau_e y^2 \tau_\eta}.$$

The term $\Xi(a)$ is thus always positive, reflecting the over-response of the equilibrium trades to private information. This pecuniary externality is different from those that originate in financial frictions in other incomplete-market economies; see, e.g., Lorenzoni (2008). It is closer in spirit to the distributive pecuniary externality of Dávila and Korinek (2018), although the mechanics and origin are different. When there is no noise in the agents' information (i.e., when $y \to \infty$), this externality vanishes. More interestingly, this externality vanishes also when $\tau_e \to \infty$, i.e., when the only noise in the agents' signals is perfectly correlated across the agents. This is because the economy is competitive and, when there is no private information the first welfare theorem applies.

Importantly, both externalities arise because of the combination of the following properties: (1) asymmetric information, (2) traders submit demand schedules, and (3) markets are incomplete. When information is symmetric, the first welfare theorem applies. Similarly, when the traders' demands do not condition on the price, there is nothing the planner can do to improve upon the traders' ability to tell apart variations in their expectations of θ driven by the fundamental value θ of the asset from those driven by noise; ex-ante welfare is below the complete-information level, but there is no inefficiency in the equilibrium usage of information (see Appendix 2 for a formal proof of this result, as well as the discussion in Subsection 3.2.4). Finally, when markets are complete, traders can fully insure against ex-post idiosyncratic variations in their consumption due to interim idiosyncratic variations in their perceptions of the fundamental value of the asset at the trading stage; again, the welfare theorems then guarantee efficiency of the equilibrium trades.[17]

Importantly, note that, whereas total welfare is higher under the efficient schedules than the equilibrium ones, moving from the latter to the former is not guaranteed to deliver a Pareto improvement: either the traders or the representative investor (but not both) can be worse off under

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[17]See, for example, Angeletos, Iovino, and La'O (2020) for a model in which the completeness of the market occurs via consumption sharing – all traders are members of the same family and share consumption equally ex-post.

the efficient schedules. This reflects the fact that the efficient allocations maximize the sum of all agents' payoffs, with each agent receiving equal weight by the planner.

3.2.3 Fictitious environment

To shed more light on the two externalities introduced above, consider a fictitious environment in which the traders are naive in that they do not recognize the information contained in the market-clearing price. Such a benchmark is similar in spirit to the (fully) cursed equilibrium of Eyster and Rabin (2005). See also Bastianello and Fontanier (2024) for recent work discussing mis-learning from prices and its interaction with mis-learning from fundamentals. The reason for considering such an environment is that it permits us to isolate the pecuniary externality by shutting down the more familiar learning externality. To facilitate the comparison to the true economy, assume that, in this fictitious environment, each trader, in addition to observing the private signal

$$s_i = \theta + \underbrace{f(y)(\eta + e_i)}_{\equiv \epsilon_i}$$

as in the true economy, also observes an exogenous public signal

$$z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$$

whose structure is the same as the one contained in the market-clearing price, but with the endogenous noise $-u/\beta a$ replaced by the exogenous one χ, with the latter drawn from a Normal distribution with mean zero and variance τ_χ^{-1} independently of all other variables (this shock is the same for all traders). Let $\tau_\zeta \equiv y\tau_\eta\tau_\chi/(\tau_\chi + y\tau_\eta)$ denote the precision of the total noise $\zeta \equiv f(y)\eta + \chi$ in the exogenous signal z. As we show in the Appendix, in the cursed equilibrium of this fictitious economy, traders submit affine demand schedules $x_i = a_{exo}^* s_i + \hat{b}_{exo}^* - \hat{c}_{exo}^* p + \hat{d}_{exo}^* z$, where the sensitivity of the traders' demands to their private information is given by

$$a_{exo}^* = \frac{1}{\lambda\Lambda(\tau_\zeta)}, \tag{14}$$

with the function $\Lambda(\cdot)$ as defined in (6). Note that the formula in (14) is similar to the one in (9) in the true economy, except for the fact that the precision $\tau_\omega(a)$ of the endogenous public signal contained in the market-clearing price is replaced by the precision τ_ζ of the exogenous public signal about θ.

Now suppose that, in this fictitious economy, the planner can control the sensitivity a of the traders' demands to their private information. However, given a, the planner must choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a and $(\hat{b}, \hat{c}, \hat{d})$ as between a_{exo}^* and $(\hat{b}_{exo}^*, \hat{c}_{exo}^*, \hat{d}_{exo}^*)$ in the

cursed equilibrium.[18] The level of a that maximizes ex-ante welfare is then equal to

$$a_{exo}^T = \frac{1}{\lambda} \frac{1}{\Lambda(\tau_\zeta) + \frac{\tau_\eta \beta (\tau_\zeta + \tau_\theta)}{\lambda \tau_e (y\tau_\eta - \tau_\zeta)}}. \tag{15}$$

Again, the formula for a_{exo}^T is similar to the one for a^T defining the efficient sensitivity to private information in the true economy, except for the fact that $\tau_\omega(a)$ is replaced by τ_ζ and the term $\Delta(a)$ in the denominator of the expression giving the socially-optimal level of a in the true economy is equal to zero, reflecting the fact that the planner recognizes that the agents do not learn from the price. Note that

$$\frac{\tau_\eta \beta (\tau_\zeta + \tau_\theta)}{\lambda \tau_e (y\tau_\eta - \tau_\zeta)}$$

has exactly the same form as the pecuniary externality $\Xi(a)$ in the true economy (except for the fact that $\tau_\omega(a)$ is replaced by τ_ζ). Hence, in this fictitious economy, the cursed-equilibrium demand schedules unambiguously feature an excessively high sensitivity to private information: $a_{exo}^* > a_{exo}^T$. Furthermore, when the precision of the exogenous public signal in the cursed economy is the same as the precision of the endogenous public signal under the efficient demand schedules of the true economy (that is, when $\tau_\zeta = \tau_\omega(a^T)$), a_{exo}^T coincides with the solution to the equation $\partial WL(a_{exo}^T, \tau_\omega(a^T))/\partial a = 0$ and $a_{exo}^T < a^T$: in the true economy, the planner recognizes the value of increasing the precision of the endogenous signal contained in the market-clearing price and thus demands that traders respond more to their private information.

3.2.4 Sign of externalities and slope of demand schedules

We now return to the economy in which both the traders and the planner account for the information contained in the market-clearing price. Whether the sensitivity of the equilibrium demand schedules to the traders' private information is excessively high or low (compared to the efficient level a^T) then depends on which of the two externalities described above prevails.

Proposition 3. *Suppose that $y_i = y$ for all i, with y exogenous. The following equalities hold:*

$$a^* - a^T \stackrel{sgn}{=} \Xi(a^T) + \Delta(a^T) \stackrel{sgn}{=} \Xi(a^*) + \Delta(a^*) \stackrel{sgn}{=} \hat{c}^* \stackrel{sgn}{=} \hat{c}^T.$$

Hence, whether the sensitivity of the equilibrium schedules to private information is excessively high or excessively low, compared to what is efficient, depends on which of the two externalities prevails under the efficient schedules. When $\Xi(a^T) + \Delta(a^T) = 0$, the two externalities cancel each other out, the efficient schedules are price-inelastic (i.e., $\hat{c}^T = 0$) and $a^* = a^T$. When, instead,

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[18]In the true economy, maintaining the same relationship between a and $(\hat{b}, \hat{c})$ is without loss of optimality for the planner (see Lemma 1 above). This need not be the case in the fictitious economy. However, imposing the restriction permits us to isolate the relevant effects.
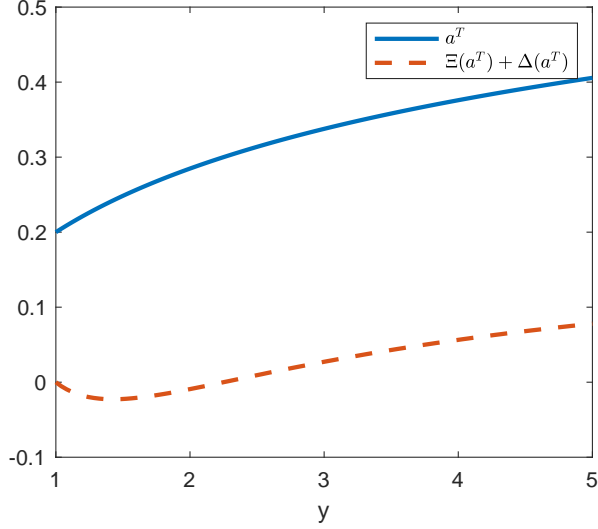
Figure 1: The blue solid line corresponds to a^T whereas the orange dashed line represents the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$. The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

$\Xi(a^T) + \Delta(a^T) > 0$, the pecuniary externality dominates, $\hat{c}^T > 0$ (the efficient schedules are downward sloping) and the equilibrium schedules feature an excessive response to the traders' private information. Finally, when $\Xi(a^T) + \Delta(a^T) < 0$, the learning externality dominates, $\hat{c}^T < 0$ (the efficient schedules are upward sloping) and the equilibrium response to private information is insufficiently low.

It is worth noting that if the traders were restricted to submitting market orders (like in a Cournot model), then the usage of information would be efficient since the two externalities would not be present (See Appendix 2 for a formal proof of this result).

Using simulations, it is possible to nail down the effect of variations in the quality y of the traders' private information on the two externalities and on the slope of the efficient demand schedules. Figure 1 depicts the sensitivity of the traders' efficient demand schedules to their private information a^T (solid blue curve) as well as the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ (dashed orange curve), as a function of the quality y of the traders' private information.

As y increases, the efficient response a^T to the traders' private information increases, reflecting the higher value of responding to more accurate private information. Furthermore, because both Ξ and Δ increase with a^T, a higher y contributes to a higher value of $\Xi(a^T) + \Delta(a^T)$ via the indirect effect that y has on the two externalities through a^T. In addition, holding a^T fixed, we have that y has a direct effect on both $\Xi(a^T)$ and $\Delta(a^T)$. Whereas $\Xi(a^T)$ is increasing in y, $\Delta(a^T)$ is decreasing. Combining the direct with the indirect effects, we then have that $\Xi(a^T)$ unambiguously increases with y, whereas $\Delta(a^T)$ is non-monotonic in y. For small values of y, the sum of the two externalities is
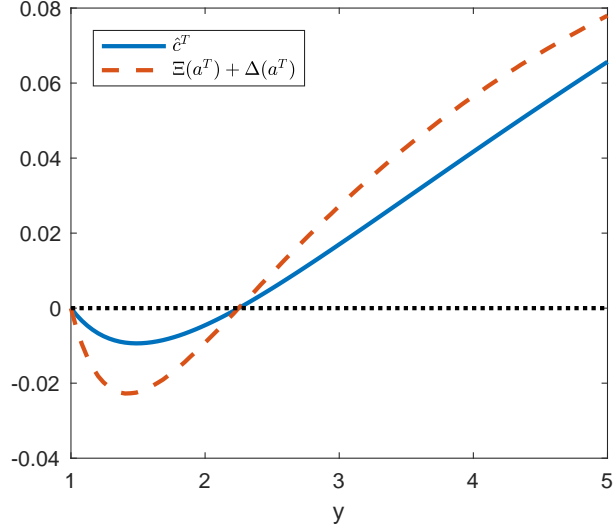
19

Figure 2: The blue solid line corresponds to $\hat{c}^T$ whereas the orange dashed line represents the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$. The two curves switch signs for the same value of y. The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \le y \le 5$.

negative and decreasing in y, whereas, for sufficiently high values of y, the sum of the two externalities is positive and increasing in y, as can be seen from Figure 1.

Next, we turn to the relationship between the two externalities and the slope of the efficient demand schedules, $\hat{c}^T$. Figure 2 depicts the sensitivity $\hat{c}^T$ of the efficient demand schedules to the price (the solid blue curve) along with the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ (the orange dashed curve), as a function of the quality y of the traders' private information. The two curves switch sign for the same value of y. As explained above, when the traders possess high-quality private information (high values of y), the marginal value of generating additional information through the price is low and the pecuniary externality dominates over the learning externality, so that $\Xi(a^T) + \Delta(a^T)$ is positive and increasing in y. In this case, $\hat{c}^T$ is positive meaning that the efficient demand schedules are downwards sloping, as they would be in an economy in which the fundamental value of the asset θ is known to the traders. When, instead, the traders possess low-quality private information, the learning externality dominates over the pecuniary externality so that $\Xi(a^T) + \Delta(a^T)$ is negative and first decreasing and then increasing in y. In this case, $\hat{c}^T$ is negative meaning that the efficient demand schedules are upwards sloping, reflecting the high sensitivity of the traders' estimates of the fundamental value of the asset θ to the price, relatively to the sensitivity of the same estimates to their private information.

