

Knowing your Lemon before You Dump it

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Motivation

- Many situations where decision to “engage” carries info about what's at stake
 - trade
 - partnerships
 - entry
 - marriage
 - ...
- **Negative inferences**
 - lemons (Akerlof)
- **Positive inferences**
 - anti-lemons (Spence)

Motivation

- Typical assumption:
 - Exogenous information

- Many problems of interest: **Endogenous information**
 - acquisition
 - attention
 - cognition

- Example: Info asset owner collects depends on mkt price

This Paper

- Generalized lemons (and anti-lemons)
 - **endogenous information**
- Information choices
 - type of strategic interaction
 - opponent's beliefs over selected information
- Two forces shaping **expectation conformity**
 - effect of information on **severity of adverse selection**
 - effect of **friendliness of opponent's reaction** on value of information
- **Expectation traps**
- **Policy implications**

- **Endogenous info in lemons problem**

- Dang (2008), Thereze (2024), Lichtig and Weksler (2023)
→ EC, ≠ bargaining game, timing, CS

- **Payoffs in lemons problem**

- Levin (2001), Bar-Isaac et al. (2018), Kartik and Zhong (2024)...
→ incentives analysis

- **Policy in mkts with adverse selection**

- Philippon and Skreta (2012), Tirole (2012), Dang et al (2017)...
→ endogenous information

- **Endogenous info in private-value bargaining**

- Ravid (2020), Ravid, Roesler, and Szentes (2021)...
→ interdependent payoffs, competitive mkt

Plan

- 1 Introduction
- 2 Model
- 3 Expectation Conformity and Expectation Traps
- 4 Policy Interventions
- 5 Flexible Information
- 6 Equilibrium under Entropy Cost
- 7 Conclusions

Model

- **Players**

- Leader
- Follower

- **Choices**

- **Leader:**

- information structure, ρ (more below)
- two actions:
 - **adverse-selection-sensitive**, $a = 1$ (“engage”)
 - adverse-selection insensitive, $a = 0$ (“not engage”)

- **Follower:**

- reaction, $r \in \mathbb{R}$ (e.g., price offer)

- **State**

- $\omega \sim$ prior G
- mean: ω_0

- **Payoffs**

- **leader**: $\delta_L(r, \omega) \equiv u_L(1, r, \omega) - u_L(0, \omega)$
 - affine in ω
 - increasing in r (higher r : friendlier reaction)
 - decreasing in ω
 - benefit of friendlier reaction (weakly) increasing in state: $\frac{\partial^2 \delta_L}{\partial \omega \partial r} \geq 0$
(benefit of higher r largest in states in which L 's value of engagement lowest)
- **follower**: $\delta_F(r, \omega) \equiv u_F(1, r, \omega) - u_F(0, \omega)$
 - affine in ω

Akerlof Example

- Leader: **seller**
 - $u_L(1, r, \omega) = r$ (price)
 - $u_L(0, r, \omega) = \omega$ (asset value)
 - $\delta_L(r, \omega) = r - \omega$

- Follower: **competitive buyer**
 - $u_F(0, \omega) = 0$
 - $u_F(1, r, \omega) = \omega + \Delta - r$
 - $\delta_F(r, \omega) = u_F(1, r, \omega)$

- **Information structures:** $\rho \in \mathbb{R}_+$
 - cdf $G(m; \rho)$ over posterior mean m (mean-preserving-contraction of G)
 - $C(\rho)$: information-acquisition cost

Definition

Information structures consistent with **MPS order** (mean-preserving spreads) if, for any $\rho' > \rho$, any $m^* \in \mathbb{R}$,

$$\int_{-\infty}^{m^*} G(m; \rho') dm \geq \int_{-\infty}^{m^*} G(m; \rho) dm$$

with $\int_{-\infty}^{+\infty} G(m; \rho') dm = \int_{-\infty}^{+\infty} G(m; \rho) dm$

- MPS order and Blackwell informativeness:
 - $G(\cdot; \rho)$ obtained from experiment $q_\rho : \Omega \rightarrow \Delta(Z)$
 - $G(\cdot; \rho')$ obtained from experiment $q_{\rho'} : \Omega \rightarrow \Delta(Z)$
 - If $\rho' > \rho$ means $q_{\rho'}$ Blackwell more informative than q_ρ , then

$$G(\cdot; \rho') \succeq_{MPS} G(\cdot; \rho)$$

(Rotations)

Model

- For any (ρ, r) , **leader engages** (i.e., $a = 1$) iff

$$m \leq m^*(r)$$

with

$$\delta_L(r, m^*(r)) = 0$$

- Truncated mean:

$$M^-(m^*; \rho) \equiv \mathbb{E}_{G(\cdot; \rho)}[m | m \leq m^*]$$

- $r(\rho)$: eq. reaction when info is ρ (assumed unique)

- **Assumption (lemons):**

$$\frac{dr(\rho)}{d\rho} \stackrel{\text{sgn}}{=} \frac{\partial}{\partial \rho} M^-(m^*(r(\rho)); \rho)$$

- Anti-lemons:

$$\frac{dr(\rho)}{d\rho} \stackrel{\text{sgn}}{=} -\frac{\partial}{\partial \rho} M^-(m^*(r(\rho)); \rho)$$

Akerlof Example

- Engagement threshold: $m^*(r) = r$
- Equilibrium price $r(\rho)$: solution to

$$r = M^-(r; \rho) + \Delta$$

- Lemons:

$$\frac{dr(\rho)}{d\rho} \stackrel{\text{sgn}}{=} \frac{\partial}{\partial \rho} M^-(r(\rho); \rho)$$

- always if $g(m; \rho)/G(m; \rho)$ decreases in m (**Monotone Hazard Rate**)

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Expectation Conformity and Expectation Traps

