Managerial Turnover in a Changing World: Supplementary Material

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October 2012

In these notes, we formally prove that the effort and retention policies that maximize the firm's profits are independent of realized cash flows.

The proof is in two steps. The first step establishes that, in any mechanism that is incentive compatible and sequentially individually rational for the managers, the firm's expected profits from each manager it hires are given by a formula analogous to that in Proposition 2 in the main text. The second step then shows that the effort and retention policies of Proposition 3 continue to solve the relaxed program in this more general environment where effort and retention decisions are allowed to depend on observed cash flows. This last result in turn implies that, when the policies of Proposition 3 are implementable (which is guaranteed in the main text by the conditions in Propositions 4 and 5), then there is no way the principal can raise the firm's payoff by conditioning effort and retention decisions on realized cash flows.

Step 1. Suppose that the firm were to offer a mechanism $\Omega \equiv \langle \xi, x, \kappa \rangle$ where the effort and retention policies $\xi_t(\theta^t, \pi^{t-1})$ and $\kappa_t(\theta^t, \pi^t)$ possibly depend on realized cash flows — note that while the recommended effort choice for each period t naturally depends only on past cash flows, the retention decision may depend also on the cash flow π_t observed at the end of the period.

Following arguments identical to those in the main text, one can then verify that, in *any* mechanism $\Omega \equiv \langle \xi, x, \kappa \rangle$ that is incentive-compatible and sequentially individually rational, each manager's intertemporal expected payoff under a truthful and obedient strategy must satisfy

$$V^{\Omega}(\theta_1) = V^{\Omega}(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_1} \mathbb{E}_{\tilde{\theta}_{>1}^{\infty}, v^{\infty}|s} \left[\sum_{t=1}^{\tau(s,\tilde{\theta}_{>1}^{\infty}, v^{\infty})} \delta^{t-1} J_1^t(s, \tilde{\theta}_{>1}^t) \psi'(\xi_t(s, \tilde{\theta}_{>1}^t, \pi^{t-1}(s, \tilde{\theta}_{>1}^{t-1}, v^{t-1}))\right] dx \quad (1)$$

where $\pi^{t-1}(\theta^{t-1}, v^{t-1})$ denotes the history of cash flows under a truthful and obedient strategy when the history of productivity realizations is θ^{t-1} and the history of transitory noise shocks is v^{t-1}.

To see this, consider a fictitious environment identical to the one defined in the main text. In this environment, after any history $h_t = (\theta^t, \hat{\theta}^{t-1}, e^{t-1}, \pi^{t-1})$, the manager can misrepresent his productivity to be $\hat{\theta}_t$ but then has to choose effort equal to

$$e_t^{\#}(\theta_t; \hat{\theta}^t, \pi^{t-1}) = \hat{\theta}_t + \xi_t(\hat{\theta}^t, \pi^{t-1}) - \theta_t \tag{2}$$

so that the distribution of period-t cash flows π_t is the same as when the manager's true period-t productivity is $\hat{\theta}_t$ and he follows the principal's recommended effort choice $\xi_t(\hat{\theta}^t, \pi^{t-1})$.

Now fix an arbitrary sequence of reports $\hat{\theta}^\infty$ and an arbitrary sequence of true productivities θ^∞ and let $C(\hat{\theta}^\infty)$ denote the net present value of the stream of payments that the manager expects to receive from the principal when the sequence of reported productivities is $\hat{\theta}^\infty$ and in each period he chooses effort according to (2). Note that, by construction, C does not depend on the true productivities θ^∞. Also note that the expectation here is over the transitory noise v^∞. For any $(\theta^\infty, \hat{\theta}^\infty)$, the manager's expected payoff in this fictitious environment is then given by

$$\mathcal{U}(\theta^\infty, \hat{\theta}^\infty) \equiv C(\hat{\theta}^\infty) + \mathbb{E}_{v^\infty}\left[\begin{array}{c} -\sum_{t=1}^{\infty} \delta^{t-1} \kappa_{t-1}(\hat{\theta}^{t-1}, \pi^{t-1}(\hat{\theta}^{t-1}, \tilde{v}^{t-1}))\psi(\hat{\theta}_t + \xi_t(\hat{\theta}^t, \pi^{t-1}(\hat{\theta}^{t-1}, \tilde{v}^{t-1})) - \theta_t) \\ + \sum_{t=1}^{\infty} \delta^{t-1} \left(1 - \kappa_{t-1}(\hat{\theta}^{t-1}, \pi^{t-1}(\hat{\theta}^{t-1}, \tilde{v}^{t-1}))\right)(1-\delta)\, U^o \end{array}\right]$$

where $\pi^{t-1}(\hat{\theta}^{t-1}, v^{t-1})$ denotes the history of cash flows that obtains when, given the reports $\hat{\theta}^{t-1}$ and the noise terms v^{t-1}, in each period $s \leq t-1$, the manager follows the behavior described by (2).

