Attention, Coordination, and Bounded Recall*

Alessandro Pavan†

Northwestern University

July 2016

Abstract

I consider a flexible framework of strategic interactions under incomplete information in which, prior to committing their actions (consumption, production, or investment decisions), agents choose the attention to allocate to an arbitrarily large number of information sources about the primitive events that are responsible for the incompleteness of information (the exogenous fundamentals). I study what type of payoff interdependencies contribute to inefficiency in the allocation of attention. The results for the case of perfect recall (in which the agents remember the influence of individual sources on their posterior beliefs) are compared to those for the case of bounded recall (in which posterior beliefs are consistent with Bayes rule but agents are unable to decompose their beliefs into the impressions they received from individual sources). The results have implications for business cycles, oligopoly, financial markets, technology adoption, and central bank disclosures.

Keywords: attention, endogenous information, strategic complementarity/substitutability, externalities, efficiency, welfare, bounded recall

JEL classification: C72, D62, D83, E50

*This paper was prepared for an invited session at the Asian Meeting of the Econometric Society. I thank the Chair of the organizing committee, Yeneng Sun, for the invitation. I also thank seminar participants at the University of California at Berkeley, University of Cambridge, the Catholic University of Milan, George Washington University, the Cornell-Penn State Macroeconomics Conference, the University of Essex, the Federal Reserve Bank of Chicago, the University of Iowa, Northwestern University, the University of Toronto, the University of Wisconsin at Madison, Iowa State University, Stanford University, the University of Zurich, the Barcelona Summer Forum on Information and Market Frictions, the EEA-ESEM Meetings, the Second Conference on Rational Inattention (Oxford, 2014), and the 2014 ESSET Conference in Gerzensee. I thank George Marios Angeletos, Simone Galperti, Filip Matejka, David Myatt, Isabel Trevino, and Chris Wallace for useful suggestions. Matteo Li Bergolis, Henrique Roscoe de Oliveira, and Sebastian Kohls provided excellent research assistance. The usual disclaimer applies.

†Department of Economics, Northwestern University; alepavan@northwestern.edu

1 Introduction

Many socio-economic interactions occur under incomplete information about relevant fundamentals affecting preferences and technology: For example, firms make real and nominal decisions under limited information about the demand for their products and/or the cost of their inputs; consumers choose consumption bundles under limited information about their own needs; traders choose portfolios under limited information about the profitability of stocks and the riskiness of bonds; voters choose candidates under limited information about their valiance and policy platforms.

Such incompleteness of information may either reflect limits on what is known to society as a whole (the long-run profitability of stocks, for example, is unknown to anyone), or individual constraints on the amount of information that each single decision maker can process. Time and cognitive capacity are limited, implying that the information that individuals use for most of their decisions is significantly less precise than what is in the public domain.

Furthermore, in most situations of interest, individuals experience difficulty in keeping track of the influence of individual sources of information on their posterior beliefs. For example, an investor reading tens, if not hundreds, of articles about the collapse of the Euro, or the UK leaving the EU, may have reasonably accurate posterior beliefs about the likelihood of such events. However, when asked about the influence of a specific source on her posterior beliefs, the investor may find it difficult to provide a precise answer. Such a difficulty is not relevant for a single decision maker; provided the posterior beliefs are a sufficient statistic for the information gathered from the different sources when it comes to predicting the exogenous events (the fundamentals), whether or not the decision maker can decompose her posterior beliefs into the different pieces does not matter. Instead, such a difficulty plays an important role in a strategic setting, for it may impact the decision maker's ability to forecast the behavior of other agents. Even if the decision maker is sophisticated and understands how the statistical properties of her posterior beliefs are affected by the attention allocated to the different sources as well as the extent to which her beliefs correlate with other agents' beliefs, the difficulty in keeping track of the influence of individual sources severely limits the agent's ability to forecast other agents' forecasts and hence ultimately other agents' actions. This inability is particularly relevant in environments in which the number of information sources is large (see, e.g., Kahneman (1973, 2011), and Kahneman, Slovic and Tversky (1982) for studies documenting such a difficulty).

In this paper, I study the allocation of attention in a strategic framework with an arbitrarily large number of information sources, where payoff interdependencies lead either to complementarity or substitutability in actions. I relate possible inefficiencies in the equilibrium allocation of attention to primitive conditions and show how the equilibrium allocation of attention, and its inefficiency, are affected by bounded recall.

Formally, bounded recall is a measurability constraint on the agents' actions. Agents allocate attention to a large number of information sources about the primitive events that are responsible for the incompleteness of information (the underlying fundamentals) and correctly use Bayes rule to update their beliefs. They are sophisticated enough to understand how their posterior beliefs, as well as other agents' beliefs, are affected by the quality of the information sources and by the equilibrium allocation of attention. However, ultimately, when it comes to committing their actions (consumption, production, or investment decisions), agents act only upon their posterior beliefs about such primitive events, instead of the various impressions they received from the individual sources of information.

While I do not formally model the dynamics of the allocation of attention, the motivation for referring to such a measurability constraint as bounded recall comes from the idea that, over time, the decision makers forget the impressions that specific sources of information left on their posterior beliefs, although their posterior beliefs remain accurate given the information processed. In this respect, the notion of bounded recall I consider is different from other notions of bounded memory considered in the literature, according to which information received in the past is time-discounted in its influence on current beliefs (e.g., Wilson (2004), Benhabou and Tirole (2004), and Kocer (2010)).

More generally, the same measurability constraint may capture various other features of the belief revision process by which impressions from individual sources are replaced by a summary statistics (as is typically assumed in the rational inattention literature). That agents experience difficulty in responding separately to individual sources of information in turn may reflect either a certain form of bounded rationality, or simply the idea that, in many environments of interest, agents do not think of the various sources of information as providing separate pieces of information. Instead, they think of them as contributing collectively to their posterior beliefs. In part, this may reflect the fact that the content of each source is often understood only after combining it with other sources. For example, a trader interested in the probability of the UK leaving the EU, rarely finds articles providing a number for such a probability. Instead, the sources he/she attains to typically provide a collection of (often, vaguely articulated) arguments that are difficult to remember in isolation but that, jointly, contribute to the trader's belief about the event of interest.

The first part of the paper considers the benchmark of perfect recall. It shows that any source that receives positive attention is characterized by a ratio between its transparency and its marginal cost of attention exceeding a critical threshold.[1] In the special case in which the attention cost depends only on the total amount of attention, the result implies that only the most transparent sources receive attention in equilibrium, thus extending a property first noticed in Myatt and Wallace (2012) to the more general structure considered here.

I then compare the equilibrium allocation of attention to the efficient allocation of attention (defined as the one that maximizes the ex-ante utility of a representative agent). I show that in economies in which information is used efficiently, possible inefficiencies in the allocation of

[1] The *transparency* of a source is the rate of return of attention to the source; that is, the extent to which additional attention to the source leads to a marginal reduction in the idiosyncratic interpretation of its content.

attention originate in the dispersion of individual actions around the mean action. In particular, the attention allocated to any given source is inefficiently low in economies in which agents suffer from the dispersion of individual actions, whereas it is inefficiently high in economies in which agents benefit from such a dispersion (for example, in beauty contests, but also in certain business-cycle models with price rigidities and dispersed information on TFP shocks). Likewise, the attention allocated to any given source is inefficiently low in economies in which the sensitivity of the complete-information equilibrium actions to fundamentals falls short of the first-best level and is inefficiently high in economies in which such a sensitivity exceeds the first-best level.

The most interesting results, however, pertain to economies in which inefficiencies in the allocation of attention originate in the discrepancy between the equilibrium and the socially-optimal degrees of coordination. When agents are excessively concerned about aligning their actions with the actions of others, they allocate too much attention to sources of high transparency and too little attention to sources that are opaque but accurate.[2]

What creates a discrepancy between the equilibrium and the efficient allocation of attention is the interaction of two forces: (i) the value that each agent assigns to reducing the dispersion of her actions around the mean action, relative to the value that the planner assigns to the same reduction (the planner takes into account externalities from the dispersion of individual actions that are not internalized in equilibrium); and (ii) the reduction in the dispersion of individual actions around the mean action that obtains when individual actions are determined by the equilibrium strategies, relative to the reduction that obtains when actions are determined by the efficient rule.

The above results are instrumental to the analysis of the effects of bounded recall, which is the subject of the second part of the paper. As mentioned above, bounded recall is a constraint on the agents' actions that imposes that the latter be measurable in the agents' posterior beliefs about the primitive sources of uncertainty (the fundamentals), as opposed to the individual impressions derived from the various sources of information.

The first insight is that, with bounded recall, the benefit that each individual agent assigns to an increase in the attention allocated to any given source of information combines the reduction in the dispersion of her action around the mean action (as in the benchmark with full recall), with a novel effect that comes from the change in the distribution of the agent's own average action around its complete-information counterpart. This second effect is absent under perfect recall, and has important implications for the equilibrium allocation of attention. Relative to the case of perfect recall, agents reallocate their attention from sources of low and high publicity (these are sources of, respectively, low and high transparency) to sources of intermediate publicity.

To understand the result, observe that sources of low publicity are sources whose ratio between transparency and accuracy is low. These sources serve the agents well in predicting the underlying fundamentals but are poor coordination devices, given that they favor idiosyncratic interpretations. In the case of perfect recall, paying a lot of attention to such sources is justified by the possibility to

[2] The *accuracy* of a source is the precision of its content.

respond separately to the impressions the decision makers obtain from such sources, thus limiting the impact of such idiosyncratic interpretations on the dispersion of individual actions around the mean action. Such a possibility is severed under bounded recall, thus inducing the agents to reduce the attention they allocate to such sources.

Sources of high publicity, instead, are sources whose ratio between transparency and accuracy is high. Such sources may be imprecise when used to predict exogenous fundamentals, but serve the agents well when used to predict the forecasts, and hence ultimately the actions, of others. With bounded recall, however, paying a lot of attention to such sources may lead to a high volatility of an agent's own expected action around its complete-information counterpart. Because such a volatility contributes negatively to payoffs, agents optimally cut the attention they allocate to such sources and redirect it towards sources of intermediate publicity.

The reason why sources of intermediate publicity receive more attention with bounded recall is that they are good *compromises*: they serve individuals relatively well both in forecasting the underlying fundamentals and in forecasting other agents' actions. This property is desirable when the possibility of responding separately to the impressions derived from the various sources is hindered by bounded recall.

I conclude by investigating how bounded recall affects the (in)efficiency of the equilibrium allocation of attention. Inefficiencies now originate not only in the discrepancy between the private and the social value of reducing the dispersion of individual actions around the mean action, but also in the discrepancy between the private and the social value of reducing the dispersion of individual expected actions around their complete-information counterparts. Despite these novel effects, economies in which agents value coordination more than the planner continue to feature an excessively high allocation of attention to sources of high publicity and an excessively low allocation of attention to sources of low publicity. The opposite property holds in economies in which agents value coordination less than the planner.

The rest of the paper is organized as follows. I briefly review the pertinent literature below. Section 2 contains all results for the case of perfect recall, while Section 3 contains the results for the case of bounded recall. Section 4 concludes. All proofs are in the Appendix at the end of the document.

1.1 Related literature

The paper belongs to the recent literature on attention and information acquisition in coordination environments. The description of the sources of information is from Myatt and Wallace (2012). The payoff structure, however, is more general, and this is essential to the analysis of the normative questions addressed in the present paper.[3] Related is also Myatt and Wallace (2015). That paper

[3] The payoff structure in Myatt and Wallace is meant to capture strategic interactions resembling Keynes' beauty contests. This particular payoff specification makes the game a "potential" game, where the potential function is social welfare (see, e.g., Monderer and Shapley (1996)). This specification is thus appropriate for positive analysis,

considers a large oligopoly game with differentiated goods. The welfare criterion is total surplus, combining producer surplus with consumer surplus. In contrast, the present paper confines attention to economies in which all players are ex-ante symmetric and act under dispersed information. When specialized to oligopoly games, the welfare criterion in the present paper corresponds to producer surplus.

The payoff structure in the present paper is the same as in Angeletos and Pavan (2007) and in Colombo, Femminis, and Pavan (2014). This structure is extensively used both in the coordination literature (see Bergemann and Morris (2013) and Vives (2016), among others, for recent developments) as well as in the rational inattention literature (see Sims (2003, 2011) for an introduction). The information structure, however, is significantly more flexible. The richer information structure considered in the present paper is essential to the analysis of the effects of bounded recall on the allocation of attention. This is the first paper to distinguish explicitly between attention and recall.

Related is also the work of Hellwig and Veldkamp (2009), Llosa and Venkateswaran (2013), Chahrour (2014), Tirole (2015), and Herskovic and Ramos (2016). Hellwig and Veldkamp investigate how complementarities in actions lead to complementarities in information acquisition. The information structure in that paper is different in that it assumes that the publicity of each source is exogenous and that the attention allocated to each source is binary. This last property has been shown to be responsible for equilibrium indeterminacy. Equilibrium indeterminacy also obtains in the model of cognitive games and cognitive traps of Tirole (2015).[4] In contrast, the (symmetric) equilibrium is unique in the present paper, as well as in each of the papers cited above. Chahrour (2014) studies optimal central bank disclosures in an economy in which processing information is costly and in which agents may miscoordinate on which sources they pay attention to. Llosa and Venkateswaran (2013) in turn compare the equilibrium acquisition of private information to the efficient acquisition of private information in three different specifications of the business cycle. Herskovic and Ramos (2016) study coordination and information acquisition in a model of network formation in which agents learn from peers.

All works mentioned above consider economies with a continuum of actions and continuous payoffs. Information acquisition in games of regime change (where payoffs are discontinuous and players have binary actions) is studied in Szkup and Trevino (2015), and in Yang (2015). The first paper considers a canonical information structure with a single perfectly private additive signal whose precision is determined in equilibrium. The latter paper considers a flexible information structure and shows how the possibility to learn asymmetrically across states (which is appealing in discontinuous games) leads to equilibrium indeterminacy. Contrary to the present paper, the above two works assume that the signals the agents receive conditional on the realized fundamental are independent. This assumption is relaxed in Denti (2016), where endogenous correlation in

but not for identifying sources of inefficiency in the equilibrium allocation of attention.

[4] For an analysis of general properties of monotone equilibria in Bayesian games of strategic complementarities, see the earlier work by Van Zandt and Vives (2007) and the more recent work by Amir and Lazzati (2014).

the noise in the agents' signals is shown to contribute to equilibrium uniqueness (see also Morris and Yang (2016) for a discussion of how equilibrium (in)determinacy in these games relates to a property of "state distinguishability").

The present paper is also related to the literature on rational inattention, as pioneered by Sims (see, e.g., Sims (2003, 2011) for an overview, Maćkowiak and Wiederholt (2009) for an influential business-cycle application, Matejka and McKay (2012, 2015) for how rational inattention provides a foundation for the multinomial logit model, and Hébert and Woodford (2016), and Stewart et al (2016) for recent extensions to dynamic problems). Among these papers, the closest is Maćkowiak and Wiederholt (2012). That paper compares the equilibrium allocation of attention to the efficient allocation of attention assuming that the decision makers can absorb any information as long as the reduction in entropy is below a given threshold. In contrast, in the present paper, the cost of attention is smooth. The information structure is also different and permits me to investigate which dimension (transparency versus accuracy) receives more weight in equilibrium, and whether the equilibrium weights are socially efficient. Importantly, while the papers in the rational inattention literature share with the model of bounded recall considered here the assumption that agents update their beliefs through a Bayesian summary statistics, none of these works studies how the allocation of attention would change if the agents were able to respond separately to the impressions they gather from the different sources of information.

The paper is also related to the literature that investigates the effects of bounded memory on individual decision making (see, e.g., Mullainathan (2002), Benabou and Tirole (2004), Wilson (2004), and Kocer (2010)). This literature does not investigate how bounded memory influences the allocation of attention in a strategic setting, or the discrepancy between the equilibrium and the efficient allocation of attention. The effects of bounded recall in settings with strategic interactions are, instead, examined in the literature on dynamic (and repeated) games with imperfect information (see, e.g., Mailath and Samuelson (2006) and the references therein). The formalization of bounded recall, as well as the questions addressed in that literature, are, however, very different from the ones in the present paper.

Finally, the paper is related to the literature that investigates how boundedly rational agents may group together different information sets into analogy-based equivalence classes when computing best-responses, as pioneered by Jehiel (2005)—see also Jehiel and Samet (2007), Jehiel and Koessler (2008), and Jehiel and Samuelson (2012). In the present paper, the coarsening of the information sets is the one corresponding to the equivalence classes defined by the agents' posterior beliefs over the primitive fundamentals.

2 Perfect Recall

2.1 Environment

Agents, Information Sources, and Attention. The economy is populated by a measure-one continuum of agents, indexed by i and uniformly distributed over $[0,1]$. Each agent i has access to $N \in \mathbb{N}$ sources of information about a primitive payoff variable θ which is responsible for the incompleteness of information (hereafter, the fundamentals). Depending on the application of interest, such variable parametrizes a technology shock, a demand shifter, or the profitability of a new investment opportunity. Agents share a common prior that θ is drawn from a Normal distribution with mean zero and precision $\pi_\theta \equiv \sigma_\theta^{-2}$ (σ_θ^2 is thus the variance of the distribution).[5] The information contained in each source $n = 1, ..., N$ is given by

$$y_n = \theta + \varepsilon_n,$$

where ε_n is normally distributed noise, independent of θ and of any ε_s, $s \neq n$, with mean zero and precision η_n. By paying attention $z^i \equiv (z_n^i)_{n=1}^N \in \mathbb{R}_+^N$ to the various sources, agent $i \in [0,1]$ then receives $x^i \equiv (x_n^i)_{n=1}^N \in \mathbb{R}^N$ *private impressions*, with each x_n^i given by

$$x_n^i = y_n + \xi_n^i,$$

where ξ_n^i is idiosyncratic noise, normally distributed, with mean zero and precision $t_n z_n^i$, independently of θ, $\varepsilon \equiv (\varepsilon_n)_{n=1}^N$, and of ξ_s^j, with $s = 1, ..., N$ for $j \neq i$, and with $s = 1, ..., n-1, n+1, ..., N$ for $j = i$. The parameter $\eta_n \in \mathbb{R}_+$ measures the source's *accuracy*, whereas the parameter t_n its *transparency* (the extent to which a marginal increase in the *attention* z_n^i allocated to the source reduces the idiosyncratic interpretation of its content). It is convenient to think of z^i as the "time" agent i devotes to the different sources of information.

Actions and Payoffs. Let $k^i \in \mathbb{R}$ denote agent i's action, $K \equiv \int_j k^j dj$ the mean action in the population, and $\sigma_k^2 \equiv \int_j \left[k^j - K\right]^2 dj$ the dispersion of individual actions around the population mean action. Each agent's payoff is given by the (expectation of the) Bernoulli utility function

$$u\left(k^i, K, \sigma_k, \theta\right) - C(z^i),$$

where $C(z^i)$ denotes the attention cost incurred by the agent. I assume that C is increasing, convex, and continuously differentiable.[6]

[5] That the prior mean is zero simplifies the formulas, without any important effect on the results.

[6] The assumption that C is convex need not be compatible with an entropy-based cost function (that is, a cost function increasing in the coefficient of mutual information between y and x^i, as assumed in certain models of rational inattention). With that type of cost-function, equilibrium uniqueness cannot be guaranteed for sufficiently high degrees of coordination. However, even in that case, social welfare continues to be concave in the allocation of attention, meaning that the efficient allocation of attention remains unique. Besides, all key results pertaining to (a) the comparison between the equilibrium allocation of attention and the efficient allocation of attention and (b) the

As is standard in the literature, I assume that u is a second-order polynomial, which can be interpreted as an approximation of some more general function. I also assume that dispersion σ_k has only a second-order non-strategic external effect, so that $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and that $u_\sigma(k, K, 0, \theta) = 0$, for all (k, K, θ).[7,8] The assumption that u is quadratic ensures the linearity of the agents' best responses and simplifies the analysis.

