Investment Subsidies with Spillovers and Endogenous Private Information: Why Pigou Got it All Right

Online Supplement

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Abstract

This document contains three sections. Section S.1 establishes that the results for the alternative production economy discussed at the end of Section 2 in the main text (where firms produce intermediate goods in a "smart" or "traditional" specification) are equivalent to those for the version of the model analyzed in the rest of the paper. Section S.2 contains an extension to a family of economies in which the firms' managers, and hence the representative household, are risk averse with a diminishing marginal utility for the consumption of the final good. Section S.3 considers an alternative economy in which spillovers affect all firms, including the non-investing ones. It establishes that results similar to those in the main text obtain when firms set prices under dispersed information (nominal rigidities) under an appropriate monetary policy that induces firms to disregard their endogenous private information when setting prices and only use it for investment purposes. All numbered items in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text.

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S.1 Supply of Inputs in "Smart" and "Traditional" Specification

Consider the production economy discussed at the end of Section 2 in the main text. Each firm must choose whether to produce the intermediate good in a traditional or in a "smart" (Industry 4.0) specification. A smart specification improves the interoperability of the inputs used in the production of the final good. Specifically, the amount of intermediate good that each firm produces is equal to

$$y_i = l_i^{\psi}, \tag{S.1}$$

where $l_i \in \mathbb{R}_+$ denotes the amount of labor employed by firm i, and $\psi \leq 1$ the labor returns to scale. The cost of producing the intermediate good in its smart specification is k. This cost is over and above the cost of employing labor l_i. Let $n_i = 1$ (alternatively, $n_i = 0$) denote the decision by firm i to produce the good in the smart (alternatively, traditional) specification, and $N = \int n_i di$ the aggregate measure of firms producing goods in the smart specification. The amount of the final good produced is equal to

$$Y = \left(\int_i ((1 + \Theta (1 + \beta N) n_i) y_i)^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}}, \tag{S.2}$$

where $v > 1$ and $\beta \geq 0$ continue to denote the elasticity of substitution between intermediate goods, and the intensity of the investment spillovers, respectively. The "fundamental" variable $\Theta \in R_+$ captures the increase in productivity in the final good sector induced by the firms' investment decisions.

The final good is produced in a competitive retail sector, taking its price P and the prices $\{p_i\}_{i \in [0,1]}$ of all the intermediate goods as given. These prices naturally depend on whether the intermediate goods are supplied in their smart or traditional specification.

Lemma 1. To see that Lemma 1 in the main text holds under this alternative production function, note that, in each state θ, the amount of the final good produced is equal to (we drop the dependence of the various functions on π_x to ease the notation)

$$Y(\theta) = \left(N(\theta) \, y_1(\theta)^{\frac{v-1}{v}} (1 + \Theta (1 + \beta N(\theta)))^{\frac{v-1}{v}} + (1 - N(\theta)) \, y_0(\theta)^{\frac{v-1}{v}} \right)^{\frac{v}{v-1}}. \tag{S.3}$$

Because $C(\theta) = Y(\theta)$, using Condition (4) in the main text, we have that the consumption of the final good in each state θ is equal to

$$C(\theta) = \left(N(\theta) \, l_1(\theta)^{\psi \frac{v-1}{v}} (1 + \Theta (1 + \beta N(\theta)))^{\frac{v-1}{v}} + (1 - N(\theta)) \, l_0(\theta)^{\psi \frac{v-1}{v}} \right)^{\frac{v}{v-1}}, \tag{S.4}$$

which coincides with the expression under the specification of Subsection 2.1 – see Conditions (A.1), (A.2), and (A.3) in the proof of Lemma 1 in the main text. It is then easy to see that all the remaining steps in the proof of Lemma 1 in the main text characterizing the policies $l_1(\theta)$, $l_0(\theta)$, $N(\theta)$, and $n(x)$ that maximize welfare apply also to this alternative production economy.

Lemma 2. We start by characterizing the equilibrium price of the final good. Recall that the final good is produced in a competitive market in which profits are equal to

$$\Pi = PY - \int p_i y_i di,$$

where Y is given by Condition (S.3) above. Note that, for each intermediate input i, the price y_i naturally depends on whether the good is provided in its smart or traditional specification. Letting p_1 denote the price for the goods provided in the smart specification and p_0 the price for the goods provided in the traditional specification, we have that the first-order conditions for the maximization of Π yield

$$p_1 = P\left(\frac{y_1}{Y}\right)^{-\frac{1}{v}} \left(1 + \Theta\left(1 + \beta N\right)\right)^{\frac{v-1}{v}}, \qquad p_0 = P\left(\frac{y_0}{Y}\right)^{-\frac{1}{v}}, \tag{S.5}$$

where we dropped the arguments of all the functions to ease the notation. The demands for the intermediate goods supplied in their smart specification are then given by

$$y_1 = \left(1 + \Theta\left(1 + \beta N\right)\right)^{v-1} \left(\frac{p_1}{p_0}\right)^{-v} y_0,$$

Using Conditions (S.3) and (S.5) above, we thus have that the amount of the final good produced in each state θ is equal to

$$Y = \left(N\left(1 + \Theta\left(1 + \beta N\right)\right)^{v-1} p_1^{1-v} + \left(1 - N\right) p_0^{1-v}\right)^{\frac{v}{v-1}} y_0 p_0^v,$$

which in turn implies that the price of the final good is equal to

$$P = \left(N\left(1 + \Theta\left(1 + \beta N\right)\right)^{v-1} p_1^{1-v} + \left(1 - N\right) p_0^{1-v}\right)^{\frac{1}{1-v}}.$$

This condition is the analog of Condition (A.21) in the main text. Notice, however, that in this economy an increase in the productivity θ of the final good reduces the price of the latter P relative to that of the inputs p_0 and p_1. Note also that, because smart inputs (that is, goods provided in the smart specification) are more productive than traditional ones, their relative price in terms of the final good is larger (by a factor of $(1+\Theta(1+\beta N))^{\varphi(1-\psi)}$) than that of the traditional inputs. To verify that the optimal policies coincide with those in the main model, we show that the extra profit (net of the subsidy) $\mathcal{R}(\theta)$ that each firm makes by choosing the smart specification takes the same form as in the proof of Lemma 2 in the main text.

Given W and P, each firm providing its input in the smart specification chooses p_1 to maximize[1]

$$\frac{p_1 y_1 - W l_1}{P} + T_1\left(\frac{p_1 y_1}{P}\right), \tag{S.6}$$

where

$$y_1 = (1+\Theta(1+\beta N))^{v-1} C\left(\frac{p_1}{P}\right)^{-v}, \tag{S.7}$$

and $l_1 = y_1^{1/\psi}$. After some algebra, the first-order condition of the above maximization problem for p_1 yields

$$\frac{1-v}{v}\frac{y_1 p_1}{P} + \frac{1}{\psi}\frac{W}{P}l_1 + \frac{1-v}{v}\frac{dT_1(p_1 y_1/P)}{dr}\frac{y_1 p_1}{P} = 0, \tag{S.8}$$

which is the analog of (A.16) in the main text.

Next, use (S.4) and (S.7), along with the fact that efficiency requires that

$$\hat{l}_1 = (1+\Theta(1+\beta N))^{\varphi}\,\hat{l}_0$$

(as shown in the proof of Lemma 1 in the main text which, as argued above, is valid also for the alternative specification of the production process considered here) and that $\hat{y}_i = \hat{l}_i^{\psi}$, to verify that, in any equilibrium implementing the efficient allocation, firms must set prices equal to

$$\hat{p}_1 = \left(\left(\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi} - 1\right)\hat{N} + 1\right)^{\frac{1}{v-1}}\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi(1-\psi)}\hat{P}, \tag{S.9}$$

and

$$\hat{p}_0 = \left(\left(\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi} - 1\right)\hat{N} + 1\right)^{\frac{1}{v-1}}\hat{P} \tag{S.10}$$

with

$$\hat{P} = \left(\hat{N}\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{v-1}\hat{p}_1^{1-v} + \left(1-\hat{N}\right)\hat{p}_0^{1-v}\right)^{\frac{1}{1-v}} \tag{S.11}$$

Equilibrium in the labor market requires that $\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}$ where $\hat{L} = \hat{l}_1\hat{N} + \hat{l}_0(1-\hat{N})$. Furthermore,

[1] We drop π^x and θ from all the formulas to ease the notation.

efficiency requires that

$$-\psi \hat{C}^{\frac{1}{v}} \left(1+\Theta\left(1+\beta \hat{N}\right)\right)^{\frac{v-1}{v}} \hat{l}_1^{\psi \frac{v-1}{v}} + \hat{L}^{\varepsilon} \hat{l}_1 = 0. \tag{S.12}$$

This condition is the analog of Condition (A.7) in the main text (after using (A.2)).
Condition (S.8) then implies that T implements the efficient allocation only if

$$T_1(r) = \frac{1}{v-1} r + s, \tag{S.13}$$

and

$$T_0(r) = \frac{1}{v-1} r \tag{S.14}$$

exactly as in the economy of Section 2 in the main text (see the proof of Lemma 2 in the main text).
Using again (S.7) above, we have that

$$\frac{y_1 p_1}{P} = y_1^{\frac{v-1}{v}} Y^{\frac{1}{v}} \left(1+\Theta\left(1+\beta N\right)\right)^{\frac{v-1}{v}}.$$

Hence, when the labor market clears, the extra profit (net of the subsidy) from producing inputs in their smart specification relative to the profits of producing them in their traditional specification is equal to

$$\mathcal{R} = \left(\frac{v-\psi(v-1)}{v-1}\right) \hat{C}(\theta)^{\frac{1}{v}} \left(\left(1+\Theta\left(1+\beta \hat{N}\right)\right)^{\frac{v-1}{v}} \hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + s(\theta) - k.$$

Using Condition (S.3) above along with the fact that $y_i = l_i^{\psi}$, the above expression can be rewritten as

$$\begin{aligned} \mathcal{R} = & \left(\frac{v-\psi(v-1)}{v-1}\right) \left(\hat{N}(\theta)\left(1+\Theta\left(1+\beta \hat{N}\right)\right)^{\frac{v-1}{v}} \hat{l}_1(\theta)^{\psi \frac{v-1}{v}} + \left(1-\hat{N}(\theta)\right) \hat{l}_0(\theta)^{\psi \frac{v-1}{v}}\right)^{\frac{1}{v}} \times \\ & \times \left(\left(1+\Theta\left(1+\beta \hat{N}\right)\right) \hat{l}_1(\theta)^{\psi \frac{v-1}{v}} - \hat{l}_0(\theta)^{\psi \frac{v-1}{v}}\right) + s(\theta) - k. \end{aligned} \tag{S.15}$$

Finally, use Equations (1) and (2) in the main text to observe that the formula for $\mathcal{R}$ at the end of the proof of Lemma 2 in the main text coincides with the one in (S.15). Hence Lemma 2 continues to hold under the specification of the production process considered here.
That Lemmas 1 and 2 hold in this alternative economy implies that all the other results in Sections 3 and 4 in the main text hold verbatim also when the production functions is given by (S.1) and (S.2).

