Abstract

This paper studies how adding direct democracy institutions to representative democracy affects politicians’ incentives and selection, and citizens’ welfare. While direct democracy allows citizens to correct politicians’ mistakes, it also reduces the incentives of elected representatives to search for good policies. This responsibility substitution reduces citizens’ ability to screen competent politicians when elections are the only means to address political agency problems. A lower cost of direct democracy induces a negative spiral on politicians incentives, which we represent by a disincentive multiplier. As a consequence, introducing initiatives or lowering their cost can make voters worse off.

Keywords: Direct Democracy, Initiatives, Political Agency, Referendum.

JEL Classification: D72, D78.

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1 Introduction

Direct democracy institutions, such as citizen initiatives, proposals, and referenda, play an important role in regimes that are otherwise based on representative democracy. In the United States, for instance, 377 initiatives have been proposed at state level between 1990 and 1999 and 371 between 2000 and 2008, of which roughly 41% have been approved (Gerber and Matsusaka, 2009). Between 1990 and 2003, referenda took place in 91 countries, including 30 European ones (Kaufmann and Waters, 2004). Since the collapse of the Soviet Union, only three out of the 30 countries that adopted a democratic constitution did not include some form of direct democracy institution; in addition, direct democracy is also being adopted in the European Union.

Theoretical critiques of direct democracy have revolved around two main themes. The first one concerns the “tyranny of the majority” and may be traced back as far as Plato’s Republic (Book VIII). The second one is based on the idea that direct democracy generates socially inferior outcomes because voters have too little expertise or wisdom to choose the best policies. However, several authors have pointed out that in modern democracies citizens can compensate their lack of technical knowledge by using “informational cues,” (e.g. newspapers columns, endorsements by citizens’ organization or interest groups, blogs) which allow them to vote as if they were fully informed.

The driving force behind the introduction of direct democracy in some European countries (most particularly, Switzerland) and American states, at the end of the nineteenth century, was the desire to give citizens the ability to control potential abuses and failures of representative democracy. This motivation, put forth by the Progressive Movement, is summarized in Theodore Roosevelt’s “Charter of Democracy” speech (1912).

I believe in the initiative and the referendum, which should be used not to destroy representative government, but to correct it when ever it becomes misrepresentative. (...)

In the great majority of cases it is far better that action on legislative matters should be taken by those specially delegated to perform the task; in other words, that the work

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1 Another famous exposition of this theory, due to Madison, is in The Federalist Papers (1788). A risk of direct democracy is that decisions could be made, in Madison’s words, “not according to the rules of justice and the rights of the minor party, but by the superior force of an interested and overbearing majority.”

2 The assumption that voters’ lack expertise is empirically supported, for example, by the analysis of Campbell et al. in their book The American Voter (1960). A similar assumption is made by Maskin and Tirole (2004).

3 For empirical evidence on the role of informational cues, see Lupia (1994) and Bowler and Donovan (1998), as well as the more nuanced findings in Garrett and McCubbins (2008).
should be done by the experts chosen to perform it. But where the men thus delegated fail to perform their duty, then it should be in the power of the people themselves to perform the duty.

Indeed, several authors have recently argued that direct democracy institutions improve policies by introducing competition in policymaking. A few papers (Boehmke and Patty, 2007; Matsusaka, 1992; Besley and Coate, 2001) study direct democracy in the context of a representative regime and, to our knowledge, only two – Matsusaka and McCarty (2001) and Gerber (1996) – analyze how the presence of these institutions affects the policies chosen by elected representatives. Their analysis, however, focuses on static agency problems and ignores the connection between political delegation and information acquisition. More generally, we are unaware of any work exploring how the presence of direct democracy affects simultaneously voters’ behavior, political selection, and the overall quality of policymaking.

