Wall Street and Silicon Valley: A Delicate Interaction*

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Abstract

This paper studies how a certain two-way feedback between the real and the financial sectors of the economy influences its response to the arrival of new, unfamiliar investment opportunities. On the one hand, financial markets look at startup activity and other real indicators for signals about the profitability of the new investment opportunities. On the other hand, entrepreneurs and CEOs are concerned about the mood in financial markets because this determines the value of IPOs and season stock offerings, the cost of external financing, and the possibility to expand. This two-way feedback is shown to increase the sensitivity of both investment and asset prices to "noise," exacerbating the disconnect from fundamentals. In addition, these phenomena are shown to be symptoms of constrained inefficiency. Appropriate taxes, or regulations, can thus improve welfare, without requiring the government to have any informational advantage vis-a-vis the market.

Keywords: mispricing, heterogeneous information, information-driven complementarities, volatility, inefficiency, beauty contests.

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1 Introduction

The arrival of new, unfamiliar investment opportunities—e.g., e-commerce in the late 90s—is often associated with spikes in investment, startup activity, IPOs, and asset prices. More generally, investment and asset prices tend to co-move in response to news about future returns. Do such fluctuations reflect the efficient response of the economy to available information about underlying fundamentals? Or could they be the product of excessive waves of optimism and pessimism?

We address this question focusing on a two-way feedback between entrepreneurial activity ("Silicon Valley") and asset markets ("Wall Street"). On the one hand, Silicon Valley is concerned about the mood in Wall Street because it determines the value of capital (e.g., the value of an IPO) and so it influences current and future investment decisions. On the other hand, Wall Street monitors entrepreneurial activity, corporate investment, and a variety of other real indicators because these variables contain valuable information about the underlying fundamentals. The first channel is the subject of a large literature. Our contribution is to add the second channel and to show how the interaction of the two channels causes an inefficiency in how the economy uses available information.

Model Preview. The real sector of our model features a large number of entrepreneurs, who make investment decisions in two stages. In the first stage, entrepreneurs decide whether or not to start a project in a new sector, or new technology. We think of this stage as the startup stage. In the second stage, those entrepreneurs who started a project can expand its scale and finance this expansion by raising funds in the financial market. They do so by selling equity shares to external investors, which we call "traders". We think of this second stage as the IPO stage. Both entrepreneurs and traders are imperfectly informed about the profitability of the new technology.

At the core of the model there is a two-way feedback between the real and the financial sectors. When entrepreneurs choose whether to start a project they know that they will have access to the financial market in the second stage, and so they base their startup decision on their expectations about future IPO prices, which will determine their capacity to expand. On the other hand, traders observe the aggregate level of startup activity in the first stage, and use it as a signal of the profitability of the new investment opportunity. Through this information channel, startup activity determines the demand for shares in the IPO market and IPO prices.[1]

Key Results. On the positive side, we show that the two-way feedback just described amplifies the response of the economy to "animal spirits." By the latter we mean waves of optimism and pessimism that are orthogonal to the underlying fundamentals. In our model, all agents are rational and such waves reflect noise-driven movements in first- or higher-order beliefs about profitability.[2] Alternatively, such waves could be modeled as the product of a correlated bias in beliefs ("irrational exuberance") or

[1] Although the information transmission from the real sector to asset prices has drawn less attention in the literature than that of the opposite direction, it is consistent with a variety of evidence about the response of asset prices to the release of sectoral and macroeconomic data (Chen, Roll and Ross, 1986, and Cutler, Poterba, and Summers, 1989).

[2] Our form of animal spirits is therefore different from that in the literature on multiple equilibria, but similar to those in Angeletos and La'O (2013), Benhabib and Farmer (2013), and Huo and Takayama (2015).

a correlated taste for start-up activity. In all these cases, the key friction is that the asset market cannot tell apart such animal spirits from true information about profitability. And its observable implication is higher non-fundamental volatility in real investment and in asset prices.

On the normative side, we show that the equilibrium is constrained inefficient and that the aforementioned non-fundamental volatility is excessive. This means that a planner who does not have superior information relative to the market can nonetheless improve on equilibrium outcomes by manipulating private incentives. Like the amplification, the inefficiency only arises when both channels of the two-way feedback are present.

Let us explain first the positive result. An entrepreneur receives various pieces of information. Some are more idiosyncratic, others are more correlated. Given her information, the entrepreneur assigns some probability to the fundamentals being high, and some probability to the presence of correlated noise, orthogonal to the fundamentals. The entrepreneur knows that aggregate startup activity will be higher in both cases, because all other entrepreneurs, like him, cannot distinguish exactly what is behind the different sources of information. He also knows that a higher level of startup activity will send a positive signal to traders about the underlying fundamentals. This gives the entrepreneur an incentive to start a project even if he does not expect the fundamentals to be particularly high, as long as he expects other entrepreneurs to do the same. The reason is that the boom in aggregate startup activity will feed into high IPO prices and will make a future expansion more profitable. In such a situation, the typical entrepreneur engages in what looks like "speculative" behavior to an outside observer: he invests because he anticipates that the financial market will be overvaluing his firm.

As all entrepreneurs do the same, their collective behavior triggers a non-fundamental spike in IPO prices. The expectation of such spike in turn reinforces the initial speculative motive of each entrepreneur. That is, the speculative motive that emerges in our setting is a source of strategic complementarity in the decisions of the entrepreneurs. This in turn explains why our mechanism amplifies the impact of noise, or random sentiment, on aggregate startup activity. Finally, because this behavior reduces the signal-to-noise ratio in aggregate startup activity, it also reduces the precision of the information upon which the financial market operates. This in turn explains why our mechanism amplifies non-fundamental volatility, not only in startup activity, but also in asset prices and in the investment that takes places at the expansion stage. In short, our mechanism is responsible for a "bubbly" comovement in both real and financial activity.

To understand the normative result, consider the problem of a planner who can control the entrepreneurs' and the traders' choices but has no informational advantage vis-a-vis the market—either in the form of additional information, or in the form of the power to centralize the information that is dispersed in the economy. We show that such a planner would dictate to the entrepreneurs to ignore the expected mispricing and, instead, base their entry decisions solely on their expectation of fundamental profitability.

Because the latter depends on the capital expansion following the IPO, the planner recognizes the importance of having the entrepreneurs conditioning their startup decisions on their expectation of their firms' market valuation. What the planner does not value is the entrepreneurs' conditioning their entry decisions on their expectation of possible mispricing at the IPO stage. This is because mispricing is just a transfer from one group of agents, the traders, to another, the entrepreneurs. Such a transfer does not contribute to welfare, creates a wedge between the private and social return to startup activity, and leads to inefficient entry.

The above argument applies even if we abstract from the welfare implications of the investments made at the project-expansion stage. Taking these implications into account only reinforces the argument. By instructing, or incentivizing, the entrepreneurs to disregard the likely mispricing in asset markets and the ensuing complementarity in their decisions, the planner also increases the informativeness of the signal that Silicon Valley sends to Wall Street, thus also increasing the efficiency of the investments made at the project-expansion stage. Internalizing this informational externality calls for an even greater reduction in the sensitivity of startup decisions to correlated noise ("animal spirits").

Policies that do not require the planner to have more information than the market and nevertheless improve welfare by better aligning investment and asset prices with the underlying fundamentals include state-contingent taxes on entrepreneurial income, taxes on financial trades, and regulations of IPO issues. We discuss these and other implications of our results in due course.

On Speculation and Beauty Contests. The kind of speculative behavior we document emerges in Silicon Valley, not in Wall Street. This is unlike a literature which focuses on speculation in asset markets and permits us to shift the focus to a different source of "bubbly" co-movement in investment and asset prices. Furthermore, our mechanism does not require the agents to be "large" and/or "strategic:" all agents are infinitesimal, with no market power. Thus, despite a certain similarity in flavor, our results are different from those in the financial microstructure literature, which, in the tradition of Kyle (1985), focuses on how large players can manipulate asset prices.[3] Rather, the speculative effects in our model are the by-product of the collective signaling by the entrepreneurs to the traders.

Our analysis also provides a particular micro-foundation for the type of abstract beauty-contest games studied, inter alia, in Morris and Shin (2002), Angeletos and Pavan (2007), and Bergemman and Morris (2013). In the equilibrium of model, the entrepreneurs' startup choices are strategic complements because higher aggregate startup activity raises asset prices, which in turn raises the individual return to start a project. In short, a beauty contest emerges because, and only because, of the collective signaling from Silicon Valley to Wall Street. Furhtermore, the strategic complementarity embedded in this game is excessive from an efficiency standpoint. An integral part of our contribution is thus to provide a justification for the *normative* properties of beauty-contest phenomena often assumed, but not always micro-founded, in the literature.

Layout. The rest of the paper is organized as follows. Section 2 discusses the related literature. . Section 3 introduces the model. Section 4 characterizes the equilibrium. Section 5 contains our positive results, whereas Section 6 contains our normative results. Section 7 discusses policy implications and the robustness of the findings. All proofs are in the Appendix at the end of the document.

[3] See, e.g., Goldstein and Guembel (2008), which emphasizes how this manipulation could distort real investment.

2 Related Literature

Allen, Morris and Shin (2006), Bacchetta and Wincoop (2005), Cespa and Vives (2012) and Kassa, Walker and Whiteman (2014) have sought to operationalize Keynes' idea that asset markets behave like a beauty contest by accommodating higher-order uncertainty in basic asset-pricing models. Our paper shifts the focus to the beauty contest that emerges in the real sector of the economy due to the two-way feedback explained above. Furthermore, this feedback is shown to be a source of inefficiency, so that the normative suggestion ("beauty contests are bad") is justified.[4]

Our paper thus adds to a literature that studies the normative properties of settings with strategic complementarity (Morris and Shin, 2002, Angeletos and Pavan, 2007, Hellwig, 2006, Lorenzoni, 2010) and informational externalities (Vives, 1997, 2017, Angeletos and Pavan, 2009, Amador and Weill, 2010, 2012). This literature typically treats the strategic complementarity and the informational spillover as two disconnected features, often studied in isolation of one another. Our paper instead identifies a strategic complementarity that is itself the product of an informational spillover.[5]

Related in this last respect are Angeletos and Lian (2018), Benhabib et al. (2015), Chahrour and Gaballo (2017), and Gaballo (2017). While the context is different—these papers study business-cycle models in the tradition of Lucas (1972), in which agents are confused between idiosyncratic and aggregate shocks—a commonality with our paper is that a strategic complementarity arises in these papers because an agent's strategy influences the signal-extraction of the others.[6]

Closely related are also Goldstein, Ozdenoren and Yuan (2013) and Albagli, Hellwig and Tsyvinski (2017), which focus on how complementarity and informational frictions impinge on financial market efficiency. In Goldstein, Ozdenoren and Yuan (2013), an inefficiency arises from a coordination motive in financial market trading, which has the flavor of a currency attack: when many traders sell a security issued by a certain firm, they end up hurting the profitability of this firm thus making selling a best response. In Albagli, Hellwig and Tsyvinski (2017), limits to arbitrage cause price fluctuations to be amplified relative to the information contained in the price itself. This creates a rent-seeking motive for incumbent shareholders that depends on the asymmetry of the distribution of the fundamental. While similar in spirits, neither paper features the mechanism investigated here, which relies on financial traders using information contained in real investment decisions.

[4]This property also differentiates the beauty contest identified here from the one that obtains from aggregate-demand externalities in the baseline business-cycle framework: although both beauty contest regard the real sector of the economy, the latter does not necessarily involve a wedge between private and social incentives (Angeletos and La'O, 2010, 2018).

[5]A related literature studies the endogeneity of private information in settings with strategic complementarity (Hellwig and Veldkamp, 2009, Myatt and Wallace, 2012, Colombo, Femminis and Pavan, 2014, Chahrour, 2014, Pavan, 2016). Although our analysis abstracts from this issue, the two-way feedback we have identified is likely to increase the entrepreneurs' incentives not only to react to sources of information with correlated noise, but also to collect such information in the first place. This would add an extra layer of inefficiency.

[6]An information-driven complementarity is also present in Goldstein, Ozdenoren and Yuan (2011), which studies a currency-attack setting where the speculators' decisions are strategic complements because a large attack causes the central bank to downgrade its assessment of the underlying fundamentals.

At the broader level, our paper adds to a large literature that studies the nexus between asset prices and real investment. One strand of this literature focuses on the importance of external financing. We embrace this point and let an informational spillover influence the cost of such financing. Another strand of this literature focuses on how the information contained in asset prices helps guide real investment (Dow and Gorton, 1997, Subrahmanyam and Titman, 1999, 2001). We instead shift the focus to the opposite informational spillover, from the real economy to asset markets.[7]

Finally, our paper shares with Chari and Kehoe (2003) and Loisel, Pommeret and Portier (2012) the broader idea that informational spillovers can generate excessive non-fundamental volatility in asset prices and investment. However, the two-way feedback at the core of our paper is new and the nature of the inefficiency is different.

3 The Model

There is a single perishable good, which can be used for either consumption or investment. There are three periods $t \in \{1, 2, 3\}$. There are two types of agents, entrepreneurs and investors, each of measure one. Entrepreneurs are born at $t = 1$, investors at $t = 2$. At $t = 1$, the entrepreneurs have the option to start new projects using internal funds; we call this period the "startup stage." At $t = 2$, the entrepreneurs who have started a project at $t = 1$ can expand it by selling shares to outside investors and using these funds to make an additional investment; we call this period the "IPO stage." In the last period, $t = 3$, the projects' profits are realized and distributed to entrepreneurs and outside investors.

Technology. Starting a project requires an initial fixed investment of one unit of the perishable good at $t = 1$. Once a project is started at $t = 1$, it requires an additional investment at date $t = 2$. The size of this investment can vary. We denote by $k_i \in \mathbb{R}_+$ the investment of entrepreneur i. The output of the project at $t = 3$ is equal to

$$q_i = \Theta f(k_i),$$

where $f(k) = k^\alpha$, with $\alpha \in (0, 1)$ parametrizing the project's returns to scale. The profitability of all projects depends on the realization of the aggregate random variable Θ, which we think of as the underlying fundamental .

Timing. Before any agent moves, Nature draws the fundamental $\theta \equiv \log \Theta$ from a Normal distribution with mean 0 and variance $\sigma_\theta^2 = 1/\pi_\theta$. This defines the common prior about θ, with π_θ parameterizing the precision of the prior. In addition, Nature draws a variety of private signals, some for the entrepreneurs and some for the investors; we specify the details of these signals in the sequel.

