Price Customization and Targeting in Many-to-Many Matching Markets

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Motivation

- Mediated (many-to-many) matching
- ad exchanges
- B2B platforms
- Media platforms

Targeting and Price Customization

- Technological progress:
  - **targeting**
- Concerns that accompanying **price customization** detrimental to consumer surplus
- Proposal: uniform-pricing obligations
- Policy debate lacks formal framework
  - third-degree + second-degree price discrimination
- Unclear how firms will respond to uniform-pricing obligations
- More generally,
  - distortions in matching mkts not well understood

This Paper

- Tractable, yer rich model, of mediated many-to-many matching
  - third-degree price discrimination
  - second degree price discrimination
- Menus of Matching Plans
  - baseline configuration
  - baseline price
  - customizing tariffs
- Targeting (structural elasticities)
- Mkt power distortions
- Uniform-pricing regulations
- Centralized vs decentralized markets

Related literature

- **One-to-One Matching Design**: Damiano and Li (2007), Johnson (2013)...
- **Many-to-Many Matching Design**: Gomes and Pavan (2016), Jeon, Kim and Menicucci (2017), Fershtman and Pavan (2016)...
- **Two-Sided Markets:** Rochet and Tirole (2006), Armstrong (2006), Weyl (2010), Jullien and Pavan (2017), Rysman (2009), Tan and Zhou (2017)...
- **Price Discrimination:** Mussa and Rosen (1978), Myerson (1981), Maskin and Riley (1984), Armstrong (1996), Wilson (1998), Aguirre et al. (2010), Bergemann, Brooks, and Morris (2015)...
- **Bundling**: Armstrong (2013), Hart and Reny (2015)...
- **Targeting**: Bergemann and Bonatti (2011)...

Plan

- Model
- Customized tariffs
- Uniform-pricing regulations
- Targeting
- Welfare under customized and uniform pricing
- Centralized vs decentralized markets

Model

- Monopolistic platform
- Two sides $k \in \{a, b\}$
- Each side: unit-mass continuum of agents $i \in [0, 1]$
- Type of agent i from side k: $\theta_k^i = (v_k^i, x_k^i)$
- $v_k^i \in V_k = [\underline{v}_k, \overline{v}_k]$: **vertical dimension**
- $x_i^k \in [0, 1]$: **horizontal dimension** (location)
- Each θ_k^i drawn independently from cdf

$$F_k \quad \textit{with support} \quad \Theta \equiv V_k \times [0, 1]$$

Model

- Type profile $\theta \equiv (\theta_k^i)_{k=A,B}^{i\in[0,1]}$

- Utility of agent i from side k from being matched to agent j from side $l \neq k$:

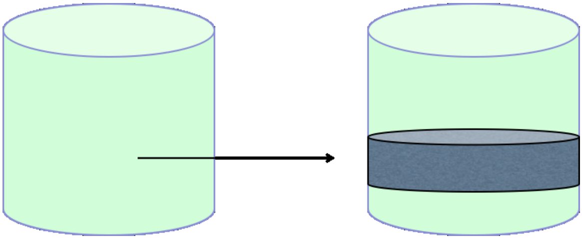
$$u_k(v_k^i, |x_k^i - x_l^j|)$$

  - increasing in v_k^i
  - decreasing in $|x_k^i - x_l^j|$

- Total utility from being matched, at a price p, to set $\mathbf{s} \subset \Theta_l$

$$\pi_k^i(\mathbf{s}, p; \theta) = \int_{\mathbf{s}} u_k\left(v_k^i, |x_k^i - x_l^j|\right) dF_l(\theta_l) - p.$$

Model



Examples

Example

(ad exchange) Platform matches advertisers (side a) with publishers (side b).

- Expected payoff $u_a(v_a, |x_a - x_b|) = v_a \cdot \phi(|x_a - x_b|)$ of advertiser of type $\theta_a = (v_a, x_a)$ from impression at publisher of type $\theta_b = (v_b, x_b)$, where
  - v_a: markup
  - ϕ: conversion probability
- Publishers indifferent wrt type of advertisement displayed (first approximation)
- Heterogeneity in publishers' payoffs reflects differences in opportunity costs

$$u_b(x_b, |x_a - x_b|) = v_b \leq 0$$

Examples

Example

(media platform) Media outlet (e.g., cable TV provider) matching viewers (side *a*) with content providers (side *b*).