Corollary 1. The result follows from the same argument as in the proof in the main text.

Lemma 3. Part 1 follows from the results in Lemma 1. As for Part 2, we show that, when production is efficient, revenues coincide with those in the main text.
Using (S.4) and (S.10), and recalling that $\hat{y}_0 = \hat{l}_0^{\psi}$, we obtain that

$$\frac{\hat{p}_0}{\hat{P}}\hat{y}_0 = \left(\left(\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\hat{l}_0^{\psi}. \tag{S.16}$$

Recalling that $\hat{l}_1 = \left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi}\hat{l}_0$, $\hat{y}_1 = \left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi\psi}\hat{l}_0^{\psi}$, and $1+\frac{\varphi}{1-v}+\varphi\psi = \varphi$, and using (S.9), we also have that

$$\frac{\hat{p}_1}{\hat{P}}\hat{y}_1 = \left(\left(\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\hat{l}_0^{\psi}\left(1+\Theta\left(1+\beta\hat{N}\right)\right)^{\varphi}. \tag{S.17}$$

To see that the revenues in (S.16) and (S.17) coincide with those in the main text, it suffices to use (A.1)-(A.3) to rewrite (A. 27) in the main text.

Propositions 1 and 2. The results follow from the same arguments as in the main text.

S.2 Non-Linear Preferences in Consumption

Consider the following economy in which the production function is the same as in Subsection 2.1 in the main text, but the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. Consistently with the rest of the pertinent literature, we assume that each manager is a member of a representative household, whose utility function is given by

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di,$$

where $R \geq 0$ is the coefficient of relative risk aversion in the consumption of the final good (the case $R = 0$ corresponds to the setup of Section 2 in the main text). The assumption that all managers are members of the same representative household is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the managers to fully insure against idiosyncratic consumption risk. The latter property, in turn, isolates the frictions (and associated inefficiencies) that originate in the interaction between (a) investment spillovers and (b) endogenous private information at the time of the investment decisions from the more familiar inefficiencies that originate in the lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i = l$ for all i), which is a consequence of the assumption that labor is homogeneous and exchanged in a competitive market. Being a member of the representative household, each manager maximizes her firm's market valuation taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each manager maximizes

$$\mathbb{E}\left[C^{-R}\left(\frac{p_i y_i - W l_i}{P} + T\right)\middle| x_i; \pi_i^x\right] - k n_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consumption of the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open (but after firms make their investment and pricing decisions). The household faces a cash-in-advance constraint according to which the maximal expenditure on the purchase of the final good cannot exceed M, that is, $PY \leq M$. The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The government maximizes the ex-ante utility of the representative household, which is given by

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x),$$

by means of a monetary policy $M(\cdot)$ and a fiscal policy $T(\cdot)$, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in Section 2 of the main text (note, in particular, that prices are set under dispersed information about θ, that is, each p_i is based on x_i instead of θ). This richer economy is consistent with most of the assumptions typically made in the pertinent literature.

S.2.1 Efficient Allocation

The following proposition characterizes the efficient allocation in this economy.

Proposition S.1. *(1) Let $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$ and $\bar{R} \equiv \frac{1+\varepsilon-\psi(v-1)\varepsilon}{(1+\varepsilon-\psi)v+\psi}$. Assume that , $v < 1 + \frac{1+\varepsilon}{\psi\varepsilon}$, and $0 \leq R \leq \bar{R}$. For any precision of private information π^x, there exists a threshold $\hat{x}(\pi^x)$ such that efficiency requires that $\hat{n}\,(x; \pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. The threshold $\hat{x}(\pi^x)$, along with* *the functions* $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$, *satisfy the following properties:*

$$\mathbb{E}\left[\left(\left(\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1-vR}{v-1}}\hat{l}_0(\theta;\pi^x)^{\psi(1-R)}\times\right.$$
$$\left.\times\left(\frac{\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}-1}{\varphi}+\frac{\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}\right)\right]-k$$

$$\hat{N}(\theta;\pi^x)=1-\Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\left(\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \tag{S.18}$$

and

$$\hat{l}_1(\theta;\pi^x)=\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}\hat{l}_0(\theta;\pi^x), \tag{S.19}$$

where $\Theta\equiv\exp(\theta)$.

(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[\frac{v\hat{C}(\theta;\pi^x)^{\frac{1-Rv}{v}}}{v-1}\left(\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}\left(\frac{\varphi\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}+1\right)-1\right)\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[\begin{array}{c}\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}\left(\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\Big)^{\varepsilon}\times\\ \times\left(\left(1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)\right)^{\varphi}\left(\varphi\frac{\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}+1\right)-1\right)\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\end{array}\right]+$$
$$-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi_x)}{d\pi_x}.$$

Proof. See Appendix S.1 in this document.

The restriction $0\leq R\leq\bar{R}$ guarantees that the marginal utility of consuming the final good does not decrease 'too quickly' with C. Along with the other restrictions in the proposition, which are the same as in Lemma 1 in the main text, this property implies that the efficient investment strategy is monotone. When, instead, $R>\bar{R}$, a higher value of θ may entail a low enough marginal utility of consumption to induce the planner to ask some firms receiving a high signal to refrain from investing. As we clarify below, our key results extend to this case, but the exposition is less transparent.

S.2.2 Equilibrium Allocation

Firms make their investment decisions under dispersed information about θ. Given these choices, they acquire labor l to meet their demands, after observing θ and aggregate investment N. In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by the same conditions as in the main text. Likewise for the labor demands. Because labor is undifferentiated and the labor market is competitive, the supply of labor is then given by

$$\frac{W}{P}C^{-R} = l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P units, starting from a level of consumption equal to C. Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R} = \left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and labor demand, respectively, of each investing firm. The corresponding functions for the firms that do not invest are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$.[2]

The above equilibrium conditions are standard. The following definition identifies the components of the equilibrium allocation that are most relevant for our analysis.

Definition S.1. Given the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then invests according to $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

The following definition clarifies what it means that $T(\cdot)$ is optimal.

Definition S.2. The fiscal policy $T^*(\cdot)$ is **optimal** if it implements the efficient acquisition and usage of information as an equilibrium. That is, if it induces all firms to choose the efficient precision of information π^{x*}, follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and set prices according to rules $\hat{p}_0(x;\pi^{x*})$ and $\hat{p}_1(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to $\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$ and result in firms employing labor according to the efficient schedules $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

[2] As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ, the measure of investing firms N depends on the precision π^x of firms' information.

The following proposition establishes the optimal fiscal policy.

Proposition S.2. *Irrespective of whether the economy satisfies the conditions in Proposition S.1, the fiscal policy*

$$T_0^*(r) = \frac{1}{v-1} r,$$

and

$$T_1^*(\theta, r) = \frac{\Theta \beta \hat{N}(\theta; \pi^{x*})}{1 + \Theta\left(1 + \beta \hat{N}(\theta; \pi^{x*})\right)} \hat{C}(\theta; \pi^{x*})^{\frac{1}{v}} \hat{y}_1(\theta; \pi^{x*})^{\frac{v-1}{v}} + \frac{1}{v-1} r,$$

is optimal.

Proof. See Appendix S.1 in this document.

The fiscal policy in the proposition guarantees that, if firms were constrained to acquire information of precision π^{x*}, they would follow the efficient rule $\hat{n}(x; \pi^{x*})$ to make their investment decisions and then set prices $\hat{p}_0(x; \pi^x)$ and $\hat{p}_1(x; \pi^x)$ that induce the efficient labor demands, and hence the efficient production of the intermediate and final goods. This is accomplished through a fiscal policy that, in addition to offsetting firms' market power with a familiar revenue subsidy $r/(v-1)$, realigns the private value of investing with the social value through an additional subsidy to the investing firms that operates as a Pigouvian correction. As in the baseline economy, the subsidy

$$s(\theta) = \hat{C}(\theta; \pi^{x*})^{\frac{1}{v}} \hat{y}_1(\theta; \pi^{x*})^{\frac{v-1}{v}} \frac{\Theta \beta \hat{N}(\theta; \pi^{x*})}{1 + \Theta\left(1 + \beta \hat{N}(\theta; \pi^{x*})\right)}$$

makes each firm internalize the marginal effect of investment on the production of the final good, in each state θ. Once this realignment is established, the value that firms assign to acquiring information coincides with its social counterpart, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

S.3 Spillovers Affecting also Non-investing Firms, Price Rigidity, and Optimal Monetary Policy

S.3.1 The economy

Consider an economy in which the technology governing the production of the intermediate goods is given by

$$y_i = \begin{cases} \gamma\Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 1 \\ \Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 0 \end{cases}, \tag{S.20}$$

with $\gamma > 1$, $\beta \geq 0$, $\alpha \geq 0$, and $\psi \leq 1$. That $\gamma > 1$ reflects the property that investment increases the amount of the good produced. The parameters α and β control for the returns to scale and the intensity of the production spillovers, respectively. Finally, the parameter ψ controls for the returns to scale of labor. Under this specification, aggregate investment N affects equally investing and non-investing firms. That non-investing firms benefit from the investment spillovers reflects the idea that investment in new technologies (e.g., AI) typically comes with the development of knowledge, auxiliary products and services (such as AI-based software) useful to all firms, including those retaining the old technologies.

