Robust Procurement Design

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Motivation

- Procurement

  central economic problem

  (a) public

  (b) private

- Difficulty in designing contracts

  Provider typically **privately informed** about cost

Motivation

- Standard approach: Bayesian
  - Buyer has conjecture/belief over

    (a) supplier's cost

    (b) gross value for procuring output
  - maximizes under conjecture
- CS (robust) approach
  - buyer has no conjecture
  - worst-case optimality

Motivation

- **Our Approach (alternative form of robustness)**
  - buyer has conjecture but does not fully trust it
  - prepares for the worst (in case conjecture is wrong)
  - uses conjecture to select optimal mechanism among worst-case optimal ones
  - Lexicographic approach

    (a) political/hierarchical constraints

    (b) attitude towards ambiguity

This Paper: Results

- Robust design of procurement contracts
- Uncertainty only over cost:
  - Baron Myerson with **quantity floor**
- Uncertainty over both cost and demand
  - **upward quantity adjustment for high cost**
  - **downward quantity adjustment for low/intermediate costs**
- Robustly optimal mechanism sensitive only to
  - conjecture over demand and cost
  - lowest admissible demand
- CS wrt buyer's uncertainty and policy recommendations

Literature – Incomplete

- Baron-Myerson, Ecma (1982), Lewis and Sappington (1988), Armstrong, JET (1999)
  - **Bayesian**
- Bergemann-Heumann-Morris, wp (2024)
  - **different robustness criterion (competitive ratio)**
- Guo-Shmaya, wp (2024)
  - **different robustness criterion (min-max regret)**
- Garrett, GEB (2014)
  - **different robustness criterion (worst-case optimality)**
- Mishra-Patel, wp (2024)
  - **undominated mechanisms**
- Dworczak-Pavan, Ecma (2023)
  - **robust information design**

Plan

1. Introduction
2. **Model**
3. Baron-Myerson
4. Short List: Worst-Case Optimality
5. Robustly Optimal Mechanisms
6. Ongoing Work
7. Conclusions

Model

Model

- **Players**
  - Buyer (principal/government)
  - Seller (agent/monopolist)
- **Choices**
  - output $q \geq 0$
  - transfer $t \geq 0$

Model

- **Payoffs**
  - Social value of q:
$$V^\star(q) = \int_0^q P^\star(s)ds$$
  - Cost: θq

    θ drawn from ab. cont. $F^\star$ with $f^\star(\theta) > 0$ over $\Theta = [\underline{\theta}, \bar{\theta}]$
  - Ex-post welfare ($\alpha \in [0, 1]$)
$$V^\star(q) - t + \alpha\,(t - \theta q)$$
  - Ex-ante welfare:
$$\int \left(V^\star(q) - t + \alpha\,(t - \theta q)\right) dF^\star(\theta)$$
- **Asymmetric information**
  - θ: monopolist's private information

Model

- **Uncertainty/Robustness**
  - government not sure about conjecture $(V^\star, F^\star)$
  - concerned demand and cost may be $(V, F) \neq (V^\star, F^\star)$
- **Admissible sets**
  - $\mathcal{V}$: set of possible consumer (gross) value fns
    - each $V \in \mathcal{V}$ strictly increasing, strictly concave, differentiable
  - $\mathcal{P}$: set of corresponding *inverse demand functions*
  - $P^\star \in \mathcal{P}$ and $V^\star \in \mathcal{V}$
  - lowest (inverse) demand function: $\underline{P}$
    - any $q \geq 0$ and $P \in \mathcal{P}$
      $$P(q) \geq \underline{P}(q)$$
    - $\underline{P}$: strictly decreasing and continuous and s.t.
      $$\lim_{q \to 0^+} \underline{P}(q) > \overline{\theta}$$