- Utility viewer of type $\theta_a = (v_a, x_a)$ derives from content provider of type $\theta_b = (v_b, x_b)$:

$$u_a(v_a, |x_a - x_b|) = \left[\alpha \cdot (v_a)^\delta + (1-\alpha) \cdot \phi\left(|x_a - x_b|\right)^\delta\right]^{\frac{1}{\delta}}$$

- Providers indifferent wrt type of viewers.
- Providers' payoff

$$u_b(v_b, |x_b - x_a|) = v_b$$

may be positive (advertising) or negative (royalties)

Information

- Vertical types are private information
- Horizontal types (i.e., locations)
  - *Scenario (i)*: Locations public on both sides
  - *Scenario (ii)*: Locations private on side a, public on side b
  - *Scenario (iii)*: Locations public on side a, private on side b
  - *Scenario (iv)*: locations private on both sides.

Reciprocity

Given tariff T_k, *matching demand* of type $\theta_k = (v_k, x_k)$ given by matching set

$$\hat{\mathbf{s}}_k(\theta_k; T_k) \in \arg\max_{\mathbf{s}_k \subseteq \Theta_l} \left\{ \int_{\Theta_l} u_k \left(v_k, |x_k - x_l| \right) dF_l(\theta_l) - T_k \left(\mathbf{s}_k \right) \right\}.$$

Definition

Tariffs T_k, $k = a, b$, are *feasible* if, for all $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l \in \{a, b\}$, $l \neq k$,

$$\theta_l \in \hat{\mathbf{s}}_k(\theta_k; T_k) \Longleftrightarrow \theta_k \in \hat{s}_l(\theta_l; T_l).$$

Customized tariffs

Definition

Tariff T_k is ***customized*** if there exists collection of *matching plans*

$$\left\{ (\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k)) : x_k \in [0,1] \right\},$$

- $\underline{\mathbf{s}}_k(x_k)$: baseline configuration
- $\underline{T}_k(x_k)$: baseline price
- $\mathbf{S}_k(x_k)$: set of possible customizations
- $\rho_k(\cdot|\cdot; x_k)$: customizing price schedules

s.t. each side-k agent selecting plan x_k and customization $\mathbf{s}_k \in \mathbf{S}_k(x_k)$ is charged

$$\underline{T}_k(x_k) + \int_0^1 \rho_k(q_{x_l}(\mathbf{s}_k)|x_l; x_k) dx_l,$$

where $q_{x_l}(\mathbf{s}_k)$ is total mass of x_l-agents under customization $\mathbf{s}_k$.

Conditions

Condition [R] Regularity: Virtual values

$$\varphi_k(\theta_k,\theta_l) \equiv u_k(v_k,|x_k-x_l|) - \frac{1-F_k^{v|x}(v_k|x_k)}{f_k^{v|x}(v_k|x_k)} \cdot \frac{\partial u_k}{\partial v}(v_k,|x_k-x_l|)$$

continuous and non-decreasing in v_k.

Condition [VD] Virtual values decreasing in distance:
$\varphi_k(\theta_k,(v_l,x_l))$ non-increasing in $|x_k-x_l|$

Condition [I$_k$] Independence on side k: $F_k(\theta_k) = F_k^x(x_k)F_k^v(v_k)$

Condition [S$_k$] Symmetry on side k: $F_k(\theta_k) = x_k F_k^v(v_k)$

Profit-maximizing tariffs

Proposition

In addition to Condition R, suppose one of following holds:

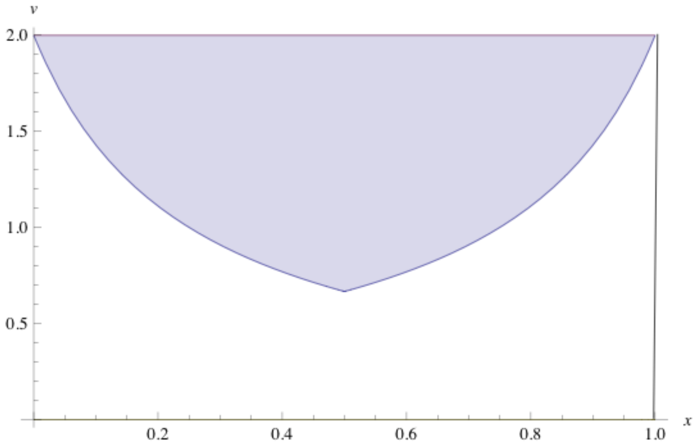
- *Scenario (i)*
- *Scenario (ii) along with Conditions VD, I_a and S_b*
- *Scenario (iii) along with Conditions VD, S_a and I_b*
- *Scenario (iv) along with Conditions VD, S_a and S_b.*

