Persuasion in Global Games with Application to Stress Testing

Nicolas Inostroza Alessandro Pavan

Northwestern University

March 1, 2018

Motivation

- Coordination: central to many socio-economic environments
- Damages to society of mis-coordination can be severe
  - Monte dei Paschi di Siena (MPS)
  - creditors with heterogenous beliefs about size of nonperforming loans
  - default by MPS: major crisis in Eurozone (and beyond)
- Government intervention
  - limited by legislation passed in 2015
- Persuasion (stress test design): **instrument of last resort**

Questions

- Structure of optimal stress tests?
- What information should be passed on to mkt?
- Optimality of
  - pass/fail policies
  - monotone rules
- Benefits to discriminatory disclosures?
- Properties of persuasion in global games?

Related literature

- **Information design:** Myerson (1986), Aumann and Maschler (1995), Calzolari and Pavan (2006,a,b), Kamenica and Gentzkow (2011),... Ely (2016) ...Bergemann and Morris (2017)...
- **Persuasion in Games:** Alonso and Camara (2013), Barhi and Guo (2016), Mathevet, Perego, Taneva (2016), Taneva (2016)......,Doval and Ely (2017)...
- **Persuasion with ex-ante heterogenous receivers:** Bergemann and Morris (2016), Kolotilin et al (2016), Laclau and Renou (2017), Chan et al (2016), Guo and Shmaya (2017)...
- **Discrimination and "Divide and Conquer":** Segal (2006), Wang (2015), Yamashita (2016)...
- **Financial Regulation and Stress Test Design:** Goldstein and Leitner (2015), Goldstein and Sapra (2014), Alvarez and Barlevy (2017), Bouvard et al. (2015), Goldstein and Huang (2016)...
- **Global Games w. Endogenous Info:** Angeletos, Hellwig and Pavan (2006, 2007), Angeletos and Pavan (2013), Edmond (2013), Iachan and Nenov (2015), Denti (2016),...

Plan

- Model
- Perfect coordination property
- Public disclosures
  - Pass/Fail Policies
  - Monotone rules
- Benefits of discriminatory disclosures
- Extensions and conclusions

Global Games of Regime Change

- Policy maker (PM)
- Agents $i \in [0,1]$
- Actions

$$a_i = \begin{cases} 1 & (attack) \\ 0 & (not\ attack) \end{cases}$$

- $A \in [0,1]$: aggregate attack
- Regime outcome: $r \in \{0,1\}$, with $r = 1$ in case of regime change (e.g., default)
- Regime rule

$$r = \begin{cases} 1 & if \quad A > \theta \\ 0 & if \quad A \leq \theta \end{cases}$$

- "fundamentals" θ parametrize amount of **performing loans**
- Dominance regions: $(-\infty, 0)$ and $[1, +\infty)$

Global Games of Regime Change

- PM's payoff

$$U^P(\theta, A) = \begin{cases} W & \text{if } r = 0 \\ L < W & \text{if } r = 1. \end{cases}$$

- Agents' payoff from attacking normalized to zero
- Agents' payoff from *not attacking*

$$u(\theta, A) = \begin{cases} g & \text{if } r = 0 \\ b & \text{if } r = 1 \end{cases}$$

with

$$g > 0 > b$$

Beliefs

- $\mathbf{x} = (x_i)_{i \in [0,1]} \in \mathbf{X}$: beliefs/signal profile with each

$$\tilde{x}_i \sim p(\cdot|\theta)$$

  i.i.d., given θ.