Startup stage. At $t = 1$, each entrepreneur is endowed with one unit of the perishable good, which he can either consume or invest into a startup. Let $n_i \in \{0, 1\}$ denote entrepreneur i's decision to start a

[7] Relatedly, while the literature on IPOs has typically treated the information of the financial traders as exogenous (Rock, 1986, Biais, Bossaerts and Rochet, 2002), our paper lets such information be influenced by the collective decisions of the entrepreneurs.

project. We refer to the entrepreneurs who decide to start a project at $t = 1$ as "active," and the rest as "inactive". Let N denote the mass of active entrepreneurs and, with some abuse of notation, let $[0, N]$ be the set of active entrepreneurs.[8]

IPO stage. At $t = 2$, the entire profile $(n_i)_{i\in[0,1]}$ of startup decisions is publicly observed. Inactive entrepreneurs have no other source of income and take no further action.[9] Active entrepreneurs receive no income at $t = 2$, so their investment k_i must be fully financed by outside investors. In particular, entrepreneurs finance their investment by selling shares in the IPO market[10] and face the budget constraint

$$k_i = p_i s_i, \tag{1}$$

where $s_i \in [0, 1]$ is the quantity of shares issued and p_i is price of a share. The microstructure of the IPO market is described in the sequel.

At $t = 3$, the fundamental θ is publicly revealed and the output of all active projects is realized. The entrepreneur receives $(1 - s_i)\Theta f(k_i)$ and the outside investors receive $s_i\Theta f(k_i)$.

Payoffs. The lifetime utility of entrepreneur i is $U_i = c_{i1} + \beta c_{i2} + \beta^2 c_{i3}$, where $\beta \in (0, 1)$ is the common discount factor. His consumption at $t = 1$ is $c_{i1} = 1 - n_i$, i.e., it is equal to the initial endowment if the entrepreneur does not start a project and zero otherwise. At $t = 2$, his consumption is 0. At $t = 3$, his consumption is zero if the entrepreneur did not start a project and $c_{i3} = (1-s_i)\Theta f(k_i)$ otherwise. Combining, the entrepreneur's lifetime utility is

$$U_i = 1 - n_i + n_i\beta^2\Pi_i, \text{ where } \Pi_i \equiv (1 - s_i)\Theta f(k_i). \tag{2}$$

All investors are identical, they behave as price takers, and are symmetrically informed. We thus represent this class of agents with a single representative investor. The following assumptions provide a microfoundation for a perfectly elastic supply of external funds by investors in the IPO market. At $t = 2$, the representative investor can produce consumption goods out of labor effort at a one-to-one rate. The consumption levels of the representative investor at $t = 2$ and $t = 3$ are, respectively,

$$c_2 = l - \int_{i\in[0,N]} p_i s_i di \qquad \text{and} \qquad c_3 = \int_{i\in[0,N]} s_i\Theta f(k_i)di,$$

where l denotes labor effort and where s_i denotes the quantity of shares bought from each active entrepreneur $i \in [0, N]$. We assume that the disutility of effort is linear so that the investor's lifetime utility is $V = c_2 - l + \beta c_3$. Combing the pieces above, the investor's lifetime utility is

$$V = \int_{i\in[0,N]} [\beta\Theta f(k_i) - p_i]\, s_i di. \tag{3}$$

[8]This amounts to a state-dependent re-ordering of the identities of the entrperneuers so that the active ones are always ordered first.

[9]This guarantees that these agents do not participate in the IPO market. We discuss how this property can be relaxed in Section 7 below.

[10]We will later provide a moral-hazard justification for why this round of investment is financed by issuing equity claims.

These assumptions imply that the supply of external funds to each active entrepreneur is perfectly elastic at a price equal to the expected profit of the entrepreneur's project.

Information. At $t = 1$, when choosing whether or not to a start a project, each entrepreneur has access to various sources of information (signals) that are not directly available to the investors. The noise in some of these signals may be mostly idiosyncratic, while, for other signals, the noise may be correlated across entrepreneurs. We capture such a possibility in the simplest possible way by assuming that each entrepreneur observes two signals. One has purely idiosyncratic noise and is given by $x_i = \theta + \xi_i$, where each ξ_i is drawn from a Normal distribution with mean zero and variance $1/\pi_x$, independently of θ and independently across entrepreneurs. The other signal is perfectly correlated among the entrepreneurs and is given by $y = \theta + \varepsilon$, where ε is drawn from a Normal distribution with mean zero and variance $1/\pi_y$, independently from θ and $\{\xi_i\}_{i\in[0,1]}$, and is the same across all entrepreneurs.

As we show below, the key role of the correlated noise ε is to introduce a source of co-movement in startup investment, financial prices, and period-2 investment that can be interpreted as a form of "market sentiment" or "exuberance."[11]

At $t = 2$, the representative external investor receives information which is not directly observed by the entrepreneurs. Any private information that the external investor may possess about the underlying technology is summarized in a signal $\omega = \theta+\eta$, where η is drawn from a Normal distribution with mean zero and variance $1/\pi_\omega$, independently of θ, ε and $(\xi_i)_{i\in[0,1]}$.[12] It follows that the information set with which the representative investor enters the IPO markets in period 2 is given by $\mathcal{I} = \{\omega, (n_j)_{j\in[0,1]}\}$, whereas the corresponding information set of an entrepreneur i is given by $\mathcal{J}_i = \{x_i, y, (n_j)_{j\in[0,1]}\}$. The information sets with which these agents exit the IPO markets contain, in addition to the above signals, the prices, the shares, and the associated levels of investment in each of the IPO markets: that is, $\mathcal{I}' \equiv \mathcal{I} \cup \mathcal{M}$ and $\mathcal{J}'_i \equiv \mathcal{J}_i \cup \mathcal{M}$, where $\mathcal{M} \equiv \{(p_i, s_i, k_i)_{i\in[0,N]}\}$ represents the market-generated information.

IPO market structure. The IPO market operates in a similar fashion as in Kyle (1985). Each entrepreneur i submits a supply correspondence $S^s_i((\tilde{p}_j)_{j\in[0,N]}, (\tilde{k}_j)_{j\in[0,N]\setminus i}|\mathcal{J}_i)$, representing the amount of shares he is willing to sell at each price $\tilde{p}_i$ conditional on the vector of prices $(\tilde{p}_j)_{j\in[0,N]\setminus i}$ and investment levels $(\tilde{k}_j)_{j\in[0,N]\setminus i}$ of all other firms. The representative investor submits a collection of demand

[11]Such a correlated error, in turn, can have various origins. Private signals about the actions of agents that moved in the past may lead in equilibrium to signals about θ with correlated errors. More broadly, network effects, social learning, and information cascades may also explain this correlation. Alternatively, as emphasized in Hellwig and Veldkamp (2009) and Myatt and Wallace (2012), strategic complementarities—like the ones that, as we will show below, emerge endogenously in our economy—by themselves generate an incentive for the entrepreneurs to collect correlated sources of information. See also Dow, Goldstein, and Guembel (2017), Froot, Scharfstein, and Stein (1992), and Veldkamp (2006) for complementary justifications.

[12]While ω is modeled here as an exogenous common signal among all the outside investors, it is straightforward to recast it as the outcome of the aggregation of information that may take place in the financial market when the outside investors have themselves dispersed private information about θ.

correspondences $\left(S_i^d(\cdot|\mathcal{I})\right)_{i\in[0,N]}$, one for each IPO market $i \in [0, N]$, with each $S_i^d((\tilde{p}_j)_{j\in[0,N]}, (\tilde{k}_j)_{j\in[0,N]}|\mathcal{I})$ specifying the amount of firm i's shares he is willing to purchase at price $\tilde{p}_i$, conditional on firm i's total investment $\tilde{k}_i$, and on the vector of prices and investment levels in all other markets be equal to $(\tilde{p}_j)_{j\in[0,N]\setminus i}$ and $(\tilde{k}_j)_{j\in[0,N]\setminus i}$, respectively. The auctioneer then selects a collection of triples $(p_i, s_i, k_i)_{i\in[0,N]}$, one for each active IPO market $i \in [0, N]$, such that each IPO market clears. That is, for each market $i \in [0, N]$,

$$s_i \in S_i^s((p_j)_{j\in[0,N]}, (k_j)_{j\in[0,N]\setminus i}|\mathcal{J}_i) \quad \text{and} \quad s_i \in S_i^d((p_j)_{j\in[0,N]}, (k_j)_{j\in[0,N]}|\mathcal{I}),$$

and the investment k_i is consistent with the price p_i and shares s_i issued, that is,

$$k_i = p_i \cdot s_i.$$

This structure has two key features. First, by allowing the orders to be contingent on prices, it allows the information of all market participants to be potentially revealed and used by other market participants. This is reminiscent to Kyle (1985), except for two differences. First, the dividend of the asset is endogenous to the level of capital raised in the IPO market. Second, the entrepreneur does not have market power.[13]

Equilibrium restrictions. We focus our analysis on perfect Bayesian equilibria that satisfy three restrictions, or refinements, defined below. The first restriction is that the equilibrium price in any given IPO market does not reveal any information not contained in the supply and demand schedules submitted by the relevant agents. This restriction is standard and requires the auctioneer to only use information coming from demand and supply schedules in choosing an allocation.

The second restriction is that the representative investor's posterior about the aggregate fundamental be log-normal. In particular, we require that (i) conditional on $\mathcal{I}$, θ is normally distributed with a variance that is state-invariant, and (ii) the investor's posterior mean, denoted by $\hat{\theta} \equiv \mathbb{E}[\theta|\mathcal{I}]$, is normally distributed with a variance that is invariant in $\mathcal{I}$. This restriction is also standard and is made for tractability, as it allows us to solve for the entrepreneurs' and the investors' inference problems in closed form.

The third restriction is that each entrepreneur i is "informationally small." In particular, we require that the representative investor's posterior beliefs about θ be independent of the startup decision n_i of any individual entrepreneur and of the realized triple (p_i, s_i, k_i) in any individual IPO market. We allow the investor's beliefs to be affected by the distribution $(p_i, s_i, k_i)_{i\in[0,N]}$ of prices, shares, and investment levels across markets, and of course we allow any given entrepreneur to affect the price in his own IPO market via his choice of a supply schedule. What we rule out is only the possibility that the actions of a single entrepreneur may move the representative investor's posterior beliefs about the aggregate fundamental θ.

[13] To see this, note that, for any level of investment k or, equivalently, for any level of the endogenous dividend, the entrepreneur chooses s by taking the the price of the shares as given.

In the next section, we show that an equilibrium satisfying the above restrictions always exists and we provide its characterization.

4 Equilibrium Characterization

We characterize the equilibrium in four steps. First, we show how investors' information determines share prices and investment levels in the IPO market in period 2. Second, we show how the entrepreneurs' expectations of share prices and fundamentals determine their startup decisions in period 1. Third, we show how aggregate startup activity conveys to the investors some of the entrepreneurs' information. In the fourth, and last, step, we close the loop by checking that the investors' information in the IPO stage is consistent with the informativeness of the endogenous signal they receive from the entrepreneurs' startup activity.

Step 1: The IPO stage

Let us start from the IPO stage at $t = 2$. Given the representative investor's preferences and given our restriction that each entrepreneur is informationally small, the representative investor's demand schedule in each market $i \in [0, N]$, conditional on each possible distribution $(p_j, s_j, k_j)_{j\in[0,N]\backslash i}$ of prices, shares, and investment levels in all IPO markets other than i's, is perfectly elastic at the price

$$p = \beta\hat{\Theta}f(k), \tag{4}$$

where

$$\hat{\Theta} \equiv \mathbb{E}[\Theta|\mathcal{I}']$$

is the representative investor's expectation of future productivity when exiting the IPO market (recall that $\mathcal{I}' = \{\omega, (n_j)_{j\in[0,1]}\} \cup \{(p_j, s_j, k_j)_{j\in[0,N]}\}$).

Next, consider each entrepreneur's choice of a supply schedule S^s. To characterize the optimal schedule, we proceed in two steps. First, we let the entrepreneur solve a relaxed problem. Next, we show that there exists a supply schedule S^s that implements the solution to the relaxed problem.

The relaxed problem is one in which the entrepreneur can condition his supply of shares directly on the representative investor's posterior beliefs about Θ, that is, on $\hat{\Theta}$. In particular, for every $\hat{\Theta}$, suppose the entrepreneur can choose any triple (p, s, k) that is consistent with the investor's demand schedule and satisfies the budget constraint $k = p \cdot s$. Given that the investor's demand schedule is characterized by (4), these constraints imply that, if the entrepreneur wants to invest k units into the project, he must sell

$$s = \frac{k}{\beta\hat{\Theta}f(k)} \tag{5}$$

shares. The entrepreneur's payoff can then be expressed as a function of Θ, $\hat{\Theta}$, and k, as follows:

$$(1 - s)\Theta f(k) = \frac{\Theta}{\beta\hat{\Theta}}\left[\beta\hat{\Theta}f(k) - k\right]. \tag{6}$$

The value of k that maximizes the right-hand side of (6) is

$$K(\hat{\Theta}) \equiv \arg\max_k \left\{ \beta\hat{\Theta}f(k) - k \right\}. \tag{7}$$

The corresponding values of p and s are then given by (4) and (5), respectively.