In addition to the above assumptions, I also assume that partial derivatives satisfy the following conditions: (i) $u_{kk} < 0$, (ii) $\alpha \equiv -u_{kK}/u_{kk} < 1$, (iii) $u_{kk} + 2u_{kK} + u_{KK} < 0$, (iv) $u_{kk} + u_{\sigma\sigma} < 0$, and (v) $u_{k\theta} \neq 0$. Condition (i) imposes concavity at the individual level, so that best responses are well defined. Condition (ii) implies that the slope of best responses is less than one, which in turn guarantees uniqueness of the equilibrium actions, for any given allocation of attention. Conditions (iii) and (iv) guarantee that the first-best allocation is unique and bounded. Finally, Condition (v) ensures that the fundamental θ affects equilibrium behavior, thus making the analysis non-trivial.

Timing. Agents simultaneously choose the attention they allocate to the various sources of information. Each agent then receives private impressions x^i. Finally, agents simultaneously commit their actions, and payoffs are realized.

Perfect Recall. As mentioned in the Introduction, the assumption of perfect recall amounts to assuming that each agent recognizes the effect that each individual impression x_n^j has on her posterior beliefs and thus can condition her action k^j separately on each x_n^j.

2.2 The equilibrium allocation of attention

First note that, under complete information about θ, the unique equilibrium features each agent taking the action $k^i = \kappa$ where $\kappa \equiv \kappa_0 + \kappa_1\theta$, with $\kappa_0 \equiv \frac{-u_k(0,0,0,0)}{u_{kk}+u_{kK}}$ and $\kappa_1 \equiv \frac{-u_{k\theta}}{u_{kk}+u_{kK}}$. Now consider the problem of an agent $j \in [0, 1]$ who allocated attention z^j to the various sources of information and received the impressions x^j. Optimality requires that the agent's action satisfies[9]

$$k^j = \mathbb{E}[(1 - \alpha)\kappa + \alpha K \mid z^j, x^j], \tag{1}$$

where $\alpha \equiv u_{kK/}|u_{kk}|$ measures the slope of individual best responses to aggregate activity.

Next, consider the agent's choice of attention. Suppose that all agents allocate attention z to the various sources of information. The (endogenous) *precision* of each source $s = 1, ..., N$ is then given by

$$\pi_s \equiv \frac{\eta_s z_s t_s}{z_s t_s + \eta_s}$$

comparison of the equilibrium with full recall and the equilibrium with bounded recall are established by looking at the *gross* private benefit of increasing the attention allocated to any given source. As such, all key results extend to a situation in which the attention cost is concave, even if in the latter case equilibrium uniqueness cannot be guaranteed.

[7] The notation u_k denotes the partial derivative of u with respect to k, whereas the notation u_{kK} denotes the cross derivative with respect to k and K. Similar notation applies to the other arguments of the utility function.

[8] In other words, u is additively separable in σ_k^2 with coefficient $u_{\sigma\sigma}/2$.

[9] This step follows from Angeletos and Pavan (2009)—Proposition 3.

and is increasing in the source's accuracy η_s, in its transparency t_s, and in the attention z_s allocated to the source.

Denote by

$$\varphi_s^j \equiv \varepsilon_s + \xi_s^i$$

the total noise in the impression agent j receives from source s, and by

$$\rho_s \equiv corr(\varphi_s^j, \varphi_s^i) = \frac{z_s t_s}{z_s t_s + \eta_s}$$

the correlation in the noise among any two different agents $i, j \in [0,1]$, $i \neq j$. Hereafter, I follow Myatt and Wallace (2013) and refer to ρ_s as the source's *endogenous publicity.* Finally, let

$$C_n'(z) \equiv \partial C(z)/\partial z_n$$

denote the marginal cost of increasing the attention allocated to the n-th source of information. The following result is then true:

Proposition 1 *There exists a unique symmetric equilibrium. In this equilibrium, the attention $\hat{z}$ that each agent $i \in [0,1]$ allocates to the various sources of information is such that, for any source $n = 1, ..., N$ that receives strictly positive attention*[10]

$$C_n'(\hat{z}) = \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n} \tag{2}$$

where

$$\gamma_n(z) \equiv \frac{\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}}{\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)}} \text{ with } \pi_s(z) = \frac{\eta_s z_s t_s}{z_s t_s + \eta_s} \text{ and } \rho_s(z) = \frac{\pi_s(z)}{\eta_s},\ s = 1, ..., N. \tag{3}$$

Given the equilibrium allocation of attention $\hat{z}$, the equilibrium actions are given by

$$k^i = k(x^i; \hat{z}) = \kappa_0 + \kappa_1 \left(\textstyle\sum_{n=1}^N \gamma_n(\hat{z}) x_n^i \right) \text{ all } i \in [0,1], \text{ all } x^i \in \mathbb{R}^N. \tag{4}$$

To understand the result, note that, when all agents follow the strategy $k(\cdot; z)$ in (4), in equilibrium, the dispersion of individual actions in the population is given by

$$Var[k - K \mid z,\ k(\cdot; z)] = \kappa_1^2 \sum_{s=1}^N \frac{\gamma_s^2(z)}{z_s t_s}.$$

[10] For any source that receives no attention

$$C_n'(\hat{z}) \geq \frac{|u_{kk}|}{2} \frac{(\kappa_1)^2 (1-\alpha)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\eta_s \hat{z}_s t_s}{(1-\alpha)\hat{z}_s t_s + \eta_s}\right]^2}.$$

Differentiating $Var[k - K \mid z,\, k(\cdot; z)]$ with respect to z_n while keeping fixed the strategy $k(\cdot; z)$ as defined in (4) (for all agents, including agent i), then reveals that the *private benefit* of increasing the attention allocated to each source n (the right-hand side in (2)) is equal to

$$\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(z))^2}{(z_n)^2\, t_n} = \frac{|u_{kk}|}{2}\left|\frac{\partial}{\partial z_n}Var[k - K \mid z,\ k(\cdot; z)]\right|. \tag{5}$$

In equilibrium, the marginal benefit that each agent assigns to paying more attention to any given source of information thus coincides with the marginal reduction in the dispersion of the individual's action around the mean action, weighted by the importance $|u_{kk}|/2$ that the individual assigns to such a reduction. Importantly, the reduction in dispersion is computed by holding fixed the strategy $k(\cdot; z)$ (From the usual envelope arguments, the agent expects the information to be used optimally once collected). As I show below, this interpretation helps understanding the sources of inefficiency in the equilibrium allocation of attention.

Also note that, fixing the equilibrium allocation of attention $\hat{z}$, the influence $\kappa_1\gamma_n$ that each source exerts on the equilibrium actions (as per (4)) increases with the source's endogenous precision π_n and increases with the source's endogenous publicity ρ_n when agents value positively aligning their actions with the actions of others (i.e., when $\alpha > 0$), whereas it decreases when they value such alignment negatively (i.e., when $\alpha < 0$). In turn, both the precision π_n and the publicity ρ_n of any given source increase with the source's accuracy η_n and with its transparency t_n. Finally, note that, when $\alpha \to 0$, the sensitivity of the equilibrium actions to each source of information converges to $\kappa_1\delta_n$ with

$$\delta_n \equiv \frac{\pi_n}{\pi_\theta + \sum_{s=1}^{N}\pi_s}.$$

This limit corresponds to a single decision maker's problem, in which case the relative influence of any two sources of information is given by their relative informativeness, as captured by the ratio between the two sources' precisions. In contrast, when $\alpha \to 1$, $\gamma_n \to 0$ for all $n = 1, ..., N$: as the agents' concern for aligning their actions with the actions of others grows large, they ignore all sources of information and base their actions on the common prior.

Let me now expand on what drives the *equilibrium allocation of attention*. To facilitate the intuition, consider the case where $\pi_\theta = 0$ (this corresponds to an improper prior over the entire real line) and where the attention cost depends on z only through the total attention $\sum_{s=1}^{N} z_s$ assigned to the various sources of information. That is, assume that there exists a strictly increasing, differentiable, convex function $c : \mathbb{R}_+ \to \mathbb{R}_+$ such that, for any $z \in \mathbb{R}_+^N$, $C(z) = c\left(\sum_{s=1}^{N} z_s\right)$. The relative attention allocated to any two sources of information $n, n' \in \{1, ..., N\}$ that receive strictly positive attention in equilibrium is then given by

$$\frac{\hat{z}_n}{\hat{z}_{n'}} = \frac{\gamma_n}{\gamma_{n'}}\sqrt{\frac{t_{n'}}{t_n}}.$$

Substituting for γ_n and $\gamma_{n'}$, one then has that

$$\hat{z}_n = \frac{\eta_n}{t_n}\left(\frac{1}{\eta_{n'}}\sqrt{t_{n'}t_n}\,\hat{z}_{n'} + \frac{\sqrt{t_n} - \sqrt{t_{n'}}}{(1-\alpha)\sqrt{t_{n'}}}\right). \tag{6}$$

Any two sources with the same transparency thus receive attention proportional to their accuracy. More generally, (6) reveals that the attention that a source receives in equilibrium is increasing in its accuracy, but nonmonotone in its transparency. The intuition is the following. When transparency is low, paying a lot of attention to a source is not worth the cost, given that the reduction in the idiosyncratic interpretation of the source's content is small. Likewise, when transparency is high, a small amount of attention suffices to almost completely eliminate any idiosyncratic interpretation of the source's content. As a result, attention is maximal for intermediate degrees of transparency.

This intuition is confirmed in the following corollary (note that, contrary to the discussion above, here we are returning to the general case in which the prior is proper and the cost function is not restricted to depending only on total attention):

Corollary 1 *There exists a threshold $R > 0$ such that, in the unique symmetric equilibrium, for any source that receives positive attention*

$$\frac{t_n}{C'_n(\hat{z})} > R,$$

whereas for any source that receives no attention $t_n / C'_n(\hat{z}) \leq R$.

When specialized to a cost function that depends only on total attention, the result in the preceding corollary thus extends to the environment under consideration here a property first noticed in Myatt and Wallace (2012) that only those sources of sufficiently high transparency receive attention in equilibrium

In general, solving for the equilibrium allocation of attention in close form can be tedious at this level of generality. Fortunately, none of the results below requires arriving at close-form solutions. However, a special case where close-form solutions can easily be arrived at is when the cost is linear and small enough that all sources receive positive attention in equilibrium.

Example 1 *Suppose that $C(z) = \bar{c} \cdot \sum_{s=1}^{N} z_s$ for some $\bar{c} \in \mathbb{R}_{++}$ and assume that $\bar{c}$ is sufficiently small that all sources receive positive attention in equilibrium. The attention that each source receives is then given by*

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \left[\frac{(1-\alpha)\kappa_1 \sqrt{\frac{|u_{kk}|}{2\bar{c}}} + \sum_{s=1}^{N} \frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s=1}^{N} \eta_s} - \frac{1}{\sqrt{t_n}} \right]. \tag{7}$$

The example illustrates the general properties discussed above that attention is increasing in accuracy but nonmonotone in transparency. It also shows that, under the assumed cost function, as the value of coordination α increases, the attention allocated to sources of low transparency decreases, whereas the attention allocated to sources of high transparency increases.[11] Finally, it

[11] Formally,

$$\frac{\partial \hat{z}_n}{\partial \alpha} < 0 \text{ if } \sqrt{t_n} \leq \left(\frac{\pi_\theta + \sum_{s=1}^{N} \eta_s}{\sum_{s=1}^{N} \frac{\eta_s}{\sqrt{t_s}}} \right) \text{ and } \frac{\partial \hat{z}_n}{\partial \alpha} > 0 \text{ if the previous inequality is reversed.}$$

shows that the total amount of attention decreases with the coordination motive, α.[12]

2.3 The efficient allocation of attention

I now turn to the allocation of attention that maximizes the ex-ante utility of a representative agent. The analysis permits me to identify payoff interdependencies that, under perfect recall, are responsible for inefficiency in the equilibrium allocation of attention. These results, when applied to specific applications, may guide policy interventions aimed at increasing the efficiency of market interactions.

First, observe that, for any allocation of attention z, the efficient use of information consists in all agents following the unique strategy $k^*(\cdot; z)$ that solves the functional equation[13]

$$k\left(x; z\right) = \mathbb{E}\left[\left(1 - \alpha^*\right)\kappa^* + \alpha^* K \ \mid z, x\right] \text{ for all } x \in \mathbb{R}^N, \tag{8}$$

where $\kappa^* = \kappa_0^* + \kappa_1^* \theta$ is the first-best allocation[14], $K = \mathbb{E}\left[k\left(x; z\right) \ \mid z, \theta, \varepsilon\right]$ is the average action, and

$$\alpha^* \equiv \frac{u_{\sigma\sigma} - 2u_{kK} - u_{KK}}{u_{kk} + u_{\sigma\sigma}} \tag{9}$$

is the socially optimal degree of coordination (that is, the level of complementarity, or substitutability, that the planner would like the agents to perceive in order for the equilibrium of the economy to coincide with the efficient allocation.) Because (8) differs from the equilibrium optimality condition (1) only by the fact that α is replaced by α^* and κ by κ^*, it is then immediate that the efficient strategy takes the linear form

$$k^*(x; z) = \kappa_0^* + \kappa_1^* \left(\textstyle\sum_{n=1}^N \gamma_n^*(z) x_n\right), \tag{10}$$

where $\gamma_n^*(z)$ is defined as $\gamma_n(z)$ but with α^* replacing α.

Next note that, for any given attention z, welfare under the efficient use of information $k^*(\cdot; z)$ can be expressed as

$$w^*(z) \equiv \mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)] - \mathcal{L}^*(z) - C(z),$$

where $\mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)]$ is expected welfare under the first-best allocation and where

$$\mathcal{L}^*(z) \equiv \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} Var[K - \kappa^* \mid z, k^*(\cdot; z)] + \frac{|u_{kk} + u_{\sigma\sigma}|}{2} Var[k - K \mid z, k^*(\cdot; z)]$$

[12] This is not immediate to see, but can be verified by differentiating $\hat{Z} = \sum_n \hat{z}_n$ with respect to α and using the property that

$$\left(\sum_{s=1}^N \frac{\eta_s}{\sqrt{t_s}}\right)^2 \leq \sum_{s=1}^N \frac{\eta_s}{t_s} \sum_{s=1}^N \eta_s.$$

[13] The characterization of the efficient use of information follows from steps similar to those in Angeletos and Pavan (2009). The contribution here is in the characterization of the efficient allocation of attention.

[14] The scalars κ_0^* and κ_1^* are given by $\kappa_0^* = \frac{u_k(0,0,0) + u_K(0,0,0)}{-(u_{kk} + 2u_{kK} + u_{KK})}$ and $\kappa_1^* = \frac{u_{k\theta} + u_{K\theta}}{-(u_{kk} + 2u_{kK} + u_{KK})}$, respectively.

combines the welfare losses originating in the volatility of the average action K around its first-best counterpart with the losses originating in the dispersion of individual actions around the mean action.

I now turn to the efficient allocation of attention. Using the envelope theorem and observing that, holding fixed the strategy $k^*(\cdot;z)$, the volatility of the aggregate action around its complete-information counterpart, $Var[K-\kappa^* \mid z, k^*(\cdot;z)]$, is independent of the allocation of attention, I then have that the *social benefit* of increasing the attention allocated to any source n (gross of its cost) is given by[15]

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\left|\frac{\partial}{\partial z_n}Var[k-K \mid z, k^*(\cdot;z)]\right| = \frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\left(\kappa_1^*\gamma_n^*(z)\right)^2}{\left(\hat{z}_n\right)^2 t_n} \tag{11}$$

where $\partial Var[k-K \mid z, k^*(\cdot;z)]/\partial z_n$ is computed holding fixed the efficient strategy $k^*(\cdot;z)$ that maps impressions x into individual actions. In other words, the social benefit of allocating more attention to any given source is given by the reduction in the dispersion of individual actions around the mean action that obtains when agents allocate more attention to that source, weighted by the social aversion to dispersion $|u_{kk}+u_{\sigma\sigma}|/2$. The following result then follows from the arguments above:

Proposition 2 *Suppose that the planner can control the agents' actions. There exists a unique allocation of attention z^* that maximizes welfare. Under such allocation, for any source n that receives positive attention,*[16]

$$C_n'(z^*) = \frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\left(\kappa_1^*\gamma_n^*(z^*)\right)^2}{\left(z_n^*\right)^2 t_n},$$

where $\kappa_1^*\gamma_n(z^*)$ represents the influence of the source on the agents' actions under the efficient use of information, with

$$\gamma_n^*(z) \equiv \frac{\frac{(1-\alpha^*)\pi_n(z)}{1-\alpha^*\rho_n(z)}}{\pi_\theta+\sum_{s=1}^N \frac{(1-\alpha^*)\pi_s(z)}{1-\alpha^*\rho_s(z)}} = \frac{\frac{(1-\alpha^*)\eta_n z_n t_n}{(1-\alpha^*)z_n t_n+\eta_n}}{\pi_\theta+\sum_{s=1}^N \frac{(1-\alpha^*)\eta_s z_s t_s}{(1-\alpha^*)z_s t_s+\eta_s}}.$$

[15] As in the equilibrium case, the expression in (11) applies to sources that receive strictly positive attention (that is, for which $z_n>0$). The marginal benefit of increasing the attention allocated to a source that receives zero attention is simply the limit of the right-hande side of (11) as $z_n \to 0$ which is equal to

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1^*)^2(1-\alpha^*)^2 t_n}{\left[\pi_\theta+\sum_{s=1}^N \frac{(1-\alpha^*)\eta_s z_s t_s}{(1-\alpha^*)z_s t_s+\eta_s}\right]^2}.$$

[16] As in the equilibrium case, for any source that receives no attention, the following condition must hold:

$$C_n'(z^*) \geq \frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1^*)^2(1-\alpha^*)^2 t_n}{\left[\pi_\theta+\sum_{s=1}^N \frac{(1-\alpha^*)\eta_s z_s t_s}{(1-\alpha^*)z_s t_s+\eta_s}\right]^2}.$$

The following conclusion can then be established by comparing the private benefit (5) to the social benefit (11) of increasing the attention allocated to any given source of information:

Corollary 2 *Let $\hat{z}$ denote the equilibrium allocation of attention. Suppose that the planner can control the agents' actions. Then, starting from $\hat{z}$, forcing the agents to pay more attention to a source n that receives positive attention in equilibrium (i.e., for which $\hat{z}_n > 0$) increases welfare if*

$$|u_{kk}|(\kappa_1\gamma_n(\hat{z}))^2 < |u_{kk} + u_{\sigma\sigma}|(\kappa_1^*\gamma_n^*(\hat{z}))^2 \tag{12}$$

and decreases it if the inequality in (12) is reversed, where $\kappa_1\gamma_n(\hat{z})$ and $\kappa_1^*\gamma_n^*(\hat{z})$ denote, respectively, the sensitivity of the equilibrium and of the efficient actions to the n-th source of information, when the attention allocated to the various sources is $\hat{z}$. Likewise, forcing the agents to pay attention to a source n that receives no attention in equilibrium (i.e., for which $\hat{z}_n = 0$) increases welfare if

$$\frac{|u_{kk} + u_{\sigma\sigma}|}{2} \frac{(\kappa_1^*)^2(1-\alpha^*)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha^*)\eta_s \hat{z}_s t_s}{(1-\alpha^*)\hat{z}_s t_s + \eta_s}\right]^2} > C_n'(\hat{z})$$

and decreases it if the inequality is reversed.

To understand the result, recall from the analysis above that both the private and the social (gross) marginal benefit of allocating more attention to any given source of information come from the marginal reduction in the dispersion of individual actions around the mean action.[17] The magnitude of this reduction depends on the sensitivity of individual actions to the source of information, which is given by $\kappa_1\gamma_n$ under the equilibrium strategy and by $\kappa_1^*\gamma_n^*$ under the efficient strategy. The weight that the planner assigns to reducing the dispersion of individual actions is $|u_{kk} + u_{\sigma\sigma}|$, whereas the weight that each individual agent assigns to reducing the dispersion of her action around the mean action is $|u_{kk}|$. The discrepancy between the two stems from the externality $u_{\sigma\sigma}$ that dispersion exerts on individual payoffs, which is not internalized in equilibrium.