Disclosure Policies

- $m : [0, 1] \to S$ *message* function
- $m_i \in S$: information disclosed to i
- $M(S)$: set of all possible message functions with range S
- *Disclosure policy* $\Gamma = (S, \pi)$

  with $\pi : \Theta \to \Delta(M(S))$
- Non discriminatory disclosures: $m_i = m_j$ all θ, all $m \in supp[\pi(\theta)]$

Timing

1. PM announces $\Gamma = (S, \pi)$ and commits to it
2. $(\theta, \mathbf{x})$ realized
3. m drawn from $\pi(\theta) \in \Delta(M(S))$
4. Information m_i disclosed to agent i, $i \in [0,1]$.
5. Agents simultaneously choose whether or not to attack
6. Regime outcome and payoffs

Solution Concept: MARP

- Robust/adversarial approach
- PM can not select agents' strategy profile
- **Most Aggressive Rationalizable Profile** (MARP):

  minimizes PM's payoff across all profiles surviving *iterated deletion of interim strictly dominated strategies* (IDISDS)
- $a^{\Gamma} \equiv (a_i^{\Gamma})_{i \in [0,1]}$: MARP consistent with Γ

Perfect Coordination Property [PCP]

Definition

$\Gamma = \{S, \pi\}$ satisfies **PCP** if, for any $(\theta, \mathbf{x})$, any message function $m \in supp(\pi(\theta))$, any $i, j \in [0,1]$, $a_i^\Gamma(x_i, m_i) = a_j^\Gamma(x_j, m_j)$, where $a^\Gamma \equiv (a_i^\Gamma)_{i \in [0,1]}$ is MARP consistent with Γ

Perfect Coordination Property [PCP]

Theorem

Given any (regular) Γ, there exists (regular) Γ^* satisfying ***PCP*** *and yielding PM a payoff weakly higher than Γ.*

- Regularity: regime outcome under MARP measurable wrp PM's information

Perfect Coordination Property [PCP]

- Policy $\Gamma^* = (S^*, \pi^*)$ removes any **strategic uncertainty**
- It preserves (and, in some cases, enhances) heterogeneity in **structural uncertainty**
- Under Γ^*, agents know actions all other agents take but not *what beliefs rationalize such actions*
- Inability to predict beliefs that rationalize other agents' actions essential to minimization of risk of regime change
- Optimal tests
  - need not create conformism in beliefs
  - but should be transparent enough to remove uncertainty about market response

PCP: Proof sketch

- Let $r(\theta, m; a^\Gamma) \in \{0,1\}$ be regime outcome at (θ, m) when agents play according to a^Γ
- Let $\Gamma^* = \{S^*, \pi^*\}$ be s.t. $S^* = S \times \{0,1\}$ and
  $\pi^*((m, r(\theta, m; a^\Gamma))|\theta) = \pi(m|\theta)$, all (θ, m) s.t. $\pi(m|\theta) > 0$
- **Key step**: $r(\theta, m; a^\Gamma) = 0 \Rightarrow$ MARP under Γ^* less aggressive than MARP under Γ, i.e.,

$$a_i^\Gamma(x_i, m_i) = 0 \Rightarrow a_i^{\Gamma^*}(x_i, (m_i, 0)) = 0, \qquad \forall i, \forall (x_i, m_i)$$

- Size of attack under Γ^* smaller than under $\Gamma \Rightarrow r(\theta, m; a^{\Gamma^*}) = 0$
- That Γ^* weakly improves upon Γ follows from
  - probability of regime change under Γ^* same as under Γ (all θ)
  - size of attack when $r = 0$ smaller under Γ^* (relevant for general payoffs).

(formal proof)

PCP: Lesson

Optimal Disclosure Policy combines:

- Discriminatory messages to different market participants
- Public **Pass/Fail** announcements
  - eliminate Strategic Uncertainty

PCP – General

Optimality of PCP extends to economies in which

- regime outcome determined by more general rule $R(\theta, A)$
- PM's payoff

$$U^P(\theta, A) = \begin{cases} W(\theta, A) & \text{if } r = 0 \\ L(\theta) & \text{if } r = 1 \end{cases}$$

  with (a) $W_A(\theta, A) \leq 0$; (b) $W(\theta, A) - L(\theta, A) > 0$ if $R(\theta, A) > 0$
- finitely many agents with asymmetric payoffs $u_i(\theta, A)$
- arbitrary collection of beliefs

$$\Lambda_i(x_i) \in \Delta(\Theta \times \mathbf{X})$$

- level-K sophistication
- PM has imperfect information about θ and agents' beliefs
- Key assumptions:
  - supermodularity of game
  - measurability of regime outcome under MARP wrt PM's info

Public Disclosures

- Designer constrained to **non discriminatory** policies $\pi{:}\Theta \to \Delta(M(S))$ s.t. $m_i = m_j$ all $m \in supp(\pi(\theta))$, all θ.