Given the solution to the above relaxed program, we can now construct the entrepreneur's supply schedule. Note that $K(\hat{\Theta})$ is strictly increasing in $\hat{\Theta}$ and that the expression on the right-hand side of (4) is strictly increasing in k. The following expression then defines a bijective relation between p and $\hat{\Theta}$:

$$p = P(\hat{\Theta}) \equiv \beta\hat{\Theta}f(K(\hat{\Theta})).$$

Given the above bijective relation, and using the fact that $s = k/p$, we have that the supply schedule that permits the entrepreneur to implement the allocation that solves the relaxed program is equal to

$$S_i^s((p_j)_{j\in[0,N]}, (k_j)_{j\in[0,N]\setminus i}|\mathcal{J}_i) = K(P^{-1}(p_i))/p_i.$$

The realized triple (p_i, s_i, k_i) is then the same across all IPO markets and is given by

$$(p_i, s_i, k_i) = (P(\hat{\Theta}), K(\hat{\Theta})/P(\hat{\Theta}), K(\hat{\Theta})).$$

Since the realized prices and quantities are the same in all IPO markets and are a known function of $\hat{\Theta}$, they reveal no information about Θ over and above what is contained in $\hat{\Theta}$ itself. This implies that representative investor does not update his beliefs about Θ after observing the realized trades in all IPO markets. We conclude that

$$\hat{\Theta} \equiv \mathbb{E}[\Theta|\mathcal{I}'] = \mathbb{E}[\Theta|\mathcal{I}].$$

Given our Cobb-Douglas technology specification, we can solve explicitly for the equilibrium prices and quantities:

$$P(\hat{\Theta}) = \alpha^{\frac{\alpha}{1-\alpha}}(\beta\hat{\Theta})^{\frac{1}{1-\alpha}}, \qquad K(\hat{\Theta}) = (\alpha\beta\hat{\Theta})^{\frac{1}{1-\alpha}}, \qquad S(\hat{\Theta}) = \alpha. \tag{8}$$

That $S(\hat{\Theta}) = \alpha$ implies that, even though we have allowed entrepreneurs to post contingent supply schedules (aka limit orders), the same equilibrium prices and allocations obtain if we assume a different market micro-structure whereby entrepreneurs commit to sell a fixed quantity of shares independently of the price (aka market orders).

Step 2: The startup stage

We now proceed to characterize the equilibrium outcomes in the startup stage. From (2), entrepreneur i finds it optimal to start the project if, and only if,

$$\beta^2\mathbb{E}_i[\Pi_i] \geq 1, \tag{9}$$

where $\mathbb{E}_i[\cdot]$ is a shortcut for $\mathbb{E}[\cdot|x_i, y]$ and where $\Pi_i \equiv (1-s_i)\Theta f(k_i)$. Using (8), we can express the realized profit Π_i as a function of Θ and $\hat{\Theta}$, as follows:

$$\Pi_i = \Pi(\Theta, \hat{\Theta}) \equiv (1-\alpha)\Theta f(K(\hat{\Theta})) = (1-\alpha)\Theta(\alpha\beta\hat{\Theta})^{\frac{\alpha}{1-\alpha}}. \tag{10}$$

We now use the restrictions that $\hat{\theta}$ and $\theta|\mathcal{I}$ are lognormal with deterministic variance. As we show in the Appendix (proof of Proposition 1), these restrictions imply that entrepreneur i finds it optimal to start a project if and only if

$$\mathbb{E}_i\left[(1-\alpha)\theta + \alpha\hat{\theta}\right] \geq C, \tag{11}$$

where $\hat{\theta} \equiv \mathbb{E}[\theta|\mathcal{I}]$ and where C is a scalar determined in equilibrium.[14] Notice that to derive this condition we are also using the restriction that each entrepreneur is informationally small, that is, that the representative investor's information $\mathcal{I}$ is independent of entrepreneur i's individual startup decision.

Condition (11) captures the first direction of the feedback mechanism at the core of our paper: when entrepreneurs expect a higher market valuation, they have a higher incentive to start a project. In particular, condition (11) shows that it is optimal for an entrepreneur to start a project if and only if he is sufficiently optimistic about a weighted average of productivity and of the market's valuation of productivity. The intuition behind this condition is simple. The entrepreneur's decision depends on his expectation of θ because it directly affects final output. The entrepreneur's decision depends on his expectation of the market's valuation $\hat{\theta}$ because that variable determines the cost of raising capital in the IPO market: the higher $\hat{\theta}$ is, the higher the IPO stock price, and hence the higher the amount of capital that the entrepreneur raises and invests at $t = 2$.

The relative importance of θ and $\hat{\theta}$ is determined by α, which reflects the strength of decreasing returns on period-2 investment. A higher α implies that profits are more sensitive to the entrepreneur's capacity to raise capital in the IPO stage and hence that they are more sensitive to variations in share prices.

Given that θ, $\hat{\theta}$ and the available signals are jointly normal, condition (11) is equivalent to

$$(1-b)x_i + by \geq c, \tag{12}$$

for some constants b and c to be determined. This condition gives us the entrepreneur's strategy in the startup stage, that is, it characterizes the startup decision as a function of the entrepreneur's information. Using (12) we can derive the aggregate level of startup activity:

$$N = \Pr\left((1-b)x_i + by \geq c \,|\, \theta, y\right) = \Phi\left(\sqrt{\pi_x}\frac{(1-b)\theta + by - c}{1-b}\right), \tag{13}$$

where Φ is the standard normal cumulative distribution function. The second equality follows from the definitions of the signal x_i and from the symmetry of the normal distribution. Since strategies, and

[14] It may be useful to clarify that $\hat{\theta} \equiv \mathbb{E}[\theta|\mathcal{I}]$ is not the same as $\log \hat{\Theta}$, where $\hat{\Theta} \equiv \mathbb{E}[\Theta|\mathcal{I}]$. The two differ by a constant that is derived in the Appendix.

hence the scalars b and c, are commonly known in equilibrium, the above implies that the observation of N conveys to the representative investor the same information as the signal

$$z \equiv (1-b)\theta + by = \theta + b\varepsilon. \tag{14}$$

The level of startup activity N is a noisy indicator of θ because the investor cannot tell apart whether, for example, a high value of N was driven by a high realization of the true fundamental θ or by a high realization of the correlated error in the entrepreneurs' beliefs.

The precision of the signal z can be derived using the last expression in (14) and is equal to

$$\pi_z = \pi_y / b^2.$$

This precision is endogenous and depends on the entrepreneurs' strategy: the signal-to-noise ratio decreases with the coefficient b, which measures the relative response of the entrepreneurs' start-up decision to the two signals x_i and y. This is a key property, whose positive and normative implications we study in the sequel.

Note that, under this simple information structure, N reveals θ perfectly to the entrepreneurs. This is a consequence of the assumption that *all* the correlated noise in the entrepreneurs' information originates in the commonly observed signal y. However, this result no longer holds with a more general information structure, for example, by adding idiosyncratic noise to the entrepreneurs' observations of y.

Step 3: Market valuation

We now derive the investors' valuations when entering the IPO market. Investors have two sources of information, the exogenous signal ω and the distribution of startup decisions $(n_j)_{j\in[0,1]}$. The restriction that individual startup decisions do not have a direct effect on the investor's expectation of Θ implies that $(n_j)_{j\in[0,1]}$ contains the same information as N. We then have

$$\hat{\Theta} = \mathbb{E}[\Theta|\omega, N].$$

This condition captures the second direction of our feedback mechanism: a high level of startup activity increases the market's valuation of investment at the IPO stage. Using the fact that the information in N is equivalent to the information in z, we can verify that the equilibrium we are constructing satisfies our distributional restrictions on θ and $\hat{\theta}$. First, we verify that $\theta|\mathcal{I}$ is normally distributed with mean

$$\hat{\theta} = \mathbb{E}[\theta|z,\omega] = \frac{\pi_\omega}{\pi}\omega + \frac{\pi_z}{\pi}z, \tag{15}$$

and (state invariant) variance $1/\pi$, where $\pi = \pi_\theta + \pi_\omega + \pi_z$ is the posterior precision, as assumed. Second, using (15), we verify that $\hat{\theta}$ is normally distributed with a variance that is state-invariant, as assumed .

Step 4: Fixed point

Now that we have an explicit expression for the investors' valuation $\hat{\theta}$ we can go back to the entrepreneurs' optimality condition (11) and use it to derive explicitly the coefficients defining their start-up decisions, as assumed in (12).

Given (14) and (15), each entrepreneur's forecast of $\hat{\theta}$ is given by

$$\mathbb{E}_i[\hat{\theta}] = \frac{\pi_\omega + \pi_z(1-b)}{\pi}\mathbb{E}_i[\theta] + \frac{\pi_z}{\pi}by.$$

Substituting $\mathbb{E}_i[\hat{\theta}]$ into the optimality condition (11), we have that each entrepreneur finds it optimal to start a project if, and only if,

$$\left(1 - \alpha + \alpha\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)\mathbb{E}_i[\theta] + \alpha\frac{\pi_z}{\pi}by \geq C. \tag{16}$$

Bayesian updating implies that

$$\mathbb{E}_i[\theta] = \delta_x x_i + \delta_y y \tag{17}$$

where

$$\delta_x \equiv \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y} \quad \text{and} \quad \delta_y \equiv \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y}.$$

Substituting δ_x and δ_y into (16) we then obtain that each entrepreneur finds it optimal to start a project if, and only if,

$$(1 - b')x_i + b'y \geq c' \tag{18}$$

where the coefficients b' and c' depend only on b and exogenous parameters (see the Appendix for details). So the problem of finding an equilibrium boils down to finding a fixed point to a known function Γ that maps $b \in \mathbb{R}$ to some $b' \in \mathbb{R}$. This function, which is defined explicitly in the Appendix, has a simple interpretation: when the investors believe the distribution of start-up decisions contains the same information as the signal $z = \theta + b\varepsilon$, for some $b \in \mathbb{R}$, the entrepreneurs respond by following a start-up decision rule that amounts to sending to the investors a signal $z' = \theta + b'\varepsilon$ with $b' = \Gamma(b)$. In equilibrium the signal received by the investors must coincide with the signal sent by the entrepreneurs. We then have the following result:

Proposition 1. *There are two functions $\Gamma : \mathbb{R} \to \mathbb{R}$ and $\Lambda : \mathbb{R} \to \mathbb{R}$ such that if b^* is a fixed point of Γ and $c^* = \Lambda(b^*)$, then there is an equilibrium in which each entrepreneur i starts a project if, and only if,*

$$(1 - b^*)x_i + b^*y \geq c^*. \tag{19}$$

Closed-form expressions for Γ and Λ are derived in the Appendix. Studying these functions permits us to reach the following existence and uniqueness results:

Proposition 2. *(i) There exists an equilibrium in which all entrepreneurs follow the start-up decision rule in (19) with $b^* \in (0, 1)$. In this equilibrium, startup investment N, the market valuation of future* *productivity $\hat{\Theta}$, and the share price $p = P(\hat{\Theta})$ are all increasing in the fundamental θ and in the noise ε. (ii) For any parametrization of the information structure, there exists a cutoff value $\bar{\alpha} > 0$ for the capital share in production, such that the equilibrium is unique for all $\alpha \leq \bar{\alpha}$. (iii) If $\alpha > \bar{\alpha}$, multiple equilibria are possible.*

The intuition for part (i) is as follows. When $b^* \in (0, 1)$, each entrepreneur's startup activity responds positively to both signals x_i and y. This immediately implies that aggregate startup activity responds positively to both the fundamental θ and the noise ε. Whenever this is the case, the investors' expectation of θ, and by implication the equilibrium asset price, also responds positively to both θ and ε. This is because the investors correctly perceive high aggregate activity as "good news" about profitability, but cannot distinguish between increases in activity driven by θ from those driven by ε.

Part (ii) guarantees that the equilibrium is unique if α is small enough, in which case the sensitivity of startup activity to the expected market valuation is not too high. When, instead, α is high, multiple equilibria can emerge. The possibility of multiple equilibria does not interfere with the main message of our paper—it only reinforces it by opening the door to sunspot volatility. At the same time, the positive and normative implications of the informational spillovers at the core of our paper are most cleanly isolated by ruling out multiple equilibria. With this in mind, in the sequel we restrict attention to the region of the parameter space that guarantees equilibrium uniqueness.

5 Positive Analysis

The role of information spillovers

To gain more insight, it is worth contrasting the equilibrium outcomes of our economy to variants in which there is no informational spillover from the real sector to the financial sector. To this purpose, suppose that investors do not learn from the observation of N. This could be either because investors do not observe N, or because their exogenous signal ω is already perfectly informative of θ, which obtains when $\pi_\omega \to \infty$. We consider both cases to emphasize that the positive results in Proposition 3 below do not depend on the amount of information in the investors' hands. What matters for the result is whether or not investors learn about aggregate profitability by looking at the entrepreneurs' activity.

When investors do not learn from N, their valuation $\hat{\theta}$ is just a linear function of their exogenous signal $\omega = \theta + \eta$. Since entrepreneurs do not possess any information about η at the startup stage, their expectation of $\hat{\theta}$ is simply a linear transformation of their expectation of the fundamental θ. It follows that, in the absence of informational spillovers, the entrepreneurs' start-up Condition (11) reduces to $\mathbb{E}_i[\theta] \geq C$, for some constant C. Using (17), we then have that each entrepreneur starts a project at $t = 1$ if, and only if,

$$(1 - \delta)x_i + \delta y \geq c$$

where $\delta = \frac{\delta_y}{\delta_x + \delta_y}$ and where c is a scalar that depends on all exogenous parameters.

The following result compares the entrepreneurs' start-up decisions in the model with investment spillovers to their counterparts in the absence of informational spillovers.

Lemma 1. *In the equilibrium in part (i) of Proposition 2, $b^* > \delta$.*

The result shows that the two-way feedback between real activity and financial prices raises the relative sensitivity of the entrepreneurs' startup activity to the correlated component of their private information, here captured by the signal y.

We provide more intuition for this result in the next subsection. First, we look at the aggregate implications of this result for investment and asset price volatility. Recall that, when the entrepreneurs follows a strategy of the form in (12), aggregate startup activity is given by

$$N = \Phi\left(\sqrt{\pi_x}\left(\frac{\theta + b\varepsilon - c}{1-b}\right)\right).$$

It follows that

$$\frac{\partial N/\partial \varepsilon}{\partial N/\partial \theta} = b.$$

That is, a higher value of b implies a higher sensitivity of aggregate startup activity to the non-fundamental noise ε relative to the fundamental θ. By the same token, if an econometrician were to observe data generated by our model and run a regression of aggregate startup activity on the fundamental θ, the R-squared of this regression—which measures the contribution of the fundamental θ to the volatility of aggregate startup activity—will be smaller the higher b is.[15] In what follows, we thus refer to b interchangeably as the sensitivity of the entrepreneurs' startup activity to y relative to x, and as a measure of the contribution of noise to aggregate volatility.