As argued in the main text, it is easy to see that if the mechanism Ω is incentive compatible and sequentially individually rational in the original environment where the manager is free to choose his effort after misreporting his type, it must also be in this fictitious environment, where he is forced to choose effort according to (2).

Next note that the assumption that ψ is differentiable and Lipschitz continuous implies that $\mathcal{U}$ is totally differentiable in θ^t, any t, and equi-Lipschitz continuous in θ^∞ in the norm

$$||\theta^\infty|| \equiv \sum_{t=1} \delta^t |\theta_t|.$$

Together with the fact that $||\theta^\infty||$ is finite (which is implied by the assumption that Θ is bounded) and that the impulse responses $J_s^t(\theta^t)$ are uniformly bounded, this means that this fictitious environment satisfies all the conditions in Proposition 3 in Pavan, Segal, and Toikka (2011). The result in that proposition in turn implies that, given any mechanism Ω that is incentive compatible and sequentially individually rational, the value function $V^\Omega(\theta_1)$ associated with the problem that consists of choosing the reports and then selecting effort according to (2) is Lipschitz continuous and, at each point of differentiability, satisfies

$$\frac{dV^\Omega(\theta_1)}{d\theta_1} = \mathbb{E}_{\tilde{\theta}^\infty_{>1}|\theta_1}\left[\sum_{t=1}^{\infty} \delta^{t-1} J_1^t(\theta_1, \tilde{\theta}^t_{>1}) \frac{\partial \mathcal{U}(\tilde{\theta}^\infty, \tilde{\theta}^\infty)}{\partial \theta_t}\right]$$

where $\partial\mathcal{U}(\theta^\infty,\theta^\infty)/\partial\theta_t$ denotes the partial derivative of $\mathcal{U}(\theta^\infty,\theta^\infty)$ with respect to the true type θ_t under truthtelling. Condition (1) then follows from the fact that

$$\frac{\partial\mathcal{U}(\theta^\infty,\theta^\infty)}{\partial\theta_t} = \mathbb{E}_{v^\infty}\left[\kappa_{t-1}(\theta^{t-1},\pi^{t-1}(\theta^{t-1},\tilde{v}^{t-1}))\psi'(\xi_t(\theta^t,\pi^{t-1}(\theta^{t-1},\tilde{v}^{t-1})))\right]$$

along with the independence between $\tilde{\theta}^\infty$ and $\tilde{v}^\infty$.

Going back to the primitive environment, we then conclude that, given any mechanism Ω that is incentive-compatible and sequentially individually rational, the firm's expected profits from each manager it hires are given by

$$\begin{array}{l} \mathbb{E}_{\tilde{\theta}^\infty,\tilde{\nu}^\infty}\left[\begin{array}{c} \sum_{t=1}^{\infty}\delta^{t-1}\kappa_{t-1}(\tilde{\theta}^{t-1},\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1})) \\ \cdot\left\{\begin{array}{c} \tilde{\theta}_t+\xi_t(\tilde{\theta}^t,\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1}))+\tilde{\nu}_t-\psi(\xi_t(\tilde{\theta}^t,\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1}))) \\ -\eta(\tilde{\theta}_1)J_1^t(\tilde{\theta}^t)\psi'(\xi_t(\tilde{\theta}^t,\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1})))-(1-\delta)U^o \end{array}\right\} \end{array}\right] \\ +U^o-V^\Omega(\underline{\theta}), \end{array}$$

or, equivalently, denoting by $\tau(\theta^\infty,v^\infty)=\inf\{t:\kappa_t(\theta^t,\pi^t(\theta^t,v^t))=0\}$ the stochastic length of the employment relationship,

$$\begin{array}{l} \mathbb{E}_{\tilde{\theta}^\infty,\tilde{v}^\infty}\left[\sum_{t=1}^{\tau(\tilde{\theta}^\infty,\tilde{v}^\infty)}\delta^{t-1}\left\{\begin{array}{c} \tilde{\theta}_t+\xi_t(\tilde{\theta}^t,\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1}))+\tilde{\nu}_t-\psi(\xi_t(\tilde{\theta}^t,\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1}))) \\ -\eta(\tilde{\theta}_1)J_1^t(\tilde{\theta}^t)\psi'(\xi_t(\tilde{\theta}^t,\pi^{t-1}(\tilde{\theta}^{t-1},\tilde{v}^{t-1})))-(1-\delta)U^o \end{array}\right\}\right] \\ +U^o-V^\Omega(\underline{\theta}). \end{array} \quad (3)$$