Theorem

*Suppose $p(x|\theta)$ is log-supermodular. Given any non-discriminatory policy Γ, there exists **binary (non-discriminatory) policy** $\Gamma^* = (S^*, \pi^*)$ in which $S^* = \{0,1\}$ satisfying PCP and yielding higher payoff than Γ.*

- Optimal non-discriminatory policy: **stochastic pass/fail test**
- Log-SM of $p(x|\theta)$ guarantees MARP is in threshold strategies: signals other than regime outcome can be dropped without affecting incentives

(Example)

Monotone Tests

Condition M:

1. $\Delta^P(\theta) \equiv W(\theta,0) - L(\theta)$ non-decreasing
2. $b(\theta, P(x|\theta))$ and $p(x|\theta)$ log-SM.
3. For any x,

$$Y(\theta;x) \equiv \frac{\Delta^P(\theta)}{p(x|\theta)|b(\theta,P(x|\theta))|}$$

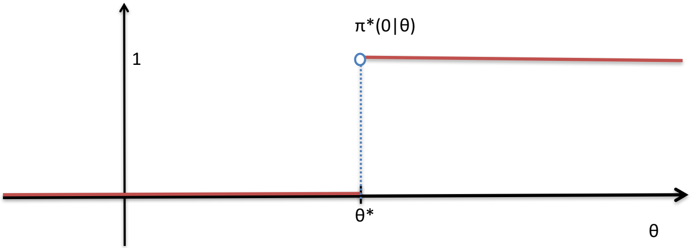
nondecreasing in θ over $[\underline{\theta}, \hat{\theta}(x)]$, with $\hat{\theta}(x)$ s.t. $R(\hat{\theta}(x), P(x|\hat{\theta}(x))) = 0$

Theorem

Suppose Condition M holds.

Given any non-discriminatory Γ, there exists ***monotone*** *non-discriminatory policy $\Gamma^* = (\{0,1\}, \pi^*)$ yielding higher payoff than Γ.*

Monotone Tests



Monotone Tests

- Optimality of monotone policies not guaranteed even in "canonical" case where W, L, b, g constant and $x_i = \theta + \sigma\varepsilon_i$ with $\varepsilon_i \sim N(0,1)$
  Counter-example

- Single receiver (same prior as PM): supermodularity of payoffs suffices (here: monotonicity of $\Delta^P(\theta)$)
  (Mensch, 2016)

Benefits of Discriminatory Disclosures

- In general, optimal stress test involves
  - public pass/fail announcement
  - discriminatory disclosures (DD)
- Benefits of DD do NOT come from possibility of tailoring information to agents' prior beliefs
- DD improve upon non-discriminatory rules even when agents' exogenous information is homogenous
- **KEY BENEFIT: Divide-and-Conquer**
- some agents find it dominant not to attack
- fraction of agents for whom not attacking dominant not CK
- iteratively dominant for all not to attack (when $s = 0$)

Optimality of non discrimination

- Conditions for optimal policy to be non discriminatory: upside risk "dominating" downside risk
  - Sensitivity of payoffs to θ higher when $r = 0$ than when $r = 1$
- Condition holds, e.g., under **equity claims** (junior/subordinated)

  - under default, liquidation value little sensitive to amount of performing loans

  - when bank survives, value of claims reflects long-term profitability
- Less precise private info $\rightarrow$ mean-preserving-spread in cross-section of beliefs $\rightarrow$ smaller attack

(NDD)

Extensions and Conclusions

- Information design in coordination games with heterogeneously informed agents
- Application: Stress Test Design
  - Perfect coordination property (“right” notion of transparency)
  - Pass/Fail tests
  - monotone rules
  - benefits to discrimination
  -
- Extension 1: PM uncertain about prior beliefs (robust-undominated design)
- Extension 2: timing of optimal disclosures
- Extension 3: Screening of banks’ balance sheets

THANKS!