Increasing the attention allocated to a source that receives positive attention in equilibrium then increases welfare if and only if the marginal reduction in the dispersion of actions under the equilibrium strategy, weighted by the importance that each agent assigns to dispersion, falls short of the marginal reduction in dispersion under the efficient strategy, weighted by the importance that the planner assigns to dispersion.

Likewise, for any source that receives no attention in equilibrium, the marginal cost exceeds the private marginal benefit of reducing dispersion. Forcing the agents to pay attention to such sources increases welfare if and only if the marginal cost falls short of the social marginal benefit of reducing dispersion.

[17] By usual envelope arguments, both marginal reductions are computed holding constant, respectively, the equilibrium and the efficient use of information, that is, the mappings $k(\cdot; z)$ and $k^*(\cdot; z)$.

Put differently, efficiency in the allocation of attention requires both (i) efficiency in the use of information and (ii) alignment between the private and the social value of reducing the dispersion of individual actions, which obtains when, and only when, $u_{\sigma\sigma} = 0$.

Now let $\#\hat{N}$ denote the cardinality of the set of sources that receive positive attention in equilibrium, and $\#N^*$ the cardinality of the set of sources that receive positive attention when the planner can control both the allocation of attention and the use of information. Let $\hat{z}$ denote the equilibrium allocation of attention and z^* the allocation of attention that maximizes welfare when the planner can control the agents' actions. The following three propositions identify primitive sources of inefficiency in the equilibrium allocation of attention. I start with economies that are efficient in their use of information.

Proposition 3 *Consider economies that are efficient in their use of information ($\kappa = \kappa^*$ and $\alpha = \alpha^*$). The attention allocated to each source is inefficiently low if $u_{\sigma\sigma} < 0$ and inefficiently high if $u_{\sigma\sigma} > 0$ (meaning that, for any n, $z_n^* \geq \hat{z}_n$ if $u_{\sigma\sigma} < 0$ and $z_n^* \leq \hat{z}_n$ if $u_{\sigma\sigma} > 0$, with the inequalities strict if $\hat{z}_n > 0$).*

Because in these economies the equilibrium use of information is efficient, the marginal reduction in the dispersion of individual actions under the equilibrium strategy coincides with the marginal reduction under the efficient strategy. This property, however, is not sufficient to guarantee that the private and the social marginal benefit of increasing the attention coincide. The reason is that the private benefit fails to take into account the direct, non-strategic effect that the dispersion of individual actions has on payoffs, as captured by $u_{\sigma\sigma}$. Because this externality has no strategic effects, it is not internalized and is thus a source of inefficiency in the allocation of attention. In particular, the attention given in equilibrium to each source falls short of the efficient level (weakly) in the presence of a negative externality from dispersion, $u_{\sigma\sigma} < 0$, while it exceeds the efficient level (weakly) in the presence of a positive externality, $u_{\sigma\sigma} > 0$.

I now turn to economies that are inefficient in their use of information and where the inefficiency originates in the value the agents assign to coordination.

Proposition 4 *Consider economies in which (a) the complete-information equilibrium actions are first-best efficient, (b) there are no externalities from dispersion, and (c) inefficiencies in the use of information originate in the discrepancy between the equilibrium and the socially optimal degrees of coordination ($\kappa = \kappa^*$, $u_{\sigma\sigma} = 0$, but $\alpha \neq \alpha^*$). When $\alpha > \alpha^*$, agents pay too much attention to sources that are transparent and too little attention to sources that are opaque (Formally, there exists $R^* > 0$ such that, if $t_n/C_n'(\hat{z}) > R^*$, then $z_n^* \leq \hat{z}_n$, whereas, if $t_n/C_n'(\hat{z}) < R^*$, then $\hat{z}_n \leq z_n^*$, with the inequalities strict if $\hat{z}_n > 0$). The opposite conclusions hold for $\alpha < \alpha^*$. Furthermore, when $C(z) = c\left(\sum_{s=1}^N z_s\right)$ with $c(\cdot)$ increasing, convex, and differentiable, the equilibrium total attention is inefficiently low and too few sources receive positive attention (meaning that $\sum_{s=1}^N \hat{z}_s \leq \sum_{s=1}^N z_s^*$ and $\#\hat{N} \leq \#N^*$) if $\alpha > \alpha^*$, whereas the opposite conclusions hold if $\alpha < \alpha^*$.*

Take an economy in which $\alpha > \alpha^*$. Because there are no direct externalities from dispersion (i.e., $u_{\sigma\sigma} = 0$), the weight that each agent assigns to a reduction in the dispersion of her action around the mean action coincides with the weight assigned by the planner. The discrepancy between the private and the social value of increasing the attention allocated to any given source then comes from the inefficiency in the equilibrium use of information. Because agents in these economies are overconcerned about coordinating with others, the equilibrium actions are too sensitive to sources that are relatively public, (i.e., for which ρ_n is high) and too little sensitive to sources that are relatively private (i.e., for which ρ_n is low). Taking into account how the publicity of a source depends on its transparency, I then show that agents pay too much attention to those sources that are highly transparent relative to their cost and too little attention to those sources that are opaque relative to their cost.

To see this last result more explicitly, consider an economy satisfying the conditions in Example 1 above, where all sources receive strictly positive attention in equilibrium. It is easy to see that, as long as the difference $\alpha - \alpha^*$ is not too high so that the planner also wants the agents to allocate attention to all sources, then the efficient allocation of attention satisfies the analog of the conditions in (7) with α^* replacing α. It is then also easy to see that $z_n^* > \hat{z}_n$ if

$$\sqrt{t_n} < \frac{\pi_\theta + \sum_{s=1}^N \eta_s}{\sum_{s=1}^N \left(\eta_s / \sqrt{t_s}\right)}$$

whereas $z_n^* < \hat{z}_n$ if the above inequality is reversed. The proposition also shows that, when the cost depends only on the total attention, as is the case in Example 1 above, (a) too few sources receive attention in equilibrium and (b) the total attention is inefficiently low. The opposite conclusions hold in economies in which agents undervalue aligning their actions, i.e., for which $\alpha < \alpha^*$.

Lastly, consider economies in which inefficiencies in the allocation of attention originate entirely in the inefficiency of the complete-information equilibrium actions.

Proposition 5 *Consider economies in which (a) there are no externalities from dispersion, and (b) the equilibrium and the socially optimal degrees of coordination coincide ($u_{\sigma\sigma} = 0$, and $\alpha = \alpha^*$). The attention allocated to each source is inefficiently low if $\kappa_1^* > \kappa_1$ and inefficiently high if $\kappa_1^* < \kappa_1$ (meaning that, for any n, $z_n^* \geq \hat{z}_n$ if $\kappa_1^* > \kappa_1$ and $z_n^* \leq \hat{z}_n$ if $\kappa_1^* < \kappa_1$, with the inequalities strict if $\hat{z}_n > 0$).*

When $\kappa_1 < \kappa_1^*$, the complete-information equilibrium actions respond too little to changes in fundamentals relative to their first-best counterparts. As a result, the agents' incentives to learn the fundamentals are inefficiently low, so that the equilibrium level of attention is also inefficiently low, for all sources. The opposite conclusion holds when $\kappa_1 > \kappa_1^*$, i.e., when the complete-information actions overrespond to changes in fundamentals.

I conclude this section by looking at how welfare changes with the attention allocated to the various sources of information (around the equilibrium level) when the agents' actions are determined by the equilibrium rule $k(\cdot;\hat{z})$ instead of the efficient rule $k^*(\cdot;\hat{z})$—that is, when the *planner can not control the agents' actions*. In the Appendix, I show that, in this case, the gross marginal benefit of inducing the agents to increase the attention allocated to any given source is

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n}+|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum\nolimits_{s=1}^{N}\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\} \qquad (13)$$
$$+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\left\{\sum\nolimits_{s=1}^{N}\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\}.$$

The first term in (13) is the direct marginal effect of a reduction in the cross-sectional dispersion of individual actions that obtains as a result of an increase in the attention z_n, holding fixed the equilibrium use of information $k(\cdot;\hat{z})$. The second term combines the marginal effects of changing the equilibrium rule $k(\cdot;\hat{z})$ on (a) the volatility of the aggregate action K around its complete-information counterpart κ and (b) the dispersion of individual actions. Finally, the last term, which is relevant only in economies that are inefficient under complete information, captures the effect of changing the rule $k(\cdot;\hat{z})$ on the way the "error" due to incomplete information $K-\kappa$ covaries with the inefficiency of the complete-information allocation. Clearly, by usual envelope arguments, these last two terms are absent in economies where the equilibrium use of information is efficient (that is, in economies where $k(\cdot;z)=k^*(\cdot;z)$, which happens if, and only if, $\alpha=\alpha^*$ and $\kappa=\kappa^*$) or, alternatively, when the planner can dictate to the agents how to use their information.

The following result is then obtained by comparing (13) to the private value (5) of increasing the attention allocated to any given source (evaluated at the equilibrium level).

Proposition 6 *Suppose the planner can not change the way the agents respond to the impressions they receive from the various sources of information.*

(a) Consider economies that are either efficient in their use of information ($\kappa=\kappa^*$ and $\alpha=\alpha^*$) or in which the inefficiency in the allocation of attention originates in the inefficiency of the complete-information equilibrium actions ($u_{\sigma\sigma}=0$, $\alpha=\alpha^*$, but $\kappa\neq\kappa^*$). The same conclusions hold as in the case where the planner can control the agents' actions (as given in Propositions 3 and 5).

(b) Consider economies that are efficient under complete information, in which there are no externalities from dispersion, and in which inefficiencies in the allocation of attention originate in the discrepancy between the equilibrium and the socially optimal degrees of coordination ($\kappa=\kappa^*$, $u_{\sigma\sigma}=0$ but $\alpha\neq\alpha^*$). There exists a threshold $M>0$ such that, starting from the equilibrium allocation of attention $\hat{z}$, inducing the agents to increase the attention to any source for which $\hat{z}_n>0$ increases welfare if

$$sign\{\alpha-\alpha^*\}=sign\left\{\frac{C_n'(\hat{z})}{t_n}-M\right\}$$

and decreases it otherwise.

Consider first part (b) and take an economy where agents are over-concerned about aligning their actions ($\alpha > \alpha^*$). In these economies, equilibrium actions are too sensitive to transparent sources and too insensitive to opaque sources. Now use Proposition 1 to observe that the sensitivity $\kappa_1 \gamma_n$ of the equilibrium actions to each source is increasing in the attention allocated to that source and decreasing in the attention allocated to any other source. By inducing the agents to reallocate their attention from sources of high transparency to sources of low transparency, the planner then also induces the agents to use their information in a more efficient manner. This effect thus reinforces the conclusions in Proposition 4 for the case where the planner can control the way agents map their impressions into their actions.

Now, consider part (a) and focus on economies in which the inefficiency in the allocation of attention originates in the inefficiency of the complete-information actions. Specifically, suppose that the complete-information equilibrium actions respond too little to variations in the fundamentals (i.e., $\kappa_1 < \kappa_1^*$). Relative to the case where the planner can control the agents' actions, the extra benefit

$$\frac{|u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2}{\pi_\theta} \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1}\right) \left\{\sum_{s=1}^N \frac{\partial \gamma_s(z)}{\partial z_n}\right\}$$

of inducing the agents to pay more attention to any given source comes from the increased sensitivity of the agents' equilibrium actions to the impressions they obtain from that source. The net effect of this adjustment is to partially correct the inefficiency of the complete-information allocation by bringing the aggregate action K, on average, closer to its first-best counterpart κ^*. This novel effect then adds to the benefit of reducing the dispersion of individual actions in the population, thus reinforcing the conclusions of Proposition 5.

3 Bounded Recall

3.1 Environment

Having compared the equilibrium to the efficient allocation of attention in an environment in which the agents, at the time of committing their actions, respond separately to the individual sources of information, I now turn to the (perhaps more realistic) case of bounded recall. To isolate the novel effects in the sharpest possible way, I consider an extreme form of bounded recall by which the agents remember only their posterior beliefs about the exogenous fundamentals (equivalently, they receive a single Bayesian summary statistics from the different sources).

As anticipated in the Introduction, this assumption may either reflect a form of bounded rationality, or the idea that the various sources of information do not provide independent pieces of information. Instead, they contribute collectively to the formation of posterior beliefs.

I capture the above considerations by dropping the individual impressions x^j and by assuming that the attention the agents assign to the various sources maps directly into their posterior beliefs about θ. Apart from this modification, the environment is the same as in the previous section. In particular, beliefs continue to be consistent with Bayes rule and agents continue to understand how their posterior beliefs are shaped by the quality of the information sources and the attention they assign to them.

Formally, given the attention $z^j = (z_n^j)_{n=1}^N$, agent j's posterior beliefs about θ continue to be Normal with mean

$$\bar{x}^j(z^j) = \sum\nolimits_{n=1}^N \delta_n(z^j)\left(\theta + \varepsilon_n + \xi_n^j\right)$$

and precision $\pi_\theta + \sum_{s=1}^N \pi_s(z^j)$, where

$$\delta_n(z^j) \equiv \frac{\pi_n(z^j)}{\pi_\theta + \sum_{s=1}^N \pi_s(z^j)}, \text{ with } \pi_s(z^j) \equiv \frac{\eta_s z_s^j t_s}{z_s^j t_s + \eta_s}.$$

Notice that such beliefs coincide with the ones that the agent would have if he received a collection of impressions $x_n^j = \theta + \varepsilon_n + \xi_n^j$, one from each source of information, as in the case of perfect recall. Under this interpretation, bounded recall amounts to assuming that the individual is unable to keep track of the influence of individual sources of information on her posterior beliefs, and hence is unable to decompose her posterior beliefs $\bar{x}^j(z^j)$ into the impressions she received from the individual sources. Equivalently, as explained above, one can think of bounded recall as an alternative specification of the belief formation process by which the agent receives a single Bayesian summary statistics

$$X^j(z^j) \equiv \theta + \sum\nolimits_{n=1}^N \frac{\pi_n(z^j)}{\pi_X(z^j)}(\varepsilon_n + \xi_n^j)$$

from the sources of information, with precision[18]

$$\pi_X(z^j) \equiv \sum\nolimits_{s=1}^N \pi_s(z^j).$$

In either case, bounded recall amounts to imposing that agent j's actions be measurable in the sigma algebra generated by the random variable $\bar{x}^j(z^j)$. In this sense, the equilibrium under bounded recall also amounts to a specific form of *analogy-based equilibrium* (as defined in Jehiel 2005) in which the coarsening of the partitions of the agents' information sets is generated by grouping together information sets corresponding to the same posterior beliefs about θ. Importantly, this restriction matters only because of strategic effects. Because $\bar{x}^j$ is a sufficient statistics for $(\bar{x}^j, x^j)$ with respect to θ, in the absence of strategic effects, actions and payoff are the same as in the case of perfect recall.

3.2 Equilibrium allocation of attention

Let

$$\rho_X(z) \equiv \sum\nolimits_{s=1}^N \frac{\pi_s(z)}{\pi_X(z)}\rho_s(z)$$

[18] Note that $X^j(z^j)$ is nothing more than a deterministic transformation of the agent's posterior mean:

$$X^j(z^j) = \frac{\bar{x}^j(z^j)}{\sum_{s=1}^N \delta_s(z^j)} = \left(\frac{\pi_\theta + \pi_X(z^j)}{\pi_X(z^j)}\right)\bar{x}^j(z^j).$$

denote the weighted average of the endogenous publicity of the various sources of information, where the weights are the relative precisions.

Proposition 7 *There is a unique symmetric equilibrium. In this equilibrium, given the attention $z^{\#}$ allocated to the various sources, individual actions are given by*

$$k^i = k^{\#}(\bar{x}^i; z^{\#}) = \kappa_0 + \kappa_1 \gamma(z^{\#}) \cdot \bar{x}^j \tag{14}$$

all $i \in [0,1]$, where

$$\gamma^{\#}(z) \equiv \left(\frac{\frac{(1-\alpha)\pi_X(z)}{1-\alpha\rho_X(z)}}{\pi_\theta + \frac{(1-\alpha)\pi_X(z)}{1-\alpha\rho_X(z)}} \right) \left(\frac{\pi_\theta + \pi_X(z)}{\pi_X(z)} \right). \tag{15}$$

Furthermore, for any $i \in [0,1]$, any source $n = 1, ..., N$ that receives strictly positive attention in equilibrium,

$$C_n'(z^{\#}) = -\frac{|u_{kk}|}{2} \frac{\partial}{\partial z_n} Var\left[k - K; z^{\#}, k^{\#}(\cdot; z^{\#})\right] - \frac{|u_{kk}|}{2}(1-\alpha)\frac{\partial}{\partial z_n} Var\left[K - \kappa; z^{\#}, k^{\#}(\cdot; z^{\#})\right] \tag{16}$$

where the derivatives are computed holding fixed the mapping $k^{\#}(\cdot; z^{\#})$ given by (14).

There are important differences relative to the case of perfect recall. First, the marginal benefit of increasing the attention allocated to each source now has two components. The first one is the marginal reduction in the dispersion of individual actions around the mean action. This component is similar to the one in the case with perfect recall and is computed holding fixed all agents' strategies by the usual envelope reasoning. Importantly, in a symmetric equilibrium, the reduction of dispersion of individual actions around the mean action is the same irrespective of whether one changes only the individual's allocation of attention or all agents' allocation of attention (this observation, which is formally proved in the Appendix, is important when comparing the equilibrium with the efficient allocation of attention).[19]

The second component reflects the fact that, with bounded recall, not only the second moment but also the first moment of the distribution of each agent's own action is affected by the allocation of attention (this is true even if one holds fixed the mapping $k^{\#}(\cdot; z^{\#})$ by usual envelope arguments). The reason is that a change in the allocation of attention changes the weights δ_n that the posterior mean $\bar{x}^j$ assigns to the different sources and hence impacts the first moment of the distribution of $\bar{x}^j$. The second component in the right-hand side of (16) thus represents the marginal benefit of bringing an agent's own expected action, which in a symmetric equilibrium coincides with the average action in the population, closer to the complete-information equilibrium action. Importantly, while the

[19] This property is also true in the benchmark with bounded recall. There, however, the result is obvious, given that the distribution of the average action K is independent of the allocation of attention. In contrast, with bounded recall, the distribution of the average action depends on the allocation of attention, even when one holds fixed the agents' strategies $k^{\#}(\cdot; z)$. The reason is that the allocation of attention impacts the weights assigned by the posterior means to the various sources of information and hence the mean of the distribution of the agents' posteriors.

weight the individual assigns to reducing the dispersion of his own action around the mean action continues to be given by the curvature of individual payoffs u_{kk}, the weight the individual assigns to reducing the volatility of his expected action around the complete-information counterpart is given by $|u_{kk}|(1-\alpha) = -(u_{kk} + u_{kK})$, which takes into account also the response of the agent's action to variations in the average action.

The next result, which is one of the key predictions of the paper, shows how the above novel effects change the allocation of attention relative to the case of perfect recall.