Step 2. Consider now the "*relaxed program*" that consists of choosing the policies $(\xi_t(\cdot),\kappa_t(\cdot))_{t=1}^\infty$ so as to maximize the sum of the profits the firm expects from each manager it hires, taking the contribution of each manager to be (3), and subject to the participation constraints of the lowest period-1 types $V^\Omega(\underline{\theta})\geq U^o$.

To see that the policies ξ^* and κ^* of Proposition 3 solve the relaxed program defined above, we then proceed as follows. First, fix an arbitrary effort and retention policies ξ and κ and note that for any (θ^∞,v^∞), any $t<\tau(\theta^\infty,\tilde{v}^\infty)$, the flow virtual surplus

$$\begin{array}{rl} & \theta_t+\xi_t(\theta^t,\pi^{t-1}(\theta^{t-1},v^{t-1}))+v_t-\psi(\xi_t(\theta^t,\pi^{t-1}(\theta^{t-1},v^{t-1}))) \\ & -\eta(\theta_1)J_1^t(\theta^t)\psi'(\xi_t(\theta^t,\pi^{t-1}(\theta^{t-1},v^{t-1})))-(1-\delta)U^o \\ \leq & \theta_t+\xi_t^*(\theta^t)+v_t-\psi(\xi_t^*(\theta^t))-\eta(\theta_1)J_1^t(\theta^t)\psi'(\xi_t^*(\theta^t))-(1-\delta)U^o. \end{array}$$

This follows immediately from the fact that, for any t, any $\theta^t\in\Theta^t$, the function

$$\theta_t+e+v_t-\psi(e)-\eta(\theta_1)J_1^t(\theta^t)\psi'(e)-(1-\delta)U^o$$

is maximized at $e=\xi_t^*(\theta^t)$, where $\xi_t^*(\theta^t)$ is as in Proposition 3.

Next let κ' be any retention policy that, given the effort policy ξ^*, in each state $(\theta^\infty, v^\infty)$ implements the same retention decisions as the original rule κ. That is, denoting by $\pi^{*t-1}(\theta^{t-1}, v^{t-1})$ the equilibrium history of cash flows under the effort policy ξ^*, κ' is such that, for any $t \geq 1$, any (θ^{t-1}, v^{t-1})

$$\kappa'_{t-1}(\theta^{t-1}, \pi^{*t-1}(\theta^{t-1}, v^{t-1})) = \kappa_{t-1}(\theta^{t-1}, \pi^{t-1}(\theta^{t-1}, v^{t-1})).$$

It is then immediate that the firm's intertemporal expected profits under the policies (κ', ξ^*) are weakly higher than under the original policies (κ, ξ). In fact, while the separation decisions are the same under the two policies κ and κ', for each manager the firm hires, and for each state $(\theta^\infty, v^\infty)$, the virtual surplus in each period t prior to separation is higher under the effort policy ξ^* than under the policy ξ.

Next, observe that, starting from the pair of policies (ξ^*, κ'), if the principal were to switch to the pair of policies (ξ^*, κ^*) of Proposition 3, then the firm's expected profits would be higher. This follows from the fact that the flow virtual surplus

$$\theta_t + \xi^*_t(\theta^t) + v_t - \psi(\xi^*_t(\theta^t)) - \eta(\theta_1) J^t_1(\theta^t) \psi'(\xi^*_t(\theta^t)) - (1-\delta)\, U^o \tag{4}$$

that the firm obtains from each manager it hires after any length t of employment and for each productivity history θ^t is the same under (ξ^*, κ') as it is under (ξ^*, κ^*), along with the fact that, by construction, the retention rule κ^* of Proposition 3 maximizes the firm's intertemporal payoff when the flow virtual surpluses are given by (4).

We conclude that the policies (ξ^*, κ^*) of Proposition 3 solve the relaxed program defined above. In turn, this implies that whenever there exists a compensation scheme x^* that implements these policies and gives the lowest period-1 type an expected payoff $V^\Omega(\underline{\theta})$ equal to his outside option, then the firm's payoff cannot be improved by switching to any other mechanism, including those where the effort and retention policies possibly depend on realized cash flows.