PCP Proof

- After receiving $m_i^+ \equiv (m_i, 0)$, agent uses Bayes' rule to update beliefs about (θ, m):

$$\partial \Lambda_i^{\Gamma^+}(\theta, m | x_i, (m_i, 0)) = \frac{1\{r(\theta, m; a^\Gamma) = 0\}}{\pi_i^\Gamma(0 | x_i, m_i)} \partial \Lambda_i^\Gamma(\theta, m | x_i, m_i)$$

where

$$\pi_i^\Gamma(0 | x_i, m_i) \equiv \int_{\{(\theta, m): r(\theta, m; a^\Gamma) = 0\}} d\Lambda_i^\Gamma(\theta, m | x_i, m_i)$$

PCP Proof

- Let $a^{\Gamma}_{(n)}$, $a^{\Gamma^+}_{(n)}$ be most aggressive profile surviving n round of IDISDS under Γ and Γ^+, respectively.

Definition

Strategy profile $a^{\Gamma^+}_{(n)}$ less aggressive than $a^{\Gamma}_{(n)}$ iff, for any $i \in [0,1]$,

$$a^{\Gamma}_{(n),i}(x_i, m_i) = 0 \Rightarrow a^{\Gamma^+}_{(n),i}(x_i, (m_i, 0)) = 0$$

Lemma

For any n, $a^{\Gamma^+}_{(n)}$ less aggressive than $a^{\Gamma}_{(n)}$

PCP Proof

- Proof by induction
- Let $a_0^{\Gamma} = a_0^{\Gamma^+}$ be strategy profile where all agents attack regardless of their (endogenous and exogenous) information
- Suppose that $a_{(n-1)}^{\Gamma^+}$ less aggressive than $a_{(n-1)}^{\Gamma}$
- Note that $r(\theta, m|a^{\Gamma}) = 1 \Rightarrow r(\theta, m|a_{(n-1)}^{\Gamma}) = 1$ ($a_{(n-1)}^{\Gamma}$ more aggressive than $a^{\Gamma} = a_{\infty}^{\Gamma}$)
- Hence, $r(\theta, m; a^{\Gamma}) = 0$ "removes" from support of agents' beliefs states (θ, m) for which regime change occurs under $a_{(n-1)}^{\Gamma}$.

PCP Proof

Because

- payoffs in case of regime change are negative
- $r(\theta, m; a^{\Gamma}) = 0$ removes from support of agents's beliefs states at which regime change occurs also under $a^{\Gamma}_{(n-1)}$

payoff from **not attacking** under Γ^{+} when agents follow $a^{\Gamma}_{(n-1)}$

$$U_i^{\Gamma^+}(x_i,(m_i,0);a^{\Gamma}_{(n-1)}) = \frac{\int_{(\theta,m)} u(\theta,A(\theta,m;a^{\Gamma}_{(n-1)}))1\{r(\theta,m;a^{\Gamma})=0\}d\Lambda_i^{\Gamma}(\theta,m|x_i,m_i)}{\pi_i^{\Gamma}(0|x_i,m_i)}$$

$$> \frac{\int_{(\omega,m)} u(\theta,A(\theta,m;a^{\Gamma}_{(n-1)}))d\Lambda_i^{\Gamma}((\theta,m)|x_i,m_i)}{\pi_i^{\Gamma}(0|x_i,m_i)}$$

$$= \frac{U_i^{\Gamma}(x_i,m_i;a^{\Gamma}_{(n-1)})}{\pi_i^{\Gamma}(0|x_i,m_i)}$$

- Hence, $U_i^{\Gamma}(x_i, m_i; a^{\Gamma}_{(n-1)}) > 0 \Rightarrow U_i^{\Gamma^+}(x_i, (m_i, 0); a^{\Gamma}_{(n-1)}) > 0$