Proposition 8 *Let $\hat{z}$ be the allocation of attention in the unique symmetric equilibrium under perfect recall. There exist thresholds ρ', ρ'' with $0 \leq \rho' \leq \rho'' \leq 1$ such that, starting from $\hat{z}$, any agent with bounded recall is better off by (a) locally increasing the attention allocated to any source for which $\rho_n(\hat{z}) \in [\rho', \rho'']$ and (b) locally decreasing the attention allocated to any source for which $\rho_n(\hat{z}) \notin [\rho', \rho'']$. Furthermore, when $\pi_\theta \to 0$, $\rho' < \rho_X(\hat{z}) < \rho''$ (with $\rho'' < 1$ for α large enough).*

Recall that the endogenous publicity of a source is given by

$$\rho_n(z) = \frac{\pi_n(z)}{\eta_n} = \frac{z_n t_n}{z_n t_n + \eta_n}$$

and that the latter measures how the total error $\varphi_n^j = \varepsilon_n + \xi_n^j$ in the source (combining the error at the origin ε_n with the error ξ_n^j in the agent's idiosyncratic interpretation of the source's content) correlates across any two agents. Sources of low publicity are thus sources whose endogenous precision $\pi_n(z)$ is small relative to the source's exogenous accuracy, η_n. A low publicity in turn may reflect either a low transparency t_n of the source or little attention z_n allocated by the agents. The information received from such sources is thus subject to significant idiosyncratic noise in the agents' interpretation of the source's content. In the case of perfect recall, the attention paid to any such source is justified by the source's high accuracy, which permits the decision maker to align well her action to the underlying fundamentals. Relative to that benchmark, under bounded recall, the benefit of allocating the same attention to any such source is reduced by the impossibility to take actions that respond separately to the noise in the source's interpretation. As a result, the decision maker reduces the attention she allocates to any such source.

Sources of high publicity are, instead, sources of potentially low accuracy but which receive significant attention under perfect recall because of their transparency. These sources thus serve primarily as coordination devices. With bounded recall, however, the coordination value of any such source is diminished by the impossibility to respond separately to the noise in the source's interpretation. As a result, the decision maker reduces the attention she allocates to any such source as well.

Finally, sources of intermediate publicity are good compromises: they permit the decision maker to align her action well both with the fundamentals and with other agents' actions. In the presence of bounded recall, the decision maker thus optimally increases her attention to such sources.

In the case in which the attention cost depends only on total attention, the monotone relationship between the publicity of the sources in the benchmark of perfect recall and their exogenous transparency then permits me to establish the following result:

Corollary 3 *Suppose that $C(z) = c\left(\sum_{s=1}^{N} z_s\right)$ with $c(\cdot)$ increasing, convex, and differentiable. Let $\hat{z}$ be the allocation of attention in the unique symmetric equilibrium under perfect recall. There exist thresholds $t', t'' \in \mathbb{R}_+$ such that, starting from $\hat{z}$, any agent with bounded recall is better off by (a) locally increasing the attention to any source for which $t_n \in [t', t'']$ and (b) locally decreasing the attention to any source for which $t_n \notin [t', t'']$.*

The results in Proposition 8 and in Corollary 3 refer to local properties of best responses, evaluated around the equilibrium allocation of attention $\hat{z}$ in the benchmark with perfect recall. Similar conclusions hold when one compares the allocation of attention in the *equilibrium* with bounded recall to its counterpart under perfect recall.

Proposition 9 *Let $C(z) = c\left(\sum_{s=1}^{N} z_s\right)$ with $c(\cdot)$ increasing, convex, and differentiable. Let $\hat{z}$ be the allocation of attention in the unique symmetric equilibrium with perfect recall and $z^{\#}$ the corresponding allocation of attention under bounded recall. There exist thresholds $t', t'' \in \mathbb{R}_{++}$ such that $z_n^{\#} > \hat{z}_n$ only if $t_n \in [t', t'']$. Furthermore for any n for which $t_n \in [t', t'']$, $z_n^{\#} < \hat{z}_n$ only if $z_n^{\#} = 0$.*

The result in Proposition 9 thus establishes that it is only those sources whose transparency is intermediate that receive more attention under bounded recall than under perfect recall. In this sense, Proposition 9 extends the results in Proposition 8 and Corollary 3 from individual best responses to equilibrium allocations. The key property in the Appendix that permits me to establish the result in the proposition is that, among those sources that do receive some attention under bounded recall, those whose transparency is the highest are also those whose publicity is the highest. Recall that this property also holds under perfect recall. In that benchmark, the monotonicity extends to all sources, implying that it is only those sources whose transparency is high enough that receive some attention in equilibrium. I could not establish this stronger property under bounded recall. In other words, I could not exclude the possibility that source n with transparency t_n receives some attention whereas source n' with transparency $t_{n'} > t_n$ receives no attention. This explains why the result in the proposition is not an "if and only if" result. However, what I could establish is that if a source of intermediate transparency receives less attention under bounded recall than under perfect recall, then it must be that it receives no attention at all.

Importantly, the result that, with bounded recall, the sources that receive an increase in attention (relative to the benchmark of full recall) are those of intermediate transparency has nothing to do with the *dimensionality* of the summary statistics of what the agents can recall. In fact, if the agents could *choose* which summary statistics to recall, they would simply pick the one corresponding to the equilibrium actions in the benchmark with full recall, in which case the allocation of attention would be unaffected by the inability to keep track of individual impressions. The notion of bounded recall considered here is meant to capture the idea that agents experience difficulty in choosing a summary statistics other than the Bayesian projection. To see this, consider the case of an improper prior ($\pi_\theta = 0$). Under bounder recall, the equilibrium actions are given by

$$k^i = \kappa_0 + \kappa_1 \left(\sum\nolimits_{n=1}^{N} \delta_n x_n^j \right),$$

whereas, under full recall, they are given by

$$k^i = \kappa_0 + \kappa_1 \left(\sum\nolimits_{n=1}^{N} \gamma_n x_n^i \right).$$

Because of strategic effects, agents would like to "summarize" the information from the various sources into the statistics $\sum_{n=1}^{N} \gamma_n x_n^j$, instead of the Bayesian projection $\sum_{n=1}^{N} \delta_n x_n^j$. Bounded recall amounts to a high cost of remembering any statistics other than the Bayesian projection. However, such an extreme cost function, while useful in illustrating the key ideas, is not essential to the results. Conclusions similar to those in the above propositions obtain also under less extreme cost functions, which may capture the idea that agents, over time, learn how to remember statistics other than their posterior beliefs.

3.3 Efficient allocation of attention

I now turn to the efficient allocation of attention under bounded recall. First note that, because the planner's problem is concave, it is never optimal to induce different agents to allocate different attention to the various sources of information. This in turn means that, for any symmetric allocation of attention z, efficiency in the agents' actions requires that, for any agent $i \in [0,1]$, almost any $\bar{x}^i$, the agent's action be given by

$$k^i = k^{**}(\bar{x}^i; z) = \kappa_0^* + \kappa_1^* \gamma^{**}(z) \bar{x}^i \tag{17}$$

with

$$\gamma^{**}(z) \equiv \left(\frac{\frac{(1-\alpha^*)\pi_X(z)}{1-\alpha^*\rho_X(z)}}{\pi_\theta + \frac{(1-\alpha^*)\pi_X(z)}{1-\alpha^*\rho_X(z)}} \right) \left(\frac{\pi_\theta + \pi_X(z)}{\pi_X(z)} \right) \tag{18}$$

where π_X and ρ_X are as defined above.[20] This implies that, for any z, the maximum welfare that can be achieved by having the agents follow the rule $k^{**}(\cdot; z)$ defined by (17) is given by

$$w^*(z) \equiv \mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)] - \mathcal{L}^*(z) - C(z), \tag{19}$$

where $u(\kappa^*, \kappa^*, 0, \theta)$ continues to denote welfare under the first-best allocation and where

$$\mathcal{L}^*(z) \equiv \frac{|u_{kk} + u_{\sigma\sigma}|}{2} Var[k - K \mid z, k^{**}(\cdot; z)] + \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} Var[K - \kappa^* \mid z, k^{**}(\cdot; z)]$$

[20] The result follows from the observation that bounded recall is mathematically equivalent to a setting in which agents receive a single signal $X = \left(\frac{\pi_\theta + \pi_X(z)}{\pi_X(z)} \right) \bar{x}$ about θ with precision $\pi_X(z)$ and correlation $\rho_X(z)$. The arguments that lead to the results below are then similar to those derived in the previous section.

continues to denote the welfare losses due to the incompleteness of information (combining the losses from the dispersion of individual actions around the mean action with the losses stemming from the volatility of the average action around its first-best counterpart). Using the fact that $|u_{kk} + 2u_{kK} + u_{KK}| = (1 - \alpha^*) |u_{kk} + u_{\sigma\sigma}|$, I then have the following result:

Proposition 10 *Suppose the planner can dictate to the agents how to map their posterior beliefs about θ into their actions. Let z^{**} denote the allocation of attention that maximizes welfare under bounded recall. Then, for any source of information that receives strictly positive attention,*

$$\begin{aligned} C_n'(z^{**}) = &-\frac{|u_{kk} + u_{\sigma\sigma}|}{2} \frac{\partial}{\partial z_n} Var[k - K \mid z^{**}, k^{**}(\cdot; z^{**})] \\ &- (1 - \alpha^*) \frac{|u_{kk} + u_{\sigma\sigma}|}{2} \frac{\partial}{\partial z_n} Var[K - \kappa^* \mid z^{**}, k^{**}(\cdot; z^{**})] \end{aligned} \tag{20}$$

where all derivatives are computed holding fixed the mapping $k^{**}(\cdot; z^{**})$ from the agents' posterior beliefs into their actions, as given by (17).

Comparing the social to the private marginal benefit of increasing the attention allocated to any given source, around the equilibrium levels, then permits me to establish the following result:

Proposition 11 *Suppose agents experience bounded recall. Let $z^{\#}$ denote the allocation of attention in the unique symmetric equilibrium and z^{**} the allocation of attention that maximizes welfare when the planner can dictate to the agents how to respond to their posterior beliefs about θ.*

(a) The same conclusions as in Proposition 3 and 5 hold for the comparison between $z^{\#}$ and z^{**}.

(b) Consider economies in which the complete-information actions are first-best efficient (i.e., $\kappa = \kappa^*$) and in which there are no externalities from the dispersion of individual actions (i.e., $u_{\sigma\sigma} = 0$). When $\alpha > \alpha^*$, there exists a critical threshold $\bar{\rho}(z^{\#}) \in [0, 1]$ such that, starting from $z^{\#}$, the planner would like the agents to reduce the attention they allocate to sources whose endogenous publicity $\rho_n(z^{\#}) > \bar{\rho}(z^{\#})$ and increase the attention they allocate to sources whose publicity $\rho_n(z^{\#}) < \bar{\rho}(z^{\#})$ and for which $z_n^{\#} > 0$. The opposite conclusion holds for economies in which $\alpha < \alpha^*$. In the limit in which $\pi_\theta \to 0$, $\bar{\rho}(z^{\#}) \to \rho_X(z^{\#})$ and $\gamma^{\#}(z^{\#}), \gamma^{**}(z^{\#}) \to 1$, implying that the planner would like the agents to allocate less attention to sources for which $(\alpha - \alpha^*) \left[\rho_n(z^{\#}) - \rho_X(z^{\#})\right] > 0$ and more attention to sources for which the inequality is reversed and $z_n^{\#} > 0$.

The results for the comparison between the equilibrium and the efficient allocation of attention under bounded recall thus parallel their counterparts in the benchmark with perfect recall. In particular, when agents value coordination more than the planner (i.e., when $\alpha > \alpha^*$), the planner would like them pay less attention to sources of high endogenous publicity and more attention to sources of low publicity, whereas the opposite is true in economies in which $\alpha < \alpha^*$. The publicity threshold $\bar{\rho}(z^{\#})$ that determines whether agents pay too much or too little attention to the various sources, however, need not coincide with the corresponding threshold under perfect recall. This is because, under bounded recall, the private and the social marginal benefit of higher attention also include the effect that the latter has on the distribution of the average actions around their complete-information counterparts, as discussed above.

4 Conclusions

In this paper, I compare the equilibrium to the efficient allocation of attention in a flexible model featuring a rich set of payoff interdependencies and an arbitrarily large number of information sources differing in their accuracy and transparency. I then examine how the allocation of attention is affected by bounded recall, defined as the difficulty to keep track of the influence of individual sources on posterior beliefs (more generally, of mapping impressions from individual sources into statistics other than the Bayesian projection).

In future work, it would be interesting to extend the analysis to a dynamic setting in which agents choose whether or not to pay attention to additional sources as a function of the information they receive from sources they visited already. A recent paper by Hébert and Woodford (2016) provides a useful step in this direction, albeit by abstracting from strategic effects.

It would also be interesting to examine how the allocation of attention interacts with the market provision of information by endogenizing the supply of information (see Galperti and Trevino (2016) for a recent paper endogenizing the supply of information). Such extension would also permit one to investigate policy interventions aimed at correcting jointly inefficiencies in the use and in the provision of information.

Appendix

Proof of Proposition 1. When all agents allocate attention z to the various sources of information, the continuation game in which the agents receive information x^j and choose their actions has a unique continuation equilibrium where all agents follow the linear strategy (4). This result follows from arguments similar to those that lead to Proposition 3 in Angeletos and Pavan (2009) — the proof is thus omitted.

Next, let

$$U^j(z^j;\hat{z}) = \mathbb{E}[u(k^j, K, \sigma_k, \theta)|z^j] - C(z^j)$$

denote agent j's expected payoff when all agents $i \neq j$ pay attention $\hat{z}$ to the different sources of information and then choose their actions according to (4), whereas agent j allocates attention z^j to the various sources and then chooses his actions optimally. It is easy to show that $U^j(z^j;\hat{z})$ is continuously right-differentiable in z_n^j, any n, any $(z^j;\hat{z})$, and that, for any $z_n^j > 0$ the derivative $\partial U^j(z^j;\hat{z})/\partial z_n^j$ coincides with the partial derivative of the agent's expected payoff holding fixed the agent's optimal strategy $k_i(\cdot;z^j;\hat{z})$ by usual envelope arguments.

Next, note that when $z^j = \hat{z}$, by symmetry, the agent's optimal strategy coincides with the one of any other agent, that is, $k_i(\cdot;z^j;\hat{z}) = k(\cdot;\hat{z})$ with $k(\cdot;\hat{z})$ given by (4). Furthermore, when all agents (including agent j) follow the linear strategy in (4), for any choice of z^j, agent j's expected payoff is given by

$$\mathbb{E}[u(K, K, \sigma_k, \theta) \mid z^j, k(\cdot;\hat{z})] + \frac{u_{kk}}{2} Var[k^j - K \mid z^j, k(\cdot;\hat{z})] - C(z^j) \tag{21}$$

where the first term in the right-hand side of (21) is the payoff the agent would obtain if his action coincided with the average action in the population in every state, while the second term is the ex-ante dispersion of the agent's own action around the mean action. Note that, when all agents follow the linear strategy in (4) — more generally, when their actions are determined by any linear mapping of their signals — the distribution of K is independent of the allocation of attention. It follows that, in any symmetric equilibrium, for any source n that receives positive attention

$$\frac{\partial U^j(\hat{z};\hat{z})}{\partial z_n^j} = \frac{u_{kk}}{2} \frac{\partial}{\partial z_n^j} Var[k^j - K \mid \hat{z}, k(\cdot;\hat{z})] - C_n'(\hat{z}) \tag{22}$$

where the derivative in the right hand side of (22) is computed holding fixed the mapping $k(\cdot;\hat{z})$ and letting such mapping be the one given by (4).

Next observe that, when all agents follow the mapping in (4),[21]

$$Var[k^j - K \mid z^j, k(\cdot;\hat{z})] = \kappa_1^2 \sum\nolimits_{n=1}^{N} \frac{(\gamma_n(\hat{z}))^2}{z_n^j t_n}.$$

[21] Note that, if $z_n^j = \hat{z}_n$, then $(\gamma_n(\hat{z}))^2/z_n^j t_n = 0$ when $\hat{z}_n = 0$, despite both the numerator and the denominator being equal to zero. The contribution of source n to the dispersion of the agent's own action around the mean action can thus be written as $(\gamma_n(\hat{z}))^2/z_n^j t_n$ for any source, irrespective of whether or not such source receives attention in equilibrium.

I conclude that, in any symmetric equilibrium, for any source of information that receives strictly positive attention, the following optimality condition must hold:

$$C_n'(\hat{z}) = \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(\hat{z}))^2}{\left(\hat{z}_n^j\right)^2 t_n}.$$

By continuity of the right-hand derivative $\partial U_+^j(z^j; \hat{z})/\partial z_n^j$, I also have that, for any source that receives no attention, the following corner condition must hold

$$C_n'(\hat{z}) \geq \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(\hat{z}))^2}{\left(\hat{z}_n^j\right)^2 t_n} = \frac{|u_{kk}|\kappa_1^2 (1-\alpha)^2 t_n}{2\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}\right]^2} = \frac{|u_{kk}|}{2} \frac{(\kappa_1)^2 (1-\alpha)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\eta_s \hat{z}_s t_s}{(1-\alpha)\hat{z}_s t_s + \eta_s}\right]^2},$$

which is equivalent to the condition that $\partial U_+^j(\hat{z}; \hat{z})/\partial z_n^j \leq 0$ at $\hat{z}_n = 0$.

Lastly, to see that the symmetric equilibrium is unique, let $\mathcal{U}$ denote the family of quadratic payoff functions satisfying all the conditions in the model setup. From arguments similar to those that lead to Proposition 2 in Angeletos and Pavan (2009), one can show that, given any $u \in \mathcal{U}$, there exists a unique $u' \in \mathcal{U}$ such that any symmetric equilibrium of the game where payoffs are given by u coincides with one of the efficient allocations for the economy with payoffs given by u'. Next observe that the efficient allocation for the economy with payoffs given by u' is unique – this follows from the fact that the planner's problem consisting in choosing a vector $z \in \mathbb{R}_+^N$ along with a function $k : \mathbb{R}^N \to \mathbb{R}$ so as to maximize the ex-ante expectation of u' is strictly concave. This in turn implies that the symmetric equilibrium for the economy with payoffs given by u is also unique, which establishes the result. Q.E.D.

Proof of Corollary 1. From Proposition 1, any source that receives strictly positive attention in equilibrium must satisfy (2). Substituting for

$$\gamma_n(\hat{z}) = \frac{\frac{(1-\alpha)\hat{z}_n t_n \eta_n}{(1-\alpha)\hat{z}_n t_n + \eta_n}}{\pi_\theta + \sum_{l=1}^N \frac{(1-\alpha)\hat{z}_l t_l \eta_l}{(1-\alpha)\hat{z}_l t_l + \eta_l}}$$

into Condition (2), I then have that

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \left\{ (1-\alpha)\sqrt{\frac{|u_{kk}|\kappa_1^2}{2C_n'(\hat{z})}} \frac{1}{M_1(\hat{z})} - \frac{1}{\sqrt{t_n}} \right\} \tag{23}$$

where

$$M_1(z) \equiv \pi_\theta + \sum_{l=1}^N \frac{(1-\alpha)\eta_l z_l t_l}{(1-\alpha)z_l t_l + \eta_l} > 0. \tag{24}$$

For the right-hand-side in (23) to be positive, it must be that

$$\frac{t_n}{C_n'(\hat{z})} > R \equiv \frac{2\left(M_1(\hat{z})\right)^2}{(1-\alpha)^2 \kappa_1^2 |u_{kk}|}, \tag{25}$$

which establishes the first claim in the Corollary.

Next, I prove that, for any source that receives no attention in equilibrium, condition (25) must be violated. To see this, suppose that, by contradiction, there exists a source n for which (25) holds and such that $\hat{z}_n = 0$. Suppose that the individual were to increase locally the attention allocated to this source. The continuity of the right-hand derivative of the agent's expected payoff $\partial U_{+}^{j}(\hat{z};\hat{z})/\partial z_n^j$ implies that the net effect on the agent's expected payoff is

$$\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2\,t_n} - C_n'(\hat{z}) = \frac{|u_{kk}|\kappa_1^2}{2}\frac{(1-\alpha)^2 t_n}{(M_1(\hat{z}))^2} - C_n'(\hat{z}) > 0,$$

contradicting the optimality of the equilibrium allocation of attention. Q.E.D.