Model

- **(Direct) mechanism** $M = (q, t)$
  - quantity schedule $q : \Theta \to \mathbb{R}_+$
  - (total) transfer schedule $t : \Theta \to \mathbb{R}$
- $M = (q, t)$ IC and IR iff, for all θ, θ',

$$t(\theta) - \theta q(\theta) \geq t(\theta') - \theta q(\theta')$$

with

$$t(\theta) - \theta q(\theta) \geq 0$$

- $\mathcal{M}$: **set of IC and IR mechanisms**

Model

- Given IC and IR mechanism $M = (q, t)$, **ex-ante welfare** under (V, F)

$$W(M; V, F) := \int w(\theta, M; V) F(\mathrm{d}\theta)$$

where **ex-post welfare:**

$$w(\theta, M; V) := V(q(\theta)) - \theta q(\theta) - (1 - \alpha) u(\theta)$$

total surplus:

$$V(q(\theta)) - \theta q(\theta)$$

"**rent**" to type θ:

$$u(\theta) := t(\theta) - \theta q(\theta)$$

Model

Definition

Given any IC and IR $M = (q, v)$, **welfare guarantee**

$$G(M) := \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F)$$

- **Short-list:**

$$\mathcal{M}^{\mathrm{SL}} := \{M \in \mathcal{M} : G(M) \geq G(M') \; \forall \; M' \in \mathcal{M}\}$$

(set of IC and IR mechanisms for which guarantee is maximal)

Model

- **Government's problem**
  - choose mechanism from $\mathcal{M}^{\mathrm{SL}}$ maximizing welfare under conjecture $(V^\star, F^\star)$

Definition

Mechanism $M \in \mathcal{M}^{\mathrm{SL}}$ **robustly optimal** iff, for every $M' \in \mathcal{M}^{\mathrm{SL}}$,

$$W(M; V^\star, F^\star) \geq W(M'; V^\star, F^\star)$$

- Robustly optimal mechanisms maximize ex-ante welfare under conjecture $(V^\star, F^\star)$ over all worst-case-optimal mechanisms

Plan

Baron Myerson

Baron Myerson

- $\partial u(\theta)$: set of all subgradients of u at θ
  - $x \in \mathbb{R}$ is subgradient iff for all θ', $u(\theta') \geq u(\theta) + x(\theta' - \theta)$

Lemma

Mechanism $M = (q, u)$ IC and IR iff u is convex, non-increasing, and s.t, for all $\theta \in \Theta$, $-q(\theta) \in \partial u(\theta)$, with $u(\overline{\theta}) \geq 0$. Equivalently,

1. q non-increasing
2. for all $\theta \in \Theta$,

$$u(\theta) = u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(z)dz$$

3. $u(\overline{\theta}) \geq 0$

Baron Myerson

- $\theta' s$ **virtual cost** (under conjecture $F^\star$):

$$z^\star(\theta) := \theta + (1-\alpha)\frac{F^\star(\theta)}{f^\star(\theta)}$$

Proposition

Assume $z^\star$ non-decreasing. (Bayesian) optimal mechanism $M^{BM} = (q^{BM}, u^{BM})$ s.t., for all θ,

$$q^{BM}(\theta) := \arg\max_q \{V^\star(q) - z^\star(\theta)q\}$$

$$u^{BM}(\theta) = \int_\theta^{\bar{\theta}} q^{BM}(z)dz$$

Baron Myerson

- **Proof**: Familiar IC analysis $\rightarrow$ ex-ante welfare under conjecture $(V^\star, F^\star)$

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \right] dF^\star(\theta)$$

- $q^{BM}(\theta)$ maximizes virtual surplus $V^\star(q) - z^\star(\theta) q$ point-wise
- $z^\star$ non-decreasing $\Rightarrow q^{BM}$ non-increasing $\Rightarrow (q^{BM}, u^{BM})$ IC and IR

Baron Myerson

- FB-efficiency (under conjecture $(V^\star, F^\star)$):