Further assume that firms set prices under their endogenous private information before observing the realization of the fundamental variable θ. Such nominal rigidities introduce a role for monetary policy, in the spirit of Correia, Nicolini, and Teles (2008), and Angeletos and La'O (2020). The purpose of the extension is twofold: it permits us to investigate the extent to which the insights are robust to the introduction of nominal rigidities; it also permits us to investigate how monetary and fiscal policy must be combined to incentivize firms to acquire and use information efficiently in the presence of investment spillovers.

To capture the role of these nominal rigidities in the simplest possible terms, we introduce a cash-in-advance constraint. The government provides the representative household with an amount of money M, and the maximal expenditure on the purchase of the final good cannot exceed M, that is

$$PY \leq M.$$

The timing of events is the same as in the main text, with the exception that prices are set under dispersed information about θ (i.e., with each p_i based on x_i instead of θ), and that the supply of money is state-dependent and governed by a monetary policy $M(\cdot)$. Each firm knows the monetary policy but does not observe the realized money supply $M(\theta)$ at the time it sets the price for its intermediate good. This economy is consistent with most of the assumptions typically made in the pertinent macroeconomic literature.

The presence of price rigidities has no implications for the efficient allocation, which continues to be characterized by the conditions in the proof of Lemmas 1 and 3 in the main text. The analysis of the equilibrium allocation, instead, must be amended to account for price rigidity. In this economy, the demands for the intermediate products, as well as the labor demands, continue to satisfy the same conditions as in the main text.

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and employment, respectively, of each investing firm. The corresponding functions for the non-investing firms are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$. Because prices are set under (endogenous) imperfect information about θ, the firms' labor demands $l_1(x,\theta;\pi^x)$ and $l_0(x,\theta;\pi^x)$ depend not only on θ and π^x but also on x.

Definition S.3. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$, and a pair of price functions $p_1(x;\pi^x)$ and $p_0(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then invests according to $n(x;\pi^x)$ and sets its price according to $p_1(x;\pi^x)$ and $p_0(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

As in the main text, the above equilibrium definition abstracts from other conditions (for wages, labor demand and supply, price of the final good) that are standard to isolate the novel and most relevant parts.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition S.4. The monetary policy $M^*(\cdot)$ and the fiscal policy $T^*(\cdot)$ are **optimal** if, jointly, they implement the efficient acquisition and usage of information as an equilibrium. They induce all firms to (1) acquire information of precision π^{x*}, (2) follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and (3) set prices (under dispersed information) according to rules $\hat{p}_1(x;\pi^{x*})$ and $\hat{p}_0(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to the efficient levels $\hat{y}_1(\theta;\pi^{x*})$ and $\hat{y}_0(\theta;\pi^{x*})$ and hence result in firms employing labor according to the efficient rules $\hat{l}_1(\theta;\pi^{x*})$ and $\hat{l}_0(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta;\pi^x)$ denote the amount of money supplied to the representative household in state θ when all firms are expected to acquire information of precision π^x. The policy $\hat{M}(\cdot;\pi^x)$ is designed so that, when all firms make their investment decisions according to the efficient rule $\hat{n}(x;\pi^x)$ and set prices according to $\hat{p}_1(x;\pi^x)$ and $\hat{p}_0(x;\pi^x)$, the resulting employment decisions coincide with the efficient ones $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ for an economy with private information of precision π^x.

The following lemma characterizes the monetary policy $\hat{M}(\cdot;\pi^x)$.

Lemma S.1. *Assume that the precision of private information is exogenously fixed at* π^x *for all firms. Any monetary policy* $\hat{M}(\cdot;\pi^x)$ *that, together with some fiscal policy* $\hat{T}(\cdot;\pi^x)$*, implements the efficient use of information as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}}$$

for all θ*, where* m *is an arbitrary positive constant. The monetary policy* $\hat{M}(\cdot;\pi^x)$ *induces all firms making the same investment decision to set the same price, irrespective of their information about* θ*.*

Proof. See Appendix S.2 in this document.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ implements the efficient allocation by inducing firms to disregard their private information about the aggregate economic conditions (the fundamental variable θ) when setting their prices, and condition the latter only on their investment decision. That prices do not respond to firms' information about θ, given their investments, is necessary to avoid allocative distortions in the induced employment and production decisions. In fact, given the firms' investments, relative prices must not vary with firms' signals about θ when the latter are imprecise. The monetary policy in Lemma S.1 is designed so that, even if firms could condition their prices on θ, thus bypassing the nominal rigidity, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supply in a way that replicates the same allocations sustained when money is constant and prices are flexible.

The result in Lemma S.1 may suggest that the monetary authority needs to know the cost of information to compute the optimal money supply in each state θ. However, as anticipated above, this is not the case. In fact, it suffices that the authority observes the cross-sectional distribution of employment and investment decisions for it to be able to compute the amount of money that needs to be supplied.

Lemma S.1 in turn permits us to verify that results analogous to those in the main text apply to the economy with price rigidities under consideration.

Appendix S.2 in this document also shows that, when information is exogenous and of precision π^x, any fiscal policy that induces efficiency in information usage must induce firms to set prices that, given the firms' investments, are invariant in the firms' signals. The only policies that satisfy this property take the form $T_0\left(r\right) = r/(v-1)$ and $T_1\left(r,\theta;\pi^x\right) = r/(v-1)+s(\theta;\pi^x)$. It then shows that, under any such fiscal policy, when the monetary policy is the one in Lemma S.1, all firms have incentives to set prices that induce them to hire the efficient amount of labor in each state. Building on these observations, the proof of Lemma S.1 then shows that, when the monetary policy takes the form in the lemma, the net private benefit that each firm with signal x expects from investing continues to be given by $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$, as in the case of flexible prices. This property, in turn, implies that the extra subsidy $s(\theta;\pi^x)$ to the investing firms must satisfy conditions analogous to those in Lemma 2 in the main text and, when information is endogenous, an additional condition analogous to the one in Lemma 3.

The above result in turn implies that a Pigouvian fiscal policy analogous to the one in Proposition 1 in the main text, in which the extra subsidy to the investing firms is equal to

$$s(\theta;\pi^{x*}) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})},$$

when paired with the monetary policy of Lemma S.1 (specialized to $\pi^x = \pi^{x*}$), continues to realign the private value from investing with its social counterpart, state by state. Once this realignment is established, the value that firms assign to information acquisition coincides with the social value, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same, as in the economy with flexible prices. Similar arguments imply that when the fiscal or monetary authorities do not know the cost of information acquisition, it remains possible to implement the efficient acquisition and usage of information but it becomes necessary to expand the contingencies in the policies, by conditioning on the cross-sectional distribution of firms' investment and employment decisions.

Appendix S.1: Proofs of the Results in Section S.2

Proof of Proposition S.1. The proof is in two parts, each corresponding to the two claims in the proposition.

Part 1. Fix the precision of private information π^x and then drop it from all expressions to ease the notation. Let $n(x)$ denote the probability that a firm receiving signal x invests, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the investing firms and by those deciding not to invest, respectively. The planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R} d\Omega(\theta) - k\int_\theta N(\theta) d\Omega(\theta) + \\ - \frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) + \\ - \int_\theta \mathcal{Q}(\theta)\left(N(\theta) - \int_x n(x)\, d\Phi(x|\theta)\right) d\Omega(\theta),$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}, \tag{S.21}$$

with

$$y_1(\theta) = A(\Theta, N(\theta))\, l_1(\theta)^{\psi}, \tag{S.22}$$

where we let $A(\Theta, N(\theta)) \equiv 1 + \Theta(1+\beta N(\theta))$ for convenience, and

$$y_0(\theta) = l_0(\theta)^{\psi}. \tag{S.23}$$

Using (S.21) and (S.22), the first-order condition of the planner's problem with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R}\left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}} A(\Theta, N(\theta))^{\frac{v-1}{v}}\, l_1(\theta)^{\psi\frac{v-1}{v}-1} \\ - \left(l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right)^{\varepsilon} = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta)), \tag{S.24}$$

and using (S.21), (S.22), and (S.23), we have that the above first-order condition reduces to

$$\psi C(\theta)^{\frac{1-vR}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta) L(\theta)^{\varepsilon}. \tag{S.25}$$

Following similar steps, the first-order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1-vR}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon}. \tag{S.26}$$

Using (S.22) and (S.23), the ratio between (S.25) and (S.26) can be written as

$$A(\Theta, N(\theta))^{\frac{v-1}{v}} \left(\frac{l_1(\theta)}{l_0(\theta)}\right)^{\psi \frac{v-1}{v}} = \frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta) = A(\Theta, N(\theta))^{\varphi} l_0(\theta). \tag{S.27}$$

Notice that (S.27) entails that, at the efficient allocation, the total labor demand, as defined in (S.24), is equal to

$$L(\theta) = l_0(\theta)\left(\left(A(\Theta, N(\theta))^{\varphi} - 1\right) N(\theta) + 1\right). \tag{S.28}$$

Using (S.22) and (S.23), we can also write aggregate consumption as

$$C(\theta) = \left(A(\Theta, N(\theta))^{\frac{v-1}{v}} l_1(\theta)^{\psi \frac{v-1}{v}} N(\theta) + l_0(\theta)^{\psi \frac{v-1}{v}} (1 - N(\theta))\right)^{\frac{v}{v-1}}.$$

Using (S.27), and the fact that

$$\frac{v-1}{v}(1 + \varphi\psi) = \varphi, \tag{S.29}$$

we can rewrite the latter expression as

$$C(\theta) = l_0(\theta)^{\psi} \left(\left(A(\Theta, N(\theta))^{\varphi} - 1\right) N(\theta) + 1\right)^{\frac{v}{v-1}}. \tag{S.30}$$

Next, use (S.27) and (S.23) to rewrite (S.26) as

$$\psi l_0(\theta)^{\psi \frac{1-vR}{v}} \left(\left(A(\Theta, N(\theta))^{\varphi} - 1\right) N(\theta) + 1\right)^{\frac{1-vR}{v-1}} l_0(\theta)^{\psi \frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon},$$

which, using (S.28), can be expressed as

$$\psi\left(\left(A\left(\Theta,N\left(\theta\right)\right)^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}l_0(\theta)^{\psi(1-R)}$$
$$=l_0(\theta)^{1+\varepsilon}\left(\left(A\left(\Theta,N\left(\theta\right)\right)^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\varepsilon}.$$

From the derivations above, we have that the efficient labor demands are given by

$$l_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\left(A\left(\Theta,N\left(\theta\right)\right)^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \tag{S.31}$$

and by (S.27).