Effect of information on adverse selection

- Truncated mean

$$M^-(m^*; \rho) \equiv \frac{\int_{-\infty}^{m^*} m dG(m; \rho)}{G(m^*; \rho)}$$

Definition

Information

- **aggravates adverse selection** if $\frac{\partial}{\partial \rho} M^-(m^*; \rho) < 0$
- **alleviates adverse selection** if $\frac{\partial}{\partial \rho} M^-(m^*; \rho) > 0$

Effect of information on adverse selection

- Let

$$G_\rho(m; \rho) \equiv \frac{\partial}{\partial \rho} G(m; \rho)$$

- **Effect of info on AS:**

$$\frac{\partial}{\partial \rho} M^-(m^*; \rho) \stackrel{\text{sgn}}{\equiv} A(m^*; \rho)$$

where

$$A(m^*; \rho) \equiv [m^* - M^-(m^*; \rho)] G_\rho(m^*; \rho) - \int_{-\infty}^{m^*} G_\rho(m; \rho) dm$$

- Two channels through which info affects AS:

- **prob. of trade:** $G_\rho(m^*; \rho)$

- **dispersion of posterior mean:** $\int_{-\infty}^{m^*} G_\rho(m; \rho) dm$

- **Adverse Selection Effect:** $A(\rho) \equiv A(m^*(r(\rho)); \rho)$

Effect of unfriendlier reactions on value of information

- L 's payoff under information ρ and reaction r :

$$\begin{aligned}\Pi(\rho; r) &\equiv \sup_{a(\cdot)} \left\{ \int_{-\infty}^{+\infty} a(m) \delta_L(r, m) dG(m; \rho) \right\} \\ &= G(m^*(r); \rho) \delta_L(r, M^-(m^*(r); \rho))\end{aligned}$$

- **Benefit of friendlier reaction** effect

- ρ : actual choice (by L)
- ρ^\dagger : anticipated choice (by F)

$$B(\rho; \rho^\dagger) \equiv -\frac{\partial^2}{\partial \rho \partial r} \Pi(\rho; r(\rho^\dagger))$$

- Starting from $r(\rho^\dagger)$, reduction in r
 - raises value of info at ρ if $B(\rho; \rho^\dagger) > 0$
 - lowers value of info at ρ if $B(\rho; \rho^\dagger) < 0$

Effect of unfriendlier reactions on value of information

- **Benefit of friendlier reaction:**

$$B(\rho; \rho^\dagger) = -\frac{\partial \delta_L(r, m^*(r(\rho^\dagger)))}{\partial r} G_\rho(m^*(r(\rho^\dagger)); \rho) \\ + \int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial^2 \delta_L(r, m)}{\partial r \partial m} G_\rho(m; \rho) dm$$

- Two channels through which, starting from $r(\rho^\dagger)$, reduction in r affects value of info at ρ :

- **prob. of trade:** $G_\rho(m^*(r(\rho^\dagger)); \rho)$

- **dispersion of posterior mean:** $\int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial^2 \delta_L(r, m)}{\partial r \partial m} G_\rho(m; \rho) dm$

Expectation Conformity

Definition

Expectation conformity holds at (ρ, ρ^\dagger) iff

$$\frac{\partial^2 \Pi(\rho; r(\rho^\dagger))}{\partial \rho \partial \rho^\dagger} > 0$$

- Complementarity between anticipated and actual investment in info

Key forces...

- $A(\rho^\dagger) \stackrel{\text{sgn}}{=} \frac{\partial}{\partial \rho} M^-(m^*(r(\rho^\dagger)); \rho^\dagger)$: **adverse-selection effect**
- $B(\rho; \rho^\dagger) = -\frac{\partial^2 \Pi(\rho; r(\rho^\dagger))}{\partial \rho \partial r}$: **benefit-of-friendlier-reactions effect**

Expectation Conformity

Proposition

Assume MPS.

- EC at (ρ, ρ^\dagger) iff $A(\rho^\dagger)B(\rho; \rho^\dagger) < 0$.
- Info aggravates AS at ρ^\dagger (i.e., $A(\rho^\dagger) < 0$) for Uniform, Pareto, Exponential, or, more generally, when it reduces prob of trade perceived by F, i.e., $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$.
- Starting from $r(\rho^\dagger)$, reduction in r raises value for info at r (i.e., $B(\rho; \rho^\dagger) > 0$) if more info reduces prob of trade perceived by L, i.e., $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$.
- Therefore EC at (ρ, ρ^\dagger) if, no matter whose perspective one takes, more info reduces prob of trade:

$$\max \left\{ G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger), G_\rho(m^*(r(\rho^\dagger)); \rho) \right\} < 0$$

- Suppose $M^-(m^*; \rho)$ decreases in ρ (Uniform, Pareto, Exponential) and $\partial^2 \delta_L(r, m) / \partial r \partial m = 0$ (e.g., Akerlof). Then, $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$ NSC for EC at (ρ, ρ^\dagger) .

EC under non-directed search in Akerlof model

- Akerlof model under non-directed search (ρ =prob. seller learns state)

$$G(m; \rho) = \begin{cases} \rho G(m) & \text{for } m < \omega_0 \\ \rho G(m) + 1 - \rho & \text{for } m \geq \omega_0 \end{cases}$$

Corollary

EC holds holds at (ρ, ρ^\dagger) iff $r(\rho^\dagger) > \omega_0$, i.e., iff gains from trade Δ large.