We conclude this subsection by highlighting the role that the common noise η in the traders' private information plays for the sign and magnitude of the two externalities identified above. Un-surprisingly, both a^* and a^T are increasing in the precision τ_η of the noise η, reflecting the fact that

responding to private information is more valuable (both for the traders and for the planner) when it is affected less by correlated noise and hence more precise. Similarly, holding a fixed, we have that the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price naturally increases with τ_η, reflecting the fact that the noise in the traders' signals becomes less correlated when τ_η increases and, as a result, washes out more at the aggregate level, making the price more informative, for given demand schedules. Furthermore, fixing a^T, the absolute value of both $\Xi(a^T)$ and $\Delta(a^T)$ increases with τ_η, reflecting the larger role that either externality plays when the noise in the private signals is less correlated. However, whereas the pecuniary externality $\Xi(a^T)$ increases with τ_η, the learning externality $\Delta(a^T)$ decreases with it. Combining all of the above effects, we then have that the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ can be non-monotonic in τ_η, depending on the other parameters' values.

3.3 Policies inducing efficient trading with exogenous information

Next, we discuss policies that correct the inefficiencies in the usage of information identified in the previous subsections, once again holding fixed the quality of the traders' information y for the time being.

Proposition 4 (**policy inducing efficient trading**). *Suppose that $y_i = y$ for all i, with y exogenous. There exists $\delta, t_p, t_0 \in \mathbb{R}$ such that the efficient use of information can be implemented with a policy that charges the traders a total tax bill equal to $T(x_i, p) = \frac{\delta}{2}x_i^2 - t_0 x_i + t_p p x_i$ where t_0, t_p and δ are functions of all exogenous parameters.*

The efficient use of information can thus be induced through a combination of a linear-quadratic tax $\frac{\delta}{2}x_i^2 - t_0 x_i$ on the individual volume of trade (equivalently on the quantity of the asset purchased), along with a (more familiar) ad-valorem tax $t_p p x_i$. The role of δ is to manipulate the traders' adjustment cost (from λ to $\lambda + \delta$). This manipulation suffices to induce the traders to submit demand schedules whose sensitivity to their exogenous private information is equal to the efficient level a^T. The role of the linear ad-valorem tax is to guarantee that, once the sensitivity a coincides with the efficient level a^T, the sensitivity $\hat{c}$ of the equilibrium demand schedules to the price coincides with the efficient level $\hat{c}^T$. In the absence of such a correction, the traders fail to submit the efficient demand schedules, even if they respond efficiently to their private information. Finally, the role of the linear tax $t_0 x_i$ on the individual volume of trade is to guarantee that the fixed part $\hat{b}$ of the demand schedules (equivalently, the unconditional volume of trade) also coincides with its efficient counterpart $\hat{b}^T$.

The result in Proposition 4 hinges on all traders being ex-ante identical. We expect similar policies to induce efficient trading in economies with multiple groups of investors, provided that each group can be subject to a group-specific tax schedule.

The tax scheme of Proposition 4 induces the traders to submit the efficient limit orders. In principle, such a scheme is simple to implement, as it only requires conditioning taxes on variables

(price and individual volume of trade) that are easy to observe. However, the scheme requires a non-linear dependence of the total tax bill on the quantity purchased. Such non-linearities, while conceptually straight-forward, are sometimes perceived as difficult to implement in practice. The question of interest is then whether a planner who is restricted to simple ad-valorem taxes such as those often discussed in the policy debate can still improve upon the laissez-faire equilibrium by choosing t_p optimally.

Proposition 5. *[**sub-optimality of ad-valorem taxes with exogenous information**] Suppose that $y_i = y$ for all i, with y exogenous. If a planner is constrained to use ad-valorem taxes (that is, a policy that, given p and x_i, charges each trader i a total tax bill equal to $T(p, x_i) = t_p p x_i$, for some $t_p \in \mathbb{R}$) then the optimal policy is such that $t_p = 0$.*

Hence, if the planner is constrained to using a tax that is linear in the expenditure px_i on the asset (equivalently, an ad-valorem tax), the optimal value of the tax is zero. This is because any such a policy fails to manipulate the relative importance that each trader attaches to his private information and the price in predicting the value of the asset. In other words, such a tax does not change the information contained in the equilibrium price. As a result, the equilibrium sensitivity to private information, a^*, is the same as in the laissez-faire equilibrium, no matter the value of t_p. Because, in the laissez-faire equilibrium, for any a, the other two elements of the equilibrium demand schedules, $\hat{b}(a)$ and $\hat{c}(a)$, are welfare-maximizing (see Lemma 1), any ad-valorem tax with $t_p \neq 0$, by changing $\hat{b}$ and $\hat{c}$ without changing a, reduces welfare.

4 Inefficiency in information acquisition and policy corrections

We now investigate how inefficiencies in information acquisition relate to inefficiencies in trading (equivalently, in information usage), and how the planner can correct them through appropriate policy interventions.

We start by establishing existence of an equilibrium in the full game of the laissez-faire economy (and its uniqueness in the family of equilibria in affine strategies) and then turn to the relation between the inefficiency in information acquisition and the inefficiency in trading, and how the planner can correct each of the two.

4.1 Equilibrium of the laissez-faire economy

Proposition 6 (**equilibrium in full game**). *There exist constants $L, M \in \mathbb{R}_{++}$ (defined in the Appendix) such that, when $\mathcal{C}'(0) \leq L$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > M$, in the full game with endogenous information acquisition, there exists one, and only one, symmetric equilibrium in affine trading strategies.*

Hence, an equilibrium of the full game with endogenous private information always exists (and is unique within the family of symmetric equilibria in affine trading strategies) when (a) the marginal cost of information is small for small y, and (b) the function $\mathcal{C}$ is sufficiently convex. The first condition guarantees existence and uniqueness of a positive quality of information y^* such that, when all other traders acquire information of quality y^* and submit the equilibrium limit orders for information of quality y^*, each agent's net private marginal benefit of increasing the quality of his information at $y_i = y^*$ is zero. The second condition guarantees that, fixing the other traders' strategies, each trader's payoff is strictly quasi-concave in the quality of information y_i, accounting for the optimal usage the trader makes of the information he collects. When $\mathcal{C}(y) = By^2/2$, the conditions in the proposition jointly hold for B large enough.

4.2 Inefficiency in information acquisition under efficient trading

Suppose now that the traders can be trusted to submit the efficient limit orders; can they be trusted to collect private information efficiently? We first consider the case where efficiency in trading is exogenous and then the case in which it is induced through the policy in Proposition 4. In both cases, we find that the traders do not acquire information efficiently. We conclude by considering richer families of policy interventions which permits us to uncover both an impossibility and a couple of possibility results.

Proposition 7 (**inefficiency in information acquisition under efficient trading**). *Let y^T denote the socially optimal quality of private information and suppose that all traders submit the efficient demand schedules for information of quality y^T (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$). When $\hat{c}^T > 0$ (i.e., when the efficient demand schedules are downward sloping), the quality of private information acquired in equilibrium is higher than y^T, whereas the opposite is true when $\hat{c}^T < 0$ (i.e., when the efficient demand schedules are upward sloping).*

Recall from the previous section that, when $\hat{c}^T > 0$, traders over-respond to private information. Forcing them to respond less to their private information then induces them to over-invest in information acquisition. When, instead, $\hat{c}^T < 0$, traders under-respond to private information. Forcing them to trade efficiently then induces them to under-invest in information acquisition.

In the special case in which $\hat{c}^T = 0$ (that is, when the efficient demand schedules are completely price-inelastic and hence can be implemented with market orders), in the absence of policy interventions, a trader endowed with information of quality y^T would trade efficiently. In this case, when information is endogenous, the trader acquires information of efficient quality y^T, even in the absence of policy interventions.

As we show in the Appendix (see the comparison between (33) and (34) in the proof of Proposition 7), the reason why efficiency in trading does not imply efficiency in the collection of information is that traders do not internalize that, fixing the asset demands at the efficient level, changes in the quality of private information affect the co-movement between the asset holdings and the various

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aggregate shocks. Because of market incompleteness, such co-movements are not priced. As a result, private incentives fail to properly account for the welfare effects of these changes. In particular, the same pecuniary and learning externalities that create a wedge between the private and social value of responding to private information when it comes to trading create a wedge between the private and social value of increasing the quality of private information under efficient trading.

Importantly, the above conclusions hinge on the traders being exposed to correlated noise in their information sources, that is, on $\tau_\eta \in (0, +\infty)$. When $\tau_\eta = 0$, the noise in the agents' private signals is infinite, making the signals worthless both for the individual traders and for the planner. When, instead, $\tau_\eta \to +\infty$, the correlated noise in the agents' private signals disappears, in which case, fixing the agents' demand schedules, we have that the aggregate volume of trade is invariant to the quality of the traders' private information. This is the case considered in most of the previous literature. In this situation, holding the demand schedules fixed, we have that the only effect of an increase in the quality of the traders' private information on welfare is through the reduction in the dispersion of individual trades around the aggregate level of trade. Because this effect is weighted equally by the planner and by the individual traders, the private and the social value of information coincide, which guarantees that the information acquired in equilibrium is efficient.

Recall that, for small y, the learning externality dominates over the pecuniary externality and the efficient demand schedules are upward sloping, whereas, for large y, the pecuniary externality dominates and the efficient demand schedules are downward sloping. The above results thus suggest that, as technological progress makes information cheaper (that is, the cost of information acquisition decreases), the economy is likely to eventually enter into a regime of over-investment in information acquisition.

To further understand the implications of the above inefficiencies for asset-price variables, it is helpful to introduce the following:

Definition 1. [market quality variables] Let market depth be the inverse of the sensitivity of the price to the supply shock u: $MD \equiv (dp/du)^{-1} = 1 + \beta \hat{c}$. Let the volatility of the price be: $\sigma_p = (Var\,[p])^{\frac{1}{2}}$. Finally, let the informativeness of the price be the precision of the endogenous signal contained in the price: τ_ω.

Figure 3 shows how the above asset-pricing variables are affected by changes in the cost of information acquisition, both under the decentralized equilibrium of the laissez-faire economy and under the efficient allocation (where both the acquisition and usage of information coincide with the welfare-maximizing levels). The figure assumes a quadratic cost of information $C(y) = By^2/2$; a reduction in the cost of information corresponds to a reduction in the parameter B. As the cost of information decreases (moving from right to left along the x-axis) market depth, price volatility, and price informativeness all move from being inefficiently low to being inefficiently high.

We could also establish the following numerical result: When $\hat{c}^T < 0$ (i.e., when the efficient demand schedules are upward sloping), the equilibrium in the absence of policy interventions is
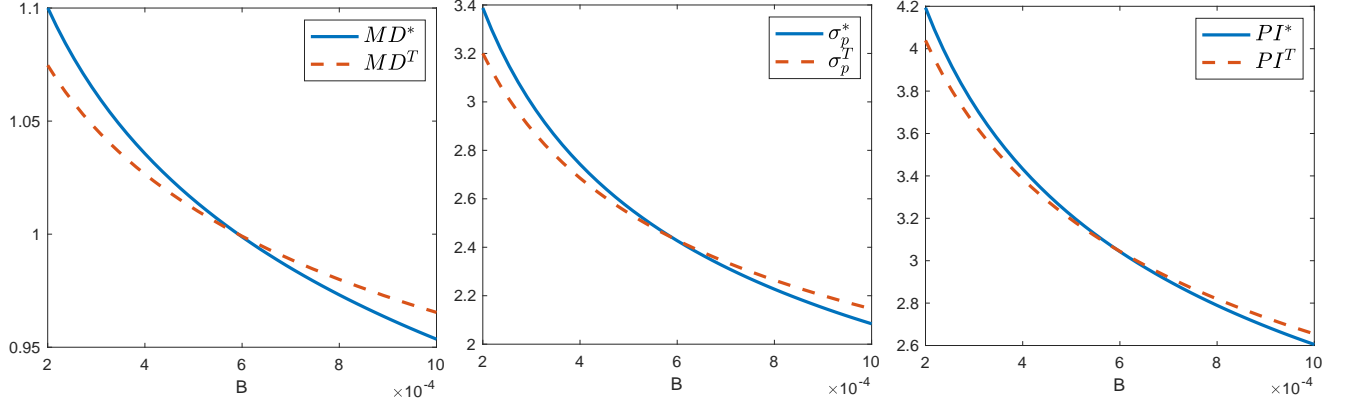
Figure 3: The first panel depicts market depth, the second one price volatility, and the third one price informativeness. In each panel, the blue solid curve corresponds to the laissez-faire economy, whereas the orange dashed curve corresponds to the solution to the planner's problem. The x-axis in all three panels represents the scalar B that parametrizes the quadratic cost of information. The other parameter values are $\lambda = \beta = 1.3$, $\tau_e = 0.8$, $\tau_\eta = 0.6$, $\tau_\theta = 0.1$, and $\tau_u = 30$.

such that the acquisition of private information, the sensitivity of the demand schedules to private information, price volatility, market depth, and price informativeness are all inefficiently low. The opposite is true when $\hat{c}^T > 0$ (i.e., when the efficient demand schedules are downward sloping). As the cost of acquiring information decreases, the economy moves from the first regime to the second.