Based on the preceding observations, we reach the following conclusion:

Proposition 3. *The informational spillovers from the real to the financial sector amplify the contribution of noise to aggregate volatility.*

Mispricing and speculation

Let us rewrite Condition (11), which characterizes the entrepreneurs' startup decisions, as follows:

$$\mathbb{E}_i[\theta] + \alpha \mathbb{E}_i[\hat{\theta} - \theta] \geq C. \tag{20}$$

Recall that $\hat{\theta}$ captures the market valuation of capital during the IPO stage, while θ identifies the true underlying profitability of capital. Therefore, the gap $\hat{\theta} - \theta$ reflects the log difference between the market price of capital and its realized value. Accordingly, we interpret $\hat{\theta} - \theta$ as a measure of the "mispricing", or "pricing error", that arises in the financial market due to the noise in the available information. Condition (20) then says that the decision of each entrepreneur to start a project depends, not only

[15] To be precise, for a linear regression to be valid, one needs to regress the transformation $\Phi^{-1}(N)$ on θ.

on his forecast of the underlying fundamental profitability of the project, but also on his forecast of the pricing error. This is because a higher pricing error in the IPO stage translates into a lower cost of raising capital and, therefore, to a higher profit for the entrepreneur. In this sense, the term $\hat{\theta} - \theta$ in (20) also captures a form of speculation that is reminiscent of the dot-com bubble: a higher startup activity obtains when entrepreneurs expect financial markets to overvalue their businesses.

The result that $b^* > \delta$ can then be interpreted as follows. Recall that the representative investor's forecast of θ is $\hat{\theta} = \frac{\pi_\omega}{\pi}\omega + \frac{\pi_z}{\pi}z$, where $z = \theta + b^*\varepsilon$ is the endogenous signal revealed by aggregate startup activity. To simplify, consider the limit case of an uninformative prior, which corresponds to $\pi_\theta \to 0$. In this case, $\frac{\pi_\omega}{\pi} + \frac{\pi_z}{\pi} = 1$, which implies that the pricing error can be rewritten as

$$\hat{\theta} - \theta = \frac{\pi_\omega}{\pi}\eta + \frac{\pi_z}{\pi}b^*\varepsilon,$$

where η is the noise in the investor's exogenous signal ω, whereas $b^*\varepsilon$ is the noise in the endogenous signal z. A high value of the common noise ε in the entrepreneurs' signals then leads to a high level of startup activity, which is perceived by investors as a favorable signal of underlying profitability θ thus leading to overpricing.

Now consider the entrepreneurs' decision at the startup stage. The entrepreneurs receive no information about η, so their expectation of η is zero. On the other hand, they possess information about ε through their correlated signal y. Their expectation of ε is given by

$$\mathbb{E}_i[\varepsilon] = y - \mathbb{E}_i[\theta] = (1 - \delta_y)y - \delta_x x.$$

The y signal contains information both about θ and about ε. A high value of y thus contributes positively to the entrepreneurs' expectation of both the primitive profitability θ of their startup activity and of the pricing error at the IPO stage. By contrast, a high value of the x_i contributes positively to the entrepreneurs' expectation of θ but negatively to their expectation of the pricing error. This explains why, in equilibrium, $b^* > \delta$, that is, the relative sensitivity of the entrepreneurs' startup activity to sources of information with correlated noise is higher than what warranted by the relative informativeness of such sources vis-a-vis the fundamental.

The argument above uses the specific signal structure assumed here. The result, however, is more general. When an entrepreneur is *idiosyncratically* optimistic about the profitability of his project (which is the case here when x_i is high relative to y), he expects the financial market to receive only a modest signal from the aggregate startup activity and hence the cost of raising capital to be relatively high, which dampens his incentive to start the project. By contrast, when the entrepreneurs are *collectively* optimistic about the profitability of their projects (which is the case here when y is high), they expect the financial market to receive a strong positive signal from aggregate startup activity and thereby the IPO prices to be high, which boosts their incentives to start a project. It is this spillover from the collective optimism of the entrepreneurs to the exuberance of the financial markets that crowds out private information and amplifies non-fundamental volatility.

Notice that the mere fact that the equilibrium startup activity responds to the expectation of financial prices is not necessarily a symptom of inefficiency. To the extent that the cost of financing (and, related, the social cost of expanding the existing projects) depend on the market valuation $\hat{\theta}$, there are efficiency gains from having the initial startup decision respond to expectations of market valuation. We postpone addressing the efficiency properties of the equilibrium to Section 6.

A beauty contest interpretation

We now offer an alternative interpretation of our result, which builds a bridge to the literature on "beauty contests" (see, among others, Morris and Shin, 2002, and Allen, Morris and Shin, 2006).

Recall that the market valuation $\hat{\theta}$ is a linear function of the endogenous signal z that the real sector sends to the financial market, and that this signal is itself a monotone transformation of the aggregate startup activity N. Using these facts, we can express the expected payoff of starting a project as a monotone function of θ and N, which yields the following result:

Proposition 4. *There exists scalars $r, c^{\#} > 0$ such that, in equilibrium, each entrepreneur starts a project if, and only if,*

$$\mathbb{E}_i[(1-r)\theta + r\Phi^{-1}(N)] \geq c^{\#}. \tag{21}$$

The equilibrium startup activity in our model can therefore be represented as the PBE of a binary-action coordination game among entrepreneurs in which the best response is given by (21). This best response is akin to the best response function in the beauty contest games studied by, inter alia, Morris and Shin (2002), and Angeletos and Pavan (2007). It follows that the amplified effect of the y signal on non-fundamental volatility in our model is akin to the amplified effect of public information in those games: in both cases, the amplification effect can be traced to strategic complementarity.

What distinguishes our framework from this other work is that the strategic complementarity is here endogenous, originating in the information spillovers from the real to the financial sector. Although each entrepreneur alone is too small to have any impact on either aggregate startup activity or on the investors' beliefs, the entrepreneurs as a group can influence the investors' beliefs and hence the price in the financial market. This leads to a complementarity in the entrepreneurs' actions: the higher the aggregate startup activity, the higher the market valuation of the new projects, and hence the lower the cost of raising capital at the IPO stage. This explains why strategic complementarity in the entrepreneurs' decisions (i.e., $r > 0$) emerges whenever a high aggregate level of startup activity is interpreted by the financial maker as "good news". By contrast, strategic complementarity vanishes (i.e., $r = 0$) when the financial market does not extract any information from the real sector's activity, as in the benchmark without spillovers considered above.

The impact of α

So far we have shown that informational spillovers lead to the amplification of non-fundamental shocks. We now show that the strength of these effect depends on the parameter α. The parameter α plays a double role in our model: it captures the growth potential of the real sector, as a larger α means a lower degree of decreasing returns; it also captures the real sector's dependence on funding from the financial market, as a larger α implies that the entrepreneurs' period-2 investments k are more sensitive to financial market valuations. In equilibrium, a larger value of α increases the entrepreneurs' incentives to rely on the correlated signal y to predict the mispricing in the IPO stage. Formally, a higher value of α shifts the Γ function in Proposition 1 upwards. As long as Γ admits a unique fixed point, this implies a higher sensitivity b^* of the entrepreneurs' startup activity to sources of information with correlated noise. We thus have the following result:

Proposition 5. *As long as the equilibrium remains unique (which is always the case for $\alpha < \bar{\alpha}$), a higher value of α implies a higher contribution of correlated noise to aggregate volatility.*

When the value of α is large, the two-way feedback between the real and the financial sector may become so strong that the equilibrium mapping Γ may admit multiple fixed points. This explains our earlier observation that a high α opens the door to multiple equilibria and sunspot volatility.[16]

Our model therefore delivers the following predictions. Sectors with high growth potential and high finance dependence are the ones that are most prone to "irrational exuberance", "manias" and "panics" if we interpret these phenomena as forms of non-fundamental volatility. This is especially true in the early stages of a new sector's life, when there is still significant uncertainty about its eventual profitability. We think these predictions square well with historical experiences such as the dot-com bubble, but we leave it for future work to formally test such predictions.

6 Welfare Analysis

The analysis in the previous section focuses on the positive properties of the equilibrium and shows that information spillovers increase the contribution of correlated noise to the volatility of startup investment. We now turn to the normative implications of this result. The questions that we address are the following: Are the positive properties derived so far a symptom of inefficiency? And, if so, how should policy try to influence startup activity?

[16] This multiplicity originates solely from the information spillover between the real and the financial sector of the economy. It is thus distinct from the one that emerges in coordination models of crises such as Diamond and Dybvig (1983) and Obstfeld (1996). Rather, it is closer to the one in Gennotte and Leland (1990) and Barlevy and Veronesi (2003). These papers also document multiplicity results that originate in information spillovers. However, these papers abstract from real economic activity and focus on spillovers that emerge *within* the financial market, between informed and uninformed traders. In our setting, instead, the multiplicity rests on the two-way feedback between the real sector and the financial sector and can manifest itself as sunspot volatility in both real investment and asset prices.

To address these questions, we proceed in two steps. We start by studying the optimization problem of a social planner who can dictate to the entrepreneurs the strategy that they should follow at the startup stage. This planning problem bypasses the details of the available policy instruments and identifies directly the normative content of the aforementioned positive properties. Once the solution to the planner's problem is at hands, we discuss some policies that can be welfare improving and policies that can implement the planner's optimum.

Consider a social planner who can choose a linear startup rule of the form

$$n_i = 1 \quad \text{if and only if} \quad (1-b)x_i + by \geq c, \tag{22}$$

for some scalars $b, c \in \mathbb{R}$ and can choose investment k_i in period 2 as a function of the signal ω and of aggregate startup activity N. By setting up the planner's problem this way, we constrain the planner to use the same information as the market economy. Requiring linearity of the startup rule is a restriction which we make for tractability. Suppose for the moment that the planner's objective is to maximize a utilitarian welfare function. We will discuss distributional issues below. Because of the linearity of individual payoffs, social welfare is given by the present value of aggregate output, net of investment costs. Therefore, social welfare is

$$\int_0^1 \left\{ n_i \left(\beta^2 \Theta f(k_i) - \beta k_i \right) + (1-n_i) \right\} di = 1 + N \left(\beta^2 \Theta f(k) - \beta k - 1 \right),$$

where we use the fact that, because of concavity, it is optimal to dictate the same level of period-2 investment to all entrepreneurs who are active in period 2. Given the startup rule (22), aggregate startup activity is given by

$$N = \Phi \left(\tfrac{\sqrt{\pi_x}}{1-b} (z-c) \right),$$

where z is the endogenous signal $z = \theta + b\varepsilon$. Since the planner can condition the choice of k on (ω, N), and observing N is equivalent to observing z, the planner's problem can be written as

$$\max_{(b,c)\in\mathbb{R}^2, \mathcal{K}\in\mathbf{C}} \mathbb{E}\left[N(z) \left(\beta^2 \Theta f(\mathcal{K}(\omega,z)) - \beta \mathcal{K}(\omega,z) - 1 \right) \right]$$
$$s.t. \quad z = \theta + b\varepsilon, \quad N(z) = \Phi\left(\tfrac{\sqrt{\pi_x}}{1-b}(z-c) \right),$$

where that **C** denotes the set of continuous functions $\mathcal{K} : \mathbb{R}^2 \to \mathbb{R}$.

Consider first the choice of $\mathcal{K}$. For almost every (ω, z), $\mathcal{K}(\omega, z)$ must maximize the expected output net of investment costs:

$$\mathcal{K}(\omega, z) = \arg\max_k \left\{ \beta \hat{\Theta} f(k) - k \right\},$$

where $\hat{\Theta} = \mathbb{E}[\Theta | \omega, z]$. Recall that the same maximization problem determines investment in the market equilibrium, therefore,

$$\mathcal{K}(\omega, z) = K(\hat{\Theta}) = (\alpha\beta\hat{\Theta})^{\frac{1}{1-\alpha}}. \tag{23}$$

This proves that the equilibrium level of investment is efficient conditional on the information available to the investors at the IPO stage. Notice, however, that the signal z may be different from the analogous signal in the market equilibrium, if the planner chooses a different startup strategy.

Thus consider next the optimal startup strategy. Given (23), the social value of a project, net of investment costs at $t = 2$, is

$$Q(\Theta, \hat{\Theta}) \equiv \beta\Theta f(K(\hat{\Theta})) - K(\hat{\Theta}) = \beta(\Theta - \alpha\hat{\Theta})(\alpha\beta\hat{\Theta})^{\frac{\alpha}{1-\alpha}}.$$

The planner's objective at the startup stage can thus be written as

$$\mathbb{E}\left[n(x,y)\left(\beta Q(\Theta,\hat{\Theta}) - 1\right)\right], \tag{24}$$

where $n(x, y)$ denotes the startup rule (22). The following proposition characterizes the optimal startup rule for the planner.

Proposition 6. *Let $(b^\diamond, c^\diamond)$ denote the coefficients of the startup rule that maximizes social welfare (24). When the equilibrium is unique (which is always the case for $\alpha < \bar{\alpha}$), $b^\diamond < b^*$, meaning that the equilibrium sensitivity of startup decisions to the signal with correlated noise is too high from a welfare perspective. Consequently, the contribution of correlated noise to aggregate volatility is inefficiently high.*

There are two reasons why the planner's startup rule differs from the market equilibrium one. First, comparing (9) and (24), one can see that the private benefit of startup investment is $\beta^2\mathbb{E}[\Pi(\Theta, \hat{\Theta})|x, y]$, while the social benefit is $\beta\mathbb{E}[Q(\Theta, \hat{\Theta}|x, y]$. Second, the planner internalizes the fact that different startup rules imply different informativeness of the endogenous signal upon which investment decisions are made at $t = 2$. The proof of Proposition 6 in the Appendix shows that both forces go in the direction of choosing a smaller b. We now discuss the two parts of this argument separately.

First, consider the discrepancy between the private and the social benefit of starting a project. To isolate this effect, consider a planner who can choose the startup rule, but only that of entrepreneur i, in which case the information revealed by N continues to be the same as in the market equilibrium. Note that

$$\begin{aligned} Q(\Theta, \hat{\Theta}) &\equiv \beta\Theta f(K(\hat{\Theta})) - K(\hat{\Theta}) = \\ &= \beta\Pi(\Theta, \hat{\Theta}) + \alpha\beta(\Theta - \hat{\Theta}) f(K(\hat{\Theta})). \end{aligned} \tag{25}$$

The last term in (25) is the difference between the true value of the shares sold to the investors at $t = 2$ (i.e., the associated cash flow) and the investors' valuation of these shares.[17] Therefore, as long as the entrepreneur's expectation $\mathbb{E}[\hat{\Theta}|x, y]$ of the investors' expectation of TFP differs from the entrepreneur's own expectation $\mathbb{E}[\Theta|x, y]$ of TFP, the social and private benefits differ:

$$\beta\mathbb{E}[Q(\Theta, \hat{\Theta})|x, y] - \beta^2\mathbb{E}[\Pi(\Theta, \hat{\Theta})|x, y] \neq 0.$$

[17] Recall that, in equilibrium, the shares sold by entrepreneur i are $s_i = \alpha$ and their price is $p_i = \hat{\Theta} f(k)$, so that $s_i \cdot \Theta f(k) = \alpha\Theta f(k)$, and $p_i \cdot s_i = \alpha\hat{\Theta} f(k)$.