EC under non-directed search in Akerlof model

- Large Δ : $r(\rho^\dagger) > \omega_0$
- Increase in info ρ^\dagger (anticipated by F)
 - seller **engages more selectively**
(when uninformed: always; when informed iff: $\omega \leq r(\rho^\dagger)$)
 - lower prob. of trade perceived by F : $G_\rho(r(\rho^\dagger); \rho^\dagger) < 0$
 - **aggravation of AS**: $A(\rho^\dagger) < 0$
 - **lower price**
 - higher cost for S of parting with valuable item
 - **higher value in learning state at ρ** : $B(\rho; \rho^\dagger) > 0$
- Hence, $A(\rho^\dagger)B(\rho; \rho^\dagger) < 0$
 - **Expectation conformity!**

EC under non-directed search in Akerlof model

- Small Δ : $r(\rho^\dagger) < \omega_0$
- S engages only when **informed** and $\omega < r(\rho^\dagger)$
- Variations in anticipated info $\rho^\dagger \rightarrow$ no effect on truncated mean

$$M^-(r(\rho^\dagger); \rho) \equiv \frac{\int_{-\infty}^{m^*} m dG(m; \rho)}{G(m^*; \rho)} = \omega_0$$

- **Adverse selection effect:** $A(\rho^\dagger) = 0$
- **No expectation conformity**

(Gains from Engagement)

Expectation Traps

Proposition

Suppose ρ_1 and $\rho_2 > \rho_1$ are eq. levels and info aggravates AS (i.e., $A(\rho) < 0$ for all $\rho \in [\rho_1, \rho_2]$). Then L better off in low-info equilibrium ρ_1 . Converse true when info alleviates AS, i.e., $A(\rho) > 0$.

(Example: Akerlof-direct-search)

(Disclosure)

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Policy Interventions

Subsidies to Trade

- Welfare (competitive F):

$$W \equiv \int_{-\infty}^{m^*} (\delta_L(r, m) + s) dG(m; \rho) - C(\rho) - (1 + \lambda)sG(m^*; \rho)$$

where

- s : **subsidy to trade**
 - λ : **cost of public funds** (DWL of taxation)
-
- Subsidy impacts:
 - engagement threshold: $m^* = r + s$
 - friendliness of F 's reaction: r
 - information: ρ

Subsidies: Akerlof

- **Eq. with subsidy s :** $(r^*(s), \rho^*(s))$
- Engagement threshold: $m^*(s) = r^*(s) + s$
- Optimality of subsidizing/taxing trade?

Proposition

Subsidizing trade optimal when

$$\frac{\partial}{\partial m^*} M^-(r^*(0); \rho^*(0)) + \frac{\partial}{\partial \rho} M^-(r^*(0); \rho^*(0)) \frac{d\rho^*(0)}{ds} > \lambda.$$

Taxing trade optimal when inequality reversed.

Subsidies: Akerlof

- Subsidies optimal when
 1. Small cost λ of public funds
 2. Information aggravates AS ($A(\rho) < 0$)
 3. CS of eq. same as BR: Subsidies disincentivize info acquisition

Subsidies: Double Dividend

- Optimal subsidy w. endogenous info: s^*
- Eq. with optimal subsidy: $(\rho^*(s^*), r^*(s^*))$
- Suppose info is exogenous and equal to $\rho^*(s^*)$
- Optimal subsidy under exogenous info $\rho^*(s^*)$: s^{**}
- Question: $s^{**} >? < s^*$

Proposition

Assume that, when $\rho = \rho^*(s^*)$, distribution of posterior mean has MHR:

$$\frac{g(m; \rho)}{G^*(m; \rho)} \quad \text{decreasing in } m.$$

Further assume info reduces prob of trade and hence aggravates AS: when $m^* = r^*(s^*) + s^*$ and $\rho = \rho^*(s^*)$,

$$G_\rho(m^*; \rho) < 0.$$

Then optimal subsidy larger with endogenous info:

$$s^{**} < s^*.$$

Subsidies: Double Dividend

- Same conditions as for EC:
 - **larger subsidy when info reduces prob. of trade**
- **Double dividend** of subsidy
 - more engagement (\Rightarrow less AS \Rightarrow higher $r \Rightarrow$ more trade)
 - less info acquisition (\Rightarrow less AS \Rightarrow higher $r \Rightarrow$ more trade)
- Implication for Gov. asset buyback programs: **more generous terms**
 - **Gov should offer to purchase assets at higher price!**

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Flexible Information

Flexible Information

- Purpose of extension:
 - 1 Robustness
 - 2 Alternative order over investments when experiments not rankable
 - 3 Eq. analysis
 - 1 expectation traps
 - 2 **(novel form of) mkt breakdown**

Flexible Information

- **Entropy:**

- ρ parametrizes MC of entropy reduction (alternatively, capacity)
- L invests in ability to process info (MC or capacity)
- then chooses experiment $q : \Omega \rightarrow \Delta(Z)$ at cost

$$C(\rho) + \frac{1}{\rho} I^q$$

where I^q is **mutual information** between z and ω

- **Max-slope:**

- ρ parametrizes max slope of stochastic choice rule $\sigma : \Omega \rightarrow [0, 1]$ specifying prob. L engages
- L chooses ρ at cost $C(\rho)$
- then selects experiment $q : \Omega \rightarrow \Delta(Z)$ and engagement strategy $a : Z \rightarrow [0, 1]$ among those inducing stochastic choice rule with slope less than ρ

- **Key insights similar to those under MPS order**

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Equilibrium Under Entropy Cost

