Keeping the Agents in the Dark: Private Disclosures in Competing Mechanisms

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Competing Mechanisms

- **Competing mechanisms**
  - oligopoly
  - insurance
  - regulation
  - taxation
  - political economy
  - auctions
  - finance
  - search
  - ...

What's a mechanism?

- Mechanism**: rule**

$$\phi : M \to \Delta(\mathcal{A})$$

  - $m \in M$: messages
  - $a \in \mathcal{A}$: allocation

- MD: **rule ϕ commonly announced to all agents**

What's a mechanism?

- Modelization: fine with single designer (Revelation Principle)

- **Also assumed in entire literature on competing principals**

What's missing? Private Disclosures

- Inform agents **asymmetrically** about $\phi : M \to \Delta(\mathcal{A})$

  (equivalently, about consequences of their actions)

Private Disclosures: Examples

- Seller informs bidders asymmetrically about
  - reserve price
- Manufacturer informs retailers asymmetrically about
  - how output supplied to other retailers depends on mkt conditions
- Insurance company informs clients asymmetrically
  - how insurance provision depends on aggregate risk

Mathematically

- **Mechanism with private disclosures**
  - set of private disclosures to agent i: S^i

    $(S \equiv S^1 \times \cdots \times S^I)$
  - joint distribution: $\sigma \in \Delta(S)$
  - augmented rule:

$$\phi : S \times M \to \Delta(\mathcal{A})$$

- Each $s \equiv (s^1, ..., s^I) \in S$ indexes standard mechanism

$$\phi(s) : M \to \Delta(\mathcal{A})$$

- (s^i, σ): hierarchy of beliefs over "effective" rule $\phi(s)$

This paper

- Private disclosures raise principals' payoff guarantees
  - **non-robustness** of eq. allocations sustained with standard mechanisms
  - non-validity of "folk theorems"
- Private disclosures permit to sustain new eq. allocations
  - **non-universality** of standard mechanisms
- Canonical game

Literature

- **Non validity of Revelation Principle**
  - McAfee (1993), Peck (1997)...
- **Universal mechanisms**
  - Epstein-Peters (1999)...
- **Folk theorems for competing-mechanism games**
  - Yamashita (2010), Peters-Troncoso Valverde (2013), Xiong (2013)...
- **Bilateral contracting**
  - Hart-Tirole (1990), McAfee-Schwartz (1994), Segal (1999), Dequiedt-Martimort (2016), Akbarpour-Li (2022)...
- **Common agency (single agent)**
  - Martimort-Stole (2002), Peters (2001), Calzolari-Pavan (2009,2010)...
- **Applications**
  - Competing auctions: McAfee (1993), Peters (1997), Virag (2010)
  - Competitive search: Guerrieri-Shimer-Wright (2010), Wright-Kircher-Julien-Guerrieri (2021)
  - Finance + insurance: Rothschild-Stiglitz (1976), Biais-Martimort-Rochet (2000), Attar-Mariotti-Salanie' (2011, 2021)
- **Information transmission between principals, MD w. aftermarkets**
  - Calzolari-Pavan (2006a,b), Dworczak (2020)...

Plan

Raising payoff guarantees

Primitive game

- Agents: A1, A2, A3
- Principals: P1 and P2
- P1's allocations $\mathcal{A}_1 = \{x_1, x_2\}$
- P2's allocations $\mathcal{A}_2 = \{y_1, y_2\}$
- A1's exogenous type $\omega^1 \in \Omega^1 = \{\omega_L, \omega_H\}$
- A2's exogenous type $\omega^2 \in \Omega^2 = \{\omega_L, \omega_H\}$
- A3: no exogenous private info
- A1's and A2's type perfectly correlated

Payoffs

- P1's and A3's payoffs constant
- Payoffs (u_{P2}, u^{A1}, u^{A2})

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$5, 8, 8$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$5, 8, 8$

- Paper considers more interesting game

Game in standard mechanisms

- $t = 0$: A1 and A2 learns ω^1 and ω^2
- $t = 1$: principals simultaneously post mechanisms
- $t = 2$: agents send messages
- $t = 3$: decisions determined by $\phi_j(m_j)$, with $m_j \equiv (m_j^1, m_j^2, m_j^3)$

(Solution concept)

Folk theorem

- D_j: set of standard DRMs (equivalently, state-contingent actions)

$$d_j : \Omega^1 \times \Omega^2 \to \mathcal{A}_j$$

- Rich message spaces: $M_j^i \supset D_j \times \Omega^i$, all i, j

Lemma 1

Suppose $M_j^i \supset D_j \times \Omega^i$, all i, j, with M finite.