Then, for $k \in \{a, b\}$:

- *profit-maximizing tariff T_k^* is customized;*
- *matching sets $\mathbf{s}_k^*$ exhibit **negative assortativeness at the margin**: there exist functions $t_k^* : \Theta_k \times [0,1] \to V_l$ s.t.*

$$\mathbf{s}_k^*(\theta_k) = \{(v_l, x_l) \in \Theta_l : \; v_l > t_k^*(\theta_k, x_l)\}$$

Negative assortativeness at the margin



Optimal Threshold

- Let
$$\triangle_k(\theta_k, \theta_l) \equiv \varphi_k(\theta_k, \theta_l) + \varphi_l(\theta_l, \theta_k)$$
denote marginal effect of linking θ_k and θ_l.
- When interior, $t_k(\theta_k, x_l)$ unique solution to
$$\triangle_k(\theta_k, (t_k(\theta_k, x_l), x_l)) = 0$$
- Conditions in proposition
  - bunching only for capacity reasons (corners)
  - only binding IC constrains are those wrt vertical dimensions

Demands

- For simplicity, locations public on both sides
- Mass of agents located at x_l demanded by θ_k:

$$\hat{q}_{x_l}(\theta_k) \in \arg \max_{q \in [0, f_l^x(x_l)]} \left\{ u_k(v_k, |x_k - x_l|) \cdot q - \rho_k(q|x_l; x_k) \right\}.$$

- Demand for q-th unit of x_l-agents by x_k-agents, at *marginal price* $\frac{d\rho_k}{dq}$:

$$D_k\left(\frac{d\rho_k}{dq}|x_l; x_k\right) = 1 - F_k^{v|x}\left(\hat{v}_{x_l}\left(\frac{d\rho_k}{dq}|x_k\right)|x_k\right)$$

where $\hat{v}_{x_l}\left(\frac{d\rho_k}{dq}|x_k\right)$ is unique solution to $u_k(v_k, |x_k - x_l|) = \frac{d\rho_k}{dq}$.

- Elasticity:

$$\varepsilon_k\left(\frac{d\rho_k}{dq}|x_l; x_k\right) \equiv -\frac{\partial D_k\left(\frac{d\rho_k}{dq}|x_l, x_k\right)}{\partial\left(\frac{d\rho_k}{dq}\right)} \cdot \frac{\frac{d\rho_k}{dq}}{D_k\left(\frac{d\rho_k}{dq}|x_l, x_k\right)}.$$

Lerner-Wilson formula

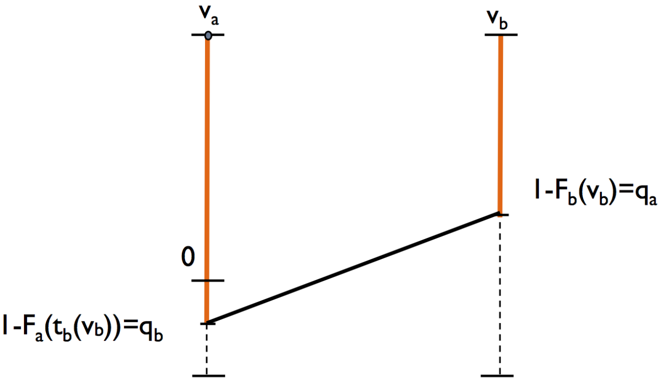
Proposition

Profit-maximizing schedules satisfy following structural conditions for all (x_a, x_b), *all* (q_a, q_b) *such that* $q_a = D_b\left(\frac{d\rho_b^*}{dq}(q_b|x_a;x_b)|x_a;x_b\right)$ *and* $q_b = D_a\left(\frac{d\rho_a^*}{dq}(q_a|x_b;x_a)|x_b;x_a\right)$*:*

$$\underbrace{\frac{d\rho_a^*}{dq}(q_a|x_b;x_a)\left(1-\frac{1}{\varepsilon_a\left(\frac{d\rho_a^*}{dq}(q_a|x_b;x_a)|x_b;x_a\right)}\right)}_{\text{net effect on side-a profits}}$$

$$+\underbrace{\frac{d\rho_b^*}{dq}(q_b|x_a;x_b)\left(1-\frac{1}{\varepsilon_b\left(\frac{d\rho_b^*}{dq}(q_b|x_a;x_b)|x_a;x_b\right)}\right)}_{\text{net effect on side-b profits}} = 0.$$