PCP Proof

- That $a^{\Gamma^+}_{(n-1)}$ less aggressive than $a^{\Gamma}_{(n-1)}$ along with supermodularity of game implies that

$$U_i^{\Gamma^+}(x_i, m_i; a^{\Gamma}_{(n-1)}) > 0 \Rightarrow U_i^{\Gamma^+}(x_i, (m_i, 0); a^{\Gamma^+}_{(n-1)}) > 0$$

- As a consequence,

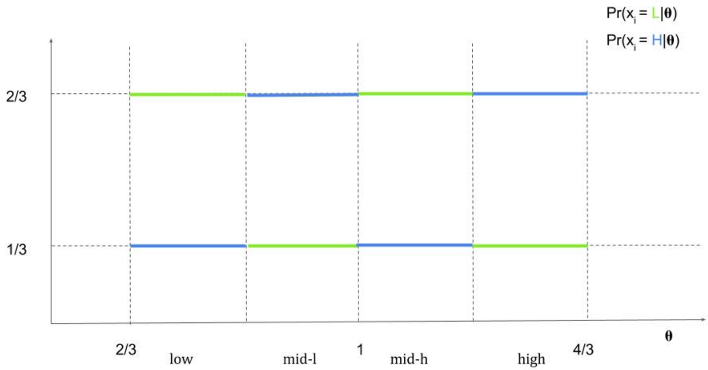
$$a^{\Gamma}_{(n),i}(x_i, m_i) = 0 \Rightarrow a^{\Gamma^+}_{(n),i}(x_i, (m_i, 0)) = 0$$

- This means that $a^{\Gamma^+}_{(n)}$ less aggressive than $a^{\Gamma}_{(n)}$.

PCP Proof

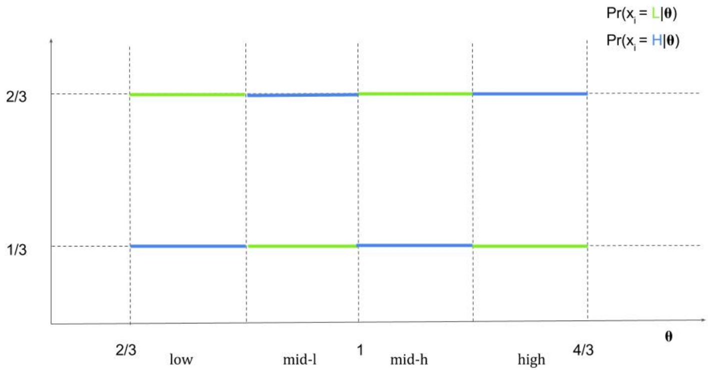
- Above lemma implies MARP under Γ^+, $a^{\Gamma^+} \equiv a^{\Gamma^+}_{(\infty)}$, less aggressive than MARP under Γ, $a^\Gamma \equiv \bar{a}^\Gamma_{(\infty)}$
- In turn, this implies that $r(\theta, m; a^\Gamma) = 0$ makes it common certainty that $r(\theta, m; a^{\Gamma^+}) = 0$
- Hence, no agent attacks after hearing $r(\theta, m; a^\Gamma) = 0$
- Similarly, $r(\theta, m; a^\Gamma) = 1$ makes it common certainty that $\theta < 1$. Under MARP, all agents attack when hearing that $r(\theta, m; a^\Gamma) = 1$
- That Γ^+ weakly improves upon Γ follows from
  - probability of regime change under Γ^+ same as under Γ (all θ)
  - size of attack when $r = 0$ smaller under Γ^+ (relevant for more general payoffs).