Proof of Example 1. Suppose that all sources receive strictly positive attention in equilibrium. The amount of attention allocated to each source n is then equal to

$$\hat{z}_n = \sqrt{\frac{|u_{kk}|\kappa_1^2}{2\bar{c}}\frac{\gamma_n(\hat{z})}{\sqrt{t_n}}}. \tag{26}$$

It follows that the influence of each source n is given by

$$\gamma_n(\hat{z}) = \sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sqrt{t_n}\hat{z}_n. \tag{27}$$

Combining the above with the fact that

$$\gamma_n(\hat{z}) = \frac{\frac{(1-\alpha)\pi_n(\hat{z})}{1-\alpha\rho_n(\hat{z})}}{\pi_\theta + \sum_{s=1}^{N}\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}} \tag{28}$$

I then have that

$$\sum_{n=1}^{N}\gamma_n(\hat{z}) = \frac{\sum_{s=1}^{N}\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}}{\pi_\theta + \sum_{s=1}^{N}\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}} = \sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sum_{n=1}^{N}\sqrt{t_n}\hat{z}_n.$$

This implies that

$$\pi_\theta + \sum_{s=1}^{N}\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})} = \frac{\pi_\theta}{1-\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sum_{s=1}^{N}\sqrt{t_s}\hat{z}_s}.$$

Replacing the latter expression into the definition of $\gamma_n(\hat{z})$ in (28) and using the fact that

$$\frac{(1-\alpha)\pi_n(\hat{z})}{1-\alpha\rho_n(\hat{z})} = \frac{(1-\alpha)\eta_n\hat{z}_n t_n}{\hat{z}_n t_n(1-\alpha)+\eta_n}$$

I then have that

$$\gamma_n(\hat{z}) = \frac{\frac{(1-\alpha)\eta_n\hat{z}_n t_n}{\hat{z}_n t_n(1-\alpha)+\eta_n}}{\frac{\pi_\theta}{1-\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sum_{s=1}^{N}\sqrt{t_s}\hat{z}_s}}.$$

Combining this expression with (26) I then have that

$$\hat{z}_n = \left[\frac{1}{\pi_\theta \sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}} - \frac{1}{\pi_\theta} \sum_{s=1}^{N} \sqrt{t_s} \hat{z}_s \right] \frac{1}{\sqrt{t_n}} \eta_n - \frac{\eta_n}{(1-\alpha) t_n}. \tag{29}$$

Multiplying both sides of (29) by $\sqrt{t_n}$, summing over n, and rearranging, I then obtain that

$$\frac{1}{\pi_\theta} \sum_{s=1}^{N} \sqrt{t_s} \hat{z}_s = \frac{\frac{\sum_{s=1}^{N} \eta_s}{\pi_\theta \sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}} - \frac{1}{(1-\alpha)} \sum_{s=1}^{N} \frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s=1}^{N} \eta_s}. \tag{30}$$

Replacing (30) into (29), I conclude that

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \left[\frac{(1-\alpha)\sqrt{\frac{|u_{kk}|\kappa_1^2}{2\bar{c}}} + \sum_{s=1}^{N} \frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s=1}^{N} \eta_s} - \frac{1}{\sqrt{t_n}} \right]$$

as claimed. Q.E.D.

Proof of Proposition 3. The proof follows directly from the results in Corollary 2. Q.E.D.

Proof of Proposition 4. Note that, in these economies, for any z, and any n, the discrepancy between the social and the private benefit of increasing the attention allocated to source n is proportional to the difference[22]

$$\frac{(\gamma_n^*(z))^2}{(z_n)^2 t_n} - \frac{(\gamma_n(z))^2}{(z_n)^2 t_n}. \tag{31}$$

Using the expressions for γ and γ^*, the difference in (31) is equal to

$$m^*(z) \frac{t_n \eta_n^2}{[(1-\alpha^*) z_n t_n + \eta_n]^2} - m(z) \frac{t_n \eta_n^2}{[(1-\alpha) z_n t_n + \eta_n]^2}$$

where

$$m(z) \equiv \frac{(1-\alpha)^2}{\left[\pi_\theta + \sum_{l=1}^{N} \frac{(1-\alpha) z_l t_l \eta_l}{(1-\alpha) z_l t_l + \eta_l}\right]^2} \text{ and } m^*(z) \equiv \frac{(1-\alpha^*)^2}{\left[\pi_\theta + \sum_{l=1}^{N} \frac{(1-\alpha^*) z_l t_l \eta_l}{(1-\alpha^*) z_l t_l + \eta_l}\right]^2}.$$

It follows that, starting from the equilibrium allocation of attention $\hat{z}$, the social benefit of increasing the attention allocated to source n exceeds the private benefit if and only if

$$\frac{(1-\alpha) \hat{z}_n t_n + \eta_n}{(1-\alpha^*) \hat{z}_n t_n + \eta_n} \geq \sqrt{\frac{m(\hat{z})}{m^*(\hat{z})}}. \tag{32}$$

Now note that, when $\alpha > \alpha^*$, $m(\hat{z}) < m^*(\hat{z})$. This means that the social benefit exceeds the private benefit for any source that receives no attention in equilibrium. The opposite conclusion holds

[22]Note that the comparison here applies also to sources that receive no attention, i.e., for which $z_n = 0$.

when $\alpha < \alpha^*$. Thus consider sources that receive strictly positive attention in equilibrium. Using (23),

$$\hat{z}_n t_n = \frac{\eta_n \sqrt{t_n}}{\sqrt{C_n'(\hat{z})}} Q(\hat{z}) - \frac{\eta_n}{(1-\alpha)} \tag{33}$$

where

$$Q(z) \equiv \sqrt{\frac{|u_{kk}|\kappa_1^2}{2}} \frac{1}{M_1(z)}$$

with the function $M_1(\cdot)$ as defined in (24). Using (33), I can rewrite the left-hand-side of (32) as follows

$$\frac{(1-\alpha)Q(\hat{z})\sqrt{\frac{t_n}{C_n'(\hat{z})}}}{(1-\alpha^*)Q(\hat{z})\sqrt{\frac{t_n}{C_n'(\hat{z})}} + \frac{\alpha^*-\alpha}{1-\alpha}}$$

which is decreasing in $t_n/C_n'(\hat{z})$ for $\alpha > \alpha^*$ and increasing in $\frac{t_n}{C_n'(\hat{z})}$ for $\alpha < \alpha^*$.

I conclude that, when $\alpha > \alpha^*$, there exists a critical value $R^* > 0$ such that, starting from the equilibrium allocation of attention $\hat{z}$, the planner would like the agents to locally increase the attention allocated to any source of information that receives positive attention in equilibrium and such that $t_n/C_n'(\hat{z}) < R^*$ and decrease the attention allocated to any source that receives positive attention and for which $t_n/C_n'(\hat{z}) > R^*$. The opposite conclusions hold for $\alpha < \alpha^*$.

Lastly, consider the case where $C(z) = c(\hat{Z})$ with $\hat{Z} \equiv \sum_{s=1}^N z_s$ and with $c(\cdot)$ strictly increasing, convex, and continuously differentiable. To prove the two claims in the proposition, note that, in these economies, the efficient allocation $(z^*, k(\cdot; z^*))$ coincides with the equilibrium allocation of another economy that differs from the original one only in the degree of coordination. It thus suffices to show that the equilibrium total attention $\hat{Z}$ (as well as the number of sources $\#\hat{N}$ that receive strictly positive attention in equilibrium) decrease with α.

Let $\hat{N}(\alpha)$ denote the subset of sources that receive strictly positive attention when the equilibrium degree of coordination is α. Now use the results in the proof of Corollary 1 to see that the attention allocated in equilibrium to each source n is given by

$$\hat{z}_n(\alpha) = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \max\left\{T(\alpha) - \frac{1}{\sqrt{t_n}}; 0\right\} \tag{34}$$

where

$$T(\alpha) \equiv (1-\alpha)\sqrt{\frac{|u_{kk}|\,\kappa_1^2}{2c'(\hat{Z}(\alpha))}} \frac{1}{M_1(\hat{z}(\alpha))} \tag{35}$$

with

$$\hat{Z}(\alpha) = \sum_{l=1}^N \hat{z}_l(\alpha) \tag{36}$$

and

$$M_1(\hat{z}(\alpha)) = \pi_\theta + \sum_{l=1}^N \frac{(1-\alpha)\eta_l \hat{z}_l(\alpha) t_l}{(1-\alpha)\hat{z}_l(\alpha) t_l + \eta_l}. \tag{37}$$

Combining (34)-(37), I then have that, for any α, $T(\alpha)$ is the unique solution to the following equation

$$\frac{T}{1-\alpha}\sqrt{c'\left(\sum_{l=1}^{N}\frac{\eta_l}{\sqrt{t_l}(1-\alpha)}\max\left\{T-\frac{1}{\sqrt{t_l}};0\right\}\right)}\left(\pi_\theta+\sum_{l=1}^{N}\frac{\eta_l\sqrt{t_l}\max\left\{T-\frac{1}{\sqrt{t_l}};0\right\}}{\sqrt{t_l}\max\left\{T-\frac{1}{\sqrt{t_l}};0\right\}+1}\right)=\kappa_1\sqrt{\frac{|u_{kk}|}{2}}. \tag{38}$$

Because the left-hand-side of (38) is increasing in both α and T, I then have that $T(\alpha)$ is decreasing in α. This means that the critical level of transparency required for each source to receive positive attention in equilibrium increases with α. In turn, this implies that that $\hat{N}(\alpha')\subset\hat{N}(\alpha)$ for any $\alpha'>\alpha$, which in turn implies that $\#\hat{N}(\alpha)$ decreases with α, as claimed.

Next, to see that the total attention $\hat{Z}(\alpha)$ also decreases with α, follow steps similar to those in the proof of Example 1 to see that, for each source $n\in\hat{N}(\alpha)$ that receives strictly positive attention,

$$\hat{z}_n(\alpha)=\frac{\eta_n}{\sqrt{t_n}(1-\alpha)}\left[\frac{(1-\alpha)\sqrt{\frac{|u_{kk}|\kappa_1^2}{2c'\left(\sum_{s\in\hat{N}(\alpha)}\hat{z}_s(\alpha)\right)}}+\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta+\sum_{s\in\hat{N}(\alpha)}\eta_s}-\frac{1}{\sqrt{t_n}}\right].$$

Summing over all $n\in\hat{N}(\alpha)$, I then have that

$$\begin{aligned}\sum_{n\in\hat{N}(\alpha)}\hat{z}_n(\alpha)=&\frac{1}{\sqrt{2c'\left(\sum_{s\in\hat{N}(\alpha)}\hat{z}_s(\alpha)\right)}}\left(\frac{\sqrt{|u_{kk}|\kappa_1^2}\sum_{n\in\hat{N}(\alpha)}\frac{\eta_n}{\sqrt{t_n}}}{\pi_\theta+\sum_{s\in\hat{N}(\alpha)}\eta_s}\right)\\&+\frac{1}{1-\alpha}\left\{\frac{\left[\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{\sqrt{t_s}}\right]^2}{\pi_\theta+\sum_{s\in\hat{N}(\alpha)}\eta_s}-\sum_{n\in\hat{N}(\alpha)}\frac{\eta_n}{t_n}\right\}.\end{aligned}$$

Holding $\hat{N}(\alpha)$ fixed, I then have that

$$\frac{\partial}{\partial\alpha}\left(\sum_{s\in\hat{N}(\alpha)}\hat{z}_s(\alpha)\right)\overset{sign}{=}\frac{\left[\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{\sqrt{t_s}}\right]^2}{\pi_\theta+\sum_{s\in\hat{N}(\alpha)}\eta_s}-\sum_{n\in\hat{N}(\alpha)}\frac{\eta_n}{t_n} \tag{39}$$

Below, I show that

$$\left(\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{\sqrt{t_s}}\right)^2-\left(\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{t_s}\right)\left(\sum_{s\in\hat{N}(\alpha)}\eta_s\right)\leq 0$$

which implies that the sign of the right-hand side of (39) is always negative.

To see this, it suffices to note that

$$\left(\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{\sqrt{t_s}}\right)^2-\left(\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{t_s}\right)\left(\sum_{s\in\hat{N}(\alpha)}\eta_s\right)=\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s^2}{t_s}+\sum_{s\in\hat{N}(\alpha)}\sum_{k\in\hat{N}(\alpha),k\neq s}\frac{\eta_s\eta_k}{\sqrt{t_s}\sqrt{t_k}}$$
$$-\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s^2}{t_s}-\sum_{s\in\hat{N}(\alpha)}\sum_{k\in\hat{N}(\alpha),k\neq s}\frac{\eta_s\eta_k}{t_s}$$
$$=\sum_{s,k\in\hat{N}(\alpha),k\neq s}\left[\eta_s\eta_k\left(\frac{2}{\sqrt{t_st_k}}-\frac{1}{t_s}-\frac{1}{t_k}\right)\right]<0.$$

Along with the property established above that $\hat{N}(\alpha')\subset\hat{N}(\alpha)$ for any $\alpha'>\alpha$, the fact that, for given $\hat{N}(\alpha)$, $\sum_{s\in\hat{N}(\alpha)}\hat{z}_s(\alpha)$ is decreasing in α implies that $\hat{Z}(\alpha)$ decreases with α. Q.E.D.

Proof of Proposition 5. The proof follows directly from the results in Corollary 2. Q.E.D.

Derivation of Condition (13). First observe that, for any given z, welfare under the equilibrium strategy $k(\cdot;z)$ is given by[23]

$$w(z)\equiv\mathbb{E}[u(k,K,\sigma_k,\theta)\mid z,k(\cdot;z)]-C(z)=\mathbb{E}[W(\kappa,0,\theta)]-\mathcal{L}(z)-C(z),\tag{40}$$

where $W(K,0,\theta)\equiv u(K,K,0,\theta)$ is the payoff that each agent obtains when all agents take the same action ($W(\kappa,0,\theta)$ is thus welfare under the complete-information equilibrium allocation $\kappa=\kappa_0+\kappa_1\theta$), whereas

$$\mathcal{L}(z)\equiv\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\cdot Var[k-K\mid z,k(\cdot;z)]+\frac{|u_{kk}+2u_{kK}+u_{KK}|}{2}\cdot Var[K-\kappa\mid z,k(\cdot;z)]$$
$$-Cov\left[K-\kappa,W_K(\kappa,0,\theta)\mid z,k(\cdot;z)\right]$$

are the welfare losses due to incomplete information. The first two terms in $\mathcal{L}$ measure the welfare losses due to, respectively, the dispersion of individual actions around the aggregate action and the volatility of the aggregate action around its complete-information counterpart. The last term captures losses (or gains) due to the correlation between the 'aggregate error' due to incomplete information, $K-\kappa$, and W_K, the social return to aggregate activity. Following steps similar to those in Angeletos and Pavan (2007) one can show that

$$Cov\left[K-\kappa,W_K(\kappa,0,\theta)\mid z,k(\cdot;z)\right]=|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\frac{\left[\sum_n\gamma_n(z)-1\right]}{\pi_\theta},$$

$$Var[K-\kappa\mid z,k(\cdot;z)]=\kappa_1^2\frac{\left[\sum_{s=1}^N\gamma_s(z)-1\right]^2}{\pi_\theta}+\sum_{s=1}^N\frac{(\kappa_1\gamma_s(z))^2}{\eta_s},$$

[23] The representation of equilibrium welfare in (40) follows from the same steps as in Angeletos and Pavan (2007); the proof is thus omitted.

and

$$Var[k - K \mid z, k(\cdot; z)] = \sum_{s=1}^{N} \frac{(\kappa_1 \gamma_s(z))^2}{z_s t_s}.$$

Welfare under the equilibrium strategy $k(\cdot; z)$ can thus be expressed as

$$\begin{aligned} w(z) = \mathbb{E}[W(\kappa, 0, \theta)] &- \frac{|u_{kk} + u_{\sigma\sigma}|\kappa_1^2}{2} \cdot \left\{ \sum_{s=1}^{N} \frac{(\gamma_s(z))^2}{z_s t_s} \right\} \\ &- \frac{|u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2}{2} \cdot \left\{ \frac{\left[\sum_{s=1}^{N} \gamma_s(z) - 1\right]^2}{\pi_\theta} + \sum_{s=1}^{N} \frac{(\gamma_s(z))^2}{\eta_s} \right\} \\ &+ |u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2 \cdot \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1} \right) \cdot \frac{\sum_{s=1}^{N} \gamma_s(z) - 1}{\pi_\theta} - C(z). \end{aligned}$$

The (gross) marginal effect on welfare of an increase in the attention allocated to the n-th source is thus equal to

$$\begin{aligned} &\frac{|u_{kk} + u_{\sigma\sigma}|}{2} \frac{(\kappa_1 \gamma_n(z))^2}{(z_n)^2 t_n} - |u_{kk} + u_{\sigma\sigma}|\kappa_1^2 \cdot \left\{ \sum_{s=1}^{N} \frac{\gamma_s(z)}{z_s t_s} \frac{\partial \gamma_s(z)}{\partial z_n} \right\} \\ &- |u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2 \left\{ \frac{\left[\sum_{s=1}^{N} \gamma_s(z) - 1\right] \left(\sum_{s=1}^{N} \frac{\partial \gamma_s(z)}{\partial z_n} \right)}{\pi_\theta} + \sum_{s=1}^{N} \frac{\gamma_s(z)}{\eta_s} \frac{\partial \gamma_s(z)}{\partial z_n} \right\} \\ &+ \frac{|u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2}{\pi_\theta} \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1} \right) \left\{ \sum_{s=1}^{N} \frac{\partial \gamma_s(z)}{\partial z_n} \right\}. \end{aligned} \tag{41}$$

Substituting $|u_{kk} + 2u_{kK} + u_{KK}| = (1 - \alpha^*)|u_{kk} + u_{\sigma\sigma}|$, I can rewrite the sum of the second and third addendum in (41) as

$$\begin{aligned} &- |u_{kk} + u_{\sigma\sigma}|\kappa_1^2 \left\{ \frac{(1 - \alpha^*) \left[\sum_{s=1}^{N} \gamma_s(z) - 1\right] \left(\sum_{s=1}^{N} \frac{\partial \gamma_s(z)}{\partial z_n} \right)}{\pi_\theta} + \sum_{s=1}^{N} \left[\frac{1 - \alpha^*}{\eta_s} + \frac{1}{z_s t_s} \right] \gamma_s \frac{\partial \gamma_s(z)}{\partial z_n} \right\} \\ &= -|u_{kk} + u_{\sigma\sigma}|\kappa_1^2 \left\{ \sum_{s=1}^{N} \left(\frac{\left[\frac{1-\alpha^*}{\eta_s} + \frac{1}{z_s t_s} \right] \frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)} - (1 - \alpha^*)}{\pi_\theta + \sum_{n=1}^{N} \frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}} \right) \frac{\partial \gamma_s(z)}{\partial z_n} \right\}. \end{aligned}$$

Using

$$\pi_s(z) \equiv \frac{\eta_s z_s t_s}{z_s t_s + \eta_s} \text{ and } \rho_s(z) = \frac{z_s t_s}{z_s t_s + \eta_s}$$

I then have that

$$\begin{aligned} &\left[\frac{1 - \alpha^*}{\eta_s} + \frac{1}{z_s t_s} \right] \frac{(1 - \alpha)\pi_s(z)}{1 - \alpha\rho_s(z)} - (1 - \alpha^*) \\ &= \frac{[(1 - \alpha^*)z_s t_s + \eta_s](1 - \alpha)}{[(1 - \alpha)z_s t_s + \eta_s]} - \frac{[(1 - \alpha)z_s t_s + \eta_s](1 - \alpha^*)}{[(1 - \alpha)z_s t_s + \eta_s]} \\ &= \frac{[(1 - \alpha^*)z_s t_s + \eta_s](1 - \alpha) - [(1 - \alpha)z_s t_s + \eta_s](1 - \alpha^*)}{[(1 - \alpha)z_s t_s + \eta_s]} \\ &= -\frac{\eta_s(\alpha - \alpha^*)}{[(1 - \alpha)z_s t_s + \eta_s]}. \end{aligned}$$

The sum of the second and third addendum in (41) can thus be rewritten as

$$|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\eta_s}{[(1-\alpha)z_st_s+\eta_s]\left[\pi_\theta+\sum_{n=1}^N\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}\right]}\right)\frac{\partial\gamma_s(z)}{\partial z_n}\right\}.$$

Next, note that

$$\frac{\eta_s}{[(1-\alpha)z_st_s+\eta_s]\left[\pi_\theta+\sum_{n=1}^N\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}\right]}=\frac{\gamma_s(z)}{(1-\alpha)z_st_s}.$$

I conclude that the gross marginal benefit of increasing the attention allocated to the n-th source is given by

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(z))^2}{(z_n)^2t_n}+|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\gamma_s(z)}{(1-\alpha)z_st_s}\right)\frac{\partial\gamma_s(z)}{\partial z_n}\right\}$$
$$+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^N\frac{\partial\gamma_s(z)}{\partial z_n}\right\}.$$

Q.E.D.