Lerner-Wilson formula



Distortions and Horizontal Differentiation

- Let

$$W = \sum_{k=a,b} \int_{\Theta_k} \int_{\hat{\mathbf{s}}_k(\theta_k; T_k)} u_k\left(v_k, |x_k - x_l|\right) dF_j(\theta_j) dF_k(\theta_k)$$

- Tariffs T_k^e, $k = a, b$, maximize welfare iff for all $\theta_k = (v_k, x_k)$ and $\theta_l = (v_l, x_l)$,

$$\theta_l \in \hat{\mathbf{s}}_k(\theta_k; T_k^e) \quad \Longleftrightarrow \quad u_k\left(v_k, |x_k - x_l|\right) + u_l\left(v_l, |x_k - x_l|\right) \geq 0.$$

Distortions and Horizontal Differentiation

Definition

Distortions on side $k \in \{a, b\}$ decrease (alternatively, increase) with distance iff

$$u_l\left(t_k^*(\theta_k, x_l), |x_l - x_k|\right) - u_l\left(t_k^e(\theta_k, x_l), |x_l - x_k|\right)$$

decreases (alternatively, increases) with $|x_k - x_l|$.

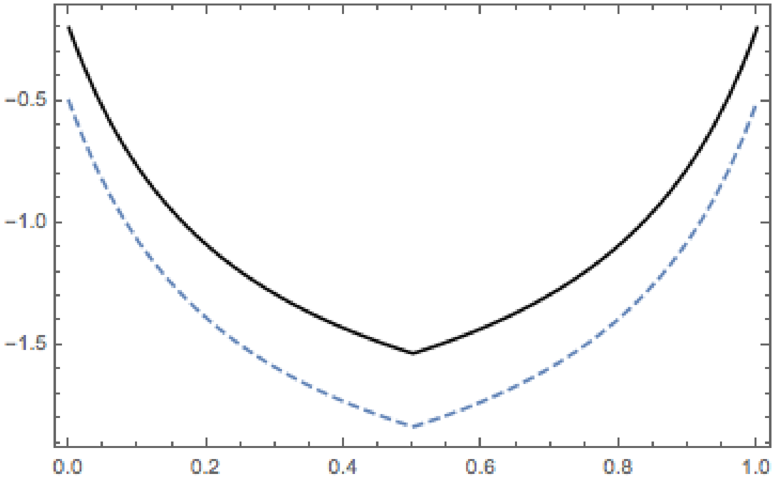
Distortions and Horizontal Differentiation

Proposition

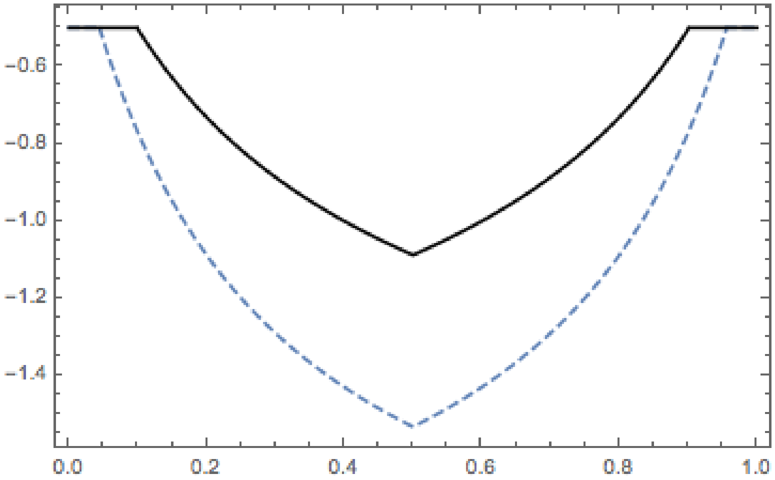
Suppose Conditions VD and I_k hold, $k = a, b$.

1. *If u_k is submodular, u_l is submodular and concave in v_l, and F_l^v has increasing hazard rate, then distortions on side k decrease with distance*
2. *If u_k is supermodular, u_l is supermodular and convex in v_l, and F_l^v has decreasing hazard rate, then distortions on side k increase with distance*

Distortions and Horizontal Differentiation



Distortions and Horizontal Differentiation



Uniform Pricing

- Growing support for uniform-pricing regulations (both US and EU)

Definition

Tariff T_k consistent with *uniform pricing* if there exists collection of *price schedules* $p_k : [0,1]^2 \to \mathbb{R}$ s.t.

$$T_k(\mathbf{s}_k) = \int_0^1 p_k(q_{x_l}(\mathbf{s}_k)|x_l)dx_l$$

where $q_{x_l}(\mathbf{s}_k)$ is mass of x_l-agents in the set $\mathbf{s}_k$.