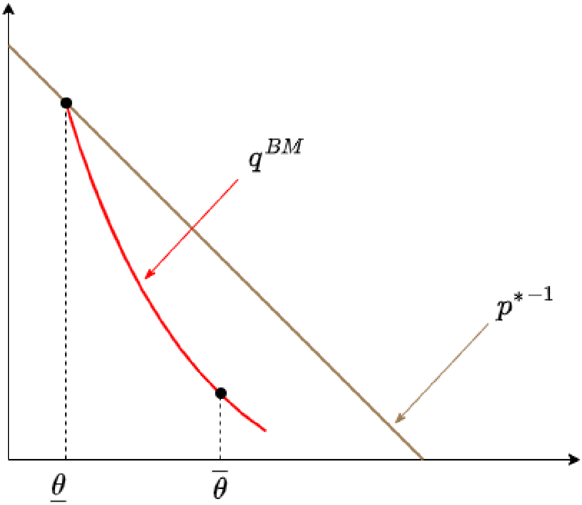
$$P^\star(q^{FB}(\theta)) = \theta$$

- BM schedule (second-best efficiency)

$$P^\star(q^{BM}(\theta)) = z^\star(\theta) := \theta + (1-\alpha)\frac{F^\star(\theta)}{f^\star(\theta)}$$

- Hence,
  - **no distortion "at top"**, i.e., for most efficient type, $\underline{\theta}$
  - **downward distortions for all** $\theta > \underline{\theta}$

Baron Myerson



Plan

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4. **Short List: Worst-Case Optimality**
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Short List

Worst-case optimality

Definition

Mechanism $M = (q, u)$ **worst-case optimal** iff, for any IC and IR $M' = (q', u') \in \mathcal{M}$,

$$G(M') := \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F) \leq \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F) := G(M)$$

Worst-case optimality

- Let
$$q_\ell := \arg\max_q \left\{ \underline{V}(q) - \bar{\theta} q \right\}$$
denote efficient output when $V = \underline{V}$ and $\theta = \bar{\theta}$

Lemma

For any IC and IR mechanism $M = (q, u) \in \mathcal{M}$
$$G(M) = \inf_{\theta \in \Theta} \{ \underline{V}(q(\theta)) - \theta q(\theta) - (1 - \alpha) u(\theta) \}$$
and
$$G(M) \leq G^* := \underline{V}(q_\ell) - \bar{\theta} q_\ell .$$

Worst-case optimality

Proof: For any IC and IR mechanism $M = (q, u) \in \mathcal{M}$

$$W(M; V, F) := \int \{V(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\} F(\mathrm{d}\theta)$$
$$\geq \int \{\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\} F(\mathrm{d}\theta)$$
$$\geq \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Hence,

$$G(M) \geq \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Because $\underline{V} \in \mathcal{V}$ and, for each θ, Dirac distribution at θ is in $\mathcal{F}$

$$G(M) \leq \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Hence,

$$G(M) = \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Because $u(\bar{\theta}) \geq 0$ and

$$\underline{V}(q(\bar{\theta})) - \bar{\theta} q(\bar{\theta}) \leq \underline{V}(q_\ell) - \bar{\theta} q_\ell := G^*$$

$G(M) \leq G^*$

Short List: Characterization

- Let

$$\mathcal{M}^{\mathrm{SL}} := \{M \in \mathcal{M} : G(M) \geq G(M') \ \forall \ M' \in \mathcal{M}\}$$

Lemma

$M = (q, u) \in \mathcal{M}^{\mathrm{SL}}$ iff q non-increasing and, for any θ

$$u(\theta) = \int_{\theta}^{\bar{\theta}} q(y) dy$$

and

$$\underline{V}(q(\theta)) - \theta q(\theta) - (1 - \alpha) \int_{\theta}^{\bar{\theta}} q(y) dy \geq G^*$$

Short List: Characterization

Proof:

- $M = (q, u)$ IC and IR:
  - (a) q nondecreasing
  - (b) $u(\theta) = u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(y)dy$, with $u(\bar{\theta}) \geq 0$
- ONLY IF:
  - $M_L = (q_L, u_L)$ w. $q_L(\theta) = q_\ell$ and $t_L(\theta) = \bar{\theta} q_\ell$ all θ
    (i) IC and IR
    (ii) $u_L(\theta) = (\bar{\theta} - \theta)q_\ell$
    (iii) $w(\theta, M_L; \underline{V}) := \underline{V}(q_\ell) - \theta q_\ell - (1 - \alpha)u_L(\theta) = G^* + \alpha(\bar{\theta} - \theta)q_\ell$
    (iv) $G(M_L) = G^*$
  - $M \in \mathcal{M}^{\mathrm{SL}}$ **only if** $G(M) = G^*$
  - Because $w(\theta, M; \underline{V}) := \underline{V}(q(\theta)) - \theta q(\theta) - (1 - \alpha)u(\theta)$
    (1) $u(\bar{\theta}) = 0$
    (2) $\underline{V}(q(\theta)) - \theta q(\theta) - (1 - \alpha) \int_{\theta}^{\bar{\theta}} q(y)dy \geq G^*$ all θ
- IF PART: immediate