Note that $l_0(\theta)>0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique critical point in (l_0,l_1) for each θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0,1]$, we have that

$$\mathcal{Q}(\theta)\equiv C(\theta)^{-R}\frac{dC(\theta)}{dN\left(\theta\right)}-k-L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed, and varying the proportion of investing firms and the amounts that each firm produces (for given investment decision) when N changes.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x)\equiv\int_{\theta}\mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right)=f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x)\overset{sgn}{=}\int_{\theta}\mathcal{Q}(\theta)f\left(\theta|x\right)d\theta=\mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all firms receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]>0$ invest, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]<0$ refrain from investing.

Next, use (S.21) to observe that

$$C(\theta)^{-R}\frac{dC(\theta)}{dN(\theta)} = \frac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) + C(\theta)^{\frac{1-vR}{v}} y_1(\theta)^{-\frac{1}{v}}\frac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta),$$

and (S.22) to note that

$$y_1(\theta)^{-\frac{1}{v}}\frac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) = \frac{\Theta\beta}{A(\Theta, N(\theta))}y_1(\theta)^{\frac{v-1}{v}}N(\theta).$$

Finally, using (S.25) and (S.26), we have that

$$\psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) = L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right).$$

We conclude that

$$\mathcal{Q}(\theta) = \left(\frac{v-\psi(v-1)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) + C(\theta)^{\frac{1-vR}{v}}y_1(\theta)^{\frac{v-1}{v}}\frac{\Theta\beta N(\theta)}{A(\Theta, N(\theta))} - k.$$

Using (S.22), (S.23), (S.27), and (S.30), after some manipulations, we have that

$$C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) = \\ = \left(\left(A(\Theta, N(\theta))^{\varphi} - 1\right)N(\theta) + 1\right)^{\frac{1-vR}{v-1}} l_0(\theta)^{\psi(1-R)}\left(A(\Theta, N(\theta))^{\varphi} - 1\right). \quad \text{(S.32)}$$

Using (S.30) and (S.22), we also have that

$$C(\theta)^{\frac{1-vR}{v}}y_1(\theta)^{\frac{v-1}{v}} = \left(\left(A(\Theta, N(\theta))^{\varphi} - 1\right)N(\theta) + 1\right)^{\frac{1-vR}{v}} A(\Theta, N(\theta))^{\varphi}\, l_0(\theta)^{\psi(1-R)}.$$

It follows that

$$\mathcal{Q}(\theta) = \left(\left(A(\Theta, N(\theta))^{\varphi} - 1\right)N(\theta) + 1\right)^{\frac{1-vR}{v-1}} l_0(\theta)^{\psi(1-R)} \times \\ \times\left(\frac{A(\Theta, N(\theta))^{\varphi} - 1}{\varphi} + \Theta\beta N(\theta)\, A(\Theta, N(\theta))^{\varphi-1}\right) - k$$

Next, recall that the optimal labor demand for the non-investing firms is given by (S.31).

Replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\mathcal{Q}(\theta) = \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \left(\left(A\left(\Theta, N\left(\theta\right)\right)^{\varphi} - 1\right) N\left(\theta\right) + 1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \times \\ \times \left(\frac{A\left(\Theta, N\left(\theta\right)\right)^{\varphi} - 1}{\varphi} + \Theta\beta N\left(\theta\right) A\left(\Theta, N\left(\theta\right)\right)^{\varphi-1} \right) - k.$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. Such a property implies that the first-best choice of N is either $N = 0$ or $N = 1$, for all θ. This observation, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ. These properties, in turn, imply that the optimal investment policy is monotone. For any $\hat{x}$, let $\bar{N}(\theta|\hat{x}) \equiv 1 - \Phi(\hat{x}|\theta)$ denote the measure of investing firms at θ when firms follow the monotone rule $n(x) = \mathbb{I}(x > \hat{x})$. Then let

$$\bar{\mathcal{Q}}(\theta|\hat{x}) = \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \left(\left(A\left(\Theta, \bar{N}\left(\theta\right)\right)^{\varphi} - 1\right) \bar{N}(\theta|\hat{x}) + 1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \times \\ \times \left(\frac{A\left(\Theta, \bar{N}\left(\theta\right)\right)^{\varphi} - 1}{\varphi} + \Theta\beta \bar{N}(\theta|\hat{x}) A\left(\Theta, \bar{N}\left(\theta\right)\right)^{\varphi-1} \right) - k,$$

denote the function $\mathcal{Q}(\theta)$ characterized above, specialized to $N(\theta) = \bar{N}(\theta|\hat{x})$, where

$$A\left(\Theta, \bar{N}\left(\theta\right)\right) \equiv 1 + \Theta\left(1 + \beta\bar{N}\left(\theta\right)\right).$$

Observe that, under the parameters' restrictions in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that $\lim_{\hat{x}\to-\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] < 0 < \lim_{\hat{x}\to+\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$. Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] = 0$ admits exactly one solution. Letting $\hat{x}$ denote the solution to this equation, we have that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] < 0$ for $x < \hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] > 0$ for $x > \hat{x}$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ such that the investment strategy $\hat{n}\left(x;\pi^x\right) = \mathbb{I}(x \geq \hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition satisfy all the first-order conditions of the planner's problem. To ease notation, let $A\left(\Theta, \hat{N}\left(\theta;\pi^x\right)\right) \equiv 1 + \Theta\left(1 + \beta\hat{N}\left(\theta;\pi^x\right)\right)$. The threshold $\hat{x}(\pi^x)$ solves

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\times\right.$$
$$\left.\times\left(\frac{A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1}{\varphi}+\Theta\beta\hat{N}\left(\theta;\pi^x\right)A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi-1}\right)\middle|\hat{x}(\pi^x),\pi^x\right]=k,$$

with $\hat{N}\left(\theta;\pi^x\right)=1-\Phi(\hat{x}(\pi^x)|\theta;\pi^x)$.

Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}\left(x;\pi^x\right)$ is monotone), any solution to the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions (S.18) and (S.19) in the proposition and $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x]>0)$, where

$$\hat{\mathcal{Q}}(\theta;\pi^x)=\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\left(A\left(\theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\times$$
$$\times\left(\frac{A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1}{\varphi}+\Theta\beta\hat{N}(\Theta;\pi^x)A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi-1}\right)-k.$$

with $\hat{N}(\theta;\pi^x)=\int_\theta\hat{n}\left(x;\pi^x\right)d\Phi\left(x|\theta,\pi^x\right)$.

Part 2. For any precision of private information π^x, use Conditions (S.28) and (S.30) in part (1) to write ex-ante welfare as

$$\mathbb{E}\left[\mathcal{W}|\pi^x\right]=$$
$$=\frac{1}{1-R}\int_\theta\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{v}{v-1}(1-R)}\hat{l}_0\left(\theta;\pi^x\right)^{\psi(1-R)}d\Omega\left(\theta\right)+$$
$$-k\int_\theta\hat{N}\left(\theta;\pi^x\right)d\Omega\left(\theta\right)-\int_\theta\frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{1+\varepsilon}d\Omega\left(\theta\right)+$$
$$-\mathcal{I}(\pi^x).$$

Using the envelope theorem, we have that the marginal effect of a variation in the precision of private information on welfare is given by

$$\frac{d\mathbb{E}\left[\mathcal{W}|\pi^x\right]}{d\pi^x}=\mathbb{E}\left[\frac{v}{v-1}\hat{C}\left(\theta;\pi^x\right)^{\frac{1-Rv}{v}}\left(\varphi A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^x\right)+A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\times\right.$$
$$\left.\times\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\hat{l}_0\left(\theta;\pi^x\right)^{\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\begin{array}{c}\hat{l}_0\left(\theta;\pi^x\right)^{1+\varepsilon}\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\varepsilon}\times\\ \times\left(\varphi A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^x\right)+A\left(\Theta,\hat{N}\left(\theta;\pi^x\right)\right)^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\end{array}\right]+$$
$$-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]-\frac{d\mathcal{I}(\pi_x)}{d\pi_x}.$$

The result in part 2 then follows from the fact that, at the optimum, the above derivative must be equal to zero. Q.E.D.

Proof of Proposition S.2. The proof is in two parts and establishes a more general result than the one in the proposition. Part 1 fixes the precision of information and identifies a condition on the fiscal policy $T(\cdot)$ that guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, firms have incentives to use information efficiently when the latter is exogenous. Part 2 identifies an additional restriction on the fiscal policy that, when combined with the condition in part 1, guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, agents have also incentives to acquire information efficiently. The arguments in parts 1 and 2 also allow us to establish that, irrespective of whether or not the economy satisfies the parameters' restrictions of Proposition S.1, when $T(\cdot)$ is the specific policy of Proposition S.2, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same.