Any payoff for P2 in feasible set $[5, 6]$ can be supported in eq.

(Folk-Th)

G^{SM}: game with **private disclosures**

- $t = 0$: A1 and A2 learns ω^1 and ω^2
- $t = 1$: principals post mechanisms and disclose s to agents
- $t = 3$: agents send messages
- $t = 4$: decisions determined by $\phi_j(s_j, m_j)$

Private Disclosures

Lemma 2

Suppose that $M^i_j \supset D_j \times \Omega^i$, all i and j, and $\left|S^1_2\right| \geq 2$, with M and S finite. In any PBE of G^{SM}, P2's payoff above $5 + K$, with $K = f(\text{primitives}) > 0$.

Lemma 2: Proof

- Wlog, assume $\{1,2\} \subset S_2^1$
- Let $\overline{\gamma}_2$ be mechanism that
  - w.p. $\alpha \in (\frac{1}{2}, 1)$ discloses $s_2^1 = 1$ to A1 and selects y_1
  - w.p. $1-\alpha$ discloses $s_2^1 = 2$ to A1 and selects y_2
  - no signal to A2 and A3
  - no dependence on messages
- No matter γ_1 and cont. eq., P2's payoff higher than $5 + K$

Lemma 2: Proof

- Decisions implemented in $\overline{\gamma}_2$ invariant to m_2
- $\Rightarrow$ no role for P1's signals

Lemma 2: Proof

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$5, 8, 8$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$5, 8, 8$

- P2's payoff $= 5 \Rightarrow x_1$ in (ω_L, ω_L) and x_2 in (ω_H, ω_H)
- (ω_L, ω_L):
    - after receiving $s_2^1 = 2$, A1 wants to min $\Pr(x_1)$
- (ω_H, ω_H):
    - after receiving $s_2^1 = 1$, A1 wants to min $\Pr(x_2)$
- So A1 must not affect P1's decision

Lemma 2: Proof

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$5, 8, 8$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$5, 8, 8$

- Because A3 does not know state, A2 must have full control over P1's decision
- Because $\Pr(y_1) > 1/2$, in state (ω_H,ω_H), A2 wants to max $\Pr(x_1)$
- No eq. giving 5 to P2

Role of Private Disclosures

- Information P2 privately discloses to A1 makes A1 "ally" of P2
- Importance of asymmetric disclosures:
  - If same information disclosed also to A2 and A3, agents can discipline each other, thus implementing IC punishments for P2

Robustness and Anti-Folk Theorem

Proposition 1

Private disclosures raise payoff guarantees.

- Non-robustness of equilibria of games in which principals restricted to standard mechanisms (no matter M)
- Non validity of folk theorems

Robustness and Anti-Folk Theorem

- Result relevant for many concrete problems
  - competition in auctions
  - manufacturer-retailer competition
  - ...
- Result extends to
  - contracts-on-contracts
  - reciprocal mechanisms
  - arbitrarily rich randomizing devices
  - alternative solution concepts (provided sequential rationality retained)
  - direct communication between principals

Plan

1. Introduction
2. Raising payoff guarantees
3. **Sustaining new allocations and payoffs**
4. Canonical game
5. Conclusions

New eq. allocations and payoffs

Non-universality of standard mechanisms

Proposition 2

Private disclosures permit to sustain allocations and payoffs that cannot be supported in any eq. of **any** *game with standard mechanisms, no matter richness of message spaces.*

Primitive Game

- Agents: $A1$ and $A2$
- Principals: P1 and P2
- P1's allocations $X = \{x_1, x_2, x_3, x_4\}$
- P2's allocations $Y = \{y_1, y_2\}$
- A2's exogenous type $\omega^2 \in \Omega^2 = \{\omega_L, \omega_H\}$, $\Pr(\omega_H) = 3/4$

Payoffs

- P1's payoff: constant
- Payoffs (u_{P2}, u^{A1}, u^{A2})