Inner Problem

- Without loss: binary experiments/recommendations ($q(1|\omega) = \mathbb{P}(a = 1|\omega)$)
- L 's **inner problem** (given ρ)

$$\sup_{q(1|\cdot):\Omega\rightarrow[0,1]} \int (r - \omega)q(1|\omega)dG(\omega) + \mathbb{E}[\omega] - \frac{I^q}{\rho}$$

where

$$I^q = \int \phi(q(1|\omega))dG(\omega) - \phi(q(1))$$

is **entropy reduction**, with

$$\phi(q) \equiv q \ln(q) + (1 - q) \ln(1 - q)$$

- $q(1) \equiv \int q(1|\omega)dG(\omega)$ is total prob of engagement

Optimal Signal

- When interior, $q(1|\cdot)$ solves **functional equation**:

$$r - \omega = \frac{1}{\rho} \left[\ln \left(\frac{q(1|\omega)}{1 - q(1|\omega)} \right) - \ln \left(\frac{q(1)}{1 - q(1)} \right) \right]$$

with $q(1) \equiv \int q(1|\omega) dG(\omega)$

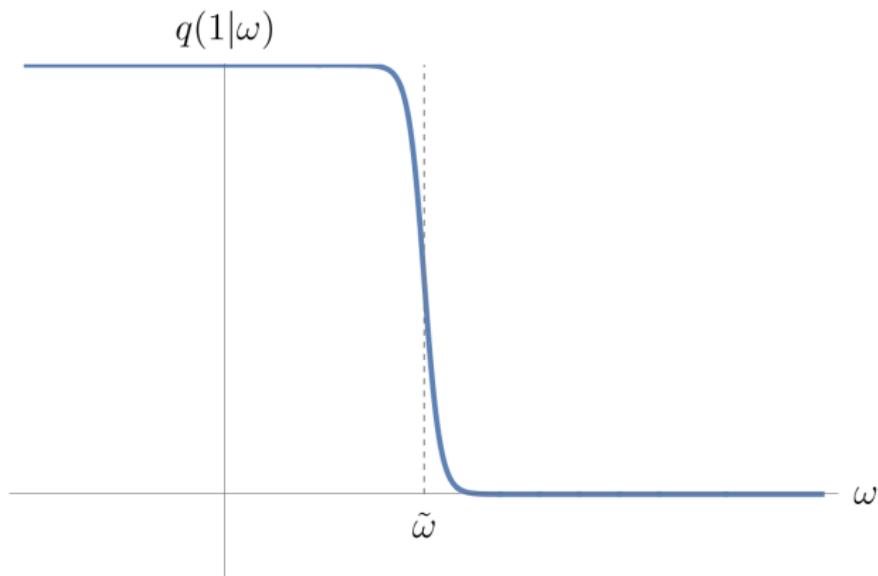
- Let $\tilde{\omega} \in \mathbb{R}$ solve the **(non-functional) equation**

$$\tilde{\omega} = r + \frac{1}{\rho} \ln \left(\frac{\int \frac{1}{1 + \exp(\rho(\omega - \tilde{\omega}))} dG(\omega)}{1 - \int \frac{1}{1 + \exp(\rho(\omega - \tilde{\omega}))} dG(\omega)} \right)$$

- There exists $\underline{r}(\rho), \bar{r}(\rho)$ s.t. seller's **optimal signal**

$$q(1|\omega) = \begin{cases} 0 & \forall \omega \text{ if } r \leq \underline{r}(\rho) \\ \frac{1}{1 + \exp(\rho(\omega - \tilde{\omega}))} & \text{if } r \in (\underline{r}(\rho), \bar{r}(\rho)) \\ 1 & \forall \omega \text{ if } r \geq \bar{r}(\rho) \end{cases}$$

Logistic Signal

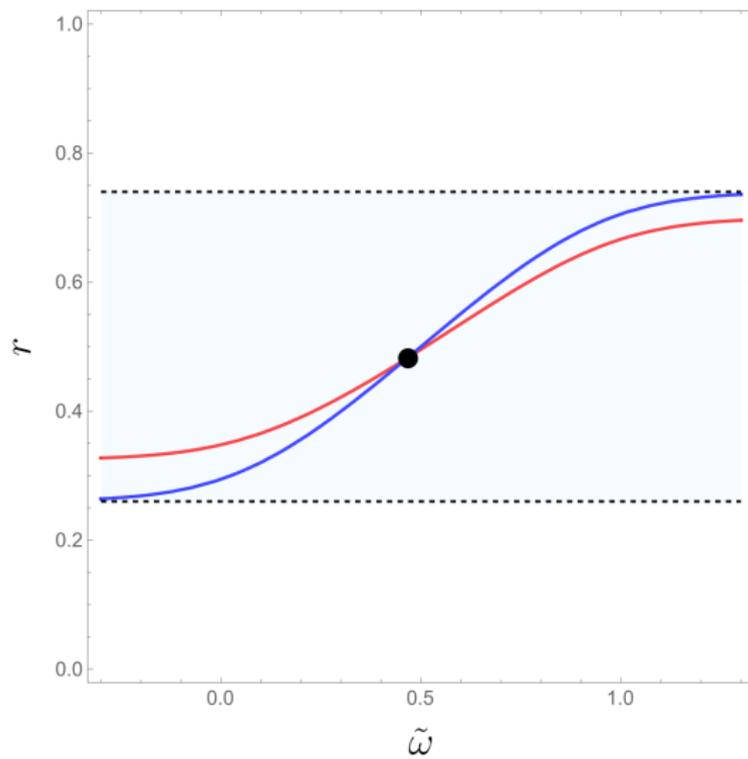


(Interior) Equilibria of Inner Game

Best-response analysis in \mathbb{R}^2

$$\left\{ \begin{array}{l} \tilde{\omega} = r + \frac{1}{\rho} \ln \left(\frac{\int \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)}{1 - \int \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)} \right) \quad (\text{seller's reaction}) \\ r = \frac{\int \frac{\omega}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)}{\int \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)} + \Delta \quad (\text{buyer's reaction}) \end{array} \right.$$

(Interior) Equilibria of Inner Game



$$\omega \sim U[0, 1], \quad \rho = 8, \quad \Delta = 0.2, \quad r^* \approx 0.5, \quad \tilde{\omega}^* \approx 0.5$$