This difference is related to the mispricing discussed in the previous section. In particular, when the entrepreneur expects $\hat{\Theta}$ to be higher than Θ, he expects his shares to be overvalued at the IPO stage, i.e., he expects to make a gain at the expenses of the outside investors. Because this speculative gain is a zero-sum transfer from the investors to the entrepreneur, it does not enter the planner's objective. As argued in the previous section, a high value of the signal y relative to x tends to increase the entrepreneur's expectation of $\hat{\Theta}$ relative to Θ, and thus increases the entrepreneur's incentive to start the project for speculative reasons. It follows that the planner, who is not driven by speculative motives, would like the entrepreneur to put less weight on y relative to x, which means a lower b.

Next, consider the informational effects of choosing a smaller value of b, when the planner can choose the startup rule for *all* entrepreneurs and thus affects the precision of the endogenous signal z at the IPO stage. The precision of z is decreasing in b and more precision is welfare improving, because it increases the efficiency of the second-period investment decisions. Formally, the ex-ante expectation of $Q(\Theta, \hat{\Theta})$ is increasing in π_z. This informational effect adds to the social benefit of reducing b to mitigate the entrepreneurs' speculative entry decisions and thus reinforces the conclusion that $b^\diamond < b^*$.

To recap, there are two reasons why the equilibrium sensitivity of startup decisions to the signal with correlated noise is inefficiently high. The first one is that the entrepreneurs can predict the difference between fundamental profitability and its market valuation, and this predictability generates a speculative wedge between their private incentives and social welfare. The second one is that there is an informational externality associated with the endogeneity of the information upon which second-period investment decisions are made. Conceptually, these sources of inefficiency are distinct. In our setting, however, they are tied together, as they both emerge from the information spillover from the real to the financial sector.

Finally, note that there also exists an inefficiency in the *level* of startup decisions. That is, even if $b^\diamond = b^*$, in general $c^\diamond \neq c^*$. This is akin to the holdup problem. The private surplus from starting a project is $\beta^2(1-\alpha)\Theta f(K)$, while the social surplus is $\beta^2 \Theta f(K) - \beta K$. Under complete information, the two coincide, but with informational frictions, their expected values may differ.

7 Discussion

In this section we first illustrate what kind of policy interventions can reduce the inefficiencies of the equilibrium identified in the previous section. We next discuss the likely robustness of our results to more realistic settings and a few additional implications that may obtain from such extensions.

Policy

What kind of policies could help improve the equilibrium use of information? Non-contingent taxes on initial entry, on subsequent investment, or on profits can control the levels of startup activity N and subsequent investment K but do not influence the relative sensitivity of such variables to different sources of information. They therefore do not help correct the key inefficiencies identified in the previous section. By contrast, state-contingent taxes can serve this goal by manipulating the entrepreneurs' entry decisions and the resulting aggregation of information.

To start with, consider a proportional tax on firms' profits, collected from the entrepreneurs at stage 3. Suppose that, at this stage, the planner can observe both the realized profits and the realized IPO prices. Suppose further that the planner is free to choose how the tax depends on such variables and can commit, as of stage 0, on any tax schedule of its choice. Because, in equilibrium, profits are a function of both Θ and $\hat{\Theta}$, whereas asset prices are a function of $\hat{\Theta}$ only, the planner can infer the pair $(\Theta, \hat{\Theta})$ from the observation of asset prices and profits and can effectively make the tax an arbitrary function of the pair $(\Theta, \hat{\Theta})$. By the same token, the net-of-taxes return of each entrepreneur i can then be expressed as a function of $(\Theta, \hat{\Theta})$ as follows

$$(1-\tau)\Pi_i = (1 - T(\Theta, \hat{\Theta}))\Pi(\Theta, \hat{\Theta})$$

where $T(\cdot)$ is the tax schedule and $\Pi(\cdot)$ is the gross profit function, as defined in Section 4. Now let $T(\cdot)$ be such that

$$\beta^2(1 - T(\Theta, \hat{\Theta}))\Pi(\Theta, \hat{\Theta}) = \beta Q(\Theta, \hat{\Theta})$$

where the function $Q(\cdot)$ captures the social gross return to entry, as defined in Section 6. It is then easy to see that, under such policies, the entrepreneurs' stage-1 entry decisions are efficient. Intuitively, this is achieved by letting the tax be an increasing function of p and a decreasing function of Π: such a tax schedule makes the entrepreneurs' startup decisions respond less to their expectations of the asset prices and more to their expectations of the underlying fundamentals. At the same time, the stage-2 investments continue to be determined according to (7), which means that they are also efficient. All in all, the proposed tax schedule therefore implements the constrained efficient allocation characterized in the previous section as a decentralized equilibrium.

Next, consider a tax on financial trades: when a trader buys s shares, her total cost is $(1+\tau)ps$, where τ is the tax rate. Suppose that this tax is collected from the traders in stage 2 and let it be an increasing function of the realized asset prices. One can think of this as a proxy for macro-prudential intervention in asset markets. Because the realized prices reveal $\hat{\Theta}$, the proposed policy is equivalent to letting the tax rate τ be a function T of $\hat{\Theta}$. Under such a policy, the equilibrium prices satisfy

$$p = \frac{\beta \hat{\Theta} f(k)}{1 + T(\hat{\Theta})}.$$

To preserve tractability, let T be such that $1 + T(\hat{\Theta}) = \hat{\Theta}^{\psi}$ for some $\psi \geq 0$. The benchmark case of zero taxes is nested as $\psi = 0$; a higher ψ corresponds to a tax that increases more steeply with a firm's market valuation. The stage-1 equilibrium investments are then given by

$$K(\hat{\Theta}) = \arg\max_k \left\{ \beta \hat{\Theta}^{1-\psi} f(k) - k \right\},$$

and each entrepreneur's realized return to entry is then given by

$$\Pi(\Theta, \hat{\Theta}) = (1-\alpha)\Theta f(K(\hat{\Theta})) = (1-\alpha)(\alpha\beta)^{\frac{\alpha}{1-\alpha}}\Theta\hat{\Theta}^{\frac{(1-\psi)\alpha}{1-\alpha}}.$$

It is then immediate that a positive ψ makes the returns to entry less sensitive to $\hat{\Theta}$, which helps improve the efficiency of the entrepreneurs' entry decisions, but also distorts the stage-2 investment decisions.[18] Unlike the tax on entrepreneurial profits studied above, the present policy therefore cannot implement the constrained efficient outcomes. In fact, with such a policy, the government faces the following trade off: raising ψ from zero to a positive level entails a cost in terms of distorting the stage-2 investment decisions conditional on $\hat{\Theta}$, and a benefit in terms of improving the efficiency of the stage-1 entry decision and of the information generated by aggregate startup activity. That said, because the stage-1 investment decisions are (constrained) efficient when $\psi = 0$, while the stage-1 entry decisions are not, a standard envelope argument guarantees that, in a neighborhood of $\psi = 0$, the benefit outweighs the cost. That is, the no-tax equilibrium can be improved upon by a "macro-prudential" intervention of the type described here.

The particular policies devised above may be difficult to implement in practice. They nevertheless illustrate the role that state-contingent policies can play in influencing the decentralized use of information and thereby in improving the efficiency of the equilibrium. Note in particular that the policies discussed above induce, not only more efficient startup decisions for given asset prices, but also a stronger correlation between asset prices (and hence the investment-expansion decisions) and the underlying fundamentals. In other words, they improve both allocative and informational efficiency.

Finally, another policy that may increase the efficiency of the equilibrium is a cap on the shares that the entrepreneurs can sell to the stock market. For such a policy to improve welfare, however, one needs to adjust the framework by making the realistic assumption that not all entrepreneurs get to do an IPO. To see this, assume that each entrepreneur gets the opportunity to access the stock market and undertake the kind of expansion described in the baseline model only with probability $\lambda \in (0,1)$; with the complementary probability $(1-\lambda)$, the entrepreneur, instead, is excluded from the IPO market in which case the return to his project is given by $q\Theta$, for some $q > 0$. This means that, in expectation, each entrepreneur's expected return to entry is equal to

$$\Pi_i = (1-\lambda)q\Theta + \lambda\Theta f(K),$$

where, as in the baseline model, K is funded with the IPO proceedings. Consider then a policy that constrains each entrepreneur to sell no more than $\bar{s} \in (0,1)$ shares, so that $\bar{s}$ is the cap on the IPO shares. It is straightforward to check that this constraint is binding if and only if $\bar{s} < \alpha$. When this is the case, the entrepreneur sells exactly $s = \bar{s}$ shares and his ex-ante expected return to entry is then given by

$$\tilde{\Pi}(\Theta, \hat{\Theta}; \bar{s}, \lambda) = (1-\lambda)q\Theta + \lambda(1-\bar{s})(\bar{s}\beta)^{\frac{\alpha}{1-\alpha}}\Theta\hat{\Theta}^{\frac{\alpha}{1-\alpha}}.$$

[18] Recall that efficiency requires that $K(\hat{\Theta}) = \arg\max_k \left\{\beta\hat{\Theta}f(k) - k\right\}$.

When both $\bar{s} = \alpha$ and $\lambda = 1$, $\tilde{\Pi}(\Theta, \hat{\Theta}; \bar{s}, \lambda)$ reduces to the return in the baseline model, that is, $\tilde{\Pi}(\Theta, \hat{\Theta}; \alpha, 1) = \Pi(\Theta, \hat{\Theta})$. When, instead, $\bar{s} < \alpha$ but $\lambda = 1$, $\tilde{\Pi}(\Theta, \hat{\Theta}; \alpha, 1)$ is different from $\Pi(\Theta, \hat{\Theta})$ but proportional to $\Pi(\Theta, \hat{\Theta})$, implying that the value of $\bar{s}$ does not affect the relative sensitivity of the entrepreneurs' payoff to Θ and $\hat{\Theta}$. By the same token, the policy can affect the level but not the relative use of different pieces of information when $\lambda = 1$. When, instead, $\lambda < 1$, the lower $\bar{s}$ is, the smaller the sensitivity of $\tilde{\Pi}$ to $\hat{\Theta}$ relative to its sensitivity to Θ. It follows that, as long as $\lambda < 1$, the proposed policy can tilt the use of information in the direction of increasing the sensitivity to fundamentals. Intuitively, requiring the entrepreneurs to keep more "skin in the game" reduces the extent to which their decisions are driven by speculative motives and can therefore help improve efficiency.

Robustness and possible extensions

We now speculate on the robustness of our results to richer, more realistic, environments and on a few additional implications that may obtain from such enrichments.

1. By assuming that all agents are rational and by abstracting from the possibility of sunspot fluctuations, we have imposed that all the non-fundamental variation in investment and asset prices is the product of correlated noise in dispersed private information. This has made clear that our amplification mechanism and its normative implications do not depend on a departure from rational expectations. Nevertheless, our insights are relevant more broadly. In particular, the kind of non-fundamental waves in beliefs, investment, and asset prices we have documented here can readily be recast as the product of "irrational exuberance" by replacing the assumed form of correlated noise in the enterprenurs' information either with a correlated bias in their beliefs about the profitability of new investment opportunities and/or asset prices, or with a correlated taste for starting up a firm. In all these cases, the key friction is that the asset market may confuse such non-fundamental variation in real activity with news about profitability.

   Such a behavioral reinterpretation of our results may be more appealing from an empirical perspective. However, it makes the welfare analysis more delicate. Our approach allows us to make a case for policy intervention *even if* the planner is neither paternalistic nor smarter than the market.

2. In our model, Θ plays a double role: it is both the aggregate productivity and the individual productivity of any particular entrepreneur. Nonetheless, what is relevant for our analysis is only the former role. In particular, our theory is concerned about the information spillovers between real and financial activity at the aggregate level, not any signaling/screening that may take place at the firm level. By the same token, our equilibrium refinement that $\hat{\Theta}$ does not depend on the individual choices of any single entrepreneur is meant to rule out the possibility that a single entrepreneur moves the market belief about the *aggregate* fundamentals, not the possibility that he can affect the market belief of his own productivity.

We have imposed the aforementioned refinement because we find it a priori appealing, yet this may not be strictly needed: the key is that the market belief of the aggregate fundamental increases with aggregate startup activity, not the details of the other sources of information that may move the market belief.

3. In our model, the only signal that Wall Street receives from Silicon Valley is the aggregate startup activity. In reality, additional such signals may include aggregate performance measures such as sales and orders. Insofar as these measures depend solely on the underlying fundamental, they are proxied by the exogenous signal ω in our model. If, instead, these measures depend also on the effort, or the early investment choices, of the entrepreneurs, they are likely to play a similar role as the endogenous signal N in our model.

4. In our model, the financial traders are risk neutral and posses the same information. To bring the asset market of our model closer to the Grossman-Stiglitz tradition, we would have to let the financial traders be risk averse and possess dispersed private information. Such an extension would let the equilibrium asset price aggregate the private information of the traders. Still, as long as the information of the traders contains correlated noise (or their demand are subject to correlated liquidity, preferences, or other shocks), the equilibrium asset price will remains sensitive to the signals sent by Silicon Valley.

   The essence of our mechanism is therefore likely to remain unaffected by such an extension. At the same time, the following additional interesting implications seem to emerge. By reducing the informativeness of the signals that Silicon Valley sends to Wall Street, our mechanism also increases the risk faced by financial traders. When the latter are risk averse, such heightened risk depresses the equilibrium price, thus also reducing the level of the second-stage investment for any value of the fundamental. Furthermore, to the extent that the heightened risk reduces the willingness of financial traders to act on their private information, our mechanism may also reduce the informativeness of asset prices.

   The mechanism sketched above is also present in our model, albeit in a more stylized manner. By reducing the informativeness of N, our mechanism reduces the correlation between p and Θ. The extension discussed above would add the effect of reducing the aggregation of the information that is dispersed among the traders. In the context of our model, this effect can be proxied by a reduction in the precision of the signal ω, because this signal itself is a proxy for the information contained in, and revealed by, asset prices.