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10}, 3, 3$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10}, 7.5, 7.5$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10}, 7.5, 5$
x_2	$\mathbf{10}, 3, 9$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

with $\zeta < 0$

G^{SM}: game with private disclosures

- No signals for P1
- Signals for P2: $S_2^1 = S_2^2 = \{1, 2\}$
- No messages for P2
- Messages for P1:
  - $M_1^1 = S_2^1$ (for A1)
  - $M_1^2 = \Omega^2 \times S_2^2$ (for A2)
- Hence,
  - P2 sends signals to both agents and asks for no messages
  - P1 sends no signals but asks for P2's signals (and ω^2)

Equilibrium outcome of G^{SM}

Lemma 3

There exists PBE of G^{SM} supporting

$$z(\omega_L) \equiv \frac{2}{3}(x_3, y_1) + \frac{1}{3}(x_4, y_2)$$
$$z(\omega_H) \equiv \frac{2}{3}(x_2, y_1) + \frac{1}{3}(x_1, y_2)$$

and giving P2 payoff of 10.

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Lemma 3

- P2 posts mechanism $\gamma_2^* = (\sigma_2^*, \phi_2^*)$ s.t.

$$\sigma_2^*(1,1) = \sigma_2^*(2,2) = \frac{1}{3}$$

$$\sigma_2^*(1,2) = \sigma_2^*(2,1) = \frac{1}{6}$$

$$\phi_2^*(s) = \begin{cases} y_1 & \text{if } s \in \{(1,1),(2,2)\} \\ y_2 & \text{if } s \in \{(1,2),(2,1)\} \end{cases}$$

- Each agent believes
  - P2 will implement y_1 with prob $\frac{2}{3}$
  - other agent received same signal as theirs with prob $\frac{2}{3}$

Proof of Lemma 3

- P1's mechanism

$$\phi_1^*(m) = \begin{cases} x_3 & \text{if } m \in \{(1,1,\omega_L),(2,2,\omega_L)\} \\ x_4 & \text{if } m \in \{(1,2,\omega_L),(2,1,\omega_L)\} \\ x_2 & \text{if } m \in \{(1,1,\omega_H),(2,2,\omega_H)\} \\ x_1 & \text{if } m \in \{(1,2,\omega_H),(2,1,\omega_H)\} \end{cases}$$

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

- Truthful reporting sequentially rational

Indispensability of Private Disclosures

- G^M: arbitrary game with standard mechanisms $\phi_j : M_j \to \Delta(\mathcal{A}_j)$

Lemma 4

No matter richness of M, there exists no PBE of G^M supporting

$$z(\omega_L) \equiv \frac{2}{3}(x_3, y_1) + \frac{1}{3}(x_4, y_2)$$
$$z(\omega_H) \equiv \frac{2}{3}(x_2, y_1) + \frac{1}{3}(x_1, y_2)$$

(more generally, no PBE giving 10 to P2)

(Proof-Lemma4)

Role of Private Disclosures

- Private disclosures: "encrypted keys"
- Correlate principals' decisions with state ω while respecting incentives
- Different from action recommendations

Non-universality of standard mechanisms

- Result implies non-universality of standard mechanisms (no **matter richness of** M)
- It extends to
  - arbitrary correlation in choice of mechanisms
  - reciprocal mechanisms
  - arbitrary correlation in agents' messages
- Private disclosures substitute for private communication between principals

Plan

Canonical Game

Canonical Mechanisms

Definition

Game G^{SM} "**large**" if

- M_j^i and S_j^i continuous Polish spaces

Definition

Canonical game, $G^{\mathring{S}\mathring{M}}$

- $\mathring{S}_j^i \equiv [0,1]$
- $\mathring{M}_j^i \equiv \Omega^i \times [0,1]^{J-1}$

Definition

Canonical eq. $(\mathring{\mu}^*, \mathring{\lambda}^*)$

- Principals' strategies **pure**
- Agents' strategies **on-path truthful**

Canonicity: universality + robustness

Theorem 1

For any eq. (μ^*, λ^*) *of* G^{SM}*, there exists a canonical eq.* $(\mathring{\mu}^*, \mathring{\lambda}^*)$ *of* $G^{\mathring{S}\mathring{M}}$ *supporting same outcome (***universality***).*

Let G^{SM} *be any large game with non-empty eq. set. For any eq.* $(\mathring{\mu}^*, \mathring{\lambda}^*)$ *of* $G^{\mathring{S}\mathring{M}}$*, there exists eq.* (μ^*, λ^*) *of* G^{SM} *supporting same outcome (***robustness***).*