Plan

Robust Optimality

Robust Optimality: **Full Program**

- Recall def of virtual cost under conjecture $(F^\star, V^\star)$

$$z^\star(\theta) := \theta + (1-\alpha)\frac{F^\star(\theta)}{f^\star(\theta)}$$

- Robustly optimal schedule q^{OPT} solves

$$\max_q \int_{\underline{\theta}}^{\bar{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta)q(\theta)\Big] dF^\star(\theta)$$

$$\text{s.t.}$$

$$q \;\; \text{non-increasing}$$

$$\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q(y)dy \geq G^* \qquad \forall\ \theta \in \Theta$$

Robust Optimality: **Relaxed Program**

- Relaxation:

$$\max_q \int_{\underline{\theta}}^{\bar{\theta}} \left[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \right] dF^\star(\theta)$$

$$\text{s.t.}$$

$$q \quad \text{non-increasing}$$

$$q(\theta) \geq q_\ell \qquad \forall \ \theta \in \Theta$$

$$q(\bar{\theta}) = q_\ell$$

Robust Optimality: **Relaxed Program**

- **Proof**: $M = (q, u) \in \mathcal{M}^{\text{SL}}$ only if, for all θ,

$$\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha) \int_{\theta}^{\bar{\theta}} q(y) dy \geq G^*$$

  - Because $G^* = \max_q \{\underline{V}(q) - \bar{\theta} q\} = \underline{V}(q_\ell) - \bar{\theta} q(q_\ell)$
  - $\Rightarrow q(\bar{\theta}) = q_\ell$
  - q non-increasing $\Rightarrow q(\theta) \geq q_\ell$ all θ

Robust Optimality: **BM with Floor**

- Let

$$q^{BM}(\theta) := \arg\max_{q}\{V^\star(q) - z^\star(\theta)q\}$$

$$q^\star(\theta) := \begin{cases} \max\{q^{\mathrm{BM}}(\theta), q_\ell\} & \text{if } \theta \neq \overline{\theta} \\ q_\ell & \text{if } \theta = \overline{\theta} \end{cases}$$

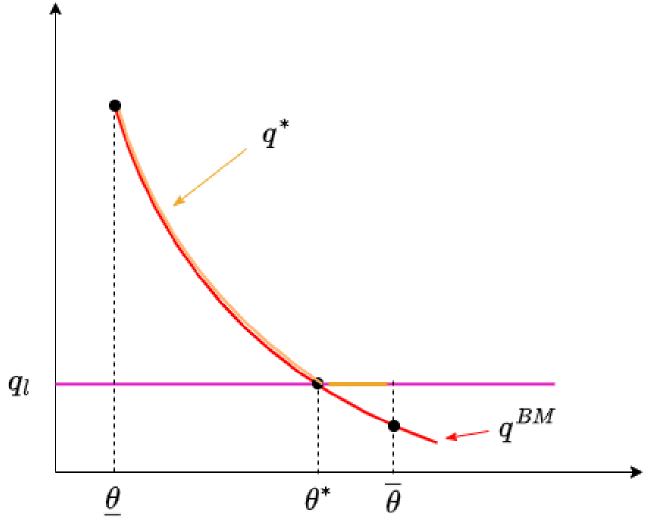
and

$$u^\star(\theta) = \int_{\theta}^{\bar{\theta}} q^\star(y)dy$$

Proposition

Suppose $z^\star$ non-decreasing and $V^\star = \underline{V}$. Mechanism $M^\star = (q^\star, u^\star)$ robustly optimal