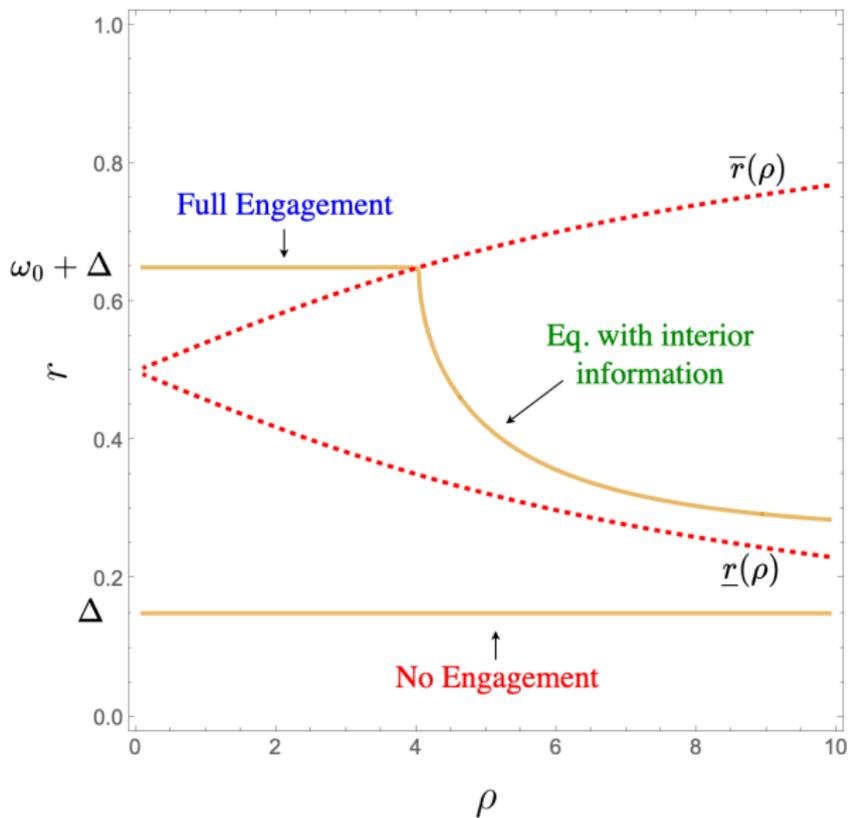
Multiple Equilibria of Inner Game

- Interior solutions can coexist with corner solutions
 - equilibria in which **no info** is acquired
- In case of no engagement, need to specify buyer's *off-path* beliefs
 - Following beliefs consistent with most refinements:

$$q^\dagger(1|\omega) = \begin{cases} 1 & \text{if } \omega = 0 \\ 0 & \text{if } \omega > 0 \end{cases}$$

- Buyer offers: $r_N \equiv \mathbb{E}[\omega|a = 1; q^\dagger] + \Delta = \Delta$
- If $\Delta < \underline{r}(\rho)$, equilibrium with no trade
- Novel form of **mkt breakdown** (with **no info** acquired on path)

Multiple Equilibria of Inner Game



Outer Game

- Seller first trains herself in processing information (formally, chooses ρ)
- Given ρ , seller selects signal flexibly
- Seller's payoff

$$\Pi(r, q; \rho) \equiv \int_{\omega} (r - \omega)q(1|\omega)dG(\omega) + \mathbb{E}[\omega] - \frac{I^q}{\rho} - C(\rho)$$

Outer Game: Interior Equilibrium

- Necessary conditions for interior equilibrium:

$$q^{\rho,r}(1|\omega) = \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} \quad \forall \omega \quad (\text{logistic signal})$$

$$\tilde{\omega} = r + \frac{1}{\rho} \ln \left(\frac{\int \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)}{1 - \int \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)} \right) \quad (\text{position parameter})$$

$$\frac{Iq^{\rho,r}}{\rho^2} = C'(\rho) \quad (\text{optimality of } \rho)$$

$$r = \frac{\int \frac{\omega}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)}{\int \frac{1}{1+\exp(\rho(\omega-\tilde{\omega}))} dG(\omega)} + \Delta \quad (\text{buyer's break even})$$