Proof idea

- **Universality**
  - correlation in agents' behavior supported by principals mixing over mechanisms and agents using realizations of principals' mixed strategies as correlation device
  - replicated by principals using signals to correlate agents' behavior
  - mixing by agents over messages
  - replicated by principals using signals collectively sent to agents as "jointly controlled lottery"
- **Robustness**
  - information used to correlate principals' decisions (on and off-path) encoded into $[0,1]^{J \times I}$
  - deviations to arbitrary mechanisms in G^{SM} punished by agents " translating" mechanisms into equivalent ones in $G^{\mathring{S}\mathring{M}}$

Proof idea

- Formal proofs uses
    - sampling variables (Aumann's trick)
    - jointly controlled lotteries
    - encryption
    - rich embeddings

Comparison with Epstein and Peters (1999)

- Result in theorem allows for
  - private disclosures
  - mixing by principals
  - mixing by agents
  - common values
  - nonexclusive competition
- Private disclosures restore canonicity of truthful-pure-strategy equilibria without need for hierarchical construction.

Long Communication

- Principals and agents exchange signals/messages over $T \in \mathbb{N} \cup \{+\infty\}$ rounds

Definition

Long-communication game G^{SMT}

- M^i_{jt} and S^i_{jt} continuous Polish spaces
- $\sigma_{jt} : \prod_{s=1}^{t-1} (S_{js} \times M_{js}) \to \Delta(S_{jt})$
- $\phi_j : \prod_{s=1}^{T} (S_{js} \times M_{js}) \to \Delta(A_j)$

Canonical extensive form

Theorem 2

For any eq. (μ^*, λ^*) *of long-communication game* G^{SMT}*, there exists a canonical eq.* $(\mathring{\mu}^*, \mathring{\lambda}^*)$ *of* $G^{\mathring{S}\mathring{M}}$ *supporting same outcome (***universality***)*

Let G^{SMT} *be any long-communication game with non-empty eq. set. For any eq.* $(\mathring{\mu}^*, \mathring{\lambda}^*)$ *of* $G^{\mathring{S}\mathring{M}}$*, there exists eq.* (μ^*, λ^*) *of* G^{SMT} *supporting same outcome (***robustness***)*

(Th2-Proof)

Theorems 1 + 2

- Equilibrium set: **same structure as in single-principal games**
  - principals do not mix on mechanisms
  - agents report truthfully on path
  - communication is short
- Canonical structure helps
  - **conceptualize strategic interactions**
  - **construct equilibria**

Conclusions

- **Private disclosures**
  - irrelevant with
    - single principal
    - competing principals with single agent (common agency)
  - **fundamental role** when multiple principals contract w. multiple agents
- **Raise payoff guarantees**
  - non-robustness of equilibria with standard mechanisms
  - non-validity of folk theorems
- **Support new eq. allocations and payoffs**
  - Non-universality of standard mechanisms
- **Canonical game**
  - truthful-pure-strategy eq.
  - short communication

Most Important Slide

THANKS!

$$\phi : S \times M \to \Delta(\mathcal{A})$$

Solution concept

Definition

Strategy profile (μ, λ), where $\lambda = (\lambda^1, \ldots, \lambda^I)$ are agents' strategies and $\mu = (\mu_1, \ldots, \mu_j)$ principals' strategies is PBE iff

1. for each mechanism profile $\gamma \in \Gamma$, $(\lambda^1(\gamma), \ldots, \lambda^I(\gamma))$ is BNE of subgame γ played by agents
2. given continuation eq. strategies λ, μ is Nash eq. of game among principals

Go back

Folk-Th

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$\mathbf{5}, \mathbf{8}, \mathbf{8}$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$\mathbf{5}, \mathbf{8}, \mathbf{8}$

- Here: show how to support 5
- Equilibrium outcome

$$z(\omega_L, \omega_L) = (x_1, y_1)\,, \quad z(\omega_H, \omega_H) = (x_2, y_2)$$

Equilibrium supporting min-max-min payoff

- On path, both P1 and P2 post recommendation mechanisms (ϕ_1^r, ϕ_2^r)