To obtain the result, we simulated the model 1,000 times drawing the parameters τ_u, τ_e, τ_η, τ_θ, λ, and β uniformly from 1 to 30. The cost function of acquiring information in the simulations is $C(y) = By^2/2$, with B drawn uniformly from 0 to 0.01. In all the simulations, the sign of $\hat{c}^T$, $y^* - y^T$, and $a^* - a^T$ is the same. Figure 4 illustrates the relationship between the cost of information, parametrized by B, the inefficiency $y^* - y^T$ in the acquisition of information, the slope $\hat{c}^T$ of the efficient demand schedules, and the inefficiency $a^* - a^T$ of the equilibrium limit orders under one of these simulations.
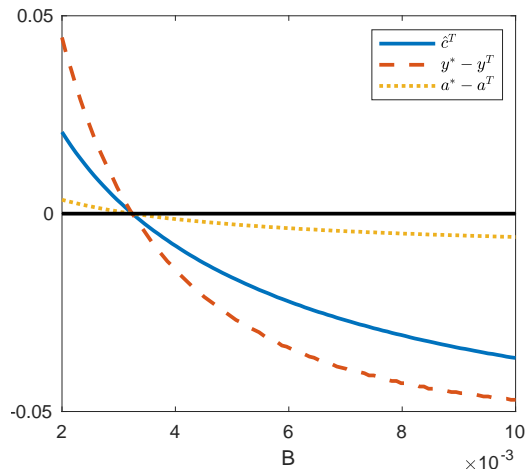
Figure 4: The blue solid curve represents the slope of the efficient demand curve. The orange dashed curve represents the inefficiency in information acquisition, where a positive number means inefficiently high acquisition, and a negative number means inefficiently low acquisition. The yellow dotted curve represents the inefficiency in the sensitivity of the demand schedules to private information, where a positive number means inefficiently high sensitivity, and a negative number means inefficiently low sensitivity. The x-axis is the value of B that parametrizes the cost of information $C(y) = BY^2/2$. The parameter values in the simulations are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, and $\tau_u = 30$.

4.3 Policy corrections

We now address the question of whether efficiency in information acquisition can be induced through an appropriate policy design. We start by considering the problem of a planner expecting the traders to submit the efficient demand schedules and then consider the more relevant problem of a planner who seeks to induce efficiency in both information acquisition and trading through the same policy.

Proposition 8. [*policy inducing efficiency in information acquisition under efficient trading*] *Let y^T denote the socially optimal quality of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules (for quality of information y^T). Suppose that, no matter the information collected, the traders submit the efficient demand schedules (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$). The planner can then induce the traders to acquire the efficient information y^T by charging them a total tax bill equal to $T(p, x_i) = \hat{t}_p p x_i$, with $\hat{t}_p > 0$ if $\hat{c}^T > 0$ (downward-sloping demands) and $\hat{t}_p < 0$ if $\hat{c}^T < 0$ (upward-sloping demands).*

Hence, a planner who expects the traders to submit the efficient demand schedules can induce the traders to collect information efficiently by using a simple "ad valorem" tax on total asset purchases similar to those discussed in the policy debate.

Next, consider the more realistic case of a planner who does not trust the market to submit the efficient limit orders and hence seeks to design a policy that induces efficiency in both information acquisition and trading. In the previous section, we showed that, when the quality of information is

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exogenous, efficiency in trading can be induced through a combination of a linear-quadratic tax on the individual volume of trade paired with an ad-valorem tax (both rebated in a lump-sum manner, if desired). Based on other results in the literature, one may conjecture that the same policy mix also induces efficiency in information acquisition. The next result shows that this is not the case. If the planner were to use the tax $T(p, x_i)$ in Proposition 4 (applied to $y = y^T$) that induces the traders to submit the efficient demand schedules when information is exogenous and such that $y_i = y^T$ for all i, then the traders would respond by acquiring information of quality different from y^T and by submitting demand schedules different from the efficient ones. More generally, the proposition shows that there exists no policy measurable in the individual volume of trade and in the price of the financial asset that induces efficiency in both trading and information acquisition.

Proposition 9 (impossibility to induce efficiency in both information acquisition and trading with standard policies). *Generically (i.e., with the exception of a set of parameters of zero Lebesgue measure), there exists no policy $T(x_i, p)$ that induces efficiency in both information acquisition and trading.*

The result is established in the Appendix by showing that any policy that induces the traders to submit the efficient demand schedules once they collect the efficient amount of private information y^T must coincide with the one in Proposition 4 (applied to $y = y^T$), except for terms that play no role for incentives. However, any such a policy induces the traders to misperceive the value of their private information (around the efficient level y^T) and hence induces them to collect an inefficient amount of private information.

We conclude with two possibility results. The first one establishes that, when information acquisition is verifiable, efficiency in both information acquisition and trading can be obtained by conditioning the total tax bill on the expenditure on information acquisition. The second result establishes that, when information acquisition is not verifiable (e.g., because it reflects the attention paid to various exogenous sources), then efficiency in both acquisition and trading can be obtained by conditioning the marginal tax rate on the aggregate volume of trade.

Proposition 10 (policy inducing efficiency in both information acquisition and trading when acquisition is verifiable). *Under conditions similar to those in Proposition 6,[19] efficiency in both information acquisition and trading can be induced through a policy*

$$T^{tot}(x_i, p, y_i) = \frac{\delta}{2} x_i^2 - t_0 x_i + t_p p x_i - A y_i$$

where (δ, t_p, t_0) are as in Proposition 4 for $y = y^T$, and $A \in \mathbb{R}$.

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[19]Formally, there exist constants $\tilde{L}, \tilde{M} \in \mathbb{R}_{++}$ such that the result in the proposition holds when $\mathcal{C}'(0) \leq \tilde{L}$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \tilde{M}$. As in Proposition 6, these conditions guarantee global quasi-concavity of the payoffs under the proposed policy.

One can show that $A < 0$ when $\hat{c}^T > 0$ whereas $A > 0$ when $\hat{c}^T < 0$. That is, expenditures on information acquisition are taxed when the efficient demand schedules are downward sloping and subsidized when they are upward sloping, reflecting the fact that, under the policy of Proposition 4, agents over-invest in information acquisition in the former case and underinvest in the latter.

Proposition 11 (policy inducing efficiency in both information acquisition and trading when acquisition is non-verifiable). *Suppose that the acquisition of information is not verifiable. Under conditions similar to those in Proposition 6,[20] there exist constants $\delta^*, t_{\tilde{x}}^*, t_0^*, t_p^* \in \mathbb{R}$ such that efficiency in both information acquisition and trading can be induced through a (linear-quadratic) tax bill of the form*

$$T^*(x_i, \tilde{x}, p) = \frac{\delta^*}{2} x_i^2 + (t_{\tilde{x}}^* \tilde{x} - t_0^*) x_i + t_p^* p x_i$$

in which the marginal tax rate $\partial T^*(x_i, \tilde{x}, p)/\partial x_i$ depends on the aggregate volume of trade $\tilde{x}$.

The dependency of the marginal tax rate on the aggregate volume of trade $\tilde{x}$ is essential to induce efficiency in both information acquisition and trading. As we show in the Appendix, with this type of policies, the planner can equalize the expected marginal tax rate

$$\frac{\partial}{\partial x_i} \mathbb{E}[T^*(x_i, \tilde{x}, p)|x_i, p; y_i, y^T]\Big|_{y_i=y^T; \, x_i=a^T s_i + \hat{b}^T - \hat{c}^T p}$$

of each individual trader who acquires information of quality $y_i = y^T$ and then submits the efficient demand schedule $a^T s_i + \hat{b}^T - \hat{c}^T$ with the discrepancy

$$\mathbb{E}[\theta|x_i, p; y_i, y^T] - p - \lambda x_i\Big|_{y_i=y^T; \, x_i=a^T s_i + \hat{b}^T - \hat{c}^T p}$$

between the private marginal benefit and the private marginal cost of expanding the individual volume of trade around the efficient level $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$. Eliminating such a discrepancy is essential to induce efficiency in trading. Importantly, the new contingency provides the planner with flexibility on how to eliminate such a discrepancy. When, instead, the policy depends only on x_i and p, there exists a unique way of eliminating such a discrepancy, as shown in the proof of Proposition 9. The extra flexibility in turn can be used to realign the marginal private value of more precise private information to its social counterpart, something that is not possible when the policy depends only on x_i and p. This can be done by exploiting the fact that, when the marginal tax rate depends on the aggregate volume of trade, because the latter is not known to the individual trader, the value of information also accounts for the benefit of forecasting the marginal tax rate. In the proof in the Appendix, we show how to exploit this extra benefit to realign incentives. In the Appendix, we also show that, when information acquisition is not verifiable, the policy that implements efficiency

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[20]Formally, there exist constants $\hat{L}, \hat{M} \in \mathbb{R}_{++}$ such that the statement in the proposition holds when $\mathcal{C}'(0) \leq \hat{L}$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \hat{M}$.

in both information acquisition and trading is in fact unique up to terms that do not matter for incentives.

As explained in the previous section, the taxes discussed in the policy debate are typically ad-valorem (that is, linear in the individual expenditures px_i). When information is exogenous, we showed in Proposition 5 that such taxes reduce welfare. The next proposition shows that this is the case also when information is endogenous.

Proposition 12 (sub-optimality of ad-valorem taxes with endogenous information). *Suppose that information is endogenous and that the planner is restricted to use ad-valorem taxes of the form $T(x_i, p) = t_p p x_i$, for some $t_p \in \mathbb{R}$. Then the optimal t_p is zero.*

The intuition for the result is similar to the one for Proposition 5. Ad-valorem taxes do not affect the equilibrium sensitivity to private information. They also do not affect the value that each trader assigns to increasing the precision of his private information. These taxes only affect the sensitivity of the equilibrium limit orders to the price and the unconditional volume of trade. However, given y and a, the sensitivity of the equilibrium schedules to the price and the unconditional volume of trade (the parameters $\hat{c}$ and $\hat{b}$ in the limit orders) are efficient under the laissez-faire allocation. As a result, these taxes only bring the equilibrium allocation further away from the efficient one, and hence reduce welfare.[21]

Propositions 5 and 12 have important implications for the debate on how to tax transactions in financial markets. They show that, no matter whether information is exogenous or endogenous, the policies that are typically proposed do more harm than good. The policy maker should instead consider more sophisticated taxes that are non-linear in the purchases of the asset and that condition marginal tax rates on the aggregate volume of trade.

In a similar vein, it is often suggested that governments can improve over the laissez-faire equilibrium by manipulating prices through asset purchases. One can show that such policies are also welfare detrimental. The reason is essentially the same as for ad-valorem taxes.

5   Conclusions

We identify inefficiency in the trading of financial assets and relate it to the information that traders collect privately before submitting their orders. We show that, if the traders' private information were exogenous, inefficiency in trading could be corrected with a combination of ad-valorem taxes with non-linear subsidies/taxes on the individual volume of asset purchases. However, when information is endogenous, there exists no policy measurable in the price of the financial asset and in the volume of individual trades that induces efficiency in both trading and information acquisition. The above impossibility result can be turned into a possibility one by conditioning the total tax bill on individual

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[21]See Dávila and Walther (2021) for a study of corrective taxation in environments where the Government's interventions are limited because some agents cannot be taxed, or certain activities cannot be regulated.

expenditures on information acquisition (when the latter are verifiable), or by conditioning the marginal tax rate on the aggregate volume of trade. In practice, authorities typically consider only simple linear *ad-valorem* taxes. We find that, if these taxes are the only instrument available, they should not be used.

In future work, it would be interesting to extend the analysis to a broader class of economies in which financial decisions interact with real decisions, and in which agents exchange multiple assets over multiple periods.

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6   Appendix 1: Proofs

Proof of Proposition 1. As explained in the main text, when the traders submit affine demand schedules with parameters $(a, \hat{b}, \hat{c})$, the market-clearing price can be expressed as follows

$$p = \frac{\alpha + \beta\hat{b}}{1 + \beta\hat{c}} + \frac{\beta a}{1 + \beta\hat{c}}z,$$

where $z \equiv \theta + \omega$ is the endogenous public signal contained in p, with noise $\omega \equiv f(y)\eta - u/(\beta a)$ of

precision $\tau_\omega(a) \equiv \left(\beta^2 a^2 y\tau_u\tau_\eta\right) / \left(\beta^2 a^2 \tau_u + y\tau_\eta\right)$.[22] In turn, this implies that the equilibrium trades $x_i = as_i + \hat{b} - \hat{c}p$ can be expressed as affine functions $x_i = as_i + b + cz$ of the traders' exogenous private information s_i and the endogenous public information z contained in the market-clearing price, with

$$b = \hat{b} - \hat{c}\frac{\alpha + \beta\hat{b}}{1 + \beta\hat{c}} \tag{16}$$

and

$$c = -\frac{\beta a\hat{c}}{1 + \beta\hat{c}}. \tag{17}$$

For each vector $(a, \hat{b}, \hat{c})$ describing the demand schedules, there exists a unique vector (a, b, c) describing the equilibrium trades and vice versa. Hereafter, we find it more convenient to characterize the equilibrium use of information in terms of the vector (a, b, c) describing the equilibrium trades. Replacing $\tilde{x} = \int x_i di = (a+c)z + u/\beta + b$ into the expression for the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$, we then have that the equilibrium price can be expressed as follows:

$$p = \alpha + \beta b + \beta(a + c)z. \tag{18}$$

Using standard projection formulas, we then have that

$$\mathbb{E}[\theta|I_i, p] = \mathbb{E}[\theta|s_i, z] = \gamma_1(\tau_\omega(a))s_i + \gamma_2(\tau_\omega(a))z$$

where, for any τ_ω,

$$\gamma_1(\tau_\omega) \equiv \frac{\tau_\epsilon y\tau_\eta (y\tau_\eta - \tau_\omega)}{y^2\tau_\eta^2 (\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega\tau_\epsilon (\tau_\theta + 2y\tau_\eta)} \tag{19}$$

and

$$\gamma_2(\tau_\omega) \equiv \left(1 - \gamma_1(\tau_\omega)\frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\omega}{\tau_\omega + \tau_\theta}. \tag{20}$$

Optimality requires that the equilibrium trades satisfy $x_i = (\mathbb{E}[\theta|s_i, z] - p)/\lambda$, which, together with the results above, is equivalent to

$$x_i = \frac{1}{\lambda} \left[\gamma_1(\tau_\omega(a))s_i - (\alpha + \beta b) + (\gamma_2(\tau_\omega(a)) - \beta(a + c)) z\right].$$

The sensitivity a^* of the equilibrium demand schedules to the traders' private information must thus satisfy $a = \gamma_1(\tau_\omega(a))/\lambda$, which is equivalent to equation (9) in the main text.