In this paper, politicians’ competence and information are endogenously determined through a political agency relationship, allowing us to study how direct democracy jointly affects policymaking, political selection, and acquisition of policy-relevant information. Our main purpose is to identify a new, unexplored channel through which direct democracy affects the functioning of a representative democracy. To isolate this channel, we develop a theoretical model of political agency in which direct democracy is, by construction, immune from the causes of earlier criticisms (tyranny of the majority, lack of expertise): in our model, voters are homogeneous and ex post fully informed. Nevertheless, direct democracy can negatively affect voters’ welfare because of a responsibility substitution effect. This effect formalizes, among other things, the common wisdom that excessive use of direct democracy allows politicians not to make decisions, and sheds light on this mechanism: direct democracy changes the ability of voters to credibly commit to reward informed decision making. As a result, elected representatives have lower incentives to acquire policy-relevant information and to act on it.

Our analysis identifies three natural conditions which, taken together, imply a potentially severe substitution effect. The first one is that politician competence affects multiple aspects of policymaking, some which may be substituted for by direct democracy, and some which may not. This means, in particular, that while citizens can correct some of the mistakes that politicians make, they cannot fully compensate the shortcomings of an incompetent politician.\footnote{For example, they cannot fully compensate for his lack of leadership.} The second assumption is that the more involved a politician is in some issue, the better citizens can assess his competence. This assumption is reminiscent of the standard single-crossing condition
in signaling models, whereby the impact of a politician’s effort on the quality of policymaking is increasing in his competence. The specific condition, which we call the “discerning power of incentives,” is not a standard single-crossing condition. It is stronger, as it implies that the output gap between a competent and incompetent politician is not only positive, but also increasing in the incentives provided to him. (See Assumption 2.) Finally, we assume that, while citizens have aligned preferences, they are unable to commit to (or coordinate on) a punishment strategy for a low-performing politician. In our model (as in most models of electoral politics), citizens reelect the incumbent if and only if their expected utility is higher with him than with his challengers, and do not take his past actions into account beyond that comparison.

Taken together, these assumptions generate an indirect effect on information acquisition and political selection, which weakens the apparent benefit of direct democracy institutions, and sometimes even dominates the direct beneficial effect, reducing citizens’ welfare. This reduction is larger, other things equal, when pure representative democracy is better able at providing incentives for politicians and at screening them.

Our model describes a form of relational contract between a principal (the voters) and an agent (the incumbent) with adverse selection and moral hazard. The mechanisms described here, such as the disincentive multiplier, responsibility substitution, and selection complementarity (see Sections 5 and 6), the structure and uniqueness of a “natural” equilibrium, and our condition regarding the discerning power of incentives could be applied to other economic settings.

Our theoretical findings also suggest a novel explanation for several facts concerning direct democracy in the last three decades: the increase in frequency of initiatives and the growing criticism that these institutions have received in recent years in United States, especially in Arizona and California; the higher reelection probability of incumbents in states with initiatives (Hugh-Jones, 2010) and the even higher reelection probability of incumbents conditional on poor economic performance (Bali and Davis, 2007); the inverse relationship between tech-

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5 The concept is introduced by legal scholars (Macauley, 1963; Macneil, 1978). Incentives in relational contracts with asymmetric information were first studied by Levin (2003).

6 One application concerns the relational contract between a manager and an employee when the manager can observe and correct the actions of the employee at some cost and only in some dimensions.

7 For example, the initiatives that restricted bilingual education in 2000, or, in 2004 and 2006, denied benefits and access to state education to undocumented immigrants.

8 See, for example, “Tyranny of the Majority” in The Economist, December 17th 2009, or David Brooks’ “The Bloody Crossroads,” published on September 7, 2009 in The New York Times. There is also a lively debate on the extent to which direct democracy is causing the current fiscal crisis in California (Matsusaka, 2005a).
nological and legal costs of proposing initiatives and their frequency.

The paper is organized as follows. Section 2 contains an informal introduction to our model and to the results of the paper. Section 3 formally describes the model and the key assumptions of the paper. Section 4 analyzes the equilibria of the model and their connection to contracting. Section 5 exposes the disincentive multiplier implied by direct democracy. Section 6 studies the impact on citizens welfare of direct democracy. Section 7 shows how the empirical implications associated with the responsibility substitution effect are consistent with the available evidence. Section 8 contains the literature review, Section 9 discusses the robustness of our assumptions, and Section 10 concludes. All proofs not in the text are relegated to the Appendix.