Part 1. We fix the precision of information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions when there is no risk of confusion.
Consider first the pricing decision of an investing firm. The firm sets p_1 to maximize

$$C^{-R}\left(\frac{p_1y_1-Wl_1}{P}+T_1\left(r_1\right)\right), \tag{S.33}$$

where $r_1=p_1y_1/P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1=C\left(\frac{P}{p_1}\right)^{v}, \tag{S.34}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{A(N)} \right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.33) with respect to p_1 is given by

$$C^{-R} \left((1-v) C P^{v-1} p_1^{-v} - \frac{W}{P} \frac{dl_1}{dp_1} + \frac{1}{P} \frac{dT_1(r_1)}{dr} \frac{d(p_1 y_1)}{dp_1} \right) = 0. \tag{S.35}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi} \frac{l_1}{p_1}, \tag{S.36}$$

$$\frac{d(p_1 y_1)}{dp_1} = (1-v) C P^{v} p_1^{-v},$$

and (S.34), we have that (S.35) can be rewritten as

$$C^{-R} \left((1-v) \frac{y_1}{P} + \frac{W}{P} \frac{v}{\psi} \frac{l_1}{p_1} + \frac{dT_1(r_1)}{dr} \frac{(1-v) y_1}{P} \right) = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\frac{1-v}{v} C^{-R} \frac{y_1 p_1}{P} + \frac{1}{\psi} C^{-R} \frac{W}{P} l_1 + \frac{1-v}{v} C^{-R} \frac{dT_1(r_1)}{dr} \frac{y_1 p_1}{P} = 0. \tag{S.37}$$

Next use (S.22), (S.23), and (S.34), along with (S.27) and (S.29), to observe that, in any equilibrium implementing the efficient allocation, firms must set prices equal to (hereafter we use "hats" to denote variables under the rules inducing the efficient allocation)

$$\hat{p}_1 = A(\hat{N})^{\frac{\varphi}{1-v}} \left(\left(A(\hat{N})^{\varphi} - 1 \right) \hat{N} + 1 \right)^{\frac{1}{v-1}} \hat{P},$$

and

$$\hat{p}_0 = \left(\left(A(\hat{N})^{\varphi} - 1 \right) \hat{N} + 1 \right)^{\frac{1}{v-1}} \hat{P},$$

with

$$\hat{P} = \left(\hat{p}_1^{1-v} \hat{N} + \hat{p}_0^{1-v} \left(1 - \hat{N} \right) \right)^{\frac{1}{1-v}}.$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to determine whether or not to invest, and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the investment decision.

Observe that clearing in the labor market requires that

$$\hat{C}^{-R}\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}, \tag{S.38}$$

and recall that, as established in the Proof of Proposition S.1,

$$\hat{L} = \hat{l}_0\left(\left(A\left(N\right)^{\varphi} - 1\right)\hat{N} + 1\right).$$

Also, consider that efficiency requires that $-\psi\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon}\hat{l}_1 = 0$. Accordingly, using Condition (S.37), we have that each investing firm finds it optimal to set the price $\hat{p}_1$ only if

$$\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}} + \frac{1-v}{v}C^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1 = 0, \tag{S.39}$$

where $\hat{r}_1 = \hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.34), we have that $\hat{y}_1^{-\frac{1}{v}} = \hat{C}^{-\frac{1}{v}}\hat{p}_1/\hat{P}$, which allows us to rewrite Condition (S.39) as

$$\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1 = 0,$$

or, equivalently,

$$\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v} + \frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right) = 0.$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr = 1/(v-1)$, the first-order condition of the firm's optimization problem with respect to its price is satisfied. Furthermore, under the proposed fiscal policy, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1 = \hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r) = \frac{r}{v-1} + s = \frac{1}{v-1}\left(\frac{p_1 y_1}{P}\right) + s,$$

where s may depend on θ but is invariant in r, the firm's objective (S.33) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}l_1 + s\left(\theta\right)\right)\middle|\, x\right].$$

Using (S.34) and (S.36), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\Bigg|x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\Bigg|x\right].$$

From the analysis above, we have that $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ when $p_1 = \hat{p}_1$. Furthermore, irrespective of x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\Bigg|x\right]=0. \tag{S.40}$$

Using (S.40), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique critical point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the non-investing firms, we have that a fiscal policy that pays to each non-investing firm a transfer equal to $T_0(r) = r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' investment choice. Hereafter, we reintroduce θ in the notation. When

$$T_0\left(r\right)=\frac{1}{v-1}r, \tag{S.41}$$

and

$$T_1\left(\theta,r\right)=s(\theta)+\frac{1}{v-1}r, \tag{S.42}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by investing, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by not investing, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Let

$$\hat{\mathcal{R}}(\theta)\equiv\hat{C}\left(\theta\right)^{-R}\left(\hat{r}_1(\theta)-\hat{r}_0(\theta)-\frac{\hat{W}(\theta)}{\hat{P}\left(\theta\right)}\left(\hat{l}_1(\theta)-\hat{l}_0(\theta)\right)+T_1\left(\theta,\hat{r}_1(\theta)\right)-T_0\left(\hat{r}_0(\theta)\right)\right)-k,$$

where $\hat{r}_1(\theta)$ and $\hat{r}_0(\theta)$ are the firm's (real) revenues when the firm follows the efficient policies, respectively, after investing and not investing. Each firm receiving signal x finds it optimal to invest if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] > 0$, and not to invest if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] < 0$. Recall from (S.34) that the Dixit and Stiglitz demand system implies that $\hat{p}_f = \hat{P}\left(\theta\right)\hat{C}\left(\theta\right)^{\frac{1}{v}}\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f\left(\theta\right) =$

$\hat{C}(\theta)^{\frac{1}{v}} \hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f = 0, 1$. Also, recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)} \hat{C}(\theta)^{-R} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}(\theta)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) - \hat{L}(\theta)^{\varepsilon} \left(\hat{l}_1(\theta) - \hat{l}_0(\theta) \right) + \\ + \hat{C}(\theta)^{-R} \left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) \right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition S.1) $\psi \hat{C}(\theta)^{\frac{1-vR}{v}} \hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta) \hat{L}(\theta)^{\varepsilon}$, and $\psi \hat{C}(\theta)^{\frac{1-vR}{v}} \hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta) \hat{L}(\theta)^{\varepsilon}$, we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) = (1-\psi) \hat{C}(\theta)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) + \hat{C}(\theta)^{-R} \left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) \right) - k.$$

Next, use (S.34) to note that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}} \hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f = 0, 1$. It follows that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1} \hat{C}(\theta)^{\frac{1}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v - \psi(v-1)}{v-1} \right) \hat{C}(\theta)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) + \hat{C}(\theta)^{-R} s(\theta) - k. \tag{S.43}$$

Recall from the proof of Proposition S.1 that efficiency requires that each firm invests if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and does not invest if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v - \psi(v-1)}{v-1} \right) \hat{C}(\theta)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) + \hat{C}(\theta)^{\frac{1-vR}{v}} \hat{y}_1(\theta)^{\frac{v-1}{v}} \frac{\Theta \beta \hat{N}(\theta)}{A\left(\hat{N}\right)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient investment rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition S.1 (see Equations (S.32) and (S.31), respectively),

$$\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)=$$
$$=\left(\left(A\left(\hat{N}\right)^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\hat{l}_0(\theta)^{\psi(1-R)}\left(A\left(\hat{N}\right)^{\varphi}-1\right),$$

and

$$\hat{l}_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\left(A\left(\hat{N}\right)^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}.$$

Using the last two expressions, we have that the first addendum in (S.43) can be rewritten as

$$\frac{v-\psi\left(v-1\right)}{v-1}\hat{C}\left(\theta\right)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)=$$
$$=\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\left(A\left(\hat{N}\right)^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\frac{A\left(\hat{N}\right)^{\varphi}-1}{\varphi}.$$

When the economy satisfies the conditions in Proposition S.1, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ in (S.43) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the parameters' restrictions in Proposition S.1, a subsidy $s(\theta)$ to the investing firms satisfying conditions (a) and (b) below guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$:

(a)
$$\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\text{ non-decreasing in }\theta;$$

(b)
$$\mathbb{E}\left[\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\middle|\,\hat{x}\right]=\mathbb{E}\left[\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}\frac{\Theta\beta\hat{N}\left(\theta\right)}{A\left(\hat{N}\right)}\middle|\,\hat{x}\right].$$

The analysis above also reveals that, when the fiscal policy takes the form in (S.41) and (S.42) with $s(\theta)=\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}\frac{\Theta\beta\hat{N}(\theta)}{A(\hat{N})}$, for all θ, then irrespective of whether or not the economy satisfies the conditions in Proposition S.1, each firm expecting all other firms to follow the efficient investment rule $\hat{n}(x)$, and setting prices according to $\hat{p}_0$ and $\hat{p}_1$ (thus inducing the efficient employment decisions), finds it optimal to do the same.

Part 2. We now show that, when the economy satisfies the conditions in Proposition S.1, the fiscal policy in (S.41) and (S.42), implements the efficient acquisition and usage of information if and only if the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) in part 1, is such that

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}\frac{\Theta\beta\hat{N}\left(\theta\right)}{A\left(\hat{N}\right)}\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and follow the efficient investment and pricing rules. Consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the proposed fiscal policy, the firm finds it optimal to set a price equal to $\hat{p}_1^*$ after investing and equal to $\hat{p}_0^*$ if it does not invest, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.

Let

$$\hat{N}^*(\theta)\equiv\hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta)\equiv\hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta)\equiv\hat{l}_1(\theta;\pi^{x*}),$$

$$A\left(N^*\right)\equiv 1+\Theta\left(1+\beta\hat{N}^*(\theta)\right),$$

$$\hat{y}_1^*(\theta)\equiv A\left(N^*\right)\hat{l}_1^*(\theta)^{\psi},$$

$$\hat{y}_0^*(\theta)\equiv\hat{l}_0^*(\theta)^{\psi},$$

$$\hat{C}^*(\theta)=\hat{Y}^*(\theta)\equiv\left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta)+\hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}},$$

$$\hat{W}^*(\theta)\equiv\hat{W}(\theta;\pi^{x*}),$$

and

$$\hat{P}^*\left(\theta\right)\equiv\left(\hat{p}_1^{*1-v}\hat{N}^*\left(\theta\right)+\hat{p}_0^{*1-v}(1-\hat{N}^*\left(\theta\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal policy, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to $\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma;\pi_i^x)$, where

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) \equiv\, & \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & + \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x),\end{aligned}$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i invests when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$, respectively in case it invests and in case it does not invest.