- Key: $p_k(q|x_l)$ do **not** depend on side-k agents' **own** profiles!

Uniform Pricing

- Aggregate demands under uniform-pricing

$$\bar{D}_a\left(\frac{dp_a}{dq}|x_b\right) \equiv \int_0^1 D_a\left(\frac{dp_a}{dq}|x_b; x_a\right) dx_a$$

- Elasticity of aggregate demand

$$\bar{\varepsilon}_a\left(\frac{dp_a}{dq}|x_b\right) \equiv -\frac{\partial \bar{D}_a\left(\frac{dp_a}{dq}|x_b\right)}{\partial\left(\frac{dp_a}{dq}\right)} \cdot \frac{\frac{dp_a}{dq}}{\bar{D}_a\left(\frac{dp_a}{dq}|x_b\right)}$$

- Let $H\left(x_a|x_b, \frac{dp_a}{dq}\right)$ be distribution over $X_a = [0,1]$ with density

$$h\left(x_a|x_b, \frac{dp_a}{dq}\right) \equiv \frac{\frac{\partial D_a\left(\frac{dp_a}{dq}|x_b; x_a\right)}{\partial\left(\frac{dp_a}{dq}\right)}}{\frac{\partial \bar{D}_a\left(\frac{dp_a}{dq}|x_b\right)}{\partial\left(\frac{dp_a}{dq}\right)}}$$

Uniform Pricing

Proposition

Under uniform pricing on side a, profit-maximizing price schedules satisfy

$$\underbrace{\frac{dp_a^u}{dq}(q|x_b)\left(1-\frac{1}{\bar{\varepsilon}_a\left(\frac{dp_a^u}{dq}(q|x_b)\right)}\right)}_{\text{net effect on side-a profits}}$$

$$+\mathbb{E}_{H\left(\tilde{x}_a|x_b,\frac{dp_a}{dq}\right)}\left[\underbrace{\frac{d\rho_b^u}{dq}(\tilde{x}_a)\left(1-\frac{1}{\varepsilon_b\left(\frac{d\rho_b^u}{dq}(\tilde{x}_a)\,|\tilde{x}_a;x_b\right)}\right)}_{\text{net effect on side-b profits}}\right]=0,$$

where

$$\frac{d\rho_b^u}{dq}(\tilde{x}_a)=\frac{d\rho_b^u}{dq}\left(D_a\left(\frac{dp_a^u}{dq}(q|x_b)|x_b;x_a\right)|\tilde{x}_a;x_b\right)$$

Uniform Pricing

- Mechanism design with novel constraint on implementing tariffs
- Under uniform pricing, types θ_a and θ_b matched iff

$$\mathbb{E}_{H(\tilde{x}_a|x_b,u_a(v_a,|x_b-x_a|))}\left[\varphi_a\left(\left(\hat{v}_{x_b}\left(u_a(v_a,|x_b-x_a|)|\tilde{x}_a\right),\tilde{x}_a\right),\theta_b\right)\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,u_a(v_a,|x_b-x_a|))}\left[\varphi_b\left(\theta_b,\left(\hat{v}_{x_b}\left(u_a(v_a,|x_b-x_a|)|\tilde{x}_a\right),\tilde{x}_a\right)\right)\right]\geq 0$$

where $\hat{v}_{x_l}\left(\frac{d\rho_k}{dq}|x_k\right)$ is unique solution to

$$u_k(v_k,|x_k-x_l|)=\frac{d\rho_k}{dq}.$$

Targeting

Definition

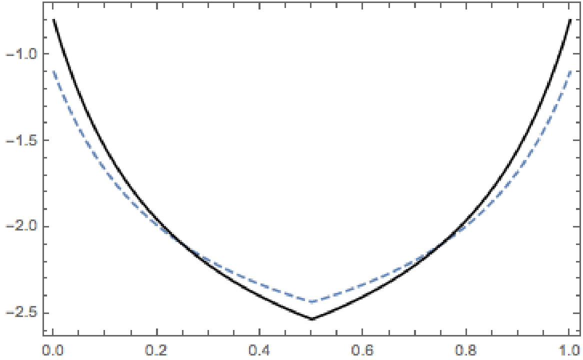
(targeting) Customized pricing (on both sides) leads to more targeting than uniform pricing (on side a) if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^*(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} < 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ > 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