---

[22]To derive $\tau_\omega(a)$ we use the fact that $f(y) = 1/\sqrt{y}$.

The sensitivity of the equilibrium trades to the endogenous public signal must satisfy

$$c = \frac{1}{\beta + \lambda} \left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a \right]. \tag{21}$$

The constant b in the equilibrium trades must satisfy

$$b = -\frac{\alpha}{\beta + \lambda}. \tag{22}$$

Inverting the relationship between b and $\hat{b}$ and c and $\hat{c}$ using (16) and (17), and using the formula for $\tau_\omega(a)$, we conclude that, given a^*, the values of $\hat{c}^*$ and $\hat{b}^*$ are given by the functions (7) and (8), as claimed in the proposition.

To complete the proof, it thus suffices to show that equation (9) admits a unique solution and that such a solution satisfies $0 < a^* < 1/\lambda$. To see this, use the fact that $\tau_\epsilon(y) = y\tau_e\tau_\eta/(\tau_e + \tau_\eta)$, along with the formula for $\tau_\omega(a)$, to observe that this equation is equivalent to

$$\lambda\beta^2 \tau_u \left(y\tau_\eta + \tau_\theta \right) a^3 + \lambda y \left[y\tau_e\tau_\eta + \tau_\theta(\tau_e + \tau_\eta) \right] a - \tau_e\tau_\eta y^2 = 0. \tag{23}$$

Clearly, because the left-hand side is strictly increasing in a, the above cubic equation has a unique real root, which is strictly positive. Furthermore, when $a = 1/\lambda$, the left-hand side is equal to

$$\frac{\beta^2 \tau_u}{\lambda^2} \left(y\tau_\eta + \tau_\theta \right) + \tau_\theta(\tau_e + \tau_\eta)y > 0.$$

We conclude that $a^* \in (0, 1/\lambda)$. Q.E.D.

Proof of Lemma 1. As explained in the proof of Proposition 1, when the traders submit demand schedules of the form $x_i = as_i + \hat{b} - \hat{c}p$, for some $(a, \hat{b}, \hat{c})$, the trades induced by market clearing can be expressed as $x_i = as_i + b + cz$, with the values of b and c given by (16) and (17). Using the fact that $\tilde{x} = a(\theta + f(y)\eta) + b + cz$, we thus have that ex-ante welfare can be expressed as follows:

$$\mathbb{E}[W] = \mathbb{E}\left[(\theta - \alpha + u)(a(\theta + f(y)\eta) + b + cz) - \tfrac{\beta}{2}(a(\theta + f(y)\eta) + b + cz)^2 - \tfrac{\lambda}{2}\int_0^1 (as_i + b + cz)^2 \, di \right].$$

Note that, given a, $\mathbb{E}[W]$ is concave in b and c . For any a, the optimal values of b and c are thus given by the FOCs $\partial\mathbb{E}[W]/\partial b = 0$ and $\partial\mathbb{E}[W]/\partial c = 0$. The values given by (21) and (22) solve these equations. Using (16) and (17) to go from the optimal trades to the demand schedules that implement them, we thus conclude that, for any choice of a, the optimal values of $\hat{c}^T$ and $\hat{b}^T$ are given by the functions (7) and (8), as claimed. Q.E.D.

Proof of Proposition 3. Observe that the function F given, for all a, by

$$\text{F}(a) = a - \frac{1}{\lambda\Lambda(\tau_\omega(a))}$$

is strictly increasing. Next, let F^T be the function given, for any a, by

$$\text{F}^T(a) = a - \frac{1}{\lambda}\frac{1}{\Lambda(\tau_\omega(a)) + \Delta(a) + \Xi(a)}.$$

Because Δ and Ξ are both increasing, F^T is also strictly increasing.

The first two equalities follow from the above monotonicities along with the fact that a^* solves $\text{F}(a^*) = 0$ whereas a^T solves $\text{F}^T(a^T) = 0$.

Next, consider the last two equalities. In the proof of Lemma 1, we established that, for any sensitivity a of the efficient trades to private information, the sensitivity of the efficient trades to the endogenous signal z contained in the market-clearing price is given by

$$c = \frac{1}{\beta + \lambda}\left[\left(1 - \lambda a - \lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right]$$

and coincides with the sensitivity of the equilibrium trades to z when the sensitivity of the equilibrium trades to private information is a. Using the formula for $\tau_\omega(a)$, we then have that $a + c > 0$. Now use Condition (17) to observe that

$$\hat{c} = -\frac{c}{\beta(a+c)}. \tag{24}$$

Because $a + c > 0$, we conclude that $sgn(\hat{c}) = -sgn(c)$. Combining this property with Condition (21), we conclude that

$$\hat{c} \stackrel{sgn}{=} \beta a - \left(1 - \lambda a - \lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}.$$

Next observe that

$$\Delta(a) + \Xi(a) = \frac{\beta\tau_\eta}{\lambda(y\tau_\eta - \tau_\omega(a))}\left(\frac{\tau_\omega(a) + \tau_\theta}{\tau_e} - \frac{\beta y^3\tau_\eta^2\tau_u\left(1 - \lambda a - \lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda\left(\beta^2 a^2\tau_u + y\tau_\eta\right)^2(\tau_\omega(a) + \tau_\theta)}\right). \tag{25}$$

Because $y\tau_\eta - \tau_\omega(a) > 0$,

$$\Delta(a) + \Xi(a) \stackrel{sgn}{=} \lambda\left(\beta^2 a^2\tau_u + y\tau_\eta\right)^2(\tau_\omega(a) + \tau_\theta)^2 - \tau_e\beta y^3\tau_\eta^2\tau_u\left(1 - \lambda a - \lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2.$$

It is then easy to see that $\Delta(a) + \Xi(a) \stackrel{sgn}{=} \hat{c}$. The above derivations hold no matter whether a is the sensitivity of the equilibrium schedules (equivalently, trades) to private information, or the sensitivity of the efficient schedules (equivalently, trades) to private information. Hence, $\hat{c}^* \stackrel{sgn}{=} \Xi(a^*) + \Delta(a^*)$

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and $\hat{c}^T \overset{sgn}{=} \Xi(a^T) + \Delta(a^T)$. Because $\Xi(a^*) + \Delta(a^*) \overset{sgn}{=} \Xi(a^T) + \Delta(a^T)$, the above results also imply that $\hat{c}^* \overset{sgn}{=} \hat{c}^T$. Q.E.D.

Proof of Proposition 2. Using Lemma 1, one can show that the welfare losses can be expressed as a function

$$
\begin{aligned}
WL(a, \tau_\omega(a)) &= \frac{\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2\left(\beta + \lambda\right)\tau_\omega(a)} + \frac{\lambda^2 a^2 + 2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{2\left(\beta + \lambda\right)y\tau_\eta} \\
&\quad + \frac{\left[1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2\left(\beta + \lambda\right)\tau_\theta} + \frac{\lambda a^2}{2y\tau_e}
\end{aligned} \tag{26}
$$

of a and $\tau_\omega(a)$. The socially optimal level of a must solve $dWL(a, \tau_\omega(a))/da = 0$ which, using the formula for $\tau_\omega(a)$, yields the condition in the proposition. One can also verify that, at $a = 1/\lambda$, $dWL(a, \tau_\omega(a))/da > 0$, whereas, at $a = 0$, $dWL(a, \tau_\omega(a))/da < 0$. Hence a^T must satisfy $0 < a^T < 1/\lambda$, as claimed. Q.E.D.

Derivation of Conditions (14) and (15). In the cursed economy, each trader receives a private signal $s_i = \theta + \underbrace{f(y)\eta + f(y)e_i}_{\equiv \epsilon_i}$ and a public signal $z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$, and believes p to be orthogonal to (θ, η). Following steps similar to those leading to Proposition 1, we have that $\mathbb{E}[\theta|s_i, z] = \bar{\gamma}_1 s_i + \gamma_2 z$, where

$$
\bar{\gamma}_1 \equiv \frac{\tau_\epsilon y\tau_\eta (y\tau_\eta - \tau_\zeta)}{y^2\tau_\eta^2(\tau_\zeta + \tau_\epsilon + \tau_\theta) - \tau_\zeta\tau_\epsilon(\tau_\theta + 2y\tau_\eta)} \text{ and } \bar{\gamma}_2 \equiv \left(1 - \bar{\gamma}_1 \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}.
$$

Because the cursed-equilibrium demand schedules $x_i = a_{exo}^* s_i + \hat{b}_{exo}^* - \hat{c}_{exo}^* p + \hat{d}_{exo}^* z$ must satisfy $x_i = (\mathbb{E}[\theta|s_i, z] - p)/\lambda$, we have that $a_{exo}^* = \bar{\gamma}_1/\lambda$, $\hat{b}_{exo}^* = 0$, $\hat{c}_{exo}^* = 1/\lambda$, and $\hat{d}_{exo}^* = \bar{\gamma}_2/\lambda$. Using the formula for $\bar{\gamma}_1$ above, we have that the formula for a_{exo}^* is equivalent to the one in (14) in the main text.

Now suppose that, given a, the planner is constrained to choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a and $(\hat{b}, \hat{c}, \hat{d})$ as between a_{exo}^* and $(\hat{b}_{exo}^*, \hat{c}_{exo}^*, \hat{d}_{exo}^*)$ in the cursed equilibrium. Following steps similar to those in the proof of Proposition 2, we then have that the value of a that minimizes the welfare must satisfy condition (15) in the main text. Q.E.D.

Proof of Proposition 4. Following the same steps as in the proof of Proposition 1, we have that the equilibrium value of a under the proposed policy is the unique solution to

$$
a = \frac{1}{\lambda + \delta} \frac{1}{\Lambda(\tau_\omega(a))}
$$

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whereas the values of b and c describing the equilibrium trades $x_i = as_i + b + cz$ are given by

$$b = \frac{t_0 - (1 + t_p)\alpha}{\lambda + \delta + (1 + t_p)\beta} \text{ and } c = \frac{\gamma_2(\tau_\omega(a)) - (1 + t_p)\beta a}{\lambda + \delta + (1 + t_p)\beta},$$

where γ_2 is the function defined in the proof of Proposition 1. Hence, the equilibrium trades under the proposed policy coincide with the efficient trades $x_i = a^T s_i + b^T + c^T z$ if and only if

$$\delta = \frac{\lambda \left[\Xi(a^T) + \Delta(a^T) \right]}{\Lambda(\tau_\omega(a^T))},$$

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda + \delta + \beta}{\beta + \lambda} \left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a \right] - \beta a^T}{\beta \left\{ \frac{1}{\beta + \lambda} \left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a \right] + a^T \right\}},$$

and

$$t_0 = (1 + t_p)\alpha - \frac{\alpha \left[\lambda + \delta + (1 + t_p)\beta \right]}{\beta + \lambda}.$$

Q.E.D.

Proof of Proposition 5. Following the same steps as in the proof of Proposition 1, one can show that, under the proposed policy, the equilibrium trades are given by $x_i = as_i + b + cz$ where a is given by the same value as in Proposition 1 whereas

$$b = -(1 + t_p)\frac{\alpha}{(1 + t_p)\beta + \lambda} \tag{27}$$

and

$$c = \frac{1}{\beta(1 + t_p) + \lambda} \left[\left(1 - \lambda a \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta} \right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - (1 + t_p)\beta a \right]. \tag{28}$$

Hence, any ad-valorem tax $t_p \neq 0$ induces the same sensitivity of the equilibrium trades to private information as in the laissez-faire equilibrium but different values of b and c. Because, given a^*, the values of b and c (equivalently, of $\hat{b}$ and $\hat{c}$) in the laissez-faire economy maximize welfare, as shown in Lemma (1), we conclude that any policy $t_p \neq 0$ results in strictly lower welfare than $t_p = 0$. Q.E.D.