$$r \in (\underline{r}(\rho), \bar{r}(\rho)) \quad (\text{interior signal})$$

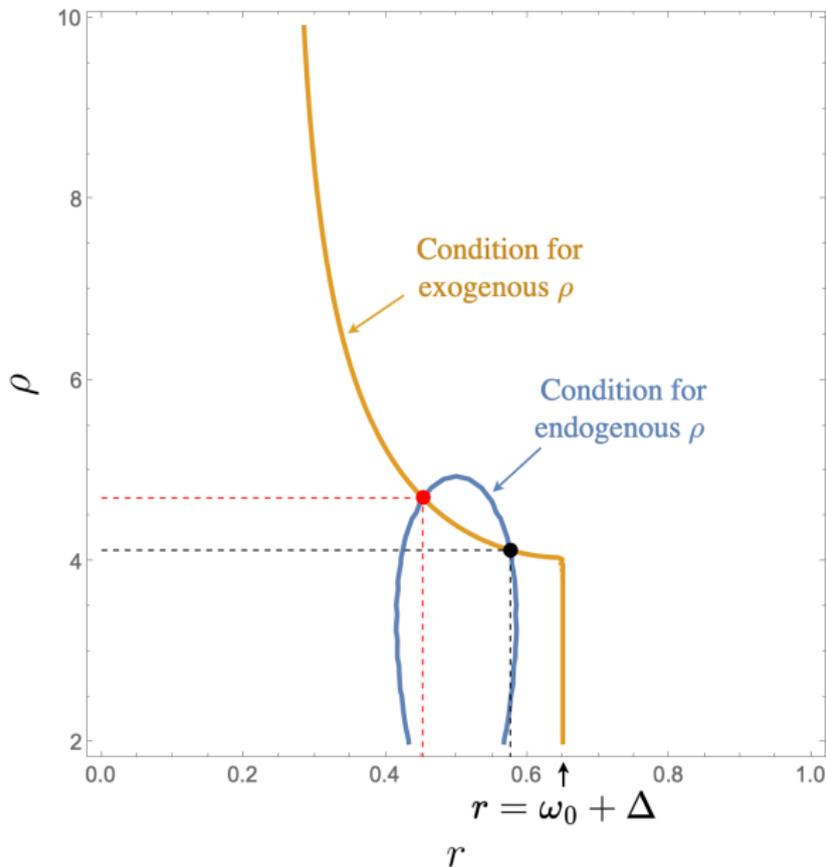
Outer Game: Numerical Example

- Assume

$$C(\rho) = \frac{a\rho^2}{2K}$$

- Graphs below: $a \approx 1.5$, $K = 1,000$, and $\Delta = 0.15$

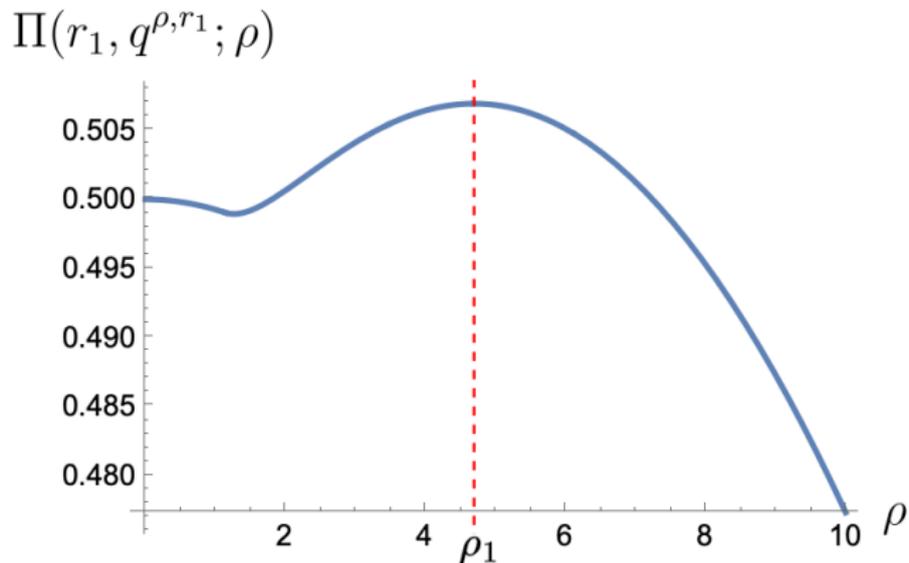
Necessary Conditions: Graphical Analysis



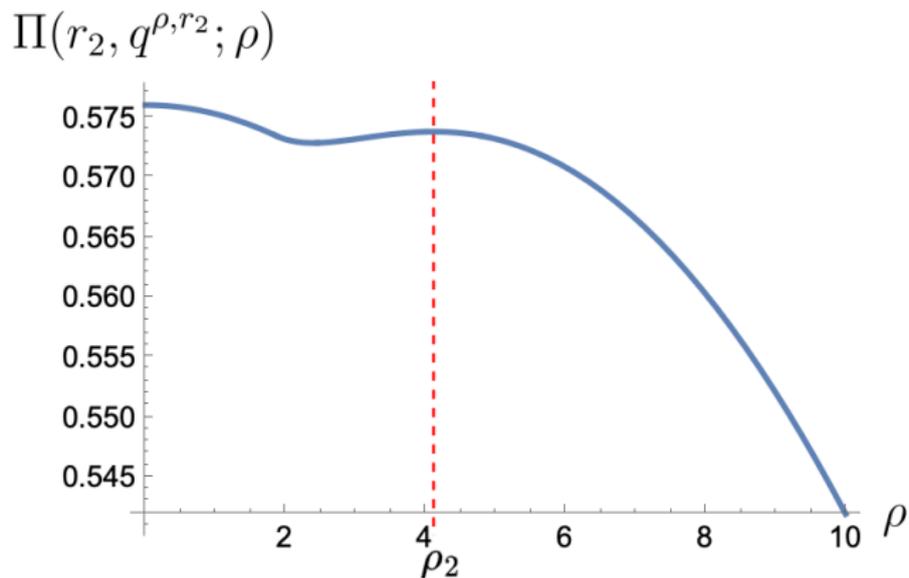
Candidate (Interior) Equilibria

- Two candidate interior equilibria:
- $\rho_1 = 4.7$, and $r_1 \approx 0.45$ (S invests a lot; B offers low price)
- $\rho_2 \approx 4.12$, and $r_2 \approx 0.58$ (S invests less; B offers higher price)