5. In our model, the entrepreneurs observe two particular signals when making their entry decisions: one with purely idiosyncratic noise, x_i, and another one with perfectly correlated noise, y. An earlier version of our paper explored how the key insights extend to richer specifications of the information structure, whereby the entrepreneurs observe a multitude of Gaussian signals with

different degrees of correlation.[19] Such an extension allows the entrepreneurs to face non-trivial uncertainty about the fundamentals after observing their entry decision, thus also allowing them to learn from the equilibrium asset price.[20] It nevertheless preserves the property that aggregate entry influences market beliefs, thus also preserving the positive and normative predictions of the starker specification studied here.

6. Our analysis takes for granted the presence of correlated noise in the entrepreneurs private information. As already noted, such noise could be a proxy for irrational impulses. But the following is also relevant. Consider an extension of our model that allows the entrepreneurs' to choose what information to collect, or pay attention to. In such an extension, the two-way feedback we have identified is likely to increase the entrepreneurs' incentives not only to react to sources of information with correlated noise, but also to collect such information in the first place.[21]

7. Consider an extension that splits the entrepreneurs into two types: those that are sufficiently good to eventually access the IPO market and those that stay private forever. This is like the extension mentioned at the end of the previous subsection, although it allows for the possibility that each entrepreneur knows a priori her type. Suppose next that the entrepreneurs that stay private also make an investment, or effort choice, in stage 2. In such an extension, the entrepreneurs who never go public are not engaging in speculation and do not care per se about the IPO market. They may nevertheless condition their stage-2 choices on the realized IPO price insofar as the latter reveals valuable information about the underlying aggregate fundamental. This in turn introduces an additional information externality, one running from the stage-1 entry decisions of all entrepreneurs to the stage-2 expansion decisions of the entrepreneurs that remain private. Clearly, this only reinforces our message.

8. Consider extensions that allow the equilibrium supply of shares to be sensitive to the entrepreneurs' private information. Recall that this is not the case in the equilibrium of our model: although entrepreneurs are free to choose the fraction of shares s they sell, in equilibrium they all pick the same s (namely $s = \alpha$) regardless of their information. This property can be relaxed by giving entrepreneurs either the possibility of financing the stage-2 investments with internal funds

[19]The model used in that earlier version abstracted from the role of the asset market in financing new investment at the IPO stage. It nevertheless contained the feature that, relative to the planner, the entrepreneurs were excessively concerned about the market valuation of their project at the entry stage.

[20]By contrast, this possibility is ruled out in our model because of three simplifying assumptions: that there are only two signals; that the average of one signal, x_i, contains no aggregate noise; and that the other signal, y, is common knowledge among the entrepreneurs. These assumptions imply that N perfectly reveals Θ to the entrepreneurs, even though it does not reveal it to the traders. Under a more general information structure, instead, the entrepreneurs need not learn Θ from N, even if they observe the latter without noise. In this case, as long as the traders possess information that is not a priori available to the entrepreneurs, such as the signal ω in our model, the asset market reveals information to the entrepreneurs.

[21]A related point is made in Froot, Scharfstein, and Stein (1992), which shows how sophisticated financial traders may have an incentive to collect correlated private information when the revelation of such information in prices is gradual.

or the option to divert part of the proceeds of the IPO to private consumption. Intuitively, an entrepreneur who believes that the market is too bullish may find it optimal to dump as many shares in the market as possible, whereas an entrepreneur who believes that the market is too bearish may opt to finance her expansion with internal funds rather than expose herself to an underpriced IPO.

In such situations, the supply of shares can be increasing in the price and decreasing in the entrepreneurs' beliefs about the fundamental. Relative to our model, this opens the door to the possibility that the asset market aggregates the information that is among the entrepreneurs over and above what is already revealed through aggregate entry. Nevertheless, as long as this kind of aggregation is imperfect (due to sufficient noise), aggregate entry may still influence the equilibrium beliefs in stage 2 and may therefore still shape the equilibrium price, leaving the essence of our insights intact.

To illustrate these ideas, consider the following modifications of our model. First, add idiosyncratic noise in the observation of the signal y, that is, replace it with a collection of correlated private signals of the form $y_i = \theta + \epsilon + \varepsilon_i$, where ϵ is common noise and ε_i is idiosyncratic noise. This implies that the observation of aggregate entry, N, does not reveal θ to the entrepreneurs, nor does it induce a common belief among them.

Next, replace the stage-2 constraint $k = sp$ with the constraint $sp = k + c_2$ and allow c_2 to be either positive or negative. A positive c_2 means that the entrepreneur consumes some of the proceeds of the IPO (or that she sells part of her own shares right after the IPO), while a negative c_2 can be interpreted as internal financing. The entrepreneur's problem in stage 2 can then be expressed as

$$\max_{s_i \in [0,1], k_i \geq 0, c_i} \mathbb{E}_{i,2} \left[\beta(1 - s_i)\Theta f(k_i) + c_2 \right]$$

$$\text{s.t.} \quad s_i p_i = k_i + c_{i,2}, \quad p_i = \beta \hat{\Theta} f(k_i),$$

where $\hat{\Theta}$ indexes the market belief (which, of course, differs from that in our baseline model due to the features added here) and $\mathbb{E}_{i,2}$ stands for the entrepreneur's stage-2 expectation operator (which is conditional, not only on the stage-1 signals, x_i and y_i, and aggregate entry, N, but also the price, p). The above can be restated as

$$\max_{s_i \in [0,1], k_i \geq 0} \left\{ \beta \left[(1 - s_i)\mathbb{E}_{i,2}[\Theta] + s_i \hat{\Theta} \right] f(k_i) - k_i \right\},$$

from which it is obvious that the solution takes the following bang-bang form:

$$\mathbb{E}_{i,2}[\Theta] > \hat{\Theta} \quad \Rightarrow \quad s_i = 0, \; k_i = K(\mathbb{E}_{i,2}[\Theta])$$

$$\mathbb{E}_{i,2}[\Theta] \leq \hat{\Theta} \quad \Rightarrow \quad s_i = 1, \; k_i = K(\hat{\Theta}),$$

where $K(z) \equiv \arg\max_k \{\beta z f(k) - k\}$. That is, the entrepreneur stays private if he believes that his firm is underpriced and sells out completely if he believes that his firm is overpriced. Provided that the aggregation of information remains imperfect, $\mathbb{E}_{i,2}[\Theta]$ follows a non-degenerate distribution in the cross-section of entrepreneurs, implying that the aggregate supply of shares is a smooth function of θ, ϵ, and p.

Finally, let each financial trader face a privately known random cost for taking a non-zero position in the IPO market, and let this cost have both an idiosyncratic and an aggregate component, so that the traders' aggregate demand for shares can be expressed as a smooth function of p, N, and u, where u denotes the aforementioned aggregate component and where N enters the demand of the traders because, and only because, it influences their beliefs conditional on p.

Under these assumptions, the equilibrium price is a function of (θ, ϵ, u), while the equilibrium aggregate entry is a function of (θ, ϵ). It follows that, except for degenerate cases, the observation of the pair (p, N) will not reveal the underlying shocks to either side of the market. Furthermore, conditional on p, we expect the aggregate demand of the traders to be an increasing function of N, because the observation of a higher N will still serve as a signal of stronger fundamentals in the eyes of the traders. We thus also expect the equilibrium price to continue to increase with aggregate entry and, as a result, the mechanism we have identified to continue to apply.

Although we have not worked out the extensions discussed above, we hope that this discussion clarifies which properties of our baseline model are likely to be robust and which are not. In particular, consider the following properties: (i) the entrepreneurs learn perfectly the fundamental in stage 2, (ii) all the entrepreneurs go public; (iii) the entrepreneurs' equilibrium supply of shares is invariant to their beliefs; and (iv) the asset market does not aggregate dispersed private information. These properties are both unrealistic and fragile. Nevertheless, relaxing them does not necessarily negate the three properties that actually drive our main results: (i) that the entrepreneurs care about the stock market in part because the latter affects the value of their capital, (ii) that the stock market in turn looks at aggregate entrepreneurial activity and other sectoral and macroeconomic data for clues about the underlying fundamentals; and (iii) that the stock market may be unable to tell apart the booms in real activity that are due to stronger fundamentals from those that are due to the collective exuberance of the entrepreneurs. Insofar as these three basic properties are preserved, we expect the essence of our insights to be robust to more realistic specifications of the environment.

Appendix: Proofs

Proof of Proposition 1. In this proof we fill in the details of the equilibrium characterization in Section 4 and provide explicit expressions for the functions Γ and Λ.

First, let us derive the optimality condition (11), which we rewrite here for convenience:

$$\mathbb{E}_i[(1-\alpha)\theta + \alpha\hat{\theta}] \geq C, \tag{26}$$

and provide an explicit expression for the constant C. In the text we argued that an entrepreneur will start up the project if and only if

$$\beta^2(1-\alpha)(\alpha\beta)^{\frac{\alpha}{1-\alpha}}\mathbb{E}_i[\Theta \cdot \hat{\Theta}^{\frac{\alpha}{1-\alpha}}] \geq 1. \tag{27}$$

As argued in the text, the distribution of θ conditional on the investors' information $\mathcal{I}$ is normally distributed with mean

$$\hat{\theta} = \frac{\pi_\omega}{\pi}\omega + \frac{\pi_z}{\pi}z \tag{28}$$

(which corresponds to (15) in the text) and variance π^{-1}, so we can compute

$$\hat{\Theta} = \mathbb{E}[e^\theta|\mathcal{I}] = \exp\left\{\hat{\theta} + \frac{1}{2}\pi^{-1}\right\}. \tag{29}$$

We can then compute the expression $\mathbb{E}_i[\Theta \cdot \hat{\Theta}^{\frac{\alpha}{1-\alpha}}]$ in (27) as follows

$$\mathbb{E}_i\left[\Theta \cdot \hat{\Theta}^{\frac{\alpha}{1-\alpha}}\right] = \mathbb{E}_i\left[\exp\left\{\frac{(1-\alpha)\,\theta + \alpha\hat{\theta}}{1-\alpha} + \frac{\alpha}{1-\alpha}\frac{1}{2}\pi^{-1}\right\}\right] =$$
$$= \exp\left\{\mathbb{E}_i\left[\frac{(1-\alpha)\,\theta + \alpha\hat{\theta}}{1-\alpha}\right] + \frac{\alpha}{1-\alpha}\frac{1}{2}\pi^{-1} + \frac{1}{2}Var_i\left[\frac{(1-\alpha)\,\theta + \alpha\hat{\theta}}{1-\alpha}\right]\right\}.$$

Substituting into (27), we obtain condition (26), where

$$C = -(1-\alpha)\ln\left(\beta^2(1-\alpha)(\alpha\beta)^{\frac{\alpha}{1-\alpha}}\right) - \frac{\alpha}{2}\pi^{-1} - \frac{1}{2(1-\alpha)}Var_i\left[(1-\alpha)\theta + \alpha\hat{\theta}\right]. \tag{30}$$

Joint normality of $(1-\alpha)\theta + \alpha\hat{\theta}$ and of the signals (x_i, y) imply that the conditional variance in the last term is a constant independent of the realization of (x_i, y).

Next, let us solve the Bayesian updating problem of entrepreneurs at the startup stage. Substituting $z = (1-b)\,\theta + by$ (which corresponds to (14) in the text) in (28) we have

$$\hat{\theta} = \frac{\pi_\omega}{\pi}(\theta + \eta) + \frac{\pi_z}{\pi}\left[(1-b)\,\theta + by\right],$$

so

$$(1-\alpha)\theta + \alpha\hat{\theta} = \left(1 - \alpha + \alpha\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)\theta + \alpha\frac{\pi_z}{\pi}by + \alpha\frac{\pi_\omega}{\pi}\eta.$$

Conditional on (x_i, y), the posterior distribution of θ has mean

$$\mathbb{E}_i[\theta] = \delta_x x_i + \delta_y y,$$

with

$$\delta_x = \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y} \quad \delta_y = \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y},$$

and variance $(\pi_\theta + \pi_x + \pi_y)^{-1}$. Moreover (x_i, y) contains perfect information on y and no information on η. Therefore, the posterior mean and variance of $(1-\alpha)\theta + \alpha\hat{\theta}$ are

$$\mathbb{E}_i\left[(1-\alpha)\theta + \alpha\hat{\theta}\right] = \left(1 - \alpha + \alpha\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)(\delta_x x_i + \delta_y y) + \alpha\frac{\pi_z}{\pi}by, \tag{31}$$

$$Var_i\left[(1-\alpha)\theta+\alpha\hat{\theta}\right]=\left(1-\alpha+\alpha\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)^2\frac{1}{\pi_\theta+\pi_x+\pi_y}+\left(\alpha\frac{\pi_\omega}{\pi}\right)^2\frac{1}{\pi_\omega}. \tag{32}$$

We can now go back to the optimality condition (26), substitute (31) and obtain

$$\left(1-\alpha+\alpha\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)(\delta_x x_i+\delta_y y)+\alpha\frac{\pi_z}{\pi}by\geq C.$$

Matching the coefficients of this condition to the coefficients of the entrepreneur's strategy (12), we obtain

$$b=\frac{\left(1-\alpha+\alpha\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)\delta_y+\alpha\frac{\pi_z}{\pi}b}{\left(1-\alpha+\alpha\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)(\delta_x+\delta_y)+\alpha\frac{\pi_z}{\pi}b}, \tag{33}$$

and

$$c=\frac{C}{\left(1-\alpha+\alpha\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)(\delta_x+\delta_y)+\alpha\frac{\pi_z}{\pi}b}. \tag{34}$$

Substituting $\pi=\pi_\theta+\pi_\omega+\pi_z$ and $\pi_z=\pi_y/b^2$ we can rewrite the right-hand side of (33) as

$$\frac{\left[(1-\alpha)\left(\pi_\theta+\pi_\omega+\frac{\pi_y}{b^2}\right)+\alpha\left(\pi_\omega+\frac{\pi_y}{b^2}(1-b)\right)\right]\delta_y+\alpha\frac{\pi_y}{b^2}b}{\left[(1-\alpha)\left(\pi_\theta+\pi_\omega+\frac{\pi_y}{b^2}\right)+\alpha\left(\pi_\omega+\frac{\pi_y}{b^2}(1-b)\right)\right](\delta_x+\delta_y)+\alpha\frac{\pi_y}{b^2}b},$$

which can be rearranged to yield

$$\Gamma(b)\equiv\frac{\left[((1-\alpha)\pi_\theta+\pi_\omega)b^2+\pi_y-\alpha\pi_y b\right]\delta_y+\alpha\pi_y b}{\left[((1-\alpha)\pi_\theta+\pi_\omega)b^2+\pi_y-\alpha\pi_y b)\right](\delta_x+\delta_y)+\alpha\pi_y b}. \tag{35}$$

This shows that the equilibrium value of b can be found finding a fixed point of the function Γ, which only involves known parameters of the model.