Robust Optimality: BM with Floor



Robust Optimality: BM with Floor

Proof

- $M^\star = (q^\star, u^\star)$ solves relaxed program
  - $q^\star(\theta)$ maximizes $V^\star(q) - z^\star(\theta)q$ under constraint $q \geq q_\ell$
  - $z^\star$ non-decreasing $\Rightarrow$ $M^\star$ IC and IR
- Ex-post welfare under $M^\star$ and $\underline{V}$

$$w(\theta; M^\star, \underline{V}) := \underline{V}(q^\star(\theta)) - \theta q^\star(\theta) - (1-\alpha) \int_{\theta}^{\bar{\theta}} q^\star(y)dy$$

- $V^\star = \underline{V} \Rightarrow w(\cdot; M^\star, \underline{V})$ non-increasing, with $w(\bar{\theta}; M^\star, \underline{V}) = \underline{V}(q_\ell) - \bar{\theta} q_\ell := G^*$
- Robustness constraints $w(\theta; M^\star, \underline{V}) \geq G^*$ satisfied
- $\Rightarrow M^\star = (q^\star, u^\star)$ **robustly optimal**

Robust Optimality: BM with Floor

- When demand known (only uncertainty over cost)
  - robust mechanism is Baron **Myerson with floor**
  - **efficiency at both bottom and top**
    - possibility that cost is less favorable than conjectured $\rightarrow$ more output
  - flat mechanisms never optimal:

$$\theta^{\star} := \inf \{\theta \in \Theta : q^{\star}(\theta) = q_{\ell}\} > \underline{\theta}$$

  - contrast w. standard "CS" approach to robustness ($q(\theta) = q_{\ell}$ all θ)

Robust Optimality: general case

- For any $M = (q, u)$ s.t. $u(\theta) = \int_{\theta}^{\bar{\theta}} q(y)dy$, ex-post welfare under lowest demand

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha) \int_{\theta}^{\bar{\theta}} q(y)dy$$

- Robustness:

$$\underline{W}(\theta, q) \geq G^* \qquad \forall\ \theta \in \Theta$$

Robust Optimality: general case

Theorem

Suppose $z^\star$ non-decreasing

(1) BM-floor mechanism $M^\star \equiv (q^\star, u^\star)$ robustly optimal iff

$$\theta_m := \sup\{\theta : \theta \in \arg\inf_{\theta'} \underline{W}(\theta', q^\star)\} = \bar{\theta}$$

(2) If $\theta_m < \bar{\theta}$, then $\theta_m < \theta^\star := \inf\{\theta \in \Theta : q^\star(\theta) = q_\ell\}$ and

(a) $q^{\mathrm{OPT}}(\theta) = q_\ell$ for all $\theta \in [\theta^\star, \bar{\theta}]$

(b) $q^{\mathrm{OPT}}(\theta) \leq q^{BM}(\theta)$ for almost all $\theta \leq \theta^\star$

(inequality strict over positive-measure $I \subseteq [\underline{\theta}, \theta^\star]$)

(3) if $\theta_m \in (\underline{\theta}, \bar{\theta})$, $\alpha = 0$, and $z^\star$ increasing, $q^{\mathrm{OPT}}(\theta) = q^{BM}(\theta)$ for $\theta \in (\underline{\theta}, \theta_m)$

Robust Optimality: general case

- (Qualitative) properties driven by

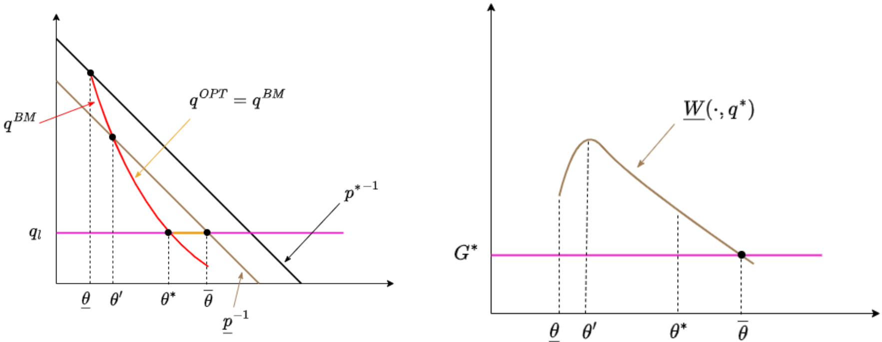
$$\underline{W}(\theta, q^\star) := \underline{V}(q^\star(\theta)) - \theta q^\star(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q^\star(y)dy$$