Example



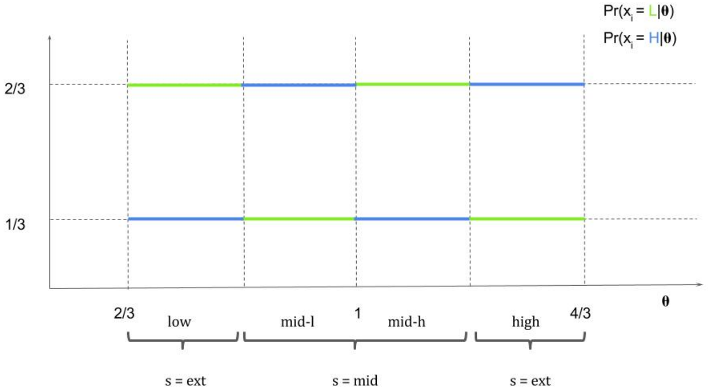
- Assume $g = -b$
- Attacking rationalizable iff $Pr(r = 1) \geq 1/2$

Example



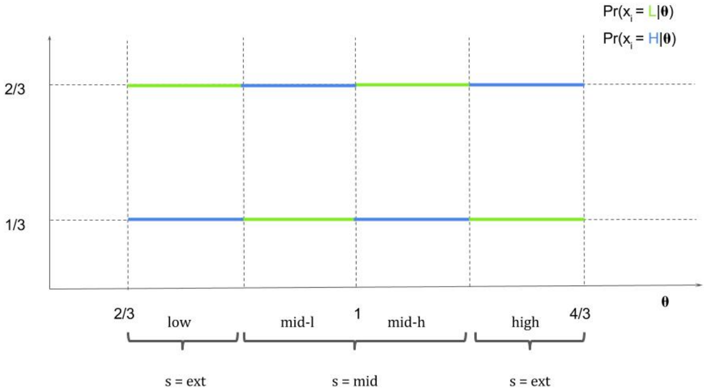
- No disclosure implies that, under MARP, $a_i^{\Gamma}(x_i) = 1$, all x_i

Example



- Suppose agents are informed whether θ is extreme or intermediate
- $a_i^\Gamma(x_i, s) = 0$, all (x_i, s)

Example



- Now suppose Γ only recommends not to attack. Then, $a_i^\Gamma(x_i, 0) = 1$ for all x_i
- **Suboptimality of P/F policies (+ failure of RP)**

Counter-example

- Suppose
  - $\Delta(\theta) = W - L$
  - $x_i = \theta + \sigma\varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}(0,1)$
  - $g = 1 - c$ and $b = -c$ with $c > 1/2$
- Observe that $\theta^{MS} = c$
- Regime change occurs iff $\theta < \theta^{\#}_{\sigma}$
- Marginal agent $x_\sigma = \theta^{\#}_{\sigma} + \sigma\Phi^{-1}(\theta^{\#}_{\sigma})$
- $lim_{\sigma \to 0^+} \theta^{\#}_{\sigma} = \theta^{MS} > 1/2$
- Hence, $x_\sigma > \theta^{\#}_{\sigma} > \theta^{\inf}_{\sigma}$ for small σ, where $\theta^{\inf}_{\sigma}$ is regime threshold under best mon. policy.
- Therefore

$$Y(\theta; x_\sigma) \equiv \frac{\Delta^P(\theta)}{p(x_\sigma|\theta)|b(\theta, P(x_\sigma|\theta))|} = \frac{W - L}{\phi(\frac{x_\sigma - \theta}{\sigma}) \cdot c}$$

strictly decreasing over $[\underline{\theta}, \theta^{\inf}_{\sigma}]$, which **implies best mon policy can be improved upon by non-monotone policy**

Optimality of NDD

- Suppose PM can engineer *any* public disclosure but constrained to *Gaussian private communications*
  - F improper uniform over $\mathbb{R}$
  - exogenous signals $x_i = \theta + \sigma_\eta \eta_i$, with $\eta_i \sim \mathcal{N}(0,1)$

$$\widetilde{m}_i = \theta + \sigma_\xi \xi_i, \text{ with } \xi_i \sim \mathcal{N}(0,1)$$