Using (S.34), we have that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$ for $f=0,1$. Hence,

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\hat{y}_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x;\varsigma)+\hat{y}_0^{*\frac{v-1}{v}}\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & + \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Using

$$\hat{y}_1^* = A\left(N^*\right)\hat{l}_1^{*\psi}, \tag{S.44}$$

$$\hat{y}_0^* = \hat{l}_0^{*\psi}, \tag{S.45}$$

and

$$\hat{l}_1^* = A\left(N^*\right)^{\varphi}\hat{l}_0^*, \tag{S.46}$$

we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ & - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] + \\ & + \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\begin{aligned}\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = {} & \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\left(\left(A\left(N^*\right)^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+ \\ & -\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\left(A\left(N^*\right)^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\hat{l}_0^*\right)\right]+ \\ & +\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*-\hat{T}_0^*\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},\end{aligned} \quad \text{(S.47)}$$

where $\partial \bar{n}(\pi_i^x;\varsigma)/\partial \pi_i^x$ is the marginal effect of varying π_i^x on the probability that the firm invests at θ, holding fixed the rule ς.

Next, recall again that, for $f=0,1$,

$$\hat{r}_f^* \equiv \frac{\hat{p}_f^*\hat{y}_f^*}{\hat{P}^*}=\hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}.$$

Using (S.44) and (S.45), we have that

$$\hat{r}_1^*-\hat{r}_0^*=\hat{C}^{*\frac{1}{v}}\left(A\left(N^*\right)^{\frac{v-1}{v}}\hat{l}_1^{*\psi\frac{v-1}{v}}-\hat{l}_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (S.46) and the structure of the proposed fiscal policy, we have that

$$\hat{T}_1^*-\hat{T}_0^*=s+\frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(A\left(N^*\right)^{\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (S.47), we obtain that

$$\begin{aligned}\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = {} & \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+ \\ & -\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\hat{l}_0^*\right)\right]+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]+ \\ & -k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.\end{aligned}$$

Next recall that, when $\pi_i^x=\pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma=\hat{n}^*$, where $\hat{n}^*(x)\equiv\hat{n}(x;\pi^{x*})$ is the efficient investment choice for a firm receiving signal x

after acquiring information of precision π^{x*}. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^*\right)\right] + \mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial \hat{N}^*/\partial \pi^x$ is the marginal change in the measure of investing firms that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding the strategy $\hat{n}^*$ fixed. Note that, in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x;\varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*};\hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the fiscal policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^*\right)\right] + \\ + \mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] = \frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(S.48)}$$

Next, use (S.38) and (S.46) to note that

$$\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*} = \left(\hat{l}_1^*\hat{N}^* + \hat{l}_0^*\left(1-\hat{N}^*\right)\right)^{\varepsilon} = \hat{l}_0^{*\varepsilon}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}.$$

It follows that (S.48) is equivalent to

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ - \mathbb{E}\left[\hat{l}_0^{*\varepsilon}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}\left(\left(A\left(N^*\right)^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^*\right)\right] + \\ + \mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] = \frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(S.49)}$$

Recall that the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[\frac{v}{v-1}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1-Rv}{v}}\left(\varphi A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)+A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\times\right.$$
$$\left.\times\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\hat{l}_0\left(\theta;\pi^{x*}\right)^{\psi\frac{v-1}{v}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]+$$
$$-\mathbb{E}\left[\begin{array}{c}\hat{l}_0\left(\theta;\pi^{x*}\right)^{1+\varepsilon}\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\times\\ \times\left(\varphi A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)+A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\end{array}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^{x}}. \tag{S.50}$$

Comparing (S.49) with (S.50), we have that, for the policy T to implement the efficient acquisition and usage of information, the subsidy s to the investing firms must satisfy the following condition

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=$$
$$=\mathbb{E}\left[\frac{v}{v-1}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1-Rv}{v}}\left(\varphi A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\hat{l}_0\left(\theta;\pi^{x*}\right)^{\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{l}_0\left(\theta;\pi^{x*}\right)^{1+\varepsilon}\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\times\right.$$
$$\left.\times\left(\varphi A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]. \tag{S.51}$$

where we reintroduce the arguments of the various functions. Taking advantage of (S.26), (S.28), and (S.23) we have

$$\hat{l}_0\left(\theta;\pi^{x*}\right)^{1+\varepsilon}\left(\left(A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}=\psi\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1-vR}{v}}\hat{l}_0\left(\theta;\pi^{x*}\right)^{\psi\frac{v-1}{v}},$$

so that (S.51) becomes

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1-Rv}{v}}\left(A\left(\Theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)\times\right.$$
$$\left.\times\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\hat{l}_0\left(\theta;\pi^{x*}\right)^{\psi\frac{v-1}{v}}\right].$$

Taking advantage of (S.22) and (S.27) we then obtain

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{\frac{1-Rv}{v}}\,\hat{y}_1(\theta;\pi^{x*})^{\frac{v-1}{v}}\,\frac{\Theta\beta\hat{N}(\theta;\pi^{x*})}{A\left(\Theta,\hat{N}(\theta;\pi^{x*})\right)}\times\right.$$
$$\left.\times\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\hat{y}_1(\theta;\pi^{x*})^{\psi\frac{v-1}{v}}\right].$$

Finally, note that, independently of whether the economy satisfies the conditions in Proposition S.1, when the subsidy to the investing firms (where we write $1+\Theta\left(1+\beta\hat{N}(\theta;\pi^{x*})\right)$ instead of $A\left(\Theta,\hat{N}(\theta;\pi^{x*})\right)$) is equal to

$$s(\theta)=\hat{C}(\theta;\pi^{x*})^{\frac{1}{v}}\,\hat{y}_1(\theta;\pi^{x*})^{\frac{v-1}{v}}\,\frac{\Theta\beta\hat{N}(\theta;\pi^{x*})}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^{x*})\right)}$$

in each state, then, as shown in part 1, the private value $\mathcal{R}$ that each firm assigns to investing coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting all other firms to do the same. This establishes the claim in the proposition. Q.E.D.

Appendix S.2: Optimal Fiscal and Monetary Policy for the Economy of Section S.3

In this Appendix, we first establish the analogs of all the results in the main text – establishing properties of optimal fiscal policies – for the economy under consideration (Subsections S.3.2-S.3.4). We then show that all these results continue to hold when prices are set under dispersed information (Subsection S.3.5).

S.3.2 Efficient Allocation (Lemma 1)

In the following, we prove the analog of Lemma 1 in the main text for the economy in Section S.3 of the Online Supplement.

Fix π^x and drop it from all expressions to ease the notation. Efficiency requires that any two firms making the same investment decision employ the same amount of labor. Letting $n(x)$ denote the probability that a firm receiving signal x invests, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the investing and the non-investing firms respectively when the fundamental variable is θ, we have that the planner's problem can be written as

$$\max_{n(\cdot),l_1(\cdot),l_0(\cdot)} \int_\theta C(\theta)d\Omega(\theta) - k\int_\theta N(\theta)d\Omega(\theta) + \\ -\frac{1}{1+\varepsilon}\int_\theta (l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta)))^{1+\varepsilon}d\Omega(\theta) + \\ -\int_\theta \mathcal{Q}(\theta)\left(N(\theta)-\int_x n(x)\Phi(x|\theta)\right)d\Omega(\theta),$$

where Ω is the cumulative distribution function of θ (with density ω), $\Phi(\cdot|\theta)$ is the cumulative distribution function of x given θ (with density $\phi(\cdot|\theta)$), $\mathcal{Q}(\theta)$ is the multiplier associated with the constraint $N(\theta)=\int_x n(x)d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}}N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}, \tag{S.52}$$

with

$$y_1(\theta) = \gamma\Theta(1+\beta N(\theta))^\alpha l_1(\theta)^\psi, \tag{S.53}$$

and

$$y_0(\theta) = \Theta(1+\beta N(\theta))^\alpha l_0(\theta)^\psi. \tag{S.54}$$

The first-order condition with respect to $l_1(\theta)$ is thus equal to

$$\psi\left(y_1(\theta)^{\frac{v-1}{v}}N(\theta)+y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}}(\gamma\Theta(1+\beta N(\theta))^\alpha)^{\frac{v-1}{v}}l_1(\theta)^{\psi\frac{v-1}{v}-1} + \\ -(l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta)))^\varepsilon = 0. \tag{S.55}$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta)), \tag{S.56}$$

and using (S.52) and (S.53), we have that the first order condition for $l_1(\theta)$ above can be expressed as

$$\psi C(\theta)^{\frac{1}{v}}y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta)L(\theta)^\varepsilon. \tag{S.57}$$

Following similar steps, the first-order condition for $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1}{v}}y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta)L(\theta)^\varepsilon. \tag{S.58}$$

Jointly, the above first-order conditions – together with (S.54) and (S.56) – yield

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon-\psi}} \left(\Theta \left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1}{1+\varepsilon-\psi}} \left(\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v\varepsilon}{(v-1)(1+\varepsilon-\psi)}}, \tag{S.59}$$

and

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{S.60}$$

Notice that (S.60) implies that, at the efficient allocation, the total labor demand, as defined in (S.56), is equal to

$$L\left(\theta\right) = l_0\left(\theta\right)\left(\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right). \tag{S.61}$$

The above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1) for any θ.
Differentiating the government's objective with respect to $N(\theta)$, we have that

$$\mathcal{Q}(\theta) = \frac{v}{v-1} C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) + \frac{\alpha\beta}{1+\beta N(\theta)} C(\theta) - k - L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right). \tag{S.62}$$

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_{\theta} \mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right) d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right) = f\left(\theta|x\right) g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x, and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_{\theta} \mathcal{Q}(\theta) f\left(\theta|x\right) d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that $n(x) = 1$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and $n(x) = 0$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$.
Use (S.57) and (S.58) to observe that

$$L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right) = \psi C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right).$$

Replacing the above expression into (S.62), we have that

$$\mathcal{Q}(\theta) = \left(\frac{v-\psi\left(v-1\right)}{v-1}\right) C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) + \frac{\alpha\beta}{1+\beta N\left(\theta\right)} C(\theta) - k.$$