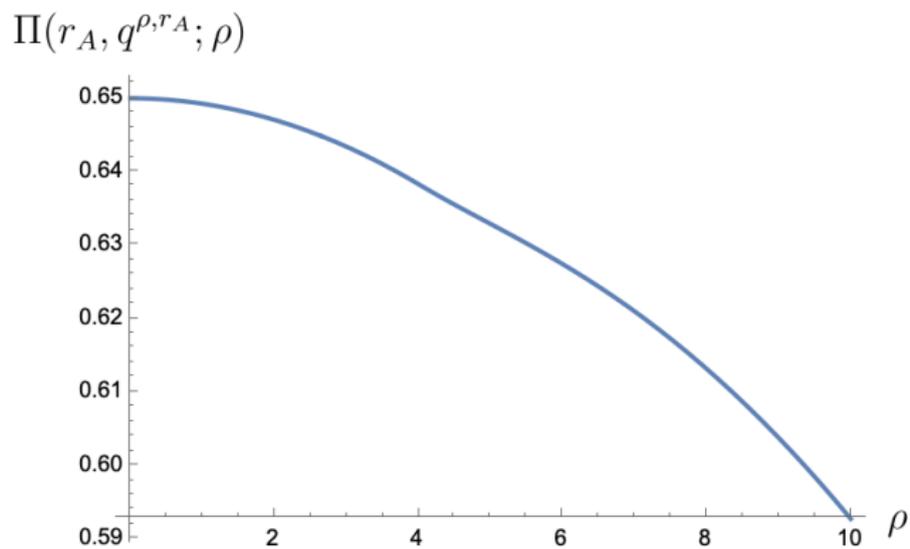
Sufficiency: low price r_1



Sufficiency: high price r_2

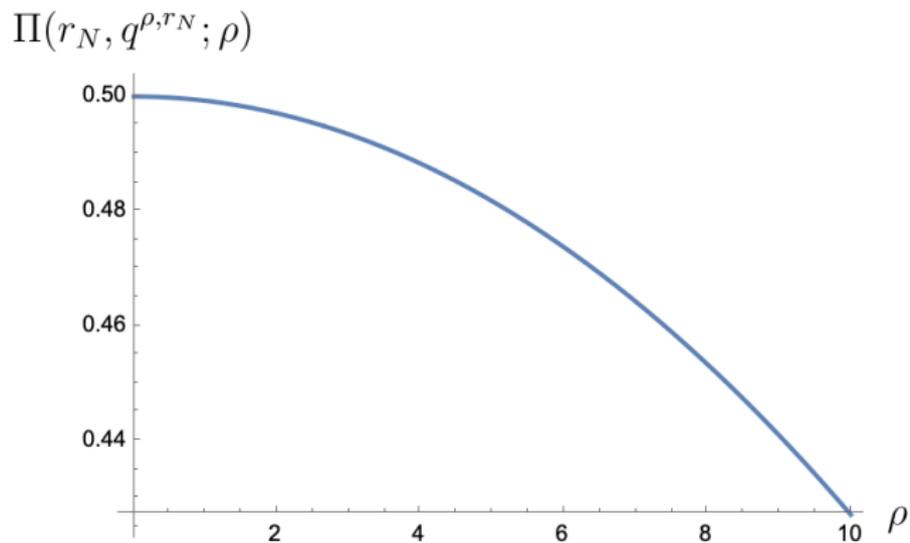


Corner 1: full engagement



$$\rho = 0, r_A = \omega_0 + \Delta = 0.65$$

Corner 2: mkt breakdown



$$\rho = 0, r_N = \Delta = 0.15$$

Multiple Equilibria: Welfare Analysis

- Three equilibria in this example
- **Interior:** $\rho^* > 0, r^* < \omega_0$
- **Corner with engagement:** $\rho_A = 0, r_A = \omega_0 + \Delta$
- **Corner with no engagement:** $\rho_N = 0, r_N = \Delta$
- Equilibria Pareto ranked:

$$(\rho_N, r_N) \prec (\rho^*, r^*) \prec (\rho_A, r_A)$$

- **Expectation traps**
- **Mkt breakdown** despite
 - zero MC when no info acquired
 - positive price when seller (off path) puts asset on sale
 - flexible info

Conclusions

- **Endogenous information** in mks with **adverse selection**
- **Expectation conformity**
 - prob of engagement decreasing in informativeness of signal
 - large gains from interaction
- Expectation traps
- Welfare and policy implications
 - endogeneous info: **larger subsidies/more generous programs**
- EC under flexible info with entropy or max-slope
- **Mkt break down under flex info**

- Future work:
 - bilateral information acquisition (**complementarity vs substitutability**)
 - implication for public information disclosures (**stress test design**)
 - ...

THANK YOU!

Definition

Info structures are **rotations** (or “simple mean-preserving spreads”) if, for any ρ , there exists rotation point m_ρ s.t.

- $G(m; \rho)$ **increasing** in ρ for $m \leq m_\rho$
- $G(m; \rho)$ **decreasing** in ρ for $m \geq m_\rho$

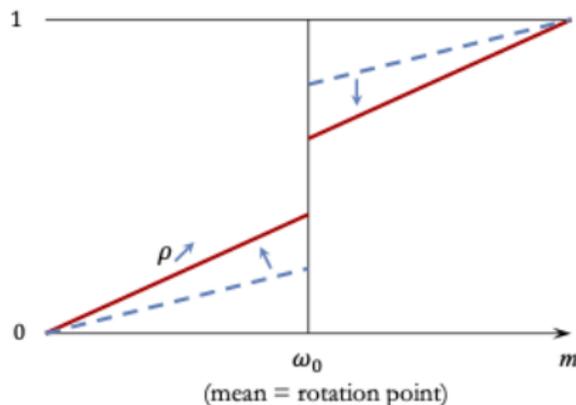
- Diamond and Stiglitz (1974), Johnston and Myatt (2006), Thereze (2022)...

Rotations Example: Non-directed Search

- L learns state with prob. ρ (nothing with prob. $1 - \rho$)

$$G(m; \rho) = \begin{cases} \rho G(m) & \text{for } m < \omega_0 \\ \rho G(m) + 1 - \rho & \text{for } m \geq \omega_0 \end{cases}$$

- Rotation point: prior mean ω_0



Rotations

- Combination of rotations need not be a rotation
- But any MPS can be obtained through sequence of rotations
- Other (notable) examples
 - G Normal and $s = \omega + \varepsilon$ with $\varepsilon \sim N(0, \rho^{-1})$
 - Pareto, Exponential, Uniform $G(\cdot; \rho)$...

Go back

Gains from Engagement

Definition

Info structures are **rotations** (or “simple mean-preserving spreads”) if, for any ρ , there exists rotation point m_ρ s.t.