Proof of Proposition 6. The proof is in four steps. Step 1 shows that, for any $y \in [0, +\infty)$, when all other agents acquire information of quality y and submit the equilibrium limit orders for information of quality y, each agent's net private marginal benefit $N(y)$ of increasing the quality of his information at $y_i = y$ (and then trade optimally) is a strictly decreasing function of y. Step 2 uses the result in step 1 to show that, when $\mathcal{C}'(0)$ is small enough, there is one, and only one, value of y for which $N(y) = 0$. Step 3 shows that, when the cost of information is sufficiently convex, then

if all other agents acquire information of quality y^* (where y^* is the unique solution to $N(y) = 0$) and then submit the equilibrium limit orders for information of quality y^*, the payoff $V^{\#}(y^*, y_i)$ that each agent obtains by acquiring information of quality y_i and then trading optimally is strictly quasi-concave in y_i. Jointly, the above properties establish the claim in the proposition.

Step 1. First observe that, when all other agents acquire information of quality y and then submit the equilibrium limit orders for information of quality y, the maximal payoff that agent i can obtain by acquiring information of quality y_i and then trading optimally is given by

$$V^{\#}(y, y_i) \equiv \sup_{g(\cdot)} \left\{ \mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))] - \mathcal{C}(y_i) \right\}$$

with

$$\mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))] \equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha + \beta b + \beta(a + c)z)\, g(s_i, z) - \frac{\lambda}{2}\, (g(s_i, z))^2\,; y_i\right],$$

where g is an arbitrary (measurable) function of the agent's private signal s_i and the public signal $z \equiv \theta + f(y)\eta - u/(\beta a)$ contained in the equilibrium price, with noise $\omega \equiv f(y)\eta - u/\beta a$ of precision $\tau_\omega(a) \equiv \beta^2(a)^2 y\tau_u\tau_\eta / \left(\beta^2(a)^2\tau_u + y\tau_\eta\right)$, describing the amount of the good traded by agent i under the limit orders he submits. Note that, in writing $\mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))]$, we used the fact that the relationship between z and the equilibrium price is given by $p = \alpha + \beta b + \beta(a + c)z$, where (a, b, c) are the coefficients describing the equilibrium trades when the quality of information is y and all agents submit the equilibrium limit orders for information of quality y – these coefficients are given by Conditions (9), (22), and (21) above. Also note that the dependence of $\mathbb{E}[\pi_i^{\#}(y, y_i; g(\cdot))]$ on y_i is through the fact that the agent's private signal is given by $s_i = \theta + f(y_i)(\eta + e_i)$. Using the envelope theorem, we then have that[23]

$$N(y) \equiv \left.\frac{\partial V^{\#}(y, y_i)}{\partial y_i}\right|_{y_i = y} = \frac{(\beta + \lambda)(a + c)\, a}{2\tau_\eta y^2} + \frac{\lambda a^2}{2y^2\tau_e} - \mathcal{C}'(y). \tag{29}$$

Next, use Conditions (5) and (21) to verify that $N(y) = F(a, y) - \mathcal{C}'(y)$, where

$$F(a, y) \equiv \frac{1}{2} a^2 \frac{a^2 \beta^2 \lambda \tau_u \tau_\theta + y\left[\lambda a^2 \beta^2 \tau_u \tau_\eta + \lambda(\tau_e + \tau_\eta)\tau_\theta + \beta^2 \tau_e \tau_u a\right]}{y^2 \tau_e \left[y\tau_\theta\tau_\eta + a^2\beta^2\tau_u\left(\tau_\theta + y\tau_\eta\right)\right]}. \tag{30}$$

As shown in the proof of Proposition 1, the equilibrium value of a (given y) is given by the unique real root to the cubic equation in (23). Equivalently, let $Z \equiv a/y$ and, for any (Z, y), let

$$R(Z, y) \equiv Z^3 y\beta^2 \lambda \tau_u\left(\tau_\theta + y\tau_\eta\right) + Z\lambda\left(\tau_e\tau_\theta + \tau_\theta\tau_\eta + y\tau_e\tau_\eta\right) - \tau_e\tau_\eta.$$

For any y, the equilibrium level of Z is given by the unique positive real solution to the equation

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[23]For the steps leading to the formula in (29), see the proof of Proposition 12 below, where we establish the result for an economy in which transactions are subject to an ad-valorem tax with rate t_p – the formula for the laissez-faire economy in (29) corresponds to the case in which $t_p = 0$.

$R(Z,y) = 0$, and is such that $Z < \tau_e/\lambda\tau_y$. Furthermore,

$$\frac{\partial}{\partial y}R(Z,y) = Z\lambda\left(\tau_e\tau_\eta + Z^2\beta^2\tau_u\tau_\theta + 2yZ^2\beta^2\tau_u\tau_\eta\right) > 0.$$

Now let $Z^*(y)$ be the equilibrium value of Z, given y. From the Implicit Function Theorem, we thus have that $Z^*(y)$ is decreasing in y.

Next, let $G(y) \equiv F(Z^*(y)y, y)$, where $F(a,y)$ is the function defined in Condition (30) above, and where we used the fact $a = Z^*(y)y$. After some algebra, one can show that

$$G(y) = \frac{1}{2}Z^*(y)\frac{\tau_e + yZ^*(y)\lambda\tau_\eta}{\tau_e(\tau_\theta + y\tau_\eta)}.$$

Note that

$$\frac{dG(y)}{dy} = \frac{1}{2}Z^*(y)\tau_\eta\frac{-\tau_e + Z^*(y)\lambda\tau_\theta}{\tau_e(\tau_\theta + y\tau_\eta)^2} + \frac{1}{2}\frac{\tau_e + 2yZ^*(y)\lambda\tau_\eta}{\tau_e(\tau_\theta + y\tau_\eta)}\frac{dZ^*(y)}{dy} < 0,$$

where the inequality follows from the fact that $Z^*(y) < \tau_e/\lambda\tau_y$ and $dZ^*(y)/dy < 0$. Because $N(y) = G(y) - \mathcal{C}'(y)$, we conclude that $N(y)$ is a strictly decreasing function of y.

Step 2. Next, consider the limit properties of $N(y)$. We have that

$$\lim_{y\to 0} N(y) = \frac{1}{2}\frac{\tau_e\tau_\eta}{\lambda\tau_\theta^2(\tau_e + \tau_\eta)} - \mathcal{C}'(0) \qquad \text{and} \qquad \lim_{y\to\infty} N(y) = -\lim_{y\to\infty}\mathcal{C}'(y) < 0.$$

Letting

$$L \equiv \frac{1}{2}\frac{\tau_e\tau_\eta}{\lambda\tau_\theta^2(\tau_e + \tau_\eta)},$$

we conclude that, when $\mathcal{C}'(0) < L$, there exists one, and only one, value of y for which $N(y) = 0$.

Step 3. Assume $\mathcal{C}'(0) < L$ and let y^* be the unique solution to $N(y) = 0$. Suppose that all other agents acquire information of quality y^* and then submit the equilibrium limit orders for information of quality y^*. Let (a^*, b^*, c^*) denote the coefficients describing the equilibrium trades under the equilibrium limit orders for information of quality y^* (these coefficients are given by Conditions (9), (22), and (21), applied to $y = y^*$). Let $\tau_\omega^* = \tau_\omega(a^*)$ denote the precision of the endogenous signal $z \equiv \theta + f(y^*)\eta - u/(\beta a^*)$ contained in the equilibrium price when all other agents acquire information of quality y^* and then submit the equilibrium limit orders for information of quality y^*.

We show that, when $\mathcal{C}$ is sufficiently convex, $V^\#(y^*, y_i)$ is strictly quasi-concave in y_i. To see this, first recall that optimality requires that, for any y_i, any (s_i, p), the trades that the agent induces through his limit orders given (s_i, p) are equal to

$$x_i = \frac{1}{\lambda}\left(\mathbb{E}[\theta|s_i, p; y_i] - p\right).$$

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Equivalently, for any y_i, the function $g^*(\cdot; y_i)$ that maximizes the agent's payoff $\mathbb{E}[\pi_i^\#(y^*, y_i; g(\cdot))] - \mathcal{C}(y_i)$ is such that, for any (s_i, z),

$$g^*(s_i, z; y_i) = \frac{1}{\lambda}\left(\mathbb{E}[\theta|s_i, z; y_i] - (\alpha + \beta b^*) - \beta(a^* + c^*)z\right),$$

where $\mathbb{E}[\theta|s_i, z; y_i] = \tilde{\gamma}_1(y_i)s_i + \tilde{\gamma}_2(y_i)z$, with

$$\tilde{\gamma}_1(y_i) \equiv \frac{\tau_e\tau_\eta\sqrt{y^*y_i}\left(\tau_\eta\sqrt{y^*y_i} - \tau_\omega^*\right)}{\tau_\eta y^*\left[\tau_e\tau_\eta y_i + (\tau_\omega^* + \tau_\theta)(\tau_e + \tau_\eta)\right] - \tau_\omega^*\tau_e\left(2\tau_\eta\sqrt{y^*y_i} + \tau_\theta\right)} \tag{31}$$

and

$$\tilde{\gamma}_2(y_i) \equiv \frac{\tau_\omega^*\tau_\eta\left[(\tau_e + \tau_\eta)y^* - \tau_e\sqrt{y^*y_i}\right]}{\tau_\eta y^*\left[\tau_e\tau_\eta y_i + (\tau_\omega^* + \tau_\theta)(\tau_e + \tau_\eta)\right] - \tau_\omega^*\tau_e\left(2\tau_\eta\sqrt{y^*y_i} + \tau_\theta\right)}, \tag{32}$$

In other words, for any y_i, the function $g^*(\cdot; y_i)$ is given by $g^*(s_i, z; y_i) = \tilde{a}(y_i)s_i + \tilde{b}(y_i) + \tilde{c}(y_i)z$, with $\tilde{a}(y_i) \equiv \tilde{\gamma}_1(y_i)/\lambda$, $\tilde{b}(y_i) \equiv -(\alpha + \beta b^*)/\lambda$, and $\tilde{c}(y_i) \equiv [\tilde{\gamma}_2(y_i) - \beta(a^* + c^*)]/\lambda$. Using the Envelope Theorem, we then have that

$$\frac{\partial V^\#(y^*, y_i)}{\partial y_i} = \frac{\partial \mathbb{E}[\pi_i^\#(y^*, y_i; g^*(\cdot; y_i))]}{\partial y_i} - \mathcal{C}'(y_i)$$
$$= -\tilde{a}(y_i)\tilde{\gamma}_2(y_i)f'(y_i)f(y^*)\frac{1}{\tau_\eta} - \lambda\tilde{a}(y_i)^2 f(y_i)f'(y_i)\left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right) - \mathcal{C}'(y_i).$$

We then have that

$$\begin{aligned}
\frac{\partial^2 V^\#(y^*, y_i)}{\partial y_i^2} &= -\tilde{a}'(y_i)\tilde{\gamma}_2(y_i)f'(y_i)f(y^*)\frac{1}{\tau_\eta}\\
&\quad -\tilde{a}(y_i)\frac{d\tilde{\gamma}_2(y_i)}{dy_i}f'(y_i)f(y^*)\frac{1}{\tau_\eta} - \tilde{a}(y_i)\tilde{\gamma}_2(y_i)f'(y_i)f(y^*)\frac{1}{\tau_\eta}\frac{f''(y_i)}{f'(y_i)}\\
&\quad -2\lambda\tilde{a}(y_i)\tilde{a}'(y_i)f(y_i)f'(y_i)\left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right)\\
&\quad -\lambda\tilde{a}(y_i)^2\left(f'(y_i)\right)^2\left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right) - \lambda\tilde{a}(y_i)^2 f(y_i)f'(y_i)\left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right)\frac{f''(y_i)}{f'(y_i)} - \mathcal{C}''(y_i).
\end{aligned}$$

Using the fact that $f(y) = 1/\sqrt{y}$ and $\tilde{a}(y_i) \equiv \tilde{\gamma}_1(y_i)/\lambda$ and letting $J : \mathbb{R}_+ \to \mathbb{R}$ be the function defined by

$$J(y_i) \equiv \frac{1}{\lambda}\left(\frac{1}{2y_i\sqrt{y_iy^*}}\frac{1}{\tau_\eta}\right)\frac{d}{dy_i}\{\tilde{\gamma}_1(y_i)\tilde{\gamma}_2(y_i)\} + \frac{1}{\lambda}\left[\frac{1}{2y_i\sqrt{y_i}}\left(\frac{1}{\tau_\eta} + \frac{1}{\tau_e}\right)\right]\frac{d}{dy_i}\left\{(\tilde{\gamma}_1(y_i))^2\frac{1}{\sqrt{y_i}}\right\},$$

we have that, at any y_i at which $\partial V^\#(y^*, y_i)/\partial y_i = 0$,

$$\frac{\partial^2 V^\#(y^*, y_i)}{\partial y_i^2} = J(y_i) - \frac{3}{2y_i}\mathcal{C}'(y_i) - \mathcal{C}''(y_i).$$

Computing all the derivatives, we can show that

$$J(y_i) = \frac{\tau_e \tau_\eta \sqrt{y^* y_i}}{4\lambda y_i^2 D^2(y_i)} \left\{ (\tau_e + \tau_\eta) y^* \tau_\eta \left[3\tau_\eta \sqrt{y_i y^*} - 2\tau_\omega^* \right] - 3\tau_\omega^* \tau_e \tau_\eta \sqrt{y^* y_i} + 2\tau_e (\tau_\omega^*)^2 \right\}$$

$$- \frac{\tau_e^2 \tau_\eta^2 y^* \left(\tau_\eta \sqrt{y^* y_i} - \tau_\omega^* \right)^2 \left[(\tau_e + \tau_\eta) y^* \tau_\eta - \tau_\omega^* \tau_e \right]}{\lambda y_i D^3(y_i)}$$

where, for any y_i,

$$D(y_i) \equiv \tau_\eta y^* \left[\tau_e \tau_\eta y_i + (\tau_\omega^* + \tau_\theta)(\tau_e + \tau_\eta) \right] - \tau_\omega^* \tau_e \left(2\tau_\eta \sqrt{y^* y_i} + \tau_\theta \right).$$

One can verify that $\lim_{y_i \to 0} J(y_i) = -\infty$, $\lim_{y_i \to +\infty} J(y_i) = 0$ and that $J(y_i)$ does not have vertical asymptotes. Hence it is bounded from above by a constant $M > 0$. Hence, when $\frac{3}{2y_i}\mathcal{C}'(y_i) + \mathcal{C}''(y_i) > M$ for all $y_i \geq 0$, the payoff is quasi-concave. Note that, when $\mathcal{C}(y) = \frac{B}{2}y^2$, the above condition becomes $B > \frac{2}{5}M$, which holds for B large enough, as claimed in the main text.