The mapping $\Lambda(b)$ is given by the right-hand side of (34), after substituting C from (30), substituting $Var_i\left[(1-\alpha)\theta+\alpha\hat{\theta}\right]$ from (32), and finally substituting $\pi=\pi_\theta+\pi_\omega+\pi_z$ and $\pi_z=\pi_y/b^2$, so as to obtain an expression that only involves known parameters and b. This shows that the equilibrium value of c can be computed by simple algebra after having found a b that solves $b=\Gamma(b)$. Q.E.D.

Proof of Proposition 2. *Part (i): Existence*. First, notice that $\Gamma(1)<1$ so there is no equilibrium with $b=1$. Now define $B\equiv b/(1-b)$ for any $b\neq 1$. Using the definition of Γ we then have that $b=B/(1+B)$, corresponds to an equilibrium if and only if

$$\begin{aligned}B &\equiv \frac{b}{1-b}=\frac{\Gamma(b)}{1-\Gamma(b)}=\frac{\alpha\pi_y b+\left[(1-\alpha)b^2\pi_\theta+b^2\pi_\omega+\pi_y-\alpha\pi_y b\right]\delta_y}{\left[(1-\alpha)b^2\pi_\theta+b^2\pi_\omega+\pi_y-\alpha\pi_y b\right]\delta_x}\\ &= \frac{\pi_y+\frac{B^2}{(1+B)^2}(\pi_\theta+\pi_\omega)+\alpha\frac{B}{1+B}\left(\pi_x+\frac{1}{1+B}\pi_\theta\right)}{\pi_y+\frac{B^2}{(1+B)^2}(\pi_\theta+\pi_\omega)-\alpha\frac{B}{1+B}\left(\pi_y+\frac{B}{1+B}\pi_\theta\right)}\frac{\delta_y}{\delta_x}.\end{aligned}$$

Letting

$$F(B)\equiv\frac{\delta_y}{\delta_x}\left\{1+\frac{\alpha(1+B)B}{[(1-\alpha)(1-\delta_x)+\Omega]B^2+(2-\alpha)\delta_y B+\delta_y}\right\} \tag{36}$$

with $\Omega \equiv \frac{\pi_\omega}{\pi_x+\pi_y+\pi_\theta} > 0$, and noting that B is a one-to-one transformation of b, we then have that the solutions to the equations $B = F(B)$ identify the solutions to the equation $b = \Gamma(b)$ and vice versa.

It is easy to see that F is well defined and continuous over $\mathbb{R}_+$, with $F(\delta_y/\delta_x) > \delta_y/\delta_x$ and $\lim_{B\to+\infty} F(B)$ finite. It follows that F has at least one fixed point $B > \delta_y/\delta_x$. Given this value of B, existence of a equilibrium then follows by letting $b = B/(1+B)$. That $B > \delta_y/\delta_x$ in turn implies that

$$b > \frac{\delta_y}{\delta_x+\delta_y} = \frac{\pi_y}{\pi_x+\pi_y} \equiv \delta.$$

That $B > 0$ in turn implies that $b < 1$. We conclude that a solution $b \in (\delta, 1)$ to $\Gamma(b) = b$ always exists, as claimed in the proposition. The implications for N, P, and $\hat{\Theta}$ follow directly from the arguments in the main text.

Part (ii): Uniqueness. To prove uniqueness, note that all equilibria must correspond to a fixed point of the mapping F defined in (36). Next, note there exists $\alpha' > 0$ such that, for any $\alpha \in [0, \alpha']$ the denominator in the fraction in the right-hand side of (36) is strictly positive, for any $B \in \mathbb{R}$. This implies that, when $\alpha \in [0, \alpha']$, the function F is defined and continuously differentiable over the entire real line, with

$$F'(B) = \alpha \frac{\delta_y}{\delta_x} \frac{[\delta_y - (1-\alpha)(1-\delta_x-\delta_y) - \Omega] B^2 + 2\delta_y B + \delta_y}{\{[(1-\alpha)(1-\delta_x)+\Omega]B^2 + (2-\alpha)\delta_y B + \delta_y\}^2}$$

Moreover,

$$\lim_{B\to-\infty} F(B) = \lim_{B\to+\infty} F(B) = F_\infty \equiv \frac{\delta_y}{\delta_x}\left[1 + \frac{\alpha}{(1-\alpha)(1-\delta_x)+\Omega}\right] > \frac{\delta_y}{\delta_x}.$$

Thus, from now on, restrict attention to $\alpha < \alpha'$. We now need to consider two cases. First, suppose $\delta_y = (1-\alpha)(1-\delta_x)+\Omega$. The function F then has a global minimum at $B = -1/2$. In this case, F is bounded from below and above, respectively, by $\underline{F} \equiv F(-1/2)$ and $\overline{F} \equiv F_\infty$. Second, suppose $\delta_y \neq (1-\alpha)(1-\delta_x)+\Omega$. Then $F'(B)$ has two zeros, respectively at $B = B_1$ and at $B = B_2$, where

$$B_1 \equiv \frac{-\delta_y - \sqrt{[(1-\alpha)(1-\delta_x-\delta_y)+\Omega]\delta_y}}{\delta_y-(1-\alpha)(1-\delta_x-\delta_y)-\Omega} \text{ and } B_2 \equiv \frac{-\delta_y + \sqrt{[(1-\alpha)(1-\delta_x-\delta_y)+\Omega]\delta_y}}{\delta_y-(1-\alpha)(1-\delta_x-\delta_y)-\Omega}.$$

When $\delta_y \neq (1-\alpha)\delta_0 + \Omega$, the function F then has a global minimum at $\underline{F} \equiv F(B_2)$ and a global maximum at $\overline{F} \equiv F(B_1)$. It is easy to check that in all the cases considered both $\underline{F}$ and $\overline{F}$ converge to δ_y/δ_x as $\alpha \to 0$. But then F converges uniformly to δ_y/δ_x as $\alpha \to 0$. It follows that for any $\varepsilon > 0$, there exists a $\hat{\alpha} \leq \alpha'$ so that, whenever $\alpha < \hat{\alpha}$, F has no fixed point outside the interval $[\delta_y/\delta_x - \varepsilon, \delta_y/\delta_x + \varepsilon]$.

Now, with a slight abuse of notation, replace $F(B)$ with $F(B;\alpha)$, to highlight the dependence of F on α. Notice that $\partial F(B;\alpha)/\partial B$ is continuous in B at $(B;\alpha) = (\delta_y/\delta_x, 0)$ and $\partial F(\delta_y/\delta_x; 0)/\partial B = 0$. It follows that there exist $\tilde{\varepsilon} > 0$ and $\tilde{\alpha} \in (0, \hat{\alpha}]$ such that $\partial F(B;\alpha)/\partial B < 1$ for all $B \in [\delta_y/\delta_x - \tilde{\varepsilon}, \delta_y/\delta_x + \tilde{\varepsilon}]$ and $\alpha \in [0, \tilde{\alpha}]$. Combining these results with the continuity of $F(\cdot;\alpha)$, we have that there exist $\bar{\varepsilon} > 0$ and $\bar{\alpha} > 0$ such that, for all $\bar{\alpha} \in [0, \bar{\alpha}]$, the following are true: for any $B \notin [\delta_y/\delta_x - \bar{\varepsilon}, \delta_y/\delta_x + \bar{\varepsilon}]$, $F(B;\alpha) \neq B$; for $B \in [\delta_y/\delta_x - \bar{\varepsilon}, \delta_y/\delta_x + \bar{\varepsilon}]$, F is continuous and differentiable in B, with $\partial F(B;\alpha)/\partial B < 1$. It follows that, if $\alpha \leq \bar{\alpha}$, F has at most one fixed point, which establishes the result.

Part (iii): Multiplicity. Consider the function $F(B;\alpha,\delta_x,\delta_y,\Omega)$ introduced in the proof of part (ii). For convenience we are highlighting here the dependence on all parameters, with $\Omega \equiv \frac{\pi_\omega}{\pi_x+\pi_y+\pi_\theta}$. Take the parameters $(\alpha,\delta_x,\delta_y,\Omega) = (.75,.2,.1,.1)$. With these parameters the function F is defined and continuous over the entire real line and $B_2 < B_1$, where B_1 and B_2 are as defined in the proof of part (ii). Moreover, at the point B_2, we have that $F(B_2;\alpha,\delta_x,\delta_y,\Omega) < B_2 < 0$. These properties, together with the properties that $F(0;\alpha,\delta_x,\delta_y,\Omega) > 0$ and $\lim_{b\to-\infty} F(B;\alpha,\delta_x,\delta_y,\Omega) > 0 > -\infty$, ensure that, in addition to a fixed point in $(\delta_y/\delta_x,+\infty)$, F admits at least one fixed point in $(-\infty,B_2)$ and one in $(B_2,0)$. Furthermore, each of these three fixed point is "strict" in the sense that $F(B)-B$ changes sign around them. Because F is continuous in $(B;\alpha,\delta_x,\delta_y,\Omega)$ in an open neighborhood of $(\alpha,\delta_x,\delta_y,\Omega) = (.75,.2,.1,.1)$, there necessarily exists an open set $D\subset(0,1)^3\times\mathbb{R}$ such that F admits at least three fixed points whenever $(\alpha,\delta_x,\delta_y,\Omega)\in D$. The result then follows by noting that for any $(\alpha,\delta_x,\delta_y,\Omega)\in D$, there corresponds a unique set of parameters $(\alpha,\pi_\theta,\pi_x,\pi_y,\pi_\omega)\in\mathbb{R}^5$. Q.E.D.

Proof of Lemma 1. The result is proved in the proof of part (i) of Proposition 2. Q.E.D.

Proof of Proposition 4. Using

$$N = \Phi\left(\tfrac{\sqrt{\pi_x}}{1-b^*}\left[(1-b^*)\theta + b^* y - c^*\right]\right)$$

we have that the endogenous signal z can be written as

$$z = \frac{1-b^*}{\sqrt{\pi_x}}\Phi^{-1}(N) + c^*$$

The investors' forecast of the fundamentals can then be expressed as follows

$$\hat{\theta} = \tfrac{\pi_\omega}{\pi_\theta+\pi_\omega+\pi_z}\omega + \tfrac{\pi_z}{\pi_\theta+\pi_\omega+\pi_z}\left(\frac{1-b^*}{\sqrt{\pi_x}}\Phi^{-1}(N) + c^*\right) \tag{37}$$

with $\pi_z = \pi_y/(b^*)^2$. Replacing (37) into (11), we have that the entrepreneurs' optimality condition can be written as

$$n_i = 1 \iff \mathbb{E}_i\left[(1-r)\theta + r\Phi^{-1}(N)\right] \geq c^{\#}$$

where

$$r \equiv \frac{\alpha\pi_y(1-b^*)}{\alpha\pi_y(1-b^*) + \left[(1-\alpha)\left(\pi_\theta(b^*)^2+\pi_y\right) + (b^*)^2\pi_\omega\right]\sqrt{\pi_x}} \tag{38}$$

and

$$c^{\#} \equiv \frac{\left[(\pi_\theta+\pi_\omega)(b^*)^2+\pi_y\right]C - \alpha\pi_y c^*}{\frac{\alpha\pi_y(1-b^*)}{\sqrt{\pi_x}} + (1-\alpha)\left(\pi_\theta(b^*)^2+\pi_y\right) + \pi_\omega(b^*)^2}$$

with C as defined in (30). From (38) we then have that $r > 0$ whenever $b < 1$, i.e., whenever N is increasing in θ, which is always the case when the equilibrium is unique, for in this case $b\in(\delta,1)$ as shown in the proof of Propositions 1 and 2. Q.E.D.

Proof of Proposition 5. Consider the function $F(B;\alpha)$ introduced in the proof of Propositions 1 and 2. For any $\alpha \in [0,\bar{\alpha})$, the function $F(\cdot;\alpha)$ is continuously differentiable over $\mathbb{R}$. Take any pair $\alpha', \alpha'' \in [0,\bar{\alpha})$ with $\alpha'' > \alpha'$, and let B' and B'' be the unique solutions to $F(B;\alpha) = B$, respectively for $\alpha = \alpha'$ and $\alpha = \alpha''$ (existence and uniqueness of such solutions follows directly from Proposition 2). Furthermore, as shown in the proof of Propositions 1 and 2, $F(B;\alpha') - B > 0$ for all $B \in [0, B')$. Simple algebra then shows that $\partial F(B;\alpha)/\partial\alpha \geq 0$ for any $B \geq 0$, with strict inequality if $B > 0$. It follows that $B'' > B'$. The result in the proposition then follows from the fact that $b \equiv B/(1+B)$ along with the fact that the contribution of noise to volatility is increasing in b. Q.E.D.

Proof of Proposition 6. Aggregate welfare is given by

$$W\left(b,\tilde{b},c\right) \equiv \mathbb{E}\left[n(x,y)\left(\beta Q\left(\Theta,\hat{\Theta}\right) - 1\right)\right],$$

where

$$Q\left(\Theta,\hat{\Theta}\right) = \beta(\Theta - \alpha\hat{\Theta})(\alpha\beta\hat{\Theta})^{\frac{\alpha}{1-\alpha}},$$

$$\hat{\Theta} = e^{\hat{\theta}+\frac{1}{2}\pi^{-1}}, \quad \hat{\theta} = \frac{\pi_\omega}{\pi}(\theta+\eta) + \frac{\pi_z}{\pi}z, \quad z = \theta + b\varepsilon, \tag{39}$$

$$\pi = \pi_\theta + \pi_\omega + \pi_z, \quad \pi_z = \pi_y/b^2, \tag{40}$$

and with the startup rule given by

$$n(x,y) = 1 \text{ iff } (1-\tilde{b})x + \tilde{b}y \geq c.$$

For convenience, we are introducing two different coefficients b and $\tilde{b}$: b captures the informativeness of the signal z, $\tilde{b}$ is the coefficient in the startup rule. Of course, for consistency, the planner must choose $b = \tilde{b}$. Therefore, the planner's optimization problem is

$$\max_{b,c} W(b,b,c).$$

We want to show that it is optimal to choose a $b < b^*$, where b^* is the coefficient in the market equilibrium (recall that the latter is unique when α is small enough). The argument is by contradiction. Suppose it is optimal to choose a pair (b,c) with $b \geq b^*$. Then we want to show that there is a (b',c'), with $b' < b$, that delivers higher welfare. We do it in two steps. First, we show that there is a pair (b',c'), with $b' < b$, such that $W(b,b',c') > W(b,b,c)$. That is, fixing the precision of the endogenous public signal, it is optimal to choose a startup rule less sensitive to the signal with correlated noise. Second, we show that a more informative endogenous signal increases welfare, that is, $W(b',b',c') > W(b,b',c')$. Combining the two inequalities we have

$$W(b',b',c') > W(b,b,c),$$

a contradiction.