Using (S.52), (S.53), (S.54), and (S.60), after some manipulations, we have that

$$C(\theta)^{\frac{1}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1}{v-1}}\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}l_0(\theta)^{\psi}\left(\gamma^{\varphi}-1\right),\tag{S.63}$$

and $C(\theta)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v}{v-1}}\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}l_0(\theta)^{\psi}$. It follows that

$$\begin{aligned}\mathcal{Q}(\theta)=\psi^{\frac{\psi}{1+\varepsilon-\psi}}\Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}}\times&\\ \times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right)-k.&\end{aligned}\tag{S.64}$$

When the parameters satisfy the regularity conditions in the main text, as well as $\gamma^{\varphi}\geq 1+\beta$ (with $\varphi\equiv(v-1)/(v-\psi(v-1))$, $\mathcal{Q}$ is increasing in both N (for given θ) and θ (for given N). That, for any θ, $\mathcal{Q}$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the firms (and hence by the planner) at the time the investment decisions are made. Such a property implies that the first-best choice of N is either $N=0$ or $N=1$, for all θ. This last property, along with the fact that $\mathcal{Q}$ is increasing in θ for any N, implies that the first-best level of N is increasing in θ. This property, in turn, implies that the efficient strategy $\hat{n}(x)$ is monotone. For any θ and $\hat{x}$, let $\bar{\mathcal{Q}}(\theta|\hat{x})$ denote the function defined in (S.64) when $N(\theta)=1-\Phi(\hat{x}|\theta)$, that is, when firms invest if and only if $x>\hat{x}$. Under the regularity conditions, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that $\lim_{\hat{x}\to-\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]<0<\lim_{\hat{x}\to+\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$. Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]=0$ admits one and only one solution. Let $\hat{x}$ denote the solution to this equation. Then note that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]<0$ for $x<\hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]>0$ for $x>\hat{x}$. We conclude that, under the assumptions in the lemma, there exists a threshold $\hat{x}$ such that the investment rule $\hat{n}(x)=\mathbb{I}(x\geq\hat{x})$, along with the employment functions $\hat{l}_1(\theta)$ and $\hat{l}_0(\theta)$ satisfying the first-order conditions above, constitute a solution to the planner's problem. Q.E.D.

S.3.3 Exogenous Information: Optimal Fiscal Policy under Flexible Prices (Lemma 2)

Here we prove the analog of Lemma 2 in the main text for the economy in Section S.3 of the Online Supplement.

We drop π^x from all formulas to ease the notation. We also drop θ when there is no risk of confusion.

Each investing firm chooses p_1 to maximize

$$\frac{p_1 y_1 - W l_1}{P} + T_1\left(\frac{p_1 y_1}{P}\right), \tag{S.65}$$

taking W and P as given. The first-order condition with respect to p_1 is given by

$$(1-v)\, C P^{v-1} p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\left(p_1 y_1 / P\right)}{dr}\frac{d\left(p_1 y_1\right)}{dp_1} = 0. \tag{S.66}$$

Using (7) in the main text together with the analog of (9), i.e.

$$l_1 = \left(\frac{y_1}{\gamma\Theta\left(1+\beta N\right)^{\alpha}}\right)^{\frac{1}{\psi}}$$

we have that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{S.67}$$

and

$$\frac{d\left(p_1 y_1\right)}{dp_1} = (1-v)\, C P^{v} p_1^{-v}. \tag{S.68}$$

Replacing (S.67) and (S.68) into (S.66), and rearranging terms, we obtain that

$$\frac{1-v}{v}\frac{y_1 p_1}{P} + \frac{1}{\psi}\frac{W}{P} l_1 + \frac{1-v}{v}\frac{dT_1\left(p_1 y_1 / P\right)}{dr}\frac{y_1 p_1}{P} = 0. \tag{S.69}$$

Next, recalling that $\varphi = \frac{v-1}{v-\psi-1}$, use (S.20), (S.52), and the demand functions, along with (S.60), to observe that, in any equilibrium implementing the efficient allocation, firms must set prices equal to (hereafter we use "hats" to denote variables under the rules inducing the efficient allocation)

$$\hat{p}_1 = \left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}} \gamma^{\frac{\varphi}{1-v}} \hat{P}, \tag{S.70}$$

and

$$\hat{p}_0 = \left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}} \hat{P}, \tag{S.71}$$

with

$$\hat{P} = \left(\hat{p}_1^{1-v}\hat{N} + \hat{p}_0^{1-v}\left(1-\hat{N}\right)\right)^{\frac{1}{1-v}}. \tag{S.72}$$

Market-clearing in the labor market requires that $\hat{W}/\hat{P} = \hat{L}^{\varepsilon}$. Use (S.56) and (S.60) to note that $\hat{L} = \hat{l}_0\left((\gamma^{\varphi}-1)\hat{N}+1\right)$. Next, use (S.57) to observe that efficiency requires that

$$-\psi \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon}\hat{l}_1 = 0. \tag{S.73}$$

Condition (S.69) then implies that T implements the efficient allocation only if

$$\frac{1}{v} = \frac{v-1}{v} \frac{dT_1\left(\hat{p}_1\hat{y}_1/\hat{P}\right)}{dr}.$$

Since $\hat{p}_1\hat{y}_1/\hat{P}$ is state dependent, we have that T_1 must be affine and satisfy

$$T_1(r) = \frac{1}{v-1}r + s, \tag{S.74}$$

with s invariant in r. Furthermore, one can show that, under the policy (S.74), the payoff of each investing firm is quasi-concave in its price, which implies that the above first-order condition is also sufficient for the firm to choose $p_1 = \hat{p}_1$.

Similar arguments imply that the transfer to the non-investing firms must be equal to

$$T_0(r) = \frac{1}{v-1}r \tag{S.75}$$

for these firms to find it optimal to set $p_0 = \hat{p}_0$.

Next, consider the decision of whether or not to invest. When the policy satisfies (S.74) and (S.75), with $s(\theta)$ possibly depending on θ, each firm finds it optimal to invest if $\mathbb{E}[\mathcal{R}(\theta)|x] > 0$ and to not invest if $\mathbb{E}[\mathcal{R}(\theta)|x] < 0$, where

$$\mathcal{R}(\theta) \equiv \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + s(\theta) - k \tag{S.76}$$

is the extra profit (net of the subsidy) from investing relative to not investing. Now note that efficiency requires that each firm invests if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and does not invest if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$, where $\mathcal{Q}(\theta)$ can be conveniently rewritten as

$$\mathcal{Q}(\theta) = \left(\tfrac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \tfrac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)} - k.$$

When the economy satisfies the regularity conditions, $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ turns from negative to positive at $x = \hat{x}$. Hence, for the policy defined by (S.74) and (S.75) to induce efficiency in investment decisions it is both necessary and sufficient that $\mathbb{E}[\mathcal{R}(\theta)|x]$ turns from negative to positive at $x = \hat{x}$. Q.E.D.

Corollary 1 in the main text holds for this version of the economy as well, with

$$\bar{s}_{\pi^x} \equiv \mathbb{E}\left[\left.\frac{\alpha\beta\hat{C}(\theta;\pi^x)}{1+\beta\hat{N}(\theta;\pi^x)}\right|\hat{x}(\pi^x),\pi^x\right].$$

Using the derivations in Subsection S.3.3, we have that

$$\mathcal{R}(\theta) = \mathcal{Q}(\theta) - \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)} + s(\theta).$$

Next observe that the function $\mathcal{Q}(\theta) - \alpha\beta\hat{C}(\theta)/\left(1+\beta\hat{N}(\theta)\right)$ is non-decreasing in θ under the regularity conditions of Definition 1. We thus have that, when $s(\theta) = \bar{s}_{\pi^x}$ for all θ, $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]$ turns from negative to positive at $x = \hat{x}$, implying that the fiscal policy T is optimal. Q.E.D.

S.3.4 Endogenous information: Optimal Fiscal Policy under Flexible Prices (Lemma 3)

Finally, we prove the analog of Lemma 3 in the main text for the economy in Section S.3 of the Online Supplement. Part 1 characterizes the efficient precision of information π^{x*}. Part 2 uses the characterization in part 1 to establish the claim in the lemma.

Part 1. For any π^x, irrespective of whether the economy is regular, *ex-ante* welfare under the efficient allocation is equal to

$$\mathcal{W} = \int_\theta \Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^\alpha \hat{l}_0(\theta;\pi^x)^\psi \left(\left(\gamma^\varphi - 1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{v}{v-1}} d\Omega(\theta) + \\ - k\int_\theta \hat{N}(\theta;\pi^x)\, d\Omega(\theta) - \int_\theta \frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left(\left(\gamma^\varphi - 1\right)\hat{N}(\theta;\pi^x)+1\right)^{1+\varepsilon} d\Omega(\theta) - \mathcal{I}(\pi^x).$$

Using the envelope theorem, we then have that π^{x*} solves

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{v\left(\gamma^\varphi - 1\right)}{(v-1)\left(\left(\gamma^\varphi-1\right)\hat{N}(\theta;\pi^{x*})+1\right)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] + \\ -k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] - \mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(\gamma^\varphi-1\right)\hat{N}(\theta;\pi^{x*})+1\right)^\varepsilon\left(\gamma^\varphi - 1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] = \frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \tag{S.77}$$

The above condition identifies the efficient precision of private information π^{x*}.

Part 2. Suppose that all firms other than i acquire information of precision π^{x*} and consider firm i's problem. Under the policy in the lemma, in each state θ, the price that maximizes firm i's profit coincides with the one that induces the efficient allocation for precision π^{x*}, irrespective of firm i's choice of π^x_i. This price is equal to $\hat{p}^*_1$ if the firm invests and $\hat{p}^*_0$ if the firm does not invest, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the functions in (S.70) and (S.71), respectively, evaluated at $\pi^x = \pi^{x*}$. Note that we use the combination between "^" and "*" to denote variables under the efficient allocation for precision π^{x*} (this notation applies not only to $\hat{p}_1^*$ and $\hat{p}_0^*$ but to all expressions below).

Dropping θ from the argument of each function to ease the notation, we have that firm i's value function is equal to $\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma;\pi_i^x)$, where

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) \equiv \mathbb{E}\left[\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] +\\ + \mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x),\end{aligned}$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i invests when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$, after investing and not investing, respectively.