The above results imply that, under the conditions in the proposition, choosing quality of information $y_i = y^*$ and then submitting the limit orders defined by the coefficients $(a^*, \hat{b}^*, \hat{c}^*)$ in Proposition 1 (for quality of information y^*) is a symmetric equilibrium in the full game. That there are no other symmetric equilibria in affine strategies follows from the uniqueness of the solution to $N(y) = 0$ established in Step 2. Q.E.D.

Proof of Proposition 7 Let y^T denote the socially optimal precision of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the precision of private information is y^T. Next, let $\mathbb{E}[W^T; \bar{y}]$ denote ex-ante gross welfare when all traders acquire information of quality $\bar{y}$ but then submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Such a welfare function is gross of the costs of information acquisition. Finally, let $\mathbb{E}[\pi_i^T; y_i, \bar{y}]$ denote the ex-ante gross profit of a trader acquiring information of quality y_i when all other traders acquire information of quality $\bar{y}$, and all traders, including i, submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ mentioned above). The payoff is again gross of the cost of information acquisition. We start by establishing the following result:

Lemma Let y^T denote the socially optimal quality of private information and suppose that all traders submit the efficient demand schedules for information of quality y^T (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$). When $\hat{c}^T > 0$ (i.e., when the pecuniary externality dominates over the learning externality so that the efficient demand schedules are downward sloping), for any $\bar{y}$,

$$\left. \frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; y_i, \bar{y}] \right|_{y_i = \bar{y}} > \frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}]$$

whereas the opposite inequality holds when $\hat{c}^T < 0$ (i.e., when the learning externality dominates over the pecuniary externality and, as a result, the efficient demand schedules are upward sloping).

Proof of Lemma 6. First observe that, for any $(\bar{y}, y_i)$,

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}\left[(\theta - p(\theta, u, \eta; \bar{y})) X_i(\theta, u, \eta, e_i; \bar{y}, y_i) - \frac{\lambda}{2} X_i^2(\theta, u, \eta, e_i; \bar{y}, y_i)\right],$$

where

$$p(\theta, u, \eta; \bar{y}) = \alpha + \beta b^T + \beta(a^T + c^T) z(\theta, u, \eta; \bar{y})$$

and

$$X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T z(\theta, u, \eta; \bar{y}),$$

where b^T and c^T are the coefficients obtained from $(a^T, \hat{b}^T, \hat{c}^T)$ using the functions (16) and (17), and where $z(\theta, u, \eta; \bar{y}) \equiv \theta + f(\bar{y})\eta - u/\beta a^T$, with $f(y) = 1/\sqrt{y}$.[24] One can then show that

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; \bar{y}, y_i]\bigg|_{y_i = \bar{y}} = -f(\bar{y})f'(\bar{y})a^T \left[\lambda \frac{a^T}{\tau_e} + (\lambda + \beta)(a^T + c^T)\frac{1}{\tau_\eta}\right]. \tag{33}$$

Next observe that

$$\mathbb{E}[W^T; \bar{y}] = \mathbb{E}\left[(\theta - \alpha + u)\tilde{X}(\theta, u, \eta; \bar{y}) - \frac{\lambda}{2}\frac{\left(a^T f(\bar{y})\right)^2}{\tau_e} - \frac{\lambda + \beta}{2}\left(\tilde{X}(\theta, u, \eta; \bar{y})\right)^2\right]$$

from which we have that

$$\frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}] = -\frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda + \beta)\left(a^T + c^T\right)^2 f'(\bar{y})f(\bar{y})\frac{1}{\tau_n}. \tag{34}$$

Comparing (33) with (34), we thus have that, when $c^T < 0$,

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; \bar{y}, y_i]\bigg|_{y_i = \bar{y}} > \frac{d}{d\bar{y}} \mathbb{E}[W^T; \bar{y}],$$

whereas the opposite inequality holds when $c^T > 0$. Finally, use Condition (17) to observe that $\hat{c}^T = -\frac{c^T}{\beta(a^T + c^T)}$ and Condition (21), along with the formula for $\tau_\omega(a)$, to observe that $a^T + c^T > 0$. Jointly, the last two conditions imply that $sgn(\hat{c}^T) = -sgn(c^T)$ which completes the proof of the lemma. ∎

Next observe that $\mathbb{E}[\pi_i^T; \bar{y}, y_i]$ and $\mathbb{E}[W^T; \bar{y}]$ are globally concave in y_i and $\bar{y}$, respectively. Because $\mathbb{E}[\pi_i^T; \bar{y}, y_i]$ is strictly concave in y_i, in equilibrium, all traders acquire information of quality y^* such that

$$\frac{\partial}{\partial y_i} \mathbb{E}[\pi_i^T; \bar{y}, y_i]\bigg|_{y_i = \bar{y} = y^*} = \mathcal{C}'(y^*).$$

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[24]Observe that the functions (16) and (17) do not depend on y and hence c^T and b^T do not depend on y.

The socially-optimal quality of information satisfies

$$\frac{d}{d\bar{y}}\mathbb{E}[W^T;\bar{y}]\Big|_{\bar{y}=y^T} = \mathcal{C}'(y^T).$$

Because $\mathbb{E}[W^T;\bar{y}]$ is strictly concave in $\bar{y}$, the result in Lemma 6 implies that, when $\hat{c}^T > 0$, $y^T < y^*$, whereas, when $\hat{c}^T < 0$, $y^T > y^*$. Q.E.D.

Proof of Proposition 8. Under the proposed policy, each trader i's ex-ante gross expected payoff when all traders other than i collect information of quality $\bar{y}$, trader i collects information of quality y_i, and all traders (including i) submit the efficient demand schedules for information of quality y^T (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$) is equal to

$$\mathbb{E}[\hat{\pi}_i^T; \bar{y}, y_i, \hat{t}_p] = \mathbb{E}\left[\theta x_i - (1 + \hat{t}_p)(\alpha - u + \beta\tilde{x})x_i - \frac{\lambda}{2}x_i^2\right]$$

with

$$x_i = X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T\left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T}\right)$$

and

$$\tilde{x} = X(\theta, u, \eta; \bar{y}) = a^T(\theta + f(\bar{y})\eta) + b^T + c^T\left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T}\right),$$

where b^T and c^T are obtained from $\hat{b}^T$ and $\hat{c}^T$ using (16) and (17).[25] It follows that

$$\frac{\partial}{\partial y_i}\mathbb{E}[\hat{\pi}_i^T; \bar{y}, y_i, \hat{t}_p] = \frac{\beta(1 + \hat{t}_p)(a^T + c^T)a^T}{2\tau_\eta y_i\sqrt{\bar{y}y_i}} + \frac{\lambda a^T}{2\tau_\eta y_i\sqrt{y_i}}\left(\frac{a^T}{\sqrt{y_i}} + \frac{c^T}{\sqrt{\bar{y}}}\right) + \frac{\lambda(a^T)^2}{2y_i^2\tau_e}.$$

Because $\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] - \mathcal{C}(y_i)$ is concave in y_i, for $y_i = \bar{y} = y^T$ to be sustained in equilibrium, it is both necessary and sufficient that

$$\frac{\partial}{\partial y_i}\mathbb{E}[\hat{\pi}_i^T; \bar{y}, y_i, \hat{t}_p]\Big|_{y_i=\bar{y}=y^T} = \mathcal{C}'(y^T)$$

which holds if and only if

$$\hat{t}_p = \frac{\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T}{\beta a^T}$$

where $\gamma_2(\tau_\omega)$ is the function defined in the proof of Proposition 1. Q.E.D.

Proof of Proposition 9. Assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ for quality of information y^T). Given any policy $T(x_i, p)$, the expected net payoff for

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[25]Note that we used the fact that $p = \alpha - u + \beta X(\theta, u, \eta; \bar{y})$.

trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$V(y^T, y_i) \equiv \sup_{g(\cdot)} \left\{ \mathbb{E}[\pi_i; y^T, y_i, g(\cdot)] - \mathcal{C}(y_i) \right\}$$

where $g : \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and the endogenous signal $z = \theta + f(y^T)\eta - u/\beta a^T$ contained in the market-clearing price, with

$$
\begin{aligned}
\mathbb{E}[\pi_i; y^T, y_i, g(\cdot)] &\equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha - u + \beta \tilde{x}) g(s_i, z) - \tfrac{\lambda}{2} (g(s_i, z))^2 \right] \\
&\quad - \mathbb{E}\left[T\left(g(s_i, z), \alpha - u + \beta \tilde{x} \right) \right].
\end{aligned}
$$

Note that, in writing V, we used the fact that p and z are related by $p = \alpha + \beta b^T + \beta(a^T + c^T)z$, where b^T and c^T are obtained from $\hat{b}^T$ and $\hat{c}^T$ using (16) and (17).

For the policy $T(x_i, p)$ to implement the efficient acquisition and usage of information, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that $\mathbb{E}[\theta | s_i, z] = \gamma_1(\tau_\omega(a^T)) s_i + \gamma_2(\tau_\omega(a^T)) z$, where γ_1 and γ_2 are the functions defined in the proof of Proposition 1, we thus have that, for the policy T to implement the efficient trades, it must be that T is differentiable in x_i and satisfy

$$
\begin{aligned}
\tfrac{\partial}{\partial x_i} &T\left(a^T s_i + b^T + c^T z, \alpha + \beta b^T + \beta(a^T + c^T)z \right) \\
&= \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T \right] \tfrac{x - b^T}{a^T} \\
&\quad + \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - \left(\gamma_1(\tau_\omega(a^T)) - \lambda a^T \right) \tfrac{c^T}{a^T} \right] \tfrac{p - \alpha - \beta b^T}{\beta(a^T + c^T)} - \left(\alpha + \beta b^T + \lambda b^T \right)
\end{aligned}
$$

for all (s_i, z), where $\tfrac{\partial}{\partial x_i} T(x_i, p)$ is the partial derivative of the tax bill with respect to the individual volume of trade. This means that $T(x_i, p)$ is a polynomial of second order of the form

$$T(x_i, p) = \frac{\delta}{2} x_i^2 + \left(t_p p - t_0 \right) x_i + \tilde{K}(p), \tag{35}$$

for some vector (δ, t_p, t_0) and some function $\tilde{K}(p)$ which plays no role for incentives and which therefore we can disregard. In the proof of Proposition 4, we showed that there exists a unique vector (δ, t_p, t_0) that induces the traders to submit the efficient demand schedules when the precision of their private information is y^T (the vector in Proposition 4 applied to $y = y^T$). Thus, if a policy T induces efficiency in both information acquisition and information usage, it must be of the form in (35) with (δ, t_p, t_0) as in Proposition 4 applied to $y = y^T$. When the policy takes this form, for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = a s_i + b + c z$, for some (a, b, c). This implies that

$$\mathbb{E}[\pi_i; y^T, y_i, g(\cdot)] = \tilde{M} - \beta(1 + t_p)(a^T + c^T)a \frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda + \delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda + \delta}{2}\frac{a^2}{y_i\tau_\eta} - \frac{\lambda + \delta}{2}\frac{a^2}{y_i\tau_e},$$

where $\tilde{M}$ is a function of all terms that do not interact with y_i. The optimal g when $y_i = y^T$ is $g(x_i, z) = a^T s_i + b^T + c^T z$. Using the Envelope Theorem, we then have that

$$\frac{\partial}{\partial y_i} V(y^T, y_i) \bigg|_{y_i = y^T} = \frac{[\beta(1 + t_p) + \lambda + \delta] (a^T + c^T) a^T}{2\tau_\eta (y^T)^2} + \frac{(\lambda + \delta) (a^T)^2}{2\tau_e (y^T)^2} - \mathcal{C}'(y^T).$$

Because the efficient y^T solves

$$\frac{(\beta + \lambda)(a^T + c^T)^2}{2\tau_\eta (y^T)^2} + \frac{\lambda (a^T)^2}{2\tau_e (y^T)^2} = \mathcal{C}'(y^T), \tag{36}$$

we have that, for the policy to implement the efficient acquisition of private information, it must be that

$$(a^T + c^T)\tau_e \left[(\beta + \lambda)c^T - (\beta t_p + \delta)a^T\right] = \delta \left(a^T\right)^2 \tau_\eta.$$

One can verify that the values of δ and t_p from Proposition 4 do not solve the above equation except for a non-generic set of parameters. Q.E.D.