Step 1. Define expected social surplus conditional on x, y as

$$S(x,y) \equiv \mathbb{E}[\beta Q(\Theta,\hat{\Theta}) - 1|x,y] = \beta^2(\alpha\beta)^{\frac{\alpha}{1-\alpha}}\mathbb{E}[\Theta\hat{\Theta}^{\frac{\alpha}{1-\alpha}}|x,y] - \beta(\alpha\beta)^{\frac{1}{1-\alpha}}\mathbb{E}[\hat{\Theta}^{\frac{1}{1-\alpha}}|x,y] - 1.$$

In the proof of Proposition 1 we showed that

$$\mathbb{E}[\Theta\cdot\hat{\Theta}^{\frac{\alpha}{1-\alpha}}|x,y] = \exp\left\{\begin{array}{c} \frac{1}{1-\alpha}\mathbb{E}[(1-\alpha)\theta + \alpha\hat{\theta}|x,y] \\ +\frac{\alpha}{2(1-\alpha)}\pi^{-1} + \frac{1}{2(1-\alpha)^2}Var\left((1-\alpha)\theta + \alpha\hat{\theta}|x,y\right)\end{array}\right\},$$

where

$$\frac{1}{1-\alpha}\mathbb{E}[(1-\alpha)\theta + \alpha\hat{\theta}|x,y] = q_x x + q_y y,$$

with

$$\begin{aligned} q_x &\equiv \frac{1}{1-\alpha}\left(1-\alpha+\alpha\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)\delta_x, \\ q_y &\equiv \frac{1}{1-\alpha}\left(1-\alpha+\alpha\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)\delta_y + \frac{1}{1-\alpha}\alpha\frac{\pi_z}{\pi}b \end{aligned}$$

and where $Var\left((1-\alpha)\theta + \alpha\hat{\theta}|x,y\right)$ is invariant in (x,y), as shown in (32).

Following similar steps, we can derive

$$\mathbb{E}[\hat{\Theta}^{\frac{1}{1-\alpha}}|x,y] = \exp\left\{\frac{1}{1-\alpha}\mathbb{E}[\hat{\theta}|x,y] + \frac{\pi^{-1}}{2(1-\alpha)} + \frac{1}{2(1-\alpha)^2}Var(\hat{\theta}|x,y)\right\},$$

where

$$\frac{1}{1-\alpha}\mathbb{E}[\hat{\theta}|x,y] = \hat{q}_x x + \hat{q}_y y,$$

with

$$\begin{aligned} \hat{q}_x &\equiv \frac{1}{1-\alpha}\left(\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)\delta_x, \\ \hat{q}_y &\equiv \frac{1}{1-\alpha}\left(\frac{\pi_\omega + \pi_z(1-b)}{\pi}\right)\delta_y + \frac{1}{1-\alpha}\frac{\pi_z}{\pi}b \end{aligned}$$

and where $Var(\hat{\theta}|x,y)$ is invariant in (x,y).

It follows that we can write expected surplus as

$$S\left(x,y\right) = Q_1 e^{q_x x + q_y y} - Q_2 e^{\hat{q}_x x + \hat{q}_y y} - 1,$$

where Q_1 and Q_2 are positive constant terms.

It will be useful to derive some properties of the coefficients $q_x, q_y, \hat{q}_x, \hat{q}_y$. The coefficients are all positive and satisfy the inequalities $\hat{q}_x < q_x$ and $\hat{q}_y/\hat{q}_x > q_y/q_x$. The first inequality follows immediately from $(\pi_\omega + \pi_z(1-b))/\pi < 1$. The second inequality follows from the fact that the expression

$$\frac{\left(1-\tilde{\alpha}+\tilde{\alpha}\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)\delta_y + \tilde{\alpha}\frac{\pi_z}{\pi}b}{\left(1-\tilde{\alpha}+\tilde{\alpha}\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)\delta_x} = \frac{\delta_y}{\delta_x} + \frac{\frac{\pi_z}{\pi}b}{\left(1/\tilde{\alpha}-1+\frac{\pi_\omega+\pi_z(1-b)}{\pi}\right)\delta_x}$$

is an increasing function of $\tilde{\alpha}$, is equal to q_y/q_x when $\tilde{\alpha} = \alpha$, and is equal to $\hat{q}_y/\hat{q}_x$ when $\tilde{\alpha} = 1$. Finally, we can prove that

$$\frac{q_y}{q_x} = F\left(\frac{b}{1-b}\right) < \frac{b}{1-b},$$

where the function F is defined in the proof of Proposition 2. The equality can be proved comparing the expressions for q_x and q_y above with the definition of F in that proof. The inequality follows from the fact that the equilibrium is unique, so the F function can only cross the 45 degree line once, from above, at $\frac{b^*}{1-b^*}$, and from the fact that, by hypothesis, we have $b > b^*$.

We can now prove two lemmas.

Lemma 2. *For each y there is a unique x such that $S(x,y) = 0$, $S(x',y) > 0$ if $x' > x$ and $S(x',y) < 0$ if $x' < x$.*

Proof of Lemma 2. Existence: Fix any $y \in (-\infty, \infty)$. Notice that $\lim_{x \to -\infty} S(x,y) = -1$. Rewrite

$$S(x,y) = e^{q_x x + q_y y}\left[Q_1 - Q_2 e^{(\hat{q}_x - q_x)x + (\hat{q}_y - q_y)y}\right] - 1,$$

and notice that $\hat{q}_x - q_x < 0$, so the term in square brackets converges to Q_1 as $x \to \infty$, so $\lim_{x \to \infty} S(x,y) = \infty$. By continuity, there exists an x such that $S(x,y) = 0$. Uniqueness: Differentiating we have

$$S_x(x,y) = q_x Q_1 e^{q_x x + q_y y} - \hat{q}_x Q_2 e^{\hat{q}_x x + \hat{q}_y y},$$

so at a (x,y) such that $S(x,y) = 0$ we have

$$S_x(x,y) = q_x - \hat{q}_x w > 0,$$

where

$$w \equiv \frac{Q_2 e^{\hat{q}_x x + \hat{q}_y y}}{1 + Q_2 e^{\hat{q}_x x + \hat{q}_y y}} \in (0,1). \tag{41}$$

An index argument implies a unique solution and a continuity argument implies the inequalities.

Lemma 3. *The slope of the locus $\{(x,y) : S(x,y) = 0\}$ satisfies*

$$\frac{dx}{dy} > -\frac{q_y}{q_x},$$

everywhere.

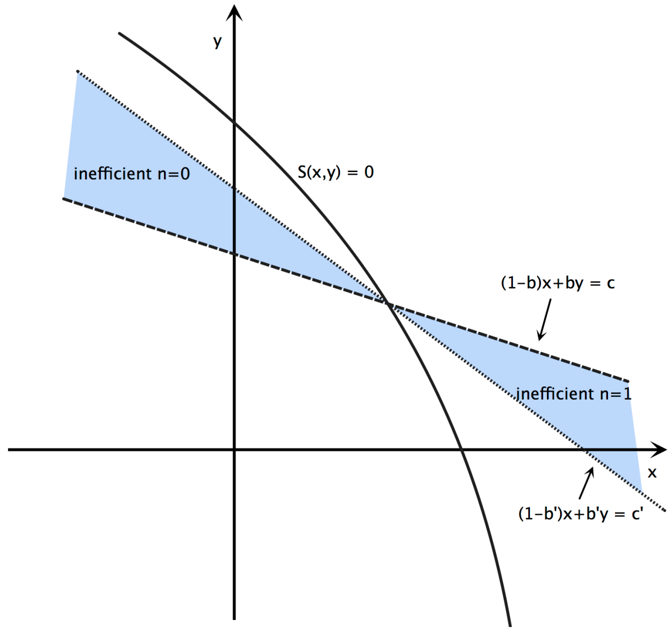
Proof of Lemma 3. Differentiating, we have

$$S_y(x,y) = q_y Q_1 e^{q_x x + q_y y} - \hat{q}_y Q_2 e^{\hat{q}_x x + \hat{q}_y y}.$$

Therefore, the slope is given by

$$\frac{dx}{dy} = -\frac{S_y}{S_x} = -\frac{q_y - \hat{q}_y w}{q_x - \hat{q}_x w} > -\frac{q_y}{q_x},$$

Figure 1: Improving the startup rule



where w is given in (41). The last inequality follows from $w > 0$, $q_x - \hat{q}_x w > 0$ and $\hat{q}_y/\hat{q}_x > q_y/q_x$.

To complete this step we use a graphical argument. In Figure 1 the locus $S(x, y) = 0$ is represented by the solid line. For values to the right of the solid line we have $S(x, y) > 0$, and for values to the left $S(x, y) < 0$. Suppose that the planner is using the conjectured optimal startup rule with $\tilde{b} = b > b^*$. Such a rule is represented by the dashed line, which represents the points where $(1 - b)x + by = c$. The dashed line must cross the locus $S(x, y) = 0$ at least at one point or it would be optimal to change c and shift the dashed line. Moreover, it can only cross it at one point, from below, given the result in Lemma 3 and the fact that $q_y/q_x < b/(1 - b)$ as shown above. Consider an alternative startup rule, given by the dotted line, which has $b' < b$ and a c' such that the two rules cross the locus $S(x, y) = 0$ in the same point. This rule delivers a higher expected welfare given that (i) it reduces the size of the region in which $n = 0$ is chosen and $S(x, y) > 0$, (ii) it reduces the size of the region in which $n = 1$ is chosen and $S(x, y) < 0$, and (iii) leaves all other choices of n unchanged. We conclude that

$$W\left(b, b', c'\right) > W\left(b, b, c\right).$$

Step 2. We want to prove that

$$W\left(b', b', c'\right) > W\left(b, b', c'\right).$$

Let us first establish a result on the benefits of basing investment decisions on a more informative signal. Let

$$z_1 = \theta + b_1\varepsilon, \quad z_2 = \theta + b_2\varepsilon$$

and assume $b_2 > b_1$. Similarly, define the random variable $\hat{\Theta}_j$ and the parameters $\pi_{z,j}$ and π_j using

(39)-(40) with $b = b_j$, for $j = 1, 2$. The unconditional version of the next lemma is just Blackwell's theorem. For our argument, however, we need to prove the conditional version stated below.

Lemma 4. *For all* (ω, z_1), *the following is true:*

$$\mathbb{E}[Q(\Theta, \hat{\Theta}_1)|z_1, \omega] > \mathbb{E}[Q(\Theta, \hat{\Theta}_2)|z_1, \omega].$$

Proof of Lemma 4. Recall that

$$Q(\Theta, \hat{\Theta}_j) = \beta\Theta f(K(\hat{\Theta}_j)) - K(\hat{\Theta}_j)$$

for $j = 1, 2$, and notice that the function $\Theta f(k) - k$ is concave in k. Then we have

$$\mathbb{E}[Q(\Theta, \hat{\Theta}_2)|z_1, \omega] \leq \mathbb{E}[Q(\Theta, \hat{\Theta}_1)|z_1, \omega] + \mathbb{E}[(\beta\Theta f'(K(\hat{\Theta}_1)) - 1)(K(\hat{\Theta}_2) - K(\hat{\Theta}_1))|z_1, \omega], \tag{42}$$

and to prove our claim we need to show that the last term on the right-hand side is negative. Recall the optimality condition for $K(\hat{\Theta}_1)$

$$\beta\hat{\Theta}_1 f'(K(\hat{\Theta}_1)) = 1.$$

Using this condition, the last term on the right-hand side of (42) simplifies to

$$(\alpha\beta)^{\frac{1}{1-\alpha}}\mathbb{E}[(\Theta/\hat{\Theta}_1 - 1)\hat{\Theta}_2^{\frac{1}{1-\alpha}}|z_1, \omega],$$

and to prove that this expression is smaller than 0 we need to prove the inequality

$$\mathbb{E}[\Theta\hat{\Theta}_2^{\frac{1}{1-\alpha}}|z_1, \omega] < \mathbb{E}[\hat{\Theta}_2^{\frac{1}{1-\alpha}}|z_1, \omega]\mathbb{E}[\Theta|z_1, \omega].$$

Because of the normality of the underlying shocks, this condition is equivalent to $Cov[\theta, \hat{\theta}_2|z_1, \omega] < 0$, which in turn is equivalent to $Cov[\theta, z_2|z_1, \omega] < 0$, given that $\hat{\theta}_2 = \frac{\pi_\omega}{\pi_2}\omega + \frac{\pi_{z,2}}{\pi_2}z_2$. The following step completes the argument

$$Cov\,[\theta, z_2|z_1, \omega] = Cov[\theta, (1 - b_2/b_1)\theta + (b_2/b_1)z_1|z_1, \omega] = -(b_2/b_1 - 1)Var[\theta|z_1, \omega] < 0.$$

Given the startup rule (b', c') we can define

$$N(z') = \Phi\left(\frac{\sqrt{\pi_x}}{1-b'}(z' - c')\right),$$

where $z' = \theta + b'\varepsilon$, and write social welfare under the two information sets for investment as

$$W(b, b', c') = \mathbb{E}[N(z')(Q(\Theta, \hat{\Theta}) - 1)], \quad W(b', b', c') = \mathbb{E}[N(z')(Q(\Theta, \hat{\Theta}') - 1)].$$

Since in the previous step we have chosen $b' < b$, we can use Lemma 4 to obtain

$$N(z')\mathbb{E}[(Q(\Theta, \hat{\Theta}) - 1)|z', \omega] < N(z')\mathbb{E}[(Q(\Theta, \hat{\Theta}') - 1)|z', \omega].$$

Taking expectations on both sides completes the proof. Q.E.D.

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