Lemma (Monotonicity)

For any positive, non-decreasing q

1. *When* $q(\theta) \leq \underline{P}^{-1}(\theta)$ *all* $\theta \in I \subset [\underline{\theta}, \theta^\star]$, $\underline{W}(\cdot, q)$ *non-increasing (decreasing if* $\alpha > 0$, *or q decreasing and* $q(\theta) < P^{-1}(\theta)$ *all* $\theta \in I$*)*
2. *When* $q(\theta) > \underline{P}^{-1}(\theta)$ *all* $\theta \in I \subset [\underline{\theta}, \theta^\star]$, *and* $\alpha = 0$, $\underline{W}(\cdot,)$ *non-decreasing (increasing is q decreasing)*

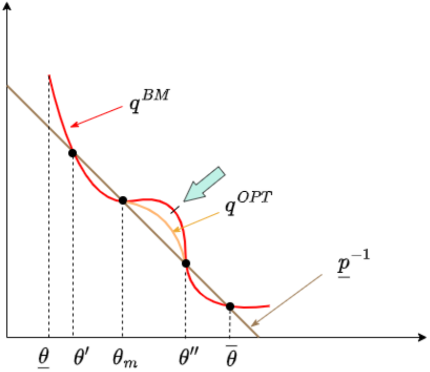
Robust Optimality: BM-floor optimal

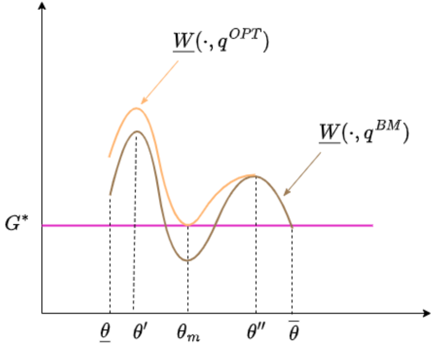


Robust Optimality: BM-floor optimal

- When $\alpha = 0$, $M^\star \equiv (q^\star, u^\star)$ robustly optimal if $\underline{W}(\underline{\theta}, q^\star) \geq G^*$
- Equivalently, when $\underline{V}(q^\star(\underline{\theta})) - \underline{\theta} q^\star(\underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} q^\star(y) dy \geq \underline{V}(q_\ell) - \bar{\theta} q_\ell$
- $q^\star(\underline{\theta}) = \arg\max_q \{V^\star(q) - \underline{\theta} q\}$
- Hence inequality holds when $|| \, V^\star, \underline{V} \, ||$ small

Robust Optimality: BM with floor and intermediate downward adjustments





Optimality of “plateau” for high costs

Lemma

Suppose $z^\star$ non-decreasing and $\theta_m \neq \bar{\theta}$. Then $q^{\mathrm{OPT}}(\theta) = q_\ell$ for all $\theta \in [\theta^\star, \bar{\theta}]$

- Minimizing quantity over $[\theta^\star, \bar{\theta}]$
  - increases $\underline{W}(\theta, q)$ for all $\theta \leq \theta^\star$ (by reducing rents)
  - possibly decreases $\underline{W}(\theta, q)$ for $\theta > \theta^\star$(by reducing $\underline{V}(q) - \theta q$)
  - However, $\underline{W}(\cdot, q^*)$ decreasing over $[\theta^\star, \bar{\theta}]$ w. $\underline{W}(\bar{\theta}, q^*) = G^*$
- For $\theta \in [\theta^\star, \bar{\theta}]$
  - q_ℓ maximizes $V^\star(q) - z^\star(\theta)q$ over $[q_\ell, +\infty)$
- Hence under any $M^{\mathrm{OPT}} = (q^{\mathrm{OPT}}, u^{\mathrm{OPT}})$

$$q^{\mathrm{OPT}}(\theta) = q_\ell \qquad \forall \theta \in [\theta^\star, \bar{\theta}]$$