Proof of Proposition 10. When all other traders acquire information of precision y^T and submit the efficient demand schedules for information of quality y^T, the maximal payoff that trader i can obtain by acquiring information of precision y_i is equal to

$$\hat{V}(y^T, y_i) \equiv \sup_{a,b,c} \left\{ \mathbb{E}[\hat{\pi}_i; y^T, y_i, a, b, c] - \mathcal{C}(y_i) + A y_i \right\},$$

where

$$\mathbb{E}[\hat{\pi}_i; y^T, y_i, a, b, c] \equiv \bar{M} - \beta(1 + t_p)(a + c)a \frac{1}{\sqrt{y^T}\sqrt{y_i \tau_\eta}} - \frac{(\lambda + \delta)ca}{\sqrt{y^T}\sqrt{y_i \tau_\eta}} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_e},$$

where $\bar{M}$ collects all variables that do not interact with y_i. Note that, in writing $\hat{V}(y^T, y_i)$, we use the fact that, for any y_i, the trader's payoff is maximized by submitting an affine demand schedule which induces trades $x_i = a s_i + b + c z$ that are affine in (s_i, z), where $z = \theta + f(y^T)\eta - u/\beta a^T$ is the endogenous signal contained in the price. Using the envelope theorem, we have that

$$\frac{\partial}{\partial y_i} \hat{V}(y^T, y_i) \bigg|_{y_i = y^T} = \frac{[\beta(1 + t_p) + \lambda + \delta] (a^T + c^T) a^T}{2\tau_\eta (y^T)^2} + \frac{(\lambda + \delta) (a^T)^2}{2\tau_e (y^T)^2} - \mathcal{C}'(y^T) + A,$$

where we use the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to $a^T s_i + b^T + c^T z$. Using the fact that y^T satisfies Condition (36) along with Condition (17) to

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express c^T as a function of $\hat{c}^T$, we thus have that the proposed policy induces the efficient acquisition of private information only if

$$A = -\frac{\left(a^T\right)^2}{2\tau_\eta \left(y^T\right)^2}\left[\frac{\beta(\beta+\lambda)\hat{c}^T}{(1+\beta\hat{c}^T)^2} + \frac{\beta t_p + \delta}{1+\beta\hat{c}^T}\right] - \frac{\delta\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2}.$$

Next note that arguments similar to those in the proof of Proposition 6 imply that there exist constants $\tilde{L}, \tilde{M} \in \mathbb{R}_{++}$ such that, when $\mathcal{C}'(0) \leq \tilde{L}$ and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \tilde{M}$, the function $\hat{V}(y^T, y_i)$ is globally quasi-concave in y_i. We conclude that, when $\mathcal{C}$ satisfies these conditions, the proposed policy implements the efficient acquisition and usage of information. Q.E.D.

Proof of Proposition 11. Assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ for quality of information y^T). Given any policy $T(x_i, \tilde{x}, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$\tilde{V}(y^T, y_i) \equiv \sup_{g(\cdot)}\left\{\mathbb{E}[\tilde{\pi}_i; y^T, y_i, g(\cdot)] - \mathcal{C}(y_i)\right\}$$

where $g : \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and z, with $z \equiv \theta + f(y^T)\eta - u/(\beta a^T)$, and where

$$\begin{aligned}\mathbb{E}[\tilde{\pi}_i; y^T, y_i, g(\cdot)] &\equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha - u + \beta\tilde{x})g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2\right]\\ &\quad -\mathbb{E}\left[T\left(g(s_i, z), \tilde{x}, \alpha - u + \beta\tilde{x}\right)\right].\end{aligned}$$

For the policy $T(x_i, \tilde{x}, p)$ to induce efficiency in both information acquisition and usage, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that

$$\mathbb{E}\left[\theta|s_i, z; y_i, y^T\right]\big|_{y_i=y^T} = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z,$$

with the functions γ_1 and γ_2 as in Proposition 1, we thus have that T must be differentiable in x_i and satisfy

$$\begin{aligned}\tfrac{\partial}{\partial x_i}&\mathbb{E}\left[T\left(a^T s_i + b^T + c^T z, \tilde{x}, \alpha - u + \beta\tilde{x}\right)|s_i, z; y_i, y^T\right]\Big|_{y_i=y^T}\\ &= \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right)\end{aligned}$$

for all (s_i, z), where $\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$, with $z \equiv \theta + f(y^T)\eta - u/(\beta a^T)$. Next recall

from the proof of Proposition 9 that, when the individual trades efficiently,

$$\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - [\alpha + \beta b^T + \beta(a^T + c^T)z] - \lambda\left(a^T s_i + b^T + c^T z\right)$$
$$= \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right]\frac{x - b^T}{a^T} + \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - \left(\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right)\frac{c^T}{a^T}\right]\frac{p - \alpha - \beta b^T}{\beta(a^T + c^T)}$$
$$- \left(\alpha + \beta b^T + \lambda b^T\right).$$

This means that $T(x_i, \tilde{x}, p)$ must be a polynomial of second order of the form

$$T(x_i, \tilde{x}, p) = \frac{\delta'}{2}x_i^2 + \left(pt'_p - t'_0 + t_{\tilde{x}}\tilde{x}\right)x_i + L'(\tilde{x}, p), \tag{37}$$

for some vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$, where $L'(\tilde{x}, p)$ is a function that does not depend on x_i, plays no role for incentives, and hence can be disregarded. Furthermore, under any such a policy,

$$\frac{\partial}{\partial x_i}\mathbb{E}\left[T(x_i, \tilde{x}, p)\,|s_i, p; y_i, y^T\right]$$
$$= \delta' x_i + pt'_p - t'_0 + \frac{t_{\tilde{x}}}{\beta}(p - \alpha) + \frac{t_{\tilde{x}}}{\beta}A^\#(y_i, y^T)s_i + \frac{t_{\tilde{x}}}{\beta}B^\#(y_i, y^T)p + \frac{t_{\tilde{x}}}{\beta}C^\#(y_i, y^T),$$

where $A^\#(y_i, y^T)$, $B^\#(y_i, y^T)$, and $C^\#(y_i, y^T)$ are the coefficients of the projection

$$\mathbb{E}\left[u|s_i, p; y_i, y^T\right] = A^\#(y_i, y^T)s_i + B^\#(y_i, y^T)p + C^\#(y_i, y^T)$$

of u on (s_i, p). When trader i acquires information of quality $y_i = y^T$ and trades efficiently,

$$\frac{\partial}{\partial x_i}\mathbb{E}\left[T(x_i, \tilde{x}, p)\,|s_i, p; y^T, y^T\right] = \delta x_i + t_p p - t_0$$

where

$$\delta = \delta' + \frac{t_{\tilde{x}}}{\beta}\hat{A}^\#, \tag{38}$$

$$t_p = t'_p + t_{\tilde{x}}\frac{1 + \hat{B}^\#}{\beta}, \tag{39}$$

and

$$t_0 = t'_0 + t_{\tilde{x}}\frac{\alpha}{\beta} - \frac{t_{\tilde{x}}}{\beta}\hat{C}^\#, \tag{40}$$

with $\hat{A}^\# \equiv A^\#(y^T, y^T)/a^T$,

$$\hat{B}^\# \equiv \left[B^\#(y^T, y^T) - \frac{A^\#(y^T, y^T)c^T}{a^T\beta(a^T + c^T)}\right],$$

and

$$\hat{C}^\# \equiv C^\#(y^T, y^T) - \frac{A^\#(y^T, y^T)b^T}{a^T} + \frac{A^\#(y^T, y^T)c^T(\alpha + \beta b^T)}{a^T\beta(a^T + c^T)}.$$

In the proof of Proposition 4, we showed that, when agents acquire information of quality y^T, for

them to trade efficiently, the values of (δ, t_p, t_0) must coincide with those in Proposition 4 (applied to $y = y^T$). Thus, for the above policy to induce efficiency in both information acquisition and information usage, it must be that the vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$ satisfies Conditions (38)-(40) with (δ, t_p, t_0) given by the values determined in Proposition 4 applied to $y = y^T$. Note that, for any $t_{\tilde{x}}$, there exist unique values of (δ', t'_p, t'_0) that solve the above three conditions. Abusing notation, denote these values by $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$.

Next, note that, when the policy takes the form in (37), for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = a s_i + b + c z$, for some (a, b, c). This implies that

$$\mathbb{E}[\tilde{\pi}_i; y^T, y_i, g(\cdot)] = \hat{M} - \left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}}))\right] \frac{a(a^T + c^T)}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda + \delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_e},$$

where $\hat{M}$ is a function of all variables that do not interact with y_i. Using the envelope theorem, we then have that

$$\left.\frac{\partial}{\partial y_i} \tilde{V}(y^T, y_i)\right|_{y_i = y^T} = \frac{\left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}})) + \lambda + \delta\right](a^T + c^T)a^T}{2\tau_\eta (y^T)^2} + \frac{(\lambda + \delta)(a^T)^2}{2\tau_e (y^T)^2} - \mathcal{C}'(y^T).$$

Once again, in writing the above derivative, we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades $a^T s_i + b^T + c^T z$. Finally, recall that y^T is defined by Condition (36). Hence, for the above policy to induce efficiency in information acquisition, it must be that

$$\frac{(\beta + \lambda)(a^T + c^T)^2}{\tau_\eta} + \frac{\lambda(a^T)^2}{\tau_e} = \frac{\left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}})) + \lambda + \delta\right](a^T + c^T)a^T}{\tau_\eta} + \frac{(\lambda + \delta)(a^T)^2}{\tau_e}. \tag{41}$$

Using (39), we have that $t'_p(t_{\tilde{x}}) = t_p - t_{\tilde{x}}\left(1 + \hat{B}^\#\right)/\beta$, with t_p given by the unique value determined in Proposition 4 applied to $y = y^T$. Because the function $\tilde{H} : \mathbb{R} \to \mathbb{R}$ given by $\tilde{H}(t_{\tilde{x}}) \equiv t_{\tilde{x}} + \beta t'_p(t_{\tilde{x}}) = \beta t_p - t_{\tilde{x}}\hat{B}^\#$ is linear, there exists a (unique) value of $t_{\tilde{x}}$ that solves (41).

Following steps similar to those in the proof of Proposition 6, one can show that there exist scalars $\hat{L}, \hat{M} \in \mathbb{R}_{++}$ such that, when when $\mathcal{C}'(0) \leq \hat{L}$, and, for all y, $\frac{3}{2y}\mathcal{C}'(y) + \mathcal{C}''(y) > \hat{M}$,, the function $\tilde{V}(y^T, y_i)$ is globally quasi-concave in y_i. We conclude that, when $\mathcal{C}$ satisfies the above properties, the policy in (37), with $t_{\tilde{x}}$ given by the unique solution to (41) and with (δ', t'_p, t'_0) given by the unique solution $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$ to Conditions (38)-(40), induces efficiency in both information acquisition and information usage. Q.E.D.

Proof of Proposition 12. We establish the result by showing that the precision of private information y acquired in equilibrium is invariant in t_p. Once this property is established, the proposition follows from what established in the proof of Proposition 5.