  - agents' payoffs depend only on $\theta : \bar{g}(\theta)$ and $\bar{b}(\theta)$
- Info disclosed to i: $m_i = (s, \widetilde{m}_i)$
- Information contained in $(x_i, \widetilde{m}_i)$ summarized by

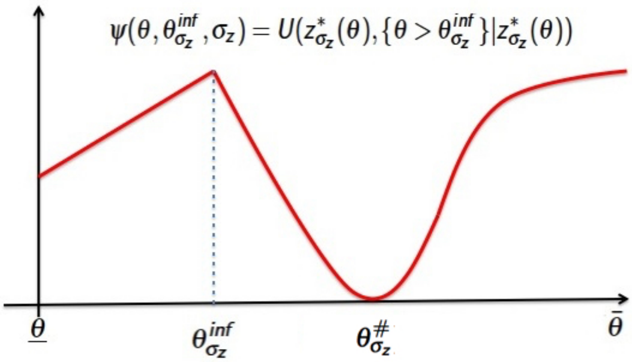
$$z_i \equiv \frac{\sigma_\xi^2 x_i + \sigma_\eta^2 \tilde{m}_i}{\sigma_\eta^2 + \sigma_\xi^2}$$

- PM's choice of discriminatory part of her policy parametrized by $\sigma_z \in (0, \sigma_\eta]$

Optimality of NDD

- MARP $a_i^{\Gamma}(x_i,(s,\tilde{m}_i)) = 1\{z_i \leq \bar{z}(s)\}$
- PCP: $\bar{z}(0) = -\infty$, and $\bar{z}(1) = +\infty$.
- Let
$$z^*_{\sigma_z}(\theta) = \theta + \sigma_z \Phi^{-1}(\theta),$$
denote "marginal agent" s.t., when agents follow cut-off strategies with cut-off $z^*_{\sigma_z}(\theta)$, regime change occurs iff $\tilde{\theta} \leq \theta$
- Let
$$\psi(\theta,\hat{\theta},\sigma_z) = U(z^*_{\sigma_z}(\theta),\{\tilde{\theta} > \hat{\theta}\}|z^*_{\sigma_z}(\theta))$$
- Define
$$\theta^{inf}_{\sigma_z} \equiv \inf\left\{\hat{\theta} :\ \psi(\theta,\hat{\theta},\sigma_z) > 0\, all\ \theta\right\}.$$
- For any $\hat{\theta} > \theta^{inf}_{\sigma_z}$, unique rationalizable profile has no agent attacking after s publicly reveals that $\theta \geq \hat{\theta}$

Optimality of NDD



Optimality of NDD

Proposition

Let

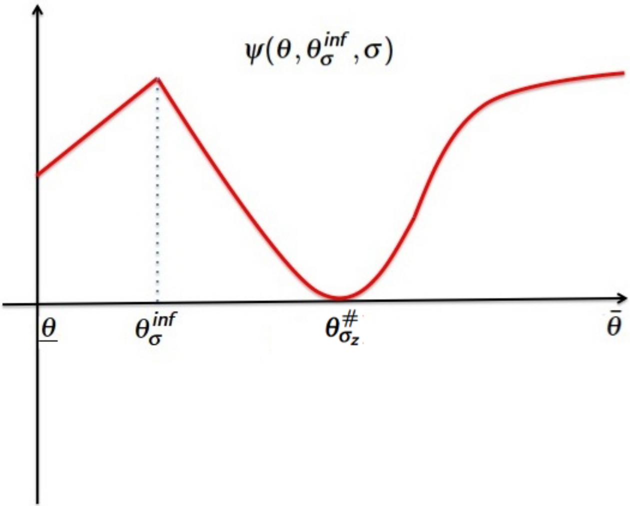
$$\sigma_z^* \equiv argmin_{\sigma_z \in (0,\sigma_\eta]} \theta_{\sigma_z}^{inf}$$

Optimal (Gaussian) policy combines public disclosure of whether or not $\theta < \theta_{\sigma_z^*}^{inf}$, with Gaussian private messages of precision