Conversely, uniform pricing on side a leads to more targeting than customized pricing on both sides if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^*(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} > 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ < 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

Targeting



Threshold function $t_a^*(\theta_a, x_b)$ under customized pricing (black solid curve) and uniform pricing $t_a^u(\theta_a, x_b)$ (dashed blue curve) when customized pricing leads to more targeting than uniform pricing

Targeting

Proposition

(comparison: targeting) *Suppose Condition VD and I_a hold.*

1. If u_a is submodular and concave in v_a, F_a^v has increasing hazard rate, and, for any θ_b, $\varphi_b(\theta_b, \theta_a)$ invariant in $|x_b - x_a|$, then uniform pricing (on side a) leads to more targeting than customized pricing (on both sides).

2. If u_a is supermodular and convex in v_a, and F_a^v has a decreasing hazard rate, then customized pricing (on both sides) leads to more targeting than uniform pricing (on side a).

Uniform Targeting: Welfare

Let

$$CD_a\left(\frac{dp_a}{dq}|x_b;x_a\right) = -\frac{\partial^2 D_a\left(\frac{dp_a^u}{dq}|x_b;x_a\right)}{\partial\left(\frac{dp_a^u}{dq}\right)^2}\frac{\frac{dp_a^u}{dq}}{\frac{\partial D_a\left(\frac{dp_a^u}{dq}|x_b;x_a\right)}{\partial\left(\frac{dp_a^u}{dq}\right)}}$$

denote convexity of x_a-agents' demand for q-th unit of x_b-agents wrt $\frac{dp_a}{dq}$.

Condition [IR] Increasing Ratio: For any $(x_a, x_b) \in [0,1]^2$, any $q \in [0, f_b^x(x_b)]$,

$$z_a\left(\frac{dp_a}{dq}|x_b;x_a\right) \equiv \frac{\frac{dp_a}{dq}}{2 - CD_a\left(\frac{dp_a}{dq}|x_b;x_a\right)}$$

nondecreasing in $\frac{dp_a}{dq}$.

Proposition

(comparison: welfare)

1. Suppose targeting is higher under uniform pricing and that convexity $CD_a\left(\frac{dp_a}{dq}|x_b;x_a\right)$ of x_a-agents' demands declines with $|x_a - x_b|$. Then welfare of side-a agents higher under uniform pricing.

2. Suppose targeting is higher under customized pricing and that convexity $CD_a\left(\frac{dp_a}{dq}|x_b;x_a\right)$ of x_a-agents' demands increases with $|x_a - x_b|$. Then welfare of side-a agents higher under uniform pricing.

- Targeting + monotonicity of convexity of demands in distance permits one to identify which matches correspond to "strong mkt" in the sense of third-degree price discrimination literature (e.g., Aguirre et al. (2010)).

Centralized vs Decentralized Markets

- Many mkts are transiting from centralized to decentralized structure (sellers post prices)
- E.g., Cable TV

Proposition

(centralized vs decentralized markets) *Suppose locations are private on side a (buyers) and public on side b (sellers) and Conditions VD, I_a, S_b, and IR hold.*

1. Suppose targeting is higher under uniform pricing and convexity $CD_a\left(\frac{dp_a}{dq}|x_b;x_a\right)$ of x_a-agents' demands declines with $|x_a-x_b|$. Then welfare of side-a agents higher in decentralized market.

2. Suppose targeting is higher under customized pricing and convexity $CD_a\left(\frac{dp_a}{dq}|x_b;x_a\right)$ of x_a-agents' demands increases with $|x_a-x_b|$. Then, again, welfare of side-a agents higher in decentralized market.

Conclusions

- Tractable model of mediated many-to-many matching
  - vertically and horizontally differentiated preferences
  - unrestricted pricing algorithms
- Customized Tariffs
  - matching plans with plan-specific customizing price schedules
- Testable (structural) predictions for profit-maximizing schedules
  - Lerner-Wilson elasticity formulas (for matching)
- Primitive conditions for distortions to be monotone in targeting
- Uniform-pricing obligations
- Transition to decentralized structure
- Applications:
  1. ad exchanges
  2. cable TV

...

THANKS!