Fix t_p, and, for any (y, y_i) denote by

$$V^\#(y, y_i) \equiv \sup_{g(\cdot)} \left\{ \mathbb{E}[\pi_i^\#; y, y_i, g(\cdot))] - \mathcal{C}(y_i) \right\}$$

the maximal payoff that trader i can obtain by selecting information of quality y_i when all other agents acquire information of quality y and then submit the equilibrium limit orders for information of quality y when the tax rate is t_p. The function $g : \mathbb{R}^2 \to \mathbb{R}$ specifies the amount of shares $x_i = g(s_i, z)$ the trader purchases as a function of s_i and the endogenous public signal z contained in the equilibrium price. Let (a, b, c) be the parameters defining the equilibrium trades when information is of quality y and the tax rate is t_p. We then have that [26]

$$\mathbb{E}[\pi_i^\#; y, y_i, g(\cdot))] \equiv \mathbb{E}\left[\theta g(s_i, z) - (1 + t_p)\left(\alpha + \beta b + \beta(a + c)z\right) g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2 |y_i \right].$$

Note that in writing $\mathbb{E}[\pi_i^\#; y, y_i, g(\cdot))]$ we used the fact that the equilibrium price is given by $p = \alpha + \beta b + \beta(a + c)z$ with $z = \theta + f(y)\eta - u/(\beta a)$. By the definition of equilibrium, if agent i acquires information of quality $y_i = y$, the limit orders that maximize his payoff must be the equilibrium ones (that is, the one corresponding to the coefficients (a, b, c)). The envelope theorem then implies that

$$N(y) \equiv \left. \frac{\partial V^\#(y, y_i)}{\partial y_i} \right|_{y_i = y} = \frac{\beta(1 + t_p)(a + c)a}{2\tau_\eta y^2} + \frac{\lambda a(a + c)}{2\tau_\eta y^2} + \frac{\lambda a^2}{2y^2 \tau_e} - \mathcal{C}'(y). \tag{42}$$

Hence, the equilibrium value of y, must satisfy $N(y) = 0$. Let $M^\#(t_p, a, c, y)$ denote the function defined by the right-hand-side of (42). Next, use the derivations in the proof of Proposition 5 to observe that, given (t_p, y), the equilibrium values of (a, b, c) are given by (9), (27), and (28). From the implicit function theorem we then have that

$$\frac{dy}{dt_p} = -\frac{\frac{\partial M^\#(t_p, a, c, y)}{\partial t_p} + \frac{\partial M^\#(t_p, a, c, y)}{\partial c}\frac{\partial c}{\partial t_p}}{\frac{\partial M^\#(t_p, a, c, y)}{\partial y} + \frac{\partial M^\#(t_p, a, c, y)}{\partial a}\frac{\partial a}{\partial y} + \frac{\partial M^\#(t_p, a, c, y)}{\partial c}\frac{\partial c}{\partial y}},$$

where we used the fact that, given any y, the equilibrium level of a is invariant in t_p. Note that $\partial c/\partial t_p$ is the partial derivative of the equilibrium level of c with respect to t_p, holding y constant, whereas $\partial a/\partial y$ and $\partial c/\partial y$ are the partial derivatives of the equilibrium levels of a and c with respect to y, holding t_p fixed. Because, for any y, $\frac{\partial}{\partial t_p} M^\#(t_p, a, c, y) = (\beta(a + c)a)/2\tau_\eta y^2$, $\frac{\partial}{\partial c} M^\#(t_p, a, c, y) = [\beta(1 + t_p) + \lambda]a/2\tau_\eta y^2$, and $\partial c/\partial t_p = -\beta(a + c)/[\beta(1 + t_p) + \lambda]$, we conclude that $dy/dt_p = 0$, as claimed. Q.E.D.

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[26] As above, given (a, b, c), the sensitivity of the equilibrium limit orders $\hat{c}$ to the price and the constant $\hat{b}$ in the equilibrium limit orders are obtained from (a, b, c) using (16) and (17).

7 Appendix 2: Market Orders

In this section, we show that, when traders are restricted to market orders, there is no inefficiency in either the collection or usage of information. As explained in the main text, that traders collect and use information efficiently when restricted to submitting market orders does not imply that welfare is higher under this microstructure than when traders can submit limit orders. This is because limit orders permit the traders to adjust their asset holding based on the information held by other traders and also respond to shocks to the supply (namely, to u). In particular, under the policies of Propositions 10 and 11, welfare is strictly higher when traders submit limit orders than when they are restricted to market orders. This is because these policies implement the efficient allocations, and the latter are obtained by maximizing welfare over a set of allocations that includes those that can be sustained with market orders.

7.1 Efficiency in usage

To start with, suppose that $y_i = y$ for all i, with y exogenous. In any symmetric equilibrium in which the price is affine in (θ, u, η), each trader's market order is an affine function of her private signal. That is,

$$x_i = as_i + b$$

for some scalars (a, b) that depend on the exogenous parameters of the model. Aggregate demand is then equal to

$$\tilde{x} = \int x_i di = a(\theta + f(y)\eta) + b.$$

Combining the above expression with the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$, we then have that the equilibrium price must satisfy

$$p = \alpha - u + \beta b + \beta a(\theta + f(y)\eta). \tag{43}$$

For each s_i, the equilibrium market order $x_i = as_i + b$ must maximize trader i's expected profits

$$\Pi_i = \mathbb{E}\left[(\theta - p)\, x_i - \lambda \frac{x_i^2}{2} | s_i; y_i\right] - \mathcal{C}(y_i),$$

where $x_i = a_i s_i + b$.

Following steps similar to those in the baseline model, we have that, for any s_i, the derivative of Π_i with respect to x_i, evaluated at $x_i = a_i s_i + b$, must be equal to zero, which yields[27]

---

[27]Note that $\mathbb{E}[u|s_i] = 0$.

$$\mathbb{E}\left[\theta|s_i\right] - \alpha - \beta b - \beta a \mathbb{E}\left[\theta + f(y)\eta|s_i\right] \;\; = \;\; \lambda(a s_i + b).$$

We conclude that the equilibrium value of b, which we denote by b^*, is equal to $b^* = -\alpha/(\beta + \lambda)$. To obtain the equilibrium value of a, which we denote by a^* we replace $\mathbb{E}\left[\theta|s_i\right] = \frac{\tau_\epsilon}{\tau_\epsilon + \tau_\theta} s_i$ and

$$\mathbb{E}\left[\eta|s_i\right] \frac{f(y)\frac{1}{\tau_\eta}\tau_\theta\tau_\epsilon}{\tau_\epsilon + \tau_\theta} s_i$$

into the above first-order condition from which we obtain that

$$a^* \;\; = \;\; \frac{\tau_\epsilon}{\lambda\left(\tau_\epsilon + \tau_\theta\right) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}}.$$

Next, we can derive the expression for the welfare losses. When the market orders are affine with coefficients a and b,

$$x_i - \tilde{x} \;\; = \;\; a(s_i - \theta - f(y)\eta)$$

from which we obtain that

$$\mathbb{E}[(x_i - \tilde{x})^2] = \mathbb{E}[a^2 f(y)^2 e_i^2] = \frac{a^2}{y\tau_e},$$

as in the baseline model. Recall that the first-best action is $x^o = (\theta - \alpha + u)/(\beta + \lambda)$. One can then show that, for any (a, b), the welfare losses are equal to

$$WL = \frac{(\beta+\lambda)\mathbb{E}[(\tilde{x}-x^o)^2]+\lambda\mathbb{E}[(x_i-\tilde{x})^2]}{2} =$$

$$\frac{1}{2(\beta+\lambda)^2}\left(\frac{(\beta a+\lambda a-1)^2}{\tau_\theta} + \frac{(\beta+\lambda)^2 a^2}{y\tau_\eta} + \frac{1}{\tau_u} + b^2(\beta + \lambda)^2 + \alpha^2 + 2\alpha b(\beta + \lambda)\right).$$

For any a, the value of b that minimizes the welfare losses is thus given by the solution to

$$\frac{\partial WL}{\partial b} = b + \frac{\alpha}{\beta + \lambda} = 0.$$

We conclude that the optimal value of b is the equilibrium one: $b^T = b^* = -\alpha/(\beta+\lambda)$. Replacing the above value of b^T into the expression for the welfare losses, we have that the latter can be expressed as a function of a as follows

$$WL(a; y) = \frac{1}{2} \left(\frac{(\beta a + \lambda a - 1)^2}{(\beta + \lambda)\tau_\theta} + \frac{(\beta + \lambda)a^2}{y\tau_\eta} + \frac{1}{(\beta + \lambda)\tau_u} + \frac{\lambda a^2}{y\tau_e} \right).$$

Differentiating $WL(a; y)$ with respect to a and setting the derivative equal to zero, we have that the socially-optimal value of a, which we denote by a^T, must satisfy

$$\frac{\partial}{\partial a} WL(a^T; y) = \frac{(\beta a^T + \lambda a^A - 1)}{\tau_\theta} + \frac{(\beta + \lambda)a^T}{y\tau_\eta} + \frac{\lambda a^T}{y\tau_e} = 0,$$

from which we obtain that

$$a^T = \frac{\tau_\epsilon}{\lambda\tau_\epsilon + \beta\tau_\epsilon + \lambda\tau_\theta + \frac{\beta\tau_\epsilon\tau_\theta}{y\tau_\eta}} = a^*.$$

We thus conclude that there is no inefficiency in the usage of information when traders are restricted to market orders.

7.2 Efficiency in information acquisition

We now characterize the equilibrium acquisition of private information and show that it is efficient. When each trader $j \neq i$ chooses $y_j = y$ and then submits the equilibrium affine market order $x_j = as_j + b$ for quality of information y, and trader i instead acquires information of quality y_i and then, after observing s_i, submits the market order x_i, his expected payoff is equal to

$$\Pi_i = \mathbb{E}\left[(\theta - p)\, x_i - \lambda \frac{x_i^2}{2} | s_i, y_i \right] - \mathcal{C}(y_i),$$

where $p = \alpha - u + \beta\tilde{x}$, with $\tilde{x} = a(\theta + f(y)\eta) + b$, with

$$a = \frac{\tau_\epsilon}{\lambda\left(\tau_\epsilon + \tau_\theta\right) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}} \tag{44}$$

and $b = -\alpha/(\beta + \lambda)$, as shown above. For any (s_i, y_i), the optimal market order for trader i is given by the solution to the first-order condition with respect to x_i which yields $x_i = a_i s_i + b$, with

$$a_i = \frac{y_i\tau_e\tau_\eta(1 - \beta a(y)) - \beta a(y)\frac{\sqrt{y_i}}{\sqrt{y}}\tau_\theta\tau_e}{\lambda\left(y_i\tau_e\tau_\eta + \tau_\theta(\tau_e + \tau_\eta)\right)} \tag{45}$$

and $b = -\alpha/(\beta + \lambda)$. That is, for any (y, y_i), trader i's expected profits when all other traders acquire information of quality y and then submit the equilibrium market orders for quality of information y, and trader i instead acquires information of quality y_i and then submits the market order that

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maximizes his payoff (the one described above) is given by

$$
\begin{aligned}
\Pi_i(y, y_i) &= \mathbb{E}\left[(\theta - \alpha + u - \beta(a\theta + af(y)\eta + b))(a_i s_i + b) - \lambda\frac{(a_i s_i + b)^2}{2}; y, y_i\right] - \mathcal{C}(y_i) \\
&= \frac{a_i - \beta a a_i}{\tau_\theta} - \frac{\beta a a_i}{\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda a_i^2}{2}\left(\frac{1}{\tau_\theta} + \frac{1}{y_i \tau_\eta} + \frac{1}{y_i \tau_e}\right) - \mathcal{C}(y_i) - \alpha b + (1-\beta)b^2
\end{aligned}
$$

where we used the fact that $s_i = \theta + f(y_i)(\eta + e_i)$.

For any (y, y_i), let $a^*(y)$ and $a_i^*(y, y_i)$ be the value in (44) and (45), respectively. Using the Envelope Theorem, we then have that

$$
\frac{\partial}{\partial y_i}\Pi_i(y, y_i) = \frac{1}{2}\frac{\beta a^*(y)a_i^*(y, y_i)}{y_i\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda\left(a_i^*(y, y_i)\right)^2}{2}\left(-\frac{1}{y_i^2 \tau_\eta} - \frac{1}{y_i^2 \tau_e}\right) - \mathcal{C}'(y_i).
$$

When y is equal to the equilibrium level, which we denote by y^*, it must be that

$$
\frac{\partial}{\partial y_i}\Pi_i(y^*, y^*) = 0
$$

which, using the fact $a_i^*(y^*, y^*) = a^*(y^*)$, yields

$$
\mathcal{C}'(y^*) = \frac{1}{2}\left(\frac{(\beta + \lambda)(a(y^*))^2}{(y^*)^2 \tau_\eta} + \frac{\lambda(a(y^*))^2}{(y^*)^2 \tau_e}\right).
$$

Following steps similar to those in the proof of Proposition 6, one can show that when $\mathcal{C}'(0)$ is small and $\mathcal{C}$ is sufficiently convex, the above equation admits one and only one solution y^*, and individual payoffs $\Pi_i(y^*, \cdot)$ are quasi-concave in y_i, implying that an equilibrium in the full game exists and is unique within the family of equilibria in affine strategies.

Next, we characterize the socially-optimal value of y. Because for any y, the socially-optimal usage of information coincides with the equilibrium, as shown above, using the Envelope Theorem, we have that the optimal value of y, which we denote by y^T is given by the condition

$$
\frac{\partial}{\partial y}WL(a(y^T); y^T) = \frac{1}{2}\left(-\frac{(\beta + \lambda)\left(a^*(y^T)\right)^2}{\left(y^T\right)^2 \tau_\eta} - \frac{\lambda\left(a^*(y^T)\right)^2}{\left(y^T\right)^2 \tau_e}\right) + \mathcal{C}'(y^T) = 0.
$$

We conclude that the optimal value of y, which we denote by y^T, is given by the solution to the following condition

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$$C'(y^T) = \frac{1}{2}\left(\frac{(\beta + \lambda)\left(a^*(y^T)\right)^2}{(y^T)^2 \tau_\eta} + \frac{\lambda\left(a^*(y^T)\right)^2}{(y^T)^2 \tau_e}\right).$$

It is immediate to see that $y^T = y^*$, implying that the equilibrium of the laissez-faire economy is efficient. Q.E.D.