Proof of Proposition 6. Consider first part (b). Using (13), note that, in these economies, the net benefit of inducing the agents to pay more attention to source n is given by

$$\frac{\partial w(\hat z)}{\partial z_n}=\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat z))^2}{(\hat z_n)^2t_n}+|u_{kk}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\gamma_s(\hat z)}{(1-\alpha)\hat z_st_s}\right)\frac{\partial\gamma_s(\hat z)}{\partial z_n}\right\}-C_n'(\hat z).$$

Using the fact that the private net marginal benefit is equal to

$$\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat z))^2}{(\hat z_n)^2t_n}-C_n'(\hat z)$$

I then have that the social benefit exceeds the private benefit if and only if

$$sign\left\{\alpha-\alpha^*\right\}=sign\left\{\sum_{s=1}^N\left(\frac{\gamma_s(\hat z)}{(1-\alpha)\hat z_st_s}\right)\frac{\partial\gamma_s(\hat z)}{\partial z_n}\right\}.$$

Next, observe that

$$\sum_{s=1}^N\left(\frac{\gamma_s(\hat z)}{(1-\alpha)\hat z_st_s}\right)\frac{\partial\gamma_s(\hat z)}{\partial z_n}$$
$$=-\sum_{s=1}^N\left\{\frac{\gamma_s(\hat z)}{(1-\alpha)\hat z_st_s}\frac{\frac{(1-\alpha)\eta_s\hat z_st_s}{(1-\alpha)\hat z_st_s+\eta_s}}{\left(\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat z_lt_l}{(1-\alpha)\hat z_lt_l+\eta_l}\right)^2}\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat z_nt_n}{(1-\alpha)\hat z_nt_n+\eta_n}\right)\right\}$$
$$+\frac{\gamma_n(\hat z)}{(1-\alpha)\hat z_nt_n}\frac{\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat z_nt_n}{(1-\alpha)\hat z_nt_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat z_lt_l}{(1-\alpha)\hat z_lt_l+\eta_l}}$$
$$=\frac{\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat z_nt_n}{(1-\alpha)\hat z_nt_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat z_lt_l}{(1-\alpha)\hat z_lt_l+\eta_l}}\left\{\frac{\gamma_n(\hat z)}{(1-\alpha)\hat z_nt_n}-\sum_{s=1}^N\left(\frac{(\gamma_s(\hat z))^2}{(1-\alpha)\hat z_st_s}\right)\right\}.$$

Clearly,

$$\frac{\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n \hat{z}_n t_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\right)}{\pi_\theta + \sum_{l=1}^{N} \frac{(1-\alpha)\eta_l \hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l+\eta_l}} > 0.$$

Hence,

$$sign\left\{\sum_{s=1}^{N}\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\} = sign\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} - \sum_{s=1}^{N}\left(\frac{(\gamma_s(\hat{z}))^2}{(1-\alpha)\hat{z}_s t_s}\right)\right\}.$$

This means that the social benefit exceeds the private benefit if and only if

$$sign\left\{\alpha - \alpha^*\right\} = sign\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} - M_0(\hat{z})\right\},$$

where $M_0(z) \equiv \sum_{s=1}^{N}\left(\frac{(\gamma_s(z))^2}{(1-\alpha)z_s t_s}\right) > 0$ does not depend on the source of information. Now observe that

$$\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} = \frac{\eta_n}{(1-\alpha)\hat{z}_n t_n + \eta_n}\frac{1}{M_1(\hat{z})}$$

where $M_1(\cdot)$ is the function defined in (24). Lastly use (2) to note that, for any source that receives positive attention in equilibrium,

$$(1-\alpha)\hat{z}_n t_n + \eta_n = M_2(\hat{z})\sqrt{\frac{t_n \eta_n^2}{C_n'(\hat{z})}}$$

where $M_2(z) \equiv \sqrt{\frac{|u_{kk}|\kappa_1^2 (M_1(z))^2 (1-\alpha)^2}{2}}$. I conclude that there exists a constant

$$M(\hat{z}) \equiv \left[M_0(\hat{z})M_1(\hat{z})M_2(\hat{z})\right]^2 > 0$$

such that

$$sign\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} - M_0(\hat{z})\right\} = sign\left\{\frac{C_n'(\hat{z})}{t_n} - M(\hat{z})\right\}.$$

The result in the proposition then follows.

Next, consider part (a). The result for economies that are efficient in their use of information follows directly from Proposition 3, given that, in these economies, the impossibility to dictate to the agents how to use their information is inconsequential. The result for economies where the inefficiency in the allocation of attention originates in the inefficiency of the complete-information actions follows from (13) along with the fact that, in these economies,[24]

$$\frac{\partial w(\hat{z})}{\partial z_n} = \frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n} + \frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^* - \kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^{N}\frac{\partial\gamma_s(z)}{\partial z_n}\right\}$$
$$- C_n'(\hat{z})$$

and the fact that

$$\sum_{s=1}^{N}\gamma_s(z) = \frac{1}{\frac{\pi_\theta}{\sum_{s=1}^{N}\frac{(1-\alpha)\eta_s z_s t_s}{(1-\alpha)z_s t_s+\eta_s}} + 1}$$

[24] If $\hat{z}_n = 0$, then interpret the derivative as the right-hand derivative.

is increasing in z_n. Q.E.D.

Proof of Proposition 7. First I prove that, when all agents allocate attention z to the various sources of information, the continuation game that starts when the agents, after observing their posterior beliefs, must choose their actions has a unique continuation equilibrium where all agents follow the linear strategy

$$k^i = k^{\#}(\bar{x}^i; z) \equiv \kappa_0 + \kappa_1 \gamma^{\#}(z)\bar{x}^i. \tag{42}$$

To see this, recall that observing the posterior mean $\bar{x}^i$ is informationally equivalent to observing the signal

$$\frac{\bar{x}^i}{\sum_{n=1}^N \delta_n(z)} = \left(\frac{\pi_X(z) + \pi_\theta}{\pi_X(z)}\right)\bar{x}^i \equiv \theta + \sum\nolimits_{n=1}^N \frac{\pi_n(z)}{\pi_X(z)}(\varepsilon_n + \xi_n^i)$$

with precision $\pi_X(z) \equiv \sum_{s=1}^N \pi_s(z)$ and with an error whose correlation across any pair of agents $i, j \in [0,1]$, $j \neq i$, is given by

$$\begin{aligned}\rho_X(z) &\equiv Corr\left(\sum\nolimits_{n=1}^N \frac{\pi_n(z)}{\pi_X(z)}(\varepsilon_n + \xi_n^j); \sum\nolimits_{n=1}^N \frac{\pi_n(z)}{\pi_X(z)}(\varepsilon_n + \xi_n^i)\right) \\ &= \sum\nolimits_{s=1}^N \frac{\pi_s(z)}{\pi_X(z)}\rho_s(z).\end{aligned}$$

This game is isomorphic to the one in Section 2, with the only difference that each agent receives a single signal. From Proposition 1 I then have that, in the unique continuation equilibrium, individual actions are given by (42).

Next, I characterize the allocation of attention in any symmetric equilibrium. To this purpose, suppose that all agents $i \neq j$ assign attention $z^i = z$ to the different sources of information and then use (42) to determine their actions. Let $U^j(z^j; z)$ denote the payoff of agent j when he assigns attention z^j to the different sources and then chooses optimally the mapping from his posterior into his actions. Using the envelope theorem, in any symmetric equilibrium, for any source for which $z_n^{\#} > 0$, $\partial U^j(z^{\#}; z^{\#})/\partial z_n^j$ must coincide with the partial derivative of the agent's expected payoff with respect to z_n^j, holding fixed the mapping $k^{\#}(\cdot; z^{\#})$ from the agent's posterior means to his actions and letting this mapping be the one in (42).[25]

Next observe that, when all agents (including agent j) follow (42), then

$$\begin{aligned}U^j(z^j; z) &= \mathbb{E}[u(K, K, \sigma_k, \theta) \mid z^j, z] + \mathbb{E}[u_k(K, K, \sigma_k, \theta)(k^j - K) \mid z^j, z] \\ &\quad + \frac{u_{kk}}{2}\mathbb{E}[\left(k^j - K\right)^2 \mid z^j, z] - C(z^j)\end{aligned}$$

where the first term in the right-hand side of (21) is the expected payoff of an agent whose action coincides with the average action in the population in every state. Importantly, note that (i) because the mapping $k^{\#}(\cdot; z)$ is kept fixed, $\mathbb{E}[u(K, K, \sigma_k, \theta) \mid z^j, z]$ is independent of the agent's

[25] Furthermore, for any source for which $z_n^{\#} = 0$, the right-hand derivative $\partial U_+^j(z^{\#}; z^{\#})/\partial z_n^j$ must coincide with the limit for $z_n \to 0^+$ of the derivative $\partial U^j((z_n, z_{-n}^{\#}); (z_n, z_{-n}^{\#}))/\partial z_n^j$ by continuity of the right-hand derivative.

own information and (ii) all expectations are computed assuming all agents' actions are determined by the linear strategy in (42).

Next observe that

$$\mathbb{E}[\left(k^j - K\right)^2 \mid z^j, z] = \mathbb{E}[\left(k^j - K^j\right)^2 + (K^j - K)^2 + 2\left(k^j - K^j\right)\left(K^j - K\right) \mid z^j, z]$$

where $K^j \equiv \mathbb{E}[k^j \mid (\theta, \varepsilon), z^j]$ denotes the agent's own average action given (θ, ε), when his attention is z^j. Using the fact that, for any z and z^j, $k^j - K^j = \kappa_1 \gamma^{\#}(z) \left\{\sum_n \delta_n(z^j)\xi_n^j\right\}$ is orthogonal to $K^j - K = \kappa_1 \gamma^{\#}(z) \left\{\sum_n (\delta_n(z^j) - \delta_n(z))(\theta + \varepsilon_n)\right\}$, I then have that

$$\begin{aligned}\frac{\partial}{\partial z_n^j}\mathbb{E}[\left(k^j - K\right)^2 \mid z, z] &= \frac{\partial}{\partial z_n^j}\mathbb{E}[\left(k^j - K^j\right)^2 \mid z, z] + \frac{\partial}{\partial z_n^j}\mathbb{E}[\left(K^j - K\right)^2 \mid z, z] \\ &= \frac{\partial}{\partial z_n^j}\mathbb{E}[\left(k^j - K^j\right)^2 \mid z, z] = \frac{\partial}{\partial z_n} Var\left[k - K \mid z, k^{\#}(\cdot; z)\right]\end{aligned}$$

where all derivatives are computed holding fixed the agents' strategies, as given by (42). Note that the second equality follows from the fact that, at a symmetric equilibrium (i.e., for $z^j = z$),

$$\frac{\partial}{\partial z_n^j}\mathbb{E}[\left(K^j - K\right)^2 \mid z, z] = 0$$

whereas the third equality uses the fact that, in a symmetric equilibrium, the dispersion of each agent's action around his own average action coincides with the dispersion of each agent's action around the mean action in the cross-section of the population (in the notation for such dispersion, I explicitly write the strategy $k^{\#}(\cdot; z)$ to make clear that the distribution of individual and aggregate actions is obtained by letting the agents follow the mapping in (42)). Importantly, note that the derivative

$$\frac{\partial}{\partial z_n} Var\left[k - K \mid z, k^{\#}(\cdot; z)\right]$$

is again computed holding fixed the agents' strategies and takes into account the fact that an increase in z_n affects the dispersion of individual actions both directly by changing the distribution of x_j and indirectly by changing the weights $\delta_s(z)$ in the agents' posterior means.

Finally, consider the term $\mathbb{E}[u_k(K, K, \sigma_k, \theta)(k^j - K) \mid z^j, z]$. Using the fact that

$$u_k(K, K, \sigma_k, \theta) = u_k(\kappa, \kappa, 0, \theta) + (u_{kk} + u_{kK})(K - \kappa),$$

along with the fact that $u_k(\kappa, \kappa, 0, \theta) = 0$ by definition of the complete-information equilibrium, I have that

$$\begin{aligned}\mathbb{E}[u_k(K, K, \sigma_k, \theta)(k^j - K) \mid z^j, z] &= (u_{kk} + u_{kK}) \cdot \mathbb{E}[(K - \kappa)(k^j - K) \mid z^j, z] \\ &= (u_{kk} + u_{kK}) \cdot \mathbb{E}[(K - \kappa)(K^j - K) \mid z^j, z]\end{aligned}$$

where the second equality uses the fact that $k^j - K^j$ is orthogonal to $K - \kappa$. Now observe that

$$\frac{\partial}{\partial z_n^j}\mathbb{E}[(K-\kappa)(K^j-K) \mid z, z] = \mathbb{E}\left[(K-\kappa)\frac{\partial(K^j-K)}{\partial z_n^j} \mid z, z\right]$$
$$= \kappa_1\gamma^{\#}(z)\mathbb{E}\left[(K-\kappa)\left(\sum_{s=1}^{N}\frac{\partial\delta_s(z)}{\partial z_n}(\theta+\varepsilon_s)\right) \mid z, z\right]$$
$$= \kappa_1^2\gamma^{\#}(z)\cdot Cov\left[\left(\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)(\theta+\varepsilon_s)-\theta\right); \left(\sum_{s=1}^{N}\frac{\partial\delta_s(z)}{\partial z_n}(\theta+\varepsilon_s)\right) \mid z, z\right]$$
$$= \kappa_1^2\gamma^{\#}(z)\cdot\left\{\left(\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)-1\right)\left(\sum_{s=1}^{N}\frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\pi_\theta}+\gamma^{\#}(z)\sum_{s=1}^{N}\left(\delta_s(z)\frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\eta_s}\right\}.$$

Let $\frac{\partial}{\partial z_n}Var\left[K-\kappa \mid z, k^{\#}(\cdot;z)\right]$ denote the marginal change in the dispersion of K around κ that obtains when one changes the attention allocated to the n-th source, holding fixed the strategy in (42). Then observe that

$$\frac{1}{2}\frac{\partial}{\partial z_n}Var\left[K-\kappa \mid z, k^{\#}(\cdot;z)\right] \tag{43}$$
$$= \frac{1}{2}\frac{\partial}{\partial z_n}Var\left[\kappa_1\left(\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)(\theta+\varepsilon_s)-\theta\right) \Bigm| z, k^{\#}(\cdot;z)\right]$$
$$= \frac{\kappa_1^2}{2}\frac{\partial}{\partial z_n}Var\left[\left(\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)-1\right)\theta+\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)\varepsilon_s\right]$$
$$= \frac{\kappa_1^2}{2}\frac{\partial}{\partial z_n}\left[\left(\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)-1\right)^2\frac{1}{\pi_\theta}+\left(\gamma^{\#}(z)\right)^2\sum_{s=1}^{N}\delta_s(z)^2\frac{1}{\eta_s}\right]$$
$$= \kappa_1^2\gamma^{\#}(z)\left\{\left(\gamma^{\#}(z)\sum_{s=1}^{N}\delta_s(z)-1\right)\left(\sum_{s=1}^{N}\frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\pi_\theta}+\gamma^{\#}(z)\sum_{s=1}^{N}\left(\delta_s(z)\frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\eta_s}\right\}$$
$$= \frac{\partial}{\partial z_n^j}\mathbb{E}[(K-\kappa)(K^j-K) \mid z, z].$$

Combining the different pieces and using the fact that $|u_{kk}|(1-\alpha) = -(u_{kk}+u_{kK})$, I conclude that

$$\frac{\partial U^j(z;z)}{\partial z_n^j} = -\frac{|u_{kk}|}{2}\frac{\partial}{\partial z_n}Var\left[k-K \mid z, k^{\#}(\cdot;z)\right] \tag{44}$$
$$-\frac{|u_{kk}|}{2}(1-\alpha)\frac{\partial}{\partial z_n}Var\left[K-\kappa \mid z, k^{\#}(\cdot;z)\right]-C_n'(z).$$

Clearly, in any symmetric equilibrium, for any source of information $n = 1, ..., N$ that receives strictly positive attention, it must be that the above derivative vanishes, which yields (16) in the main text.

Finally, the uniqueness of the symmetric equilibrium follows from arguments similar to those that establish uniqueness in the model with perfect recall; the proof is thus omitted for brevity. Q.E.D.

Proof of Proposition 8. The proof is in two steps. Step 1 computes the gross private benefit (under bounded recall) of increasing the attention to any source of information, starting from any level of attention z. Step 2 then uses the characterization from Step 1 to examine how the marginal benefit of increasing the attention to any given source, starting from the equilibrium allocation of attention $\hat{z}$ in the benchmark with perfect recall, depends on the endogenous publicity of the source.

Step 1. Under bounded recall, the marginal benefit of increasing the attention allocated to any source n is given by (44). Below, I express the various terms in (44) as a function of the parameters of the information structure. To simplify the exposition, I drop z from the arguments of the various functions, when there is no risk of confusion.