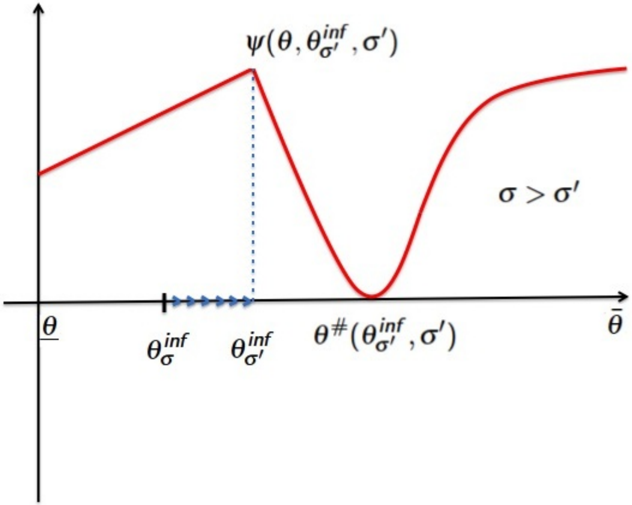
$$\sigma_\xi^{-2} = [\sigma_\eta^2 - (\sigma_z^*)^2]/(\sigma_z^*)^2\sigma_\eta^2$$

- Precision σ_ξ^{-2} guarantees that sufficient statistics has precision $1/\sigma_z^*$

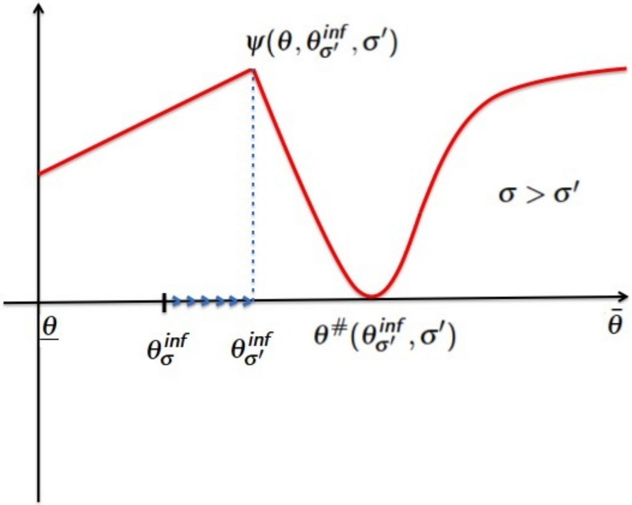
Optimality of NDD



Optimality of NDD



Optimality of NDD



Optimality of NDD

- Suppose agents' payoffs given by $\bar{b}(\theta)$ and $\bar{g}(\theta)$
- Let $\theta^{\#}_{\sigma_z}$ and $z^{\#}_{\sigma_z}$ denote regime threshold and "marginal agent" under optimal policy when information has precision σ_z^{-2}.

Proposition

Suppose that, for any $\sigma_z \in [0, \sigma_\eta]$,

$$\frac{\mathbb{E}[\bar{g}'(\theta)(\theta - \theta^{\#}_{\sigma_z})|z^{\#}_{\sigma_z}, \theta \geq \theta^{\#}_{\sigma_z}]}{\mathbb{E}[\bar{g}(\theta)|z^{\#}_{\sigma_z}, \theta \geq \theta^{\#}_{\sigma_z}]} > \frac{\mathbb{E}[\bar{b}'(\theta)(\theta - \theta^{\#}_{\sigma_z})|z^{\#}_{\sigma_z}, \theta \in (\theta^{inf}_{\sigma_z}, \theta^{\#}_{\sigma_z})]}{\mathbb{E}[|\bar{b}(\theta)||z^{\#}_{\sigma_z}, \theta \in (\theta^{inf}_{\sigma_z}, \theta^{\#}_{\sigma_z})]}$$

Optimal (Gaussian) policy is non-discriminatory.

- Condition says upside risk "dominates" downside risk