  Given messages $m_j = (d_j, \omega^i)_{j=1}^J$,

$$\phi_j^r(m_j^1, \ldots, m_j^I) \equiv \begin{cases} \hat{d}_j(\omega^1, \ldots, \omega^I) & \text{if } \left|\{i : m_j^i = (\hat{d}_j, \omega^i)\}\right| \geq I - 1 \\ \bar{a}_j & \text{otherwise} \end{cases}$$

Equilibrium supporting min-max-min payoff

- In subgame (ϕ_1^r, ϕ_2^r), all agents recommend DRMs

$$d_1^*(\omega) \equiv \begin{cases} x_1 & \text{if } \omega = (\omega_L, \omega_L) \\ x_2 & \text{otherwise} \end{cases} \qquad d_2^*(\omega) \equiv \begin{cases} y_1 & \text{if } \omega = (\omega_L, \omega_L) \\ y_2 & \text{otherwise} \end{cases}$$

and A1 and A2 report truthfully to both principals

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	**5, 8, 8**	5, 1, 1
x_2	6, 4.5, 4.5	6, 4.5, 4.5

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	6, 4.5, 4.5	6, 4.5, 4.5
x_2	5, 1, 1	**5, 8, 8**

Equilibrium supporting min-max-min payoff

- Suppose P2 deviates to $\phi_2 : M_2 \to \Delta(Y)$

- Let $p(m_2) = \Pr(y_1|m_2)$

$$\overline{p} \equiv p(\overline{m}_2^1, \overline{m}_2^2, \overline{m}_2^3) \geq p(m_2) \qquad \forall m_2$$

$$\underline{p} \equiv p(\underline{m}_2^1, \underline{m}_2^2, \overline{m}_2^3) \leq p(m_2^1, m_2^2, \overline{m}_2^3) \qquad \forall (m_2^1, m_2^2)$$

Equilibrium supporting min-max-min payoff

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$\mathbf{5}, \mathbf{8}, \mathbf{8}$	$5, 1, 1$
x_2	$6, 4.5, 4.5$	$6, 4.5, 4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6, 4.5, 4.5$	$6, 4.5, 4.5$
x_2	$5, 1, 1$	$\mathbf{5}, \mathbf{8}, \mathbf{8}$

- Case 1: $\overline{p} \geq 1/2$
  - all agents recommend $d_1^*(\omega) \equiv \begin{cases} x_1 & \text{if } \omega = (\omega_L, \omega_L) \\ x_2 & \text{otherwise} \end{cases}$
  - Each agent sends $\overline{m}_2^i$
    - (ω_L, ω_L): $8\overline{p} + (1 - \overline{p}) \geq 4.5 \Rightarrow$ truthful reporting $+ \overline{m}_2^i$ is BR
    - (ω_H, ω_H): no agent can unilaterally change P1's decision
  - P2's payoff: 5

Equilibrium supporting min-max-min payoff

$\omega = (\omega_L, \omega_L)$

	y_1	y_2
x_1	$\mathbf{5,8,8}$	$5,1,1$
x_2	$6,4.5,4.5$	$6,4.5,4.5$

$\omega = (\omega_H, \omega_H)$

	y_1	y_2
x_1	$6,4.5,4.5$	$6,4.5,4.5$
x_2	$5,1,1$	$\mathbf{5,8,8}$

- Case 2: $\overline{p} < 1/2$
  - all agents recommend $d_1(\omega) \equiv \begin{cases} x_2 & \text{if } \omega = (\omega_H, \omega_H) \\ x_1 & \text{otherwise} \end{cases}$
  - A3 sends $\overline{m}_2^3$, A1 and A2 send $\underline{m}_2^1$ and $\underline{m}_2^2$
    - (ω_L, ω_L): no agent can unilaterally change P1's decision
    - (ω_H, ω_H): $\underline{p} + 8(1 - \underline{p}) \geq 4.5 \Rightarrow$ truthful reporting + $\underline{m}_2^i$ is BR
  - P2's payoff: 5

Go back

Proof-Lemma4

- Let $\mu \in \Delta(\Phi_1 \times \Phi_2)$ and $\lambda = (\lambda^1, \lambda^2)$ continuation eq. for G^M
- **Step 1**: For μ-almost all $\phi \in supp[\mu]$, $\lambda(\phi)$-almost all (m_1, m_2),

$$(\phi_1(m_1), \phi_2(m_2)) \in \overline{\mathrm{Int}\Delta(X)} \times \overline{\mathrm{Int}\Delta(Y)}$$