First observe that

$$\frac{\partial}{\partial z_n} Var\left[k - K \mid z, k^{\#}(\cdot; z)\right] = \left(\kappa_1 \gamma^{\#}\right)^2 \frac{\partial}{\partial z_n} Var\left(\sum_{s=1}^{N} \delta_s \xi_s\right).$$

Next observe that

$$\frac{\partial}{\partial z_n} Var\left(\sum_{s=1}^{N} \delta_s \xi_s\right) = \frac{\partial}{\partial z_n}\left[\sum_{s=1}^{N} \frac{\delta_s^2}{t_s z_s}\right] = \sum_{s=1}^{N} \frac{2\delta_s}{t_s z_s} \frac{\partial \delta_s}{\partial z_n} - \frac{\delta_n^2}{t_n z_n^2} = 2\sum_{s\neq n}^{N} \frac{\delta_s}{t_s z_s} \frac{\partial \delta_s}{\partial z_n} + 2\frac{\delta_n}{t_n z_n}\frac{\partial \delta_n}{\partial z_n} - \frac{\delta_n^2}{t_n z_n^2}$$
$$= -2\sum_{s\neq n}^{N} \frac{\delta_s}{t_s z_s} \frac{\pi_s \frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2} + 2\frac{\delta_n}{t_n z_n}\left(-\frac{\pi_n \frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2} + \frac{\frac{\partial \pi_n}{\partial z_n}}{\pi_\theta + \pi_X}\right) - \frac{\delta_n^2}{t_n z_n^2}$$
$$= -2\frac{\frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2} \sum_{s=1}^{N} \frac{\delta_s \pi_s}{t_s z_s} + 2\frac{\delta_n}{t_n z_n}\frac{\frac{\partial \pi_n}{\partial z_n}}{\pi_\theta + \pi_X} - \frac{\delta_n^2}{t_n z_n^2}$$
$$= -2\frac{\frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2}\left(\sum_{s=1}^{N} \frac{\delta_s \pi_s}{t_s z_s} - \frac{\delta_n}{t_n z_n}(\pi_\theta + \pi_X)\right) - \frac{\delta_n^2}{t_n z_n^2}$$
$$= -2\frac{\frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2}\left(\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N} \frac{\pi_s}{\pi_X}\frac{\pi_s}{t_s z_s} - \frac{\pi_n}{t_n z_n}\right) - \frac{\pi_n}{(\pi_\theta + \pi_X)^2}\frac{\pi_n}{t_n z_n}\frac{1}{z_n}.$$

Now recall that $\pi_s = \frac{\eta_s z_s t_s}{z_s t_s + \eta_s}$, which means that $\frac{\pi_s}{t_s z_s} = \frac{\eta_s}{z_s t_s + \eta_s}$ and that $\frac{\partial \pi_n}{\partial z_n} = \frac{\pi_n}{z_n}\frac{\pi_n}{t_n z_n}$. I thus have that

$$\frac{\partial}{\partial z_n} Var\left(\sum_{s=1}^{N} \delta_s \xi_s\right) = \frac{-2}{(\pi_\theta + \pi_X)^2}\frac{\pi_n}{t_n z_n}\frac{\pi_n}{z_n}\left[\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\pi_s}{t_s z_s} - \frac{\pi_n}{t_n z_n}\right] - \frac{1}{(\pi_\theta + \pi_X)^2}\frac{\pi_n}{t_n z_n}\frac{\pi_n}{z_n}$$
$$= -\frac{1}{(\pi_\theta + \pi_X)^2}\frac{\eta_n^2 t_n}{(z_n t_n + \eta_n)^2}\left\{1 + 2\left[\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\eta_s}{z_s t_s + \eta_s} - \frac{\eta_n}{z_n t_n + \eta_n}\right]\right\}.$$

This means that

$$\frac{\partial}{\partial z_n} Var\left[k - K \mid z, k^{\#}(\cdot; z)\right] \tag{45}$$
$$= -\frac{\left(\kappa_1 \gamma^{\#}\right)^2}{(\pi_\theta + \pi_X)^2}\frac{\eta_n^2 t_n}{(z_n t_n + \eta_n)^2}\left\{1 + 2\left[\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\eta_s}{z_s t_s + \eta_s} - \frac{\eta_n}{z_n t_n + \eta_n}\right]\right\}.$$

Next, use (43) to observe that

$$
\begin{aligned}
&\frac{1}{2}\frac{\partial}{\partial z_n} Var\left[K-\kappa \mid z, k^{\#}(\cdot; z)\right] \\
&= \kappa_1^2\gamma^{\#}\left\{\left(\gamma^{\#}\sum_{s=1}^{N}\delta_s - 1\right)\left(\sum_{s=1}^{N}\frac{\partial \delta_s}{\partial z_n}\right)\frac{1}{\pi_\theta} + \gamma^{\#}\sum_{s=1}^{N}\left(\delta_s\frac{\partial \delta_s}{\partial z_n}\right)\frac{1}{\eta_s}\right\}.
\end{aligned}
$$

Note that

$$
\sum_{s=1}^{N}\frac{\partial \delta_s}{\partial z_n} = \frac{\frac{\partial \pi_n}{\partial z_n}\pi_\theta}{\left(\pi_X+\pi_\theta\right)^2}
$$

and that

$$
\gamma^{\#}\sum_{s=1}^{N}\delta_s - 1 = \frac{\gamma^{\#}\pi_X}{\pi_X+\pi_\theta} - 1 = -\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\pi_X+\pi_\theta}.
$$

Hence

$$
\left(\gamma^{\#}\sum_{s=1}^{N}\delta_s - 1\right)\left(\sum_{s=1}^{N}\frac{\partial \delta_s}{\partial z_n}\right)\frac{1}{\pi_\theta} = -\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)^3}\frac{\partial \pi_n}{\partial z_n}.
$$

Also note that

$$
\gamma^{\#}\sum_{s=1}^{N}\left(\delta_s\frac{\partial \delta_s}{\partial z_n}\right)\frac{1}{\eta_s} = \gamma^{\#}\sum_{s=1}^{N}\left(\frac{\partial \delta_s}{\partial z_n}\right)\frac{\rho_s}{\pi_\theta+\pi_X} = -\gamma^{\#}\frac{\partial \pi_n}{\partial z_n}\sum_{s=1}^{N}\left(\frac{\pi_s\rho_s}{\left(\pi_\theta+\pi_X\right)^3}\right) + \gamma^{\#}\frac{\partial \pi_n}{\partial z_n}\frac{\rho_n}{\left(\pi_\theta+\pi_X\right)^2}.
$$

It follows that

$$
\begin{aligned}
&\frac{\partial}{\partial z_n} var\left[K-\kappa \mid z, k^{\#}(\cdot; z)\right] \qquad (46)\\
&= 2\kappa_1^2\gamma^{\#}\left\{-\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)^3}\frac{\partial \pi_n}{\partial z_n} - \gamma^{\#}\frac{\partial \pi_n}{\partial z_n}\sum_{s=1}^{N}\left(\frac{\pi_s\rho_s}{\left(\pi_\theta+\pi_X\right)^3}\right) + \gamma^{\#}\frac{\partial \pi_n}{\partial z_n}\frac{\rho_n}{\left(\pi_\theta+\pi_X\right)^2}\right\} \\
&= -2\left(\kappa_1\gamma^{\#}\right)^2\frac{\frac{\partial \pi_n}{\partial z_n}}{\left(\pi_X+\pi_\theta\right)^2}\left\{\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)\gamma^{\#}} - \left(\rho_n - \frac{\pi_X}{\pi_\theta+\pi_X}\rho_X\right)\right\} \\
&= -2\frac{\left(\kappa_1\gamma^{\#}\right)^2}{\left(\pi_X+\pi_\theta\right)^2}\frac{\eta_n^2 t_n}{\left(z_n t_n+\eta_n\right)^2}\left\{\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)\gamma^{\#}} + \frac{\pi_X}{\pi_\theta+\pi_X}\rho_X - \rho_n\right\}.
\end{aligned}
$$

Substituting (45) and (46) into (44), I conclude that, for any source n and any allocation of attention z,

$$
\begin{aligned}
\frac{\partial U^j(z;z)}{\partial z_n} &= \frac{|u_{kk}|\left(\kappa_1\gamma^{\#}\right)^2}{\left(\pi_\theta+\pi_X\right)^2}\frac{\eta_n^2 t_n}{\left(z_n t_n+\eta_n\right)^2}\left\{\frac{1}{2} + \frac{\pi_X}{\pi_\theta+\pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\eta_s}{z_s t_s+\eta_s} - \frac{\eta_n}{z_n t_n+\eta_n}\right\} \qquad (47)\\
&+ \frac{|u_{kk}|(1-\alpha)\left(\kappa_1\gamma^{\#}\right)^2}{\left(\pi_X+\pi_\theta\right)^2}\frac{\eta_n^2 t_n}{\left(z_n t_n+\eta_n\right)^2}\left\{\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)\gamma^{\#}} + \frac{\pi_X}{\pi_\theta+\pi_X}\rho_X - \rho_n\right\} - C_n'(z).
\end{aligned}
$$

Equivalently, (47) can be simplified to

$$
\begin{aligned}
\frac{\partial U^j(z;z)}{\partial z_n} &= \frac{|u_{kk}|\left(\kappa_1\gamma^{\#}\right)^2}{\left(\pi_\theta+\pi_X\right)^2}\frac{\eta_n^2 t_n}{\left(z_n t_n+\eta_n\right)^2}\left\{-\frac{1}{2} + \frac{\pi_X}{\pi_\theta+\pi_X} + \frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)\gamma^{\#}}\right\} \qquad (48)\\
&- \alpha\frac{|u_{kk}|\left(\kappa_1\gamma^{\#}\right)^2}{\left(\pi_X+\pi_\theta\right)^2}\frac{\eta_n^2 t_n}{\left(z_n t_n+\eta_n\right)^2}\left\{\frac{(1-\gamma^{\#})\pi_X+\pi_\theta}{\left(\pi_X+\pi_\theta\right)\gamma^{\#}} + \frac{\pi_X}{\pi_\theta+\pi_X}\rho_X - \rho_n\right\} - C_n'(z).
\end{aligned}
$$

Step 2. From the proof of Proposition 1, one can see that, with perfect recall, the gross benefit of increasing (locally) the attention to any source of information, around the equilibrium level $\hat{z}$, is given by (again, I drop the dependence of the various functions on z when there is no risk of confusion):

$$\begin{aligned}
&\frac{|u_{kk}|(\kappa_1)^2}{2}\frac{1}{(\hat{z}_n)^2 t_n}\left(\frac{\frac{(1-\alpha)\hat{\pi}_n}{1-\alpha\hat{\rho}_n}}{\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\hat{\pi}_s}{1-\alpha\hat{\rho}_s}}\right)^2 \qquad (49)\\
&=\frac{|u_{kk}|(\kappa_1)^2}{2}\frac{(1-\alpha)^2(\eta_n\hat{z}_n t_n)^2}{(\hat{z}_n t_n+\eta_n)^2(\hat{z}_n)^2 t_n}\frac{1}{(1-\alpha\hat{\rho}_n)^2\left(\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\hat{\pi}_s}{1-\alpha\hat{\rho}_s}\right)^2}\\
&=\frac{|u_{kk}|(\kappa_1)^2(1-\alpha)^2}{2}\frac{\eta_n^2 t_n}{(\hat{z}_n t_n+\eta_n)^2}\frac{1}{(1-\alpha\hat{\rho}_n)^2\left(\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\hat{\pi}_s}{1-\alpha\hat{\rho}_s}\right)^2}
\end{aligned}$$

where

$$\hat{\pi}_s\equiv\frac{\eta_s\hat{z}_s t_s}{\hat{z}_s t_s+\eta_s}\text{ and }\hat{\rho}_s\equiv\frac{\hat{\pi}_s}{\eta_s}.$$

From (48), one can also see that, starting from $\hat{z}$, the gross benefit of increasing (locally) the attention to the n-th source for an agent with bounded recall is given by

$$\begin{aligned}
&\frac{|u_{kk}|\left(\kappa_1\hat{\gamma}^{\#}\right)^2}{(\pi_\theta+\pi_X)^2}\frac{\eta_n^2 t_n}{(\hat{z}_n t_n+\eta_n)^2}\cdot \qquad (50)\\
&\cdot\left\{-\frac{1}{2}+\frac{\hat{\pi}_X}{\pi_\theta+\hat{\pi}_X}+\frac{(1-\alpha)(1-\hat{\gamma}^{\#})\hat{\pi}_X+\pi_\theta}{(\hat{\pi}_X+\pi_\theta)\hat{\gamma}^{\#}}-\alpha\frac{\hat{\pi}_X}{\pi_\theta+\hat{\pi}_X}\hat{\rho}_X+\alpha\hat{\rho}_n\right\}
\end{aligned}$$

where $\hat{\pi}_X\equiv\sum_{s=1}^N\hat{\pi}_s$, $\hat{\rho}_X\equiv\sum_{s=1}^N\frac{\hat{\pi}_s}{\hat{\pi}_X}\hat{\rho}_s$, and $\hat{\gamma}^{\#}=\gamma^{\#}(\hat{z})$.

Comparing (49) with (50), it is then easy to see that the gross benefit is larger in the presence of bounded recall if

$$\begin{aligned}
&2\alpha\left[\hat{\rho}_n-\frac{\hat{\pi}_X}{\pi_\theta+\hat{\pi}_X}\hat{\rho}_X\right]> \qquad (51)\\
&\frac{(\pi_\theta+\hat{\pi}_X)^2(1-\alpha)^2}{(\hat{\gamma}^{\#})^2(1-\alpha\hat{\rho}_n)^2\left(\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\hat{\pi}_s}{1-\alpha\hat{\rho}_s}\right)^2}+1-2\frac{\hat{\pi}_X}{\pi_\theta+\hat{\pi}_X}-\frac{2(1-\alpha)(1-\hat{\gamma}^{\#})\hat{\pi}_X+\pi_\theta}{(\hat{\pi}_X+\pi_\theta)\hat{\gamma}^{\#}}
\end{aligned}$$

and lower if the inequality is reversed. Next note that the inequality in (51) can be rewritten as

$$2\alpha\hat{\rho}_n>\hat{L}_1\frac{1}{(1-\alpha\hat{\rho}_n)^2}+\hat{A}_1 \qquad (52)$$

where

$$\hat{A}_1\equiv 1-2\frac{\hat{\pi}_X}{\pi_\theta+\hat{\pi}_X}-\frac{2(1-\alpha)(1-\hat{\gamma}^{\#})\hat{\pi}_X+\pi_\theta}{(\hat{\pi}_X+\pi_\theta)\hat{\gamma}^{\#}}+2\alpha\frac{\hat{\pi}_X}{\pi_\theta+\hat{\pi}_X}\hat{\rho}_X$$

and

$$\hat{L}_1\equiv\frac{(\pi_\theta+\hat{\pi}_X)^2(1-\alpha)^2}{(\hat{\gamma}^{\#})^2\left(\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\hat{\pi}_s}{1-\alpha\hat{\rho}_s}\right)^2}$$

The right-hand side of (52) is convex in $\hat{\rho}_n$ whereas the left-hand side is linear in $\hat{\rho}_n$. This means that there exist $\rho', \rho'' \in [0,1]$ with $0 \leq \rho' \leq \rho'' \leq 1$ such that the inequality in (51) holds if and only if $\hat{\rho}_n \in [\rho', \rho'']$ whereas the opposite inequality holds if and only if $\hat{\rho}_n \notin [\rho', \rho'']$.

Finally, observe that, when $\pi_\theta \to 0$, $\hat{\gamma}^{\#} \to 1$, $\hat{A}_1 \to -1 + 2\alpha\hat{\rho}_X$ and

$$\hat{L}_1 \to \frac{1}{\left(\sum_{s=1}^{N} \frac{\hat{\pi}_s}{\hat{\pi}_X} \frac{1}{1-\alpha\hat{\rho}_s}\right)^2}.$$

The inequality in (52) then reduces to

$$1 + 2\alpha[\hat{\rho}_n - \hat{\rho}_X] > \frac{1}{(1-\alpha\hat{\rho}_n)^2 \left(\sum_{s=1}^{N} \frac{\hat{\pi}_s}{\hat{\pi}_X} \frac{1}{1-\alpha\hat{\rho}_s}\right)^2}. \tag{53}$$

Because the function defined by

$$f(\rho) \equiv \frac{1}{1-\alpha\rho}$$

is convex, by Jensen inequality,

$$\sum_{s=1}^{N} \frac{\hat{\pi}_s}{\hat{\pi}_X} \frac{1}{1-\alpha\hat{\rho}_s} > \frac{1}{1-\alpha\hat{\rho}_X}.$$

If follows that the inequality in (53) always holds for sources for which $|\hat{\rho}_n - \hat{\rho}_X|$ is small, implying that, in this case, $\rho' < \hat{\rho}_X < \rho''$. Also note that, when $\alpha = 1$, the inequality in (53) is reversed for ρ close to 1. By continuity, one then has that $\rho'' < 1$ for α large enough. Likewise, one can verify that, for α large enough, the inequality in (53) can be reversed when evaluated at ρ_n close to zero. This means that there can be situations in which $\rho' > 0$ for α large enough. Q.E.D.

Proof of Corollary 3. The result follows from Proposition 8 along with the fact that, when the attention cost depends only on total attention, then under perfect recall, there is an increasing relationship between the exogenous transparency of the sources and their endogenous publicity. Namely, if $t_n > t_{n'}$, then $\rho_n(\hat{z}) \geq \rho_{n'}(\hat{z})$. To see this, use the results in the proof of Proposition 4 to note that, for any source n,

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \max\left\{T(\alpha) - \frac{1}{\sqrt{t_n}}; 0\right\} \tag{54}$$

where $T(\alpha)$ is as defined in the proof of Proposition 4. Clearly, for all sources for which $t_n \leq 1/[T(\alpha)]^2$, $\hat{z}_n = 0$ and hence $\rho_n(\hat{z}) = 0$. On the other hand, for all sources for which $t_n > 1/[T(\alpha)]^2$,

$$\rho_n(\hat{z}) = \frac{\hat{z}_n t_s}{\hat{z}_n t_n + \eta_n} = \frac{\sqrt{t_n}T(\alpha) - 1}{\sqrt{t_n}T(\alpha) - \alpha}$$

which is increasing in t_n. Q.E.D.

Proof of Proposition 9. Use (48) to observe that, when $C(z) = c(\sum_{l=1}^{N} z_l)$, in the equilibrium with *bounded* recall, for any source n for which $z_n^{\#} > 0$, the following condition must hold

$$z_n^{\#} = \frac{\eta_n}{\sqrt{t_n}} \left\{A^{\#}\sqrt{B^{\#} + \alpha\rho_n^{\#}} - \frac{1}{\sqrt{t_n}}\right\}$$

with

$$A^{\#} \equiv \sqrt{\frac{|u_{kk}| \left(\kappa_1 \gamma^{\#}\right)^2}{c'(Z^{\#}) \left(\pi_\theta + \pi_X^{\#}\right)^2}} \text{ and } B^{\#} \equiv -\frac{1}{2} + \frac{\pi_X^{\#}}{\pi_\theta + \pi_X^{\#}} + \frac{(1-\alpha)(1-\gamma^{\#})\pi_X^{\#} + \pi_\theta}{\left(\pi_X^{\#} + \pi_\theta\right)\gamma^{\#}} - \alpha \frac{\pi_X^{\#}}{\pi_\theta + \pi_X^{\#}} \rho_X^{\#},$$

where $\gamma^{\#}$, $\pi_n^{\#}$, $\rho_n^{\#}$, $\pi_X^{\#}$, $\rho_X^{\#}$, and $Z^{\#}$ are shortcuts for $\gamma^{\#}(z^{\#})$, $\pi_n(z^{\#})$, $\rho_n(z^{\#})$, $\pi_X(z^{\#})$, $\rho_X(z^{\#})$, and $Z^{\#} \equiv \sum_{l=1}^N z_l^{\#}$, respectively. Furthermore, for any source that receives no attention in equilibrium, the following condition must hold

$$c'(Z^{\#}) \geq \frac{|u_{kk}| \left(\kappa_1 \gamma^{\#}\right)^2}{\left(\pi_\theta + \pi_X^{\#}\right)^2} t_n B^{\#}.$$

Next, use the results in the proof of Proposition 4 to observe that, when $C(z) = c(\sum_{l=1}^N z_l)$, in the equilibrium with *full* recall, for any source n,

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \max\left\{\hat{T} - \frac{1}{\sqrt{t_n}}; 0\right\}$$

where

$$\hat{T} \equiv (1-\alpha)\sqrt{\frac{|u_{kk}| \left(\kappa_1\right)^2}{2c'(\hat{Z})}} \frac{1}{M_1(\hat{z})}$$

with

$$\hat{Z} = \sum_{l=1}^N \hat{z}_l$$

and

$$M_1(\hat{z}) \equiv \pi_\theta + \sum_{l=1}^N \frac{(1-\alpha)\eta_l \hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l + \eta_l}.$$

From the above observations, I conclude that

$$z_n^{\#} > \hat{z}_n \;\Rightarrow\; A^{\#}\sqrt{B^{\#} + \alpha\rho_n^{\#}} + \frac{\alpha}{1-\alpha}\frac{1}{\sqrt{t_n}} > \frac{\hat{T}}{1-\alpha} \text{ and} \tag{55}$$

$$0 < z_n^{\#} < \hat{z}_n \;\Rightarrow\; A^{\#}\sqrt{B^{\#} + \alpha\rho_n^{\#}} + \frac{\alpha}{1-\alpha}\frac{1}{\sqrt{t_n}} < \frac{\hat{T}}{1-\alpha}.$$

Finally, use the definition of the publicity of a source to observe that, for any source n for which $z_n^{\#} > 0$,

$$\rho_n^{\#} = 1 - \frac{1}{A^{\#}\sqrt{t_n}\sqrt{B^{\#} + \alpha\rho_n^{\#}}}. \tag{56}$$

That is, the publicity $\rho_n^{\#}$ of any source that receives some attention in the equilibrium with bounded recall must solve the following equation:

$$\left[1 - \rho_n^{\#}\right]\sqrt{B^{\#} + \alpha\rho_n^{\#}} = \frac{1}{A^{\#}\sqrt{t_n}}. \tag{57}$$

Next observe that the left-hand-side of Condition (57) is decreasing in ρ when $\alpha \leq 0$. In this case, Condition (57) implicitly defines an increasing function $\rho^{\#}(t)$ between the transparency t and the publicity $\rho^{\#}$ of those sources that receive attention in the equilibrium with bounded recall. The same is true when $\alpha > 0$. To see this, fix $B^{\#}$, let

$$\underline{\rho}^{\#} \equiv \begin{cases} 0 \text{ if } B^{\#} \geq 0 \\ -\frac{B^{\#}}{\alpha} \text{ if } B^{\#} < 0 \end{cases}$$

and note that the function

$$h(\rho) \equiv (1-\rho)\sqrt{B^{\#} + \alpha\rho}$$

defined by the left-hand-side of Condition (57) (a) is defined over $[\underline{\rho}^{\#}, 1]$, (b) is non-negative, (c) satisfies $h(1) = 0$ and $h(\underline{\rho}^{\#}) = 0$ when $\underline{\rho}^{\#} > 0$ and $h(\underline{\rho}^{\#}) > 0$ when$\underline{\rho}^{\#} = 0$, and (d) is concave. The above properties imply that $h(\cdot)$ is either decreasing over $[\underline{\rho}^{\#}, 1]$, or it inverted U-shaped with a stationary point $\rho^s \in (\underline{\rho}^{\#}, 1)$. In this case, Condition (57) may admit two solutions. However, when this is the case, it is always the largest one that identifies the equilibrium publicity of the source. To see this, observe that, when h is decreasing over $[\underline{\rho}^{\#}, 1]$, the unique solution to the equation defined by Condition (57) is such that $h(\rho) > \frac{1}{A^{\#}\sqrt{t_n}}$ for $\rho < \rho_n^{\#}$ and $h(\rho) < \frac{1}{A^{\#}\sqrt{t_n}}$ for $\rho > \rho_n^{\#}$. This means that, for the agent's payoff to reach at a global maximum at $z_n = z_n^{\#}$ or, equivalently, for $\rho_n^{\#}$ to be the equilibrium publicity of the source, it must be that h is locally decreasing at the equilibrium level $\rho_n^{\#}$.