- $G(m; \rho)$ **increasing** in ρ for $m \leq m_\rho$
- $G(m; \rho)$ **decreasing** in ρ for $m \geq m_\rho$

Proposition

Suppose info structures are rotations and L 's payoff is $\delta_L(m, r) = \tilde{\delta}_L(m, r) + \theta$. For all (ρ, ρ^\dagger) , there exists $\theta^(\rho, \rho^\dagger)$ s.t., for all $\theta \geq \theta^*(\rho, \rho^\dagger)$, EC holds at (ρ, ρ^\dagger) .*

- EC more likely when gains from engagement are large.

Gains from Engagement

- Result driven by AS
- Fixing r ,

$$\frac{\partial^2 \Pi}{\partial \theta \partial \rho} = G_{\rho}(m^*(r, \theta); \rho)$$

- Hence, marginal value of info **decreases with gains from engagement** under suff. condition for EC

$$G_{\rho}(m^*(r(\rho^{\dagger}); \theta), \theta; \rho) < 0$$

- Larger gains \rightarrow **smaller benefit from learning state**

Example: Akerlof-direct-search

- ρ : prob Seller learns state
- G uniform over $[0, 1]$
- $C(\rho) = \rho^2/20$
- $\Delta = 0.25$
- Eq. conditions

$$r = M^-(r; \rho) + \Delta$$
$$- \int_r^{+\infty} G_\rho(m; \rho) dm = C'(\rho)$$

- Two equilibria:

$$\rho_1 \approx 0.48 \quad r_1 \approx 0.69$$

$$\rho_2 \approx 0.88 \quad r_2 \approx 0.58$$

- For any $m^* > \omega_0 = .5$, $G_\rho(m^*; \rho) < 0$
- Hence, $A(\rho) < 0$ for all $\rho \in [\rho_1, \rho_2]$ (**info aggravates AS**)
- Seller better off in low-information eq.

Disclosure

- Suppose L can prove informativeness of her signal exceeds $\hat{\rho}$
 - **hard information**
- $\hat{\rho}(\rho^*)$: hard information disclosed in eq. supporting ρ^*

Definition (regularity)

Eq. supporting ρ^* regular if, after disclosing $\hat{\rho} < \hat{\rho}(\rho^*)$, informativeness of L 's signal expected by F (weakly) below ρ^*

- Monotone equilibrium selection

Proposition

Assume info aggravates AS ($A(\rho^\dagger) < 0$ for all ρ^\dagger)

- Any pure-strategy eq. ρ of no-disclosure game also eq. level of disclosure game
- Largest and smallest equilibrium levels in regular set of disclosure game also eq. levels of no-disclosure game.
- Result driven by AS effect
 - disclosing less than eq. level \rightarrow inconsequential
 - disclosing more \rightarrow unfriendlier reactions
- Without regularity, there exist eq. in disclosure game supporting
$$\rho^* > \sup\{\text{eq. } \rho \text{ no disclosure game}\}$$
 - sustained by F expecting larger ρ when L discloses $\hat{\rho} < \hat{\rho}(\rho^*)$

- L 's cost $C(\rho; \xi)$ decreasing in ξ

Corollary

Suppose L can acquire information cheaply (ξ_H) or expensively (ξ_L) and can disclose only ξ_H (IQ interpretation) or only ξ_L (work load). Further assume that, in eq., player F 's reaction is decreasing in posterior that $\xi = \xi_H$. Then L poses as "information puppy dog", i.e., does not disclose in IQ interpretation and discloses in work load one.

- $q^{\rho,r}(1|\omega)$: prob. signal recommends $a = 1$ at ω
- $q^{\rho,r}(1)$: tot prob. signal recommends $a = 1$
- Optimal (interior) signal for **entropy**:

$$\delta_L(r, \omega) = \frac{1}{\rho} \left[\ln \left(\frac{q^{\rho,r}(1|\omega)}{1 - q^{\rho,r}(1|\omega)} \right) - \ln \left(\frac{q^{\rho,r}(1)}{1 - q^{\rho,r}(1)} \right) \right]$$

- Optimal (interior) signal for **max-slope**:

$$q^{\rho,r}(1|\omega) = \begin{cases} 1 & \text{if } \omega \leq m^*(r) - \frac{1}{2\rho} \\ \frac{1}{2} - \rho(\omega - m^*(r)) & \text{if } m^*(r) - \frac{1}{2\rho} < \omega \leq m^*(r) + \frac{1}{2\rho} \\ 0 & \text{if } \omega > m^*(r) + \frac{1}{2\rho} \end{cases}$$

Proposition

Fix (ρ, ρ^\dagger) .

(i) EC holds at (ρ, ρ^\dagger) iff $A(\rho^\dagger)B(\rho; \rho^\dagger) < 0$.

(ii) Info aggravates AS at ρ^\dagger if $q^{\rho, r(\rho^\dagger)}(1|\omega)/q^{\rho, r(\rho^\dagger)}$ increasing in ρ for $\omega < m^*(r(\rho^\dagger))$, decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, at $\rho = \rho^\dagger$.

(iii) Reduction in r at $r(\rho^\dagger)$ raises L 's value of info at ρ if condition in (ii) holds and $q^{\rho, r(\rho^\dagger)}(1)$ non-increasing in ρ .

(iv) Suppose $M^-(m^*(r(\rho^\dagger)); \rho)$ decreasing in ρ at $\rho = \rho^\dagger$ and $\partial^2 \delta_L(r, m)/\partial r \partial m = 0$ (e.g., Akerlof). Then $q^{\rho, r(\rho^\dagger)}(1)$ decreasing in ρ at $\rho = \rho^\dagger$ NSC for EC at (ρ, ρ^\dagger) .