  - deterministic response to messages

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Lemma 4

- **Step 2**: For μ-almost all $\phi = (\phi_1, \phi_2)$, IC for A2 requires that

$$\Pr(x_3, y_1|\omega_L; \phi, \lambda) = 1 - \Pr(x_4, y_2|\omega_L; \phi, \lambda) = 2/3$$

$$\Pr(x_2, y_1|\omega_H; \phi, \lambda) = 1 - \Pr(x_1, y_2|\omega_H; \phi, \lambda) = 2/3$$

- Else ω_H can draw m_1^2 from $\lambda^2(\omega_H|\phi)$ and m_2^2 from $\lambda^2(\omega_L|\phi)$ to "de-correlate" the two principals' decisions and do strictly better

$\omega^2 = \omega_L$

	y_1	y_2
x_1	$\zeta, 4, 1$	$\zeta, 8, 3.5$
x_2	$\zeta, 2, 5$	$\zeta, 9, 8$
x_3	$\mathbf{10, 3, 3}$	$\zeta, 5.5, 3.5$
x_4	$\zeta, 1, 3.5$	$\mathbf{10, 7.5, 7.5}$

$\omega^2 = \omega_H$

	y_1	y_2
x_1	$\zeta, 1, 6$	$\mathbf{10, 7.5, 5}$
x_2	$\mathbf{10, 3, 9}$	$\zeta, 5.5, 6$
x_3	$\zeta, 8, 7$	$\zeta, 4.5, 7$
x_4	$\zeta, 9, 6$	$\zeta, 3, 9$

Proof of Lemma 4

- **Step 3**: For μ-almost all ϕ, there exists no pair of behavioral strategies inducing

$$\Pr(x_3, y_1|\omega_L; \phi, \lambda) = 1 - \Pr(x_4, y_2|\omega_L; \phi, \lambda) = 2/3$$

$$\Pr(x_2, y_1|\omega_H; \phi, \lambda) = 1 - \Pr(x_1, y_2|\omega_H; \phi, \lambda) = 2/3$$

  - messages A2 sends in state ω_H must have no bite
    - else ω_L can draw twice from $\lambda^2(\omega_H|\phi)$, send m_1^2 from first draw and m_2^2 from second draw, invert correlation between principals' decisions while preserving marginals and do strictly better
  - ...but then A1 has profitable deviation

Go back

Th2-Proof

Definition

Auxiliary long-communication game, $G^{\mathring{S}\mathring{M}T}$

- $\mathring{S}^i_{jt} \equiv [0,1]$
- $\mathring{M}^i_{j1} \equiv \Omega^i \times [0,1]^{J-1}$
- $\mathring{M}^i_{jt} \equiv [0,1]^{J-1},\ t > 1$

Proof-Th2

- WLOG, restrict to eq. of auxiliary long-communication game in which
  - principals' strategies: pure
  - agents' strategies: (on path) truthfully **at all rounds**
  - signals: drawn from $[0, 1]$, **independently** across agents and rounds

Proof-Th2

- Reduction of dimensionality
  - vector

$$\xi \equiv \left(\xi^{i}_{jt}\right)_{j=1,\ldots,J, i=1,\ldots,I, t=1,\ldots,T}$$

generated by uni-dimensional $\xi_0 \sim U[0,1]$ via interlacing

Proof-Th2

- Jointly controlled lottery
  - variable ξ^0 generated by each principal drawing signal $\xi_j^i \sim U[0,1]$ for each agent s.t.

    (a) in isolation, ξ_j^i carries no information about ξ_0

    (b) given $\xi \equiv \left(\xi_{jt}^i\right)_{j=1,\ldots,J,i=1,\ldots,I}$, $\xi_0 = g(\xi)$

    (c) no principal can manipulate distribution of ξ_0

Proof-Th2

- From long communication to short-communication
  - only relevant signals: drawn at $t = 1$
  - agents' long communication strategies: embedded into

$$\mathring{M}^i_{j1} \equiv \Omega^i \times [0,1]^{J-1}$$

  - interim vs ex-ante BNE

Proof-Th2

- From non-canonical eq. of $G^{\mathring{S}\mathring{M}}$ to canonical eq. of $G^{\mathring{S}\mathring{M}}$
    - Theorem 1

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