I conclude that, irrespective of the sign of α, Condition (57) identifies an increasing relationship $\rho^{\#}(\cdot)$ between the endogenous publicity $\rho_n^{\#}$ and the transparency t_n of the sources that receive attention in equilibrium, with the relationship $\rho^{\#}(\cdot)$ given by the highest solution to the equation in Condition (57).

Now let $[\underline{t}, +\infty)$ denote the set of transparency levels for which the equation in Condition (57) admits a solution. Then observe that, over $[\underline{t}, +\infty)$, the highest solution to the equation in Condition (57) identifies a differentiable function with

$$\frac{\partial \rho^{\#}(t)}{\partial t} = \frac{\left[B^{\#} + \alpha\rho^{\#}(t)\right]\left[1 - \rho^{\#}(t)\right]}{t\left\{2\left[B^{\#} + \alpha\rho^{\#}(t)\right] - \left[1 - \rho^{\#}(t)\right]\alpha\right\}}.$$

Then, for ant $t \in [\underline{t}, +\infty)$, let $\Lambda(t)$ be the function defined by

$$\Lambda(t) = A^{\#}\sqrt{B^{\#} + \alpha\rho^{\#}(t)} + \frac{\alpha}{1-\alpha}\frac{1}{\sqrt{t}} = \frac{1}{\left[1 - \rho^{\#}(t)\right]\sqrt{t}} + \frac{\alpha}{1-\alpha}\frac{1}{\sqrt{t}} \tag{58}$$

with $\rho^{\#}(t)$ denoting the increasing function implicitly defined by the highest solution to the equation in Condition (57). The function $\Lambda(\cdot)$ is differentiable over $[\underline{t}, +\infty)$ with

$$\begin{aligned}\Lambda'(t) &= \frac{1}{\left[1 - \rho^{\#}(t)\right]^2 t}\left\{\frac{\partial \rho^{\#}(t)}{\partial t}\sqrt{t} - \frac{1 - \rho^{\#}(t)}{2\sqrt{t}}\right\} - \frac{\alpha}{(1-\alpha)2t\sqrt{t}} \\ &= \frac{\alpha}{2t\sqrt{t}}\left\{\frac{1}{2B^{\#} - \alpha + 3\alpha\rho^{\#}(t)} - \frac{1}{1-\alpha}\right\}.\end{aligned}$$

Note that, irrespective of the sign of α, because $\rho^{\#}(t)$ is non-decreasing, $\Lambda(t)$ is quasi-concave, meaning that either $\Lambda'(t)$ is of constant sign, or there exists $t^{\#}$ such that $\Lambda'(t) > 0$ for $t < t^{\#}$ and $\Lambda'(t) < 0$ if $t > t^{\#}$. The quasi-concavity of $\Lambda(t)$ is clearly preserved when the function $\Lambda(t)$ is restricted to the set $\left\{t_n : \ n = 1, ..., N \text{ and } z_n^{\#} > 0\right\}$. Because $\Lambda(t)$ coincides with the left-hand side of the inequalities in (55) that are responsible for whether $z_n^{\#} > \hat{z}_n$ or $0 < z_n^{\#} < \hat{z}_n$, I then conclude that, among those sources that receive attention under bounded recall, one of the following must be true: (a) $z_n^{\#} > \hat{z}_n$ for all n; (b) $z_n^{\#} < \hat{z}_n$ for all n; (c) there exists t_1 such that $z_n^{\#} > \hat{z}_n$ for those n for which $t_n < t_1$ and $z_n^{\#} < \hat{z}_n$ for those n for which $t_n > t_1$; (d) there exists t_2 such that $z_n^{\#} > \hat{z}_n$ for those n for which $t_n > t_2$ and $z_n^{\#} < \hat{z}_n$ for those n for which $t_n < t_2$; (e) there exist thresholds t_1 and t_2 such that $z_n^{\#} > \hat{z}_n$ for those n for which $t_n \in (t_1, t_2)$ and $z_n^{\#} < \hat{z}_n$ for those n for which $t_n \notin (t_1, t_2)$. All these various cases can be summarized concisely by saying that there exist thresholds t' and t'' such that the properties in the proposition hold. Q.E.D.

Proof of Proposition 10. The result follows directly from applying the envelope theorem to the welfare function under the efficient actions, as given in (19). Q.E.D.

Proof of Proposition 11. Fix the equilibrium allocation of attention $z^{\#}$. Using the results in Propositions 7 and 10, I have that that, starting from $z^{\#}$, the private and the social marginal benefits of increasing the attention to source n are given by, respectively,

$$PB_n(z^{\#}) = -\frac{|u_{kk}|}{2}\frac{\partial}{\partial z_n}Var\left[k - K \mid z^{\#}, k^{\#}(\cdot; z^{\#})\right] - \frac{|u_{kk}|}{2}(1-\alpha)\frac{\partial}{\partial z_n}Var\left[K - \kappa \mid z^{\#}, k^{\#}(\cdot; z^{\#})\right]$$

and

$$\begin{aligned} SB_n(z^{\#}) = &-\frac{|u_{kk} + u_{\sigma\sigma}|}{2}\frac{\partial}{\partial z_n}Var\left[k - K \mid z^{\#}, k^{**}(\cdot; z^{\#})\right] \\ &- \frac{|u_{kk} + u_{\sigma\sigma}|}{2}(1-\alpha^*)\frac{\partial}{\partial z_n}Var\left[K - \kappa \mid z^{\#}, k^{**}(\cdot; z^{\#})\right] \end{aligned}$$

where $k^{\#}(\cdot; z^{\#})$ and $k^{**}(\cdot; z^{\#})$ are, respectively, the equilibrium and the efficient strategy, with bounded recall.

Next, use the results in the Proof of Proposition 8, along with the observation that the efficient strategy $k^{**}(\cdot; z^{\#})$ can be obtained from the equilibrium strategy $k^{\#}(\cdot; z^{\#})$ by replacing (κ_0, κ_1) with (κ_0^*, κ_1^*) and α with α^*, to express $PB_n(z^{\#})$ and $SB_n(z^{\#})$ as follows

$$PB_n(z^{\#}) = \frac{|u_{kk}|\left(\kappa_1\gamma^{\#}(z^{\#})\right)^2}{\left(\pi_\theta + \pi_X(z^{\#})\right)^2}\frac{\eta_n^2 t_n}{\left(z_n^{\#}t_n + \eta_n\right)^2}\left\{-\frac{1}{2} + \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} + \frac{(1-\gamma^{\#}(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{\left(\pi_X(z^{\#}) + \pi_\theta\right)\gamma^{\#}(z^{\#})}\right\}$$

$$-\alpha\frac{|u_{kk}|\left(\kappa_1\gamma^{\#}(z^{\#})\right)^2}{\left(\pi_\theta + \pi_X(z^{\#})\right)^2}\frac{\eta_n^2 t_n}{\left(z_n^{\#}t_n + \eta_n\right)^2}\left\{\frac{(1-\gamma^{\#}(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{\left(\pi_X(z^{\#}) + \pi_\theta\right)\gamma^{\#}(z^{\#})} + \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})}\rho_X(z^{\#}) - \rho_n(z^{\#})\right\}$$

and

$$SB_n(z^{\#}) = \frac{|u_{kk} + u_{\sigma\sigma}| \left(\kappa_1^* \gamma^{**}(z^{\#})\right)^2}{(\pi_\theta + \pi_X(z^{\#}))^2} \frac{\eta_n^2 t_n}{\left(z_n^{\#} t_n + \eta_n\right)^2} \left\{ -\frac{1}{2} + \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} + \frac{(1-\gamma^{**}(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{(\pi_X(z^{\#}) + \pi_\theta)\,\gamma^{**}(z^{\#})} \right\}$$

$$-\alpha^* \frac{|u_{kk} + u_{\sigma\sigma}| \left(\kappa_1^* \gamma^{**}(z^{\#})\right)^2}{(\pi_\theta + \pi_X(z^{\#}))^2} \frac{\eta_n^2 t_n}{\left(z_n^{\#} t_n + \eta_n\right)^2} \left\{ \frac{(1-\gamma^{**}(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{(\pi_X(z^{\#}) + \pi_\theta)\,\gamma^{**}(z^{\#})} + \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} \rho_X(z^{\#}) - \rho_n(z^{\#}) \right\}$$

It is then immediate that the same conclusions as in Propositions 3 and 5 hold for the comparison between $z^{\#}$ and z^{**}, which establishes part (a) in the proposition.

Next, consider part (b). When $(\kappa_0, \kappa_1) = (\kappa_0^*, \kappa_1^*)$ and $u_{\sigma\sigma} = 0$, I have that

$$\begin{aligned} &SB_n(z^{\#}) - PB_n(z^{\#}) \overset{sign}{=} Q(\pi_X(z^{\#}), \rho_X(z^{\#}), \pi_\theta, \gamma^{**}(z^{\#}), \alpha^*, \gamma^{\#}(z^{\#}), \alpha) \\ &+ \left[\alpha^* \left(\gamma^{**}(z^{\#})\right)^2 - \alpha \left(\gamma^{\#}(z^{\#})\right)^2 \right] \rho_n(z^{\#}) \end{aligned}$$

where

$$\begin{aligned} &Q(\pi_X(z^{\#}), \rho_X(z^{\#}), \pi_\theta, \gamma^{**}(z^{\#}), \alpha^*, \gamma^{\#}(z^{\#}), \alpha) \\ &\equiv \left(\gamma^{**}(z^{\#})\right)^2 \left\{ \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} - \frac{1}{2} + \frac{(1-\gamma^{**}(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{(\pi_X(z^{\#}) + \pi_\theta)\,\gamma^{**}(z^{\#})} \right\} \\ &- \alpha^* \left(\gamma^{**}(z^{\#})\right)^2 \left\{ \frac{(1-\gamma^{**}(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{(\pi_X(z^{\#}) + \pi_\theta)\,\gamma^{**}(z^{\#})} + \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} \rho_X(z^{\#}) \right\} \\ &- \left(\gamma(z^{\#})\right)^2 \left\{ \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} - \frac{1}{2} + \frac{(1-\gamma(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{(\pi_X(z^{\#}) + \pi_\theta)\,\gamma(z^{\#})} \right\} \\ &+ \alpha \left(\gamma(z^{\#})\right)^2 \left\{ \frac{(1-\gamma(z^{\#}))\pi_X(z^{\#}) + \pi_\theta}{(\pi_X(z^{\#}) + \pi_\theta)\,\gamma(z^{\#})} + \frac{\pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})} \rho_X(z^{\#}) \right\} \\ &= (1-\alpha^*)\gamma^{**}(z^{\#}) - (1-\alpha)\gamma(z^{\#}) - \frac{1}{2} \left[\left(\gamma^{**}(z^{\#})\right)^2 - \left(\gamma^{\#}(z^{\#})\right)^2 \right] \\ &+ \left[\alpha^* \left(\gamma^{**}(z^{\#})\right)^2 - \alpha \left(\gamma^{\#}(z^{\#})\right)^2 \right] \frac{\left[1 - \rho_X(z^{\#})\right] \pi_X(z^{\#})}{\pi_\theta + \pi_X(z^{\#})}. \end{aligned}$$

The result in part (b) in the proposition then follows by observing that (i) Q is the same across all sources of information, (ii) when $\alpha > \alpha^*$ and $\alpha^* \left(\gamma^{**}(z^{\#})\right)^2 > \alpha \left(\gamma^{\#}(z^{\#})\right)^2$, then $Q > 0$, (iii) when $\alpha < \alpha^*$ and $\alpha^* \left(\gamma^{**}(z^{\#})\right)^2 < \alpha \left(\gamma^{\#}(z^{\#})\right)^2$, $Q < 0$. Finally note that, when $\pi_\theta \to 0$, $\gamma^{\#}(z^{\#}), \gamma^{**}(z^{\#}) \to 1$, in which case

$$Q \to (\alpha - \alpha^*)\, \rho_X(z^{\#})$$

implying that $\bar{\rho} \to \rho_X(z^{\#})$. Q.E.D.

References

[1] Angeletos, G.M. and A. Pavan, 2007, "Efficient Use of Information and Social Value of Information," *Econometrica*, 75, 1103-1142.

[2] Angeletos, G.M. and A. Pavan, 2009, "Policy with Dispersed Information," *Journal of the European Economic Association*, 11-60.

[3] Amir, R. and N. Lazzati, 2014, "Endogenous Information Acquisition in Bayesian Games with Strategic Complementarities," mimeo, University of Iowa.

[4] Benabou R., and J. Tirole, 2004, "Willpower and Personal Rules," *Journal of Political Economy*, 112(4), 848-886.

[5] Bergemann, D. and S. Morris, 2013, "Robust Predictions in Games with Incomplete Information," *Econometrica*, 81(4), 1251-1308.

[6] Chahrour, R., 2014, "Public Communication and Information Acquisition," *American Economic Journal: Macroeconomics*, 6-3, 73-101.

[7] Colombo, L., G. Femminis, and A. Pavan, 2014, "Information Acquisition and Welfare," *Review of Economic Studies,* 81, 1438-1483.

[8] Denti. T., 2016, "Unrestricted Information Acquisition" mimeo Massachusetts Institute of Technology.

[9] Galperti, S. and I. Trevino, 2016, "Competition for Attention, Information Supply, and Coordination," mimeo University of California at San Diego.

[10] Hellwig, C. and L. Veldkamp, 2009, "Knowing What Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies,* 76, 223-251.

[11] Hébert, B. and M. Woodford, 2016,"Rational Inattention with Sequential Information Sampling" mimeo Columbia and Stanford Universities.

[12] Herskovic, B. and J. Ramos, 2016, "Acquiring Information Through Peers" mimeo UCLA and USC Marshall.

[13] Jehiel, P., 2005, "Analogy-Based Expectation Equilibrium," *Journal of Economic Theory* 123, 81–104.

[14] Jehiel, P. and D. Samet, 2007, "Valuation Equilibrium," *Theoretical Economics* 2, 163–185.

[15] Jehiel, P. and F. Koessler, 2008, "Revisiting Games of Incomplete Information with Analogy-based Expectations," *Games and Economic Behavior* 62, 533–557.

[16] Jehiel, P. and L. Samuelson, 2012, "Reputation with Analogical Reasoning," *Quarterly Journal of Economics*, 1-43.

[17] Kahneman, D., 1973, "Attention and Effort," Englewood Cliffs, NJ: Prentice-Hall.

[18] Kahneman, D., 2011, "Thinking, Fast and Slow," New York: Farrar, Strauss, Giroux.

[19] Kahneman, D., Slovic, P., and A. Tversky, 1982, "Judgment Under Uncertainty: Heuristics and Biases," New York: Cambridge University Press.

[20] Kocer, Y., 2010, "Endogenous Learning with Bounded Memory," Job Market Paper, *mimeo*, Princeton University.

[21] Llosa, L. G. and V. Venkateswaran, 2013, "Efficiency under Endogenous Information Choice," *mimeo*, New York University.

[22] Maćkowiak, B. and M. Wiederholt, 2009, "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, 99, 769-803.

[23] Maćkowiak, B. and M. Wiederholt, 2012, "Inattention to Rare Events," *mimeo*, European Central Bank.

[24] Mailath, G. And L. Samuelson, 2006, "Repeated Games and Reputations: Long-Run Relationships," Oxford University Press.

[25] Matejka, F. and A. McKay, 2012, "Simple Market Equilibria with Rationally Inattentive Consumers," *American Economic Review: Papers & Proceedings* 102(3), 24-29.

[26] Matejka, F. and A. McKay, 2014, "Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model," *American Economic Review*, 105 (1), 272-298.

[27] Myatt, D. P. and C. Wallace, 2012, "Endogenous Information Acquisition in Coordination Games," *Review of Economic Studies*, 79, 340-374.

[28] Myatt, D. P. and C. Wallace, 2015, "Cournot Competition and the Social Value of Information," *Journal of Economic Theory*, 158(B), 466-506.

[29] Monderer D. and L. Shapley, 1995, "Potential Games," *Games and Economic Behavior*, 14, 124–143.

[30] Morris, S. and H.S. Shin, 2002, "The Social Value of Public Information," *American Economic Review*, 92, 1521-1534.

[31] Morris, S. and M. Yang, 2016, "Coordination and the Relative Cost of Distinguishing Nearby States," mimeo Duke and Princeton Universities.

[32] Mullainathan S., 2002, "A Memory-Based Model of Bounded Rationality," *Quarterly Journal of Economics*, 117(3), 735-774.

[33] Sims, C.A., 2003, "Implications of Rational Inattention," *Journal of Monetary Economics*, 50, 665-690.

[34] Sims, C.A., 2011, "Rational Inattention and Monetary Economics," Handbook of Monetary Economics, 3A, 155-182.

[35] Stewart, C., J. Steiner, and F. Matejka, 2016, "Rational Inattention Dynamics: Inertia and Delay in Decision-Making" mimeo University of Toronto and CERGE-EI.

[36] Szkup, M. and I. Trevino, 2015, "Information Acquisition in Global Games of Regime Change," *Journal of Economic Theory*, 2015, 160, 387-428.

[37] Tirole, J., 2015, "Cognitive Games and Cognitive Traps," mimeo Toulouse School of Economics.

[38] Van Zandt, T., and X. Vives, 2007, "Monotone Equilibria in Bayesian Games of Strategic Complementarities, *Journal of Economic Theory*, 134, 339-360.

[39] Vives, X., 2016, "Endogenous Public Information and Welfare in Market Games," *mimeo* IESE.

[40] Wilson, A., 2004, "Bounded Memory and Biases in Information Processing," *mimeo* NYU.

[41] Yang, M., 2015, "Coordination with Flexible Information Acquisition," *Journal of Economic Theory*, 158(B), 721–738.