Suboptimality of upward adjustments for low costs

Lemma

Suppose $z^\star$ *non-decreasing and* $\theta_m \neq \bar{\theta}$*. Then* $q^{\mathrm{OPT}}(\theta) \leq q^{BM}(\theta)$ *for almost all* $\theta \in [\underline{\theta}, \theta^\star)$*, with inequality strict over positive-measure* $I \subseteq [\underline{\theta}, \theta^\star)$

Sub-optimality of upward adjustments for low costs

- Take any $M = (q, u) \in \mathcal{M}^{\mathrm{SL}}$ s.t.
  - $q(\theta) > q^\star(\theta) = q^{BM}(\theta)$ over positive-measure $I \subseteq [\underline{\theta}, \theta^\star)$
  - $q(\theta) = q_\ell$ for all $\theta \geq \theta^\star$
- Take $\tilde{M} = (\tilde{q}, \tilde{u})$ s.t.

$$\tilde{q}(\theta) := \min\{q^\star(\theta), q(\theta)\}$$

  and $\tilde{u}(\theta) = \int_{\theta}^{\bar{\theta}} \tilde{q}(y)dy$
- Clearly,
  - $\tilde{M}$ is IC and IR
  - Higher payoff under $\tilde{M}$ than M: $\tilde{q}$ closer to q^{BM}which maximizes virtual surplus
- $\tilde{M} \in \mathcal{M}^{\mathrm{SL}}$?

Sub-optimality of upward adjustments for low costs

- $\tilde{M} \in \mathcal{M}^{\mathrm{SL}} \Leftrightarrow$

$$\underline{W}(\theta, \tilde{q}) := \underline{V}(\tilde{q}(\theta)) - \underline{\theta}\tilde{q}(\theta) - \int_{\theta}^{\bar{\theta}} \tilde{q}(y)dy \geq G^* \qquad \forall\theta$$

- Clearly so when
  - $\tilde{q}(\theta) = q(\theta)$ — smaller rents, same TS
  - $\underline{P}^{-1}(\theta) \leq \tilde{q}(\theta) < q(\theta)$ —smaller rents, higher TS

    [recall $\underline{P}^{-1}(\theta)$ maximizes $\underline{V}(q) - \underline{\theta}q$)]
- $\tilde{q}(\theta) < \min\{q(\theta), \underline{P}^{-1}(\theta)\}$: **not clear**

Sub-optimality of upward adjustments for low costs

- Because $q^\star(\bar{\theta}) = q_\ell = \underline{P}^{-1}(\bar{\theta})$, there exists $\theta' > \theta$ s.t.

$$\tilde{q}(y) := \min\{q(y), q^\star(\theta)\} \leq \underline{P}^{-1}(y) \qquad \forall y \in [\theta, \theta']$$

$$\tilde{q}(\theta') = \min\{\underline{P}^{-1}(\theta'), q(\theta')\}$$

- Monotonicity lemma $\Rightarrow$ $\underline{W}(\cdot, \tilde{q})$ non-increasing over $[\theta, \theta']$
- Previous lemma $\Rightarrow$ $\underline{W}(\theta', \tilde{q}) \geq G^*$
- Hence $\underline{W}(\theta, \tilde{q}) \geq G^*$ Q.E.D.

Plan

Ongoing Work

Ongoing work

- **Local robustness**
  - arbitrary set of distributions $\mathcal{F}$
- Effects of **changes in optimist/pessimism**
  - variations in conjecture $(F^\star, V^\star)$
  - variation in demand lower bound $\underline{V}$ (equivalently, $\underline{P}$)
- **Characterization of optimal schedule** when $q^{\mathrm{OPT}}(\theta) < q^{BM}(\theta)$
  - novel constraint

$$\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q(y)dy \geq G^*$$

- **Comparison with other robustness criteria**

Plan

Conclusions

Conclusions

- **Novel approach to robustness**
  - government has conjecture but does not trust it
  - first protects itself against worst-case
  - then uses conjecture to optimize over worst-case optimal set
- When only uncertainty is over cost
  - robustly optimal mechanism is Baron Myerson with floor
  - efficiency at top and bottom
- Robustness
  - upward quantity adjustment for high cost
  - downward output adjustment for low/intermediate costs
- ...more to be done!

Most Important Slide

THANK YOU