Substituting

$$\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}, \tag{S.78}$$

$f=0,1$, into $\Pi_i(\varsigma;\pi_i^x)$ and using (S.20), we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +\\ &- \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] +\\ &+ \mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly,

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +\\ &- \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right] +\\ &+ \mathbb{E}\left[\left(\hat{T}_1^*-\hat{T}_0^*\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}.\end{aligned} \tag{S.79}$$

Replacing

$$\hat{T}_1^* - \hat{T}_0^* = s + \frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}$$

into (S.79), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x} l_0^{*\psi\frac{v-1}{v}}\right]+ \\ -\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right]+\mathbb{E}\left[s\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}. \quad \text{(S.80)}$$

Recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+ \\ -\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+\mathbb{E}\left[s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial\hat{N}^*/\partial\pi^x$ is the marginal change in the measure of investing firms that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding $\hat{n}^*$ fixed. For the proposed policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. This requires that

$$\mathbb{E}\left[\frac{v\,(\gamma^{\varphi}-1)\,\hat{C}\,(\theta;\pi^{x*})}{(v-1)\left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^{x*})+1\right)}\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]+ \\ -\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^{x*})+1\right)^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]+ \\ +\mathbb{E}\left[s(\theta)\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]=\frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}, \quad \text{(S.81)}$$

where we reintroduce all the arguments of the various functions to make the result consistent with the claim in the main text.

Comparing (S.81) with (S.77) in part 1, we thus have that a fiscal policy induces the firms to acquire the efficient precision of private information only if, in addition to $s(\theta)$ satisfying the property in the previous section, it also satisfies above condition. Q.E.D.

S.3.5 Price Rigidity: Optimal Monetary Policy

Proof of Lemma S.1. We drop π^x from all formulas to ease the notation. Using (S.57) and (S.58), we have that $\hat{l}_1(\theta)\hat{L}\,(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}$, and $\hat{l}_0(\theta)\hat{L}\,(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}$, with

$\hat{L}(\theta)$ defined by (S.56). The Dixit and Stiglitz demand system implies that $y_i = C\left(P/p_i\right)^v$. Hence, efficiency requires that the prices set by any two firms making the same investment decision coincide, which means that they must be independent of the signal x, conditional on the investment decision. Let $\hat{p}_1$ be the (state-invariant) price set by the investing firms and $\hat{p}_0$ the price set by the non-investing firms. Let $\hat{P}(\theta)$ denote the price of the final good in state θ when all firms follow the efficient rules. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)\left(\hat{P}(\theta)/\hat{p}_1\right)^{v-1}, \tag{S.82}$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)\left(\hat{P}(\theta)/\hat{p}_0\right)^{v-1}, \tag{S.83}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (S.60), implies that $\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0$. The price of the final good is then equal to

$$\hat{P}(\theta) = \left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{1}{1-v}}\hat{p}_0. \tag{S.84}$$

Combining (S.83) with the cash-in-advance constraint $M = PC$, we have that, in each state θ,

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)\hat{P}(\theta)^{v-2}\hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{v-2}{1-v}}\hat{p}_0^{-1},$$

where we also used (S.84) to express $\hat{P}(\theta)$ as a function of $\hat{N}(\theta)$ and $\hat{p}_0$. Finally, using (S.61), we obtain that, in each state θ, the money supply must be given by

$$\hat{M}(\theta) = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}}\hat{p}_0.$$

It is immediate to verify that the same conclusion can be obtained starting from (S.82). Because $\hat{p}_0$ can be taken to be arbitrary, the result in the lemma obtains by setting $m = \frac{1}{\psi}\hat{p}_0$. Q.E.D.

We conclude by showing that all the properties of optimal fiscal policies identified above for flexible prices remain valid when prices are set under dispersed information. Formally:

Proposition S.3. *All the results about the structure of the optimal fiscal policy in Subsec-*

tion S.3 for the case of flexible prices carry over to the economy with price rigidities under consideration.

Proof. The proof is in two parts. Part 1 shows that, when information is exogenous and the monetary policy is the one in Lemma S.1 (which, by virtue of the lemma, is the only one that can induce efficiency in information usage), any optimal fiscal policy must take the form $T_0(r) = r/(v-1)$ and $T_1(r) = r/(v-1) + s$, for some s that is invariant in r. The reason why this result is not implied by the results above and requires a separate proof is that the information upon which the firms set their prices is different from the one considered above; this implies that, in principle, the way the government provides incentives to the firms may be different from what established for flexible prices. Part 2 then uses the result in Part 1 to establish the conclusions in the proposition.

Part 1. Fix the precision of private information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions below when there is no risk of confusion. Consider first the pricing decision of an investing firm. The firm sets p_1 to maximize

$$\mathbb{E}\left[\frac{p_1 y_1 - W l_1}{P} + T_1(r_1)\middle| x\right], \tag{S.85}$$

where $r_1 = p_1 y_1/P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^v, \tag{S.86}$$

and that the amount of labor that the firm will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta(1+\beta N)^\alpha}\right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.85) with respect to p_1 is given by

$$\mathbb{E}\left[(1-v)\,CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1(r_1)}{dr}\frac{d(p_1 y_1)}{dp_1}\middle| x\right] = 0. \tag{S.87}$$

Using $\frac{dl_1}{dp_1} = -\frac{v l_1}{\psi p_1}$, $\frac{dp_1 y_1}{dp_1} = (1-v)\,CP^v p_1^{-v}$, and (S.86), we have that (S.87) can be rewritten as

$$\mathbb{E}\left[(1-v)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT_1(r_1)}{dr}\frac{(1-v)\,y_1}{P}\middle| x\right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}\frac{y_1 p_1}{P}+\frac{1}{\psi}\frac{W}{P}l_1+\frac{1-v}{v}\frac{dT_1\left(r_1\right)}{dr}\frac{y_1 p_1}{P}\right|x\left.\vphantom{\frac{1}{1}}\right]=0. \tag{S.88}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to make their investment decisions and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend on the signals x only through the effect that the latter has on firms' investment decisions, as in the proof of Lemma S.1. Consistently with the notation used above, we add "hats" to all relevant variables to highlight that these are computed under the efficient rules. Observe that market clearing in the labor market requires that $\hat{W}/\hat{P}=\hat{L}^{\varepsilon}$, and recall that $\hat{L}=\hat{l}_0\left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)$. Also, observe that efficiency requires that $-\psi\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1=0$. Accordingly, using Condition (S.88), we have that each investing firm finds it optimal to set the price $\hat{p}_1$ that sustains the efficient allocation only if

$$\mathbb{E}\left[\frac{1-v}{v}\hat{r}_1+\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\left.\vphantom{\frac{1}{1}}\right]=0, \tag{S.89}$$

where $\hat{r}_1=\hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.86), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (S.89) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{r}_1+\hat{r}_1+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\left.\vphantom{\frac{1}{1}}\right]=0,$$

or, equivalently, as

$$\mathbb{E}\left[\hat{r}_1\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\right|x\left.\vphantom{\frac{1}{1}}\right]=0.$$

It follows that, to induce the firm to set the efficient price $\hat{p}_1$ irrespective of his signal x, the fiscal policy must satisfy $dT_1\left(r_1\right)/dr=1/(v-1)$ for all r_1. Furthermore, one can verify that, when $dT_1\left(r_1\right)/dr=1/(v-1)$ for all r_1, the firm's payoff is quasi-concave in p_1, which implies that setting the price $p_1=\hat{p}_1$ is indeed optimal for all x. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient rules and

$$T_1(r)=\frac{r}{v-1}+s=\frac{1}{v-1}\frac{p_1 y_1}{P}+s,$$

where s is invariant in r, the firm's objective (S.85) is equal to

$$\mathbb{E}\left[\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}l_1+s\right|x\left.\vphantom{\frac{1}{1}}\right].$$

Using (S.86) and the fact that $dl_1/dp_1 = -vl_1/\psi p_1$, the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[-v\frac{y_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\middle| x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{1}{p_1}\left(v^2\frac{y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\middle| x\right].$$

From the analysis above, when $p_1 = \hat{p}_1$, $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ. Furthermore, irrespective of x, the derivative of the firm's objective function with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[-v\frac{\hat{y}_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\middle| x\right] = 0. \tag{S.90}$$

Using (S.90), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique critical point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the non-investing firms, we have that any fiscal policy that induces efficiency in information usage must pay to each non-investing firm a transfer equal to $T_0(r_0)$ such that $dT_0(r_0)/dr = 1/(v-1)$, and that any such policy indeed induces these firms to set a price equal to $\hat{p}_0$ irrespective of the signals x. Thus, we conclude that any policy inducing efficiency in information usage must have the structure

$$T_0\left(r\right) = \frac{1}{v-1}r, \tag{S.91}$$

and

$$T_1\left(\theta, r\right) = \frac{1}{v-1}r + s(\theta), \tag{S.92}$$

where we reintroduce the dependence of s on θ in light of the analysis below.

Part 2. Observe that, under any monetary and fiscal policy that implement the efficient allocation, the real revenues, i.e., the revenues expressed in terms of the consumption of the final good, must be the same as under flexible prices. This follows from the fact that the equilibrium in the market for intermediate goods implies that $\hat{y}_f = \hat{C}\left(\hat{P}/\hat{p}_f\right)^v$, for $f = 0, 1$, which means that $\hat{p}_f/\hat{P}$ – and hence $\hat{r}_f = (\hat{p}_f\hat{y}_f)/\hat{P}$ – is uniquely pinned down by the efficient allocation. Because the transfers to the firms are in terms of real revenues, and because real wages are also uniquely pinned down by the efficient allocation (as $\hat{W}/\hat{P} = \hat{L}^{\varepsilon}$), the value of investing and of acquiring information must coincide with their counterparts under flexible prices. In turn, this implies that the subsidy to the investing firms $s(\theta)$ must satisfy the same conditions as identified above for the case of flexible prices. Finally, that the analogs of Propositions 1 and 2 in the main text (but for the economy under consideration) hold follows directly from the same arguments as in the proofs of these propositions. Q.E.D.