2 Responsibility Substitution and the Disincentive Multiplier

In our model, politicians affect voters’ welfare along two dimensions. One is a policy dimension which voters can amend, at some cost, through direct democracy. The second dimension is a valence term, representing a combination of personal qualities and other policy aspects for which amendment by voters is infeasible. The first period starts with an incumbent politician who exerts some privately-observed effort to improve the expected quality of his (binary) decision making along the policy dimension, and whose competence affects both the cost of this effort and the quality of his performance along the second dimension.

At the end of the first period, citizens observe the incumbent’s decision along the policy dimension and whether it was better than the alternative. Citizens use this information to update their belief about the incumbent’s competence. Based on their posterior belief, they decide whether to reelect him over his challengers. If the incumbent is reelected, his past efforts carry over to the second and last period of the model.

Direct democracy is introduced as follows. Citizens’ cost for amending the policy chosen by the incumbent is reduced. As a result, citizens care relatively less about a politician’s role along that policy dimension, and relatively more about the second “valence” dimension. Hence, they care less about the incumbent’s past efforts and more about his competence per se. Their

9An example of valence is the politician’s ability to attract public funds or private investment. Another example where direct democracy is infeasible concerns the choice of more detailed aspects arising in the implementation of a law.
posterior about the incumbent’s competence must therefore be higher, other things equal, for them to be willing to reelect him.

The determination of this posterior is the key for the responsibility substitution effect. In equilibrium, a politician risks not being reelected only if he makes a mistake, and citizens’ posterior conditional on a mistake (the “reelection posterior threshold”) determines their indifference condition for reelection. As one increases the politician’s incentive for searching for a good policy, there is more effort separation across competence types, by our “discerning power of incentives” condition. This means that a mistake makes it more likely that the politician was incompetent. Therefore, for the posterior conditional on a mistake to be higher, the politician must have less incentive to exert effort. However, less search by the incumbent has two consequences. First, mistakes are more likely to occur, which involves more frequent costly correction by citizens. Second, citizens are less able to screen competent types. (At the extreme, if citizens care only about the second dimension, the incumbent makes no effort at all.) As a result, citizens not only have to propose costly initiatives more frequently, but also cannot learn about the incumbent’s competence, which destroys their ability to use reelection for screening competent politicians.

Moreover, lower search effort reduces the incumbent’s advantage relative to challengers, since the effort is carried over to the second period. The reelection posterior threshold must therefore be higher. This implies less selection and less search, creating a negative spiral and a disincentive multiplier, which describes how a small cost reduction in direct democracy can have an amplified disincentive on politician’s effort and selection. The size of his amplifying effect is increasing in the initial level of search. As a consequence, while direct democracy can help ‘fixing’ a poorly performing representative democracy, it will also prevent citizens from enjoying the benefits of an improved political process.

Cheap citizen initiatives thus worsen political agency problems and, potentially, citizen’s welfare. These effects do not occur if any of the three key assumptions is relaxed. With commitment, for example, citizens could enforce a large range of search level, independently of the cost of initiatives, and would only enjoy the benefit from direct democracy. Similarly, if politician abilities were not correlated across policy dimensions, citizens would not use policies to infer the incumbent’s competence level. Finally, without a second dimension, voters’ reelection behavior would not depend on amendment costs.
3 Model

Overview. The model has two periods. In the first period, citizens face an incumbent politician of privately-known competence whose goal is to be reelected for a second term. Before the election, the incumbent chooses some costly effort to learn a state $\tilde{h} \in \{h, h'\}$ that affects citizens’ utility in the first period. Upon observing a signal about the state, the incumbent chooses either the default policy, $x$, or a reform, $x'$. Citizens then observe the policy and the state and decide, based on these observations, whether to reelect the incumbent or to elect a challenger. In the second and final period, a new independently distributed state of the world arises, which the elected politician makes no additional effort to learn. However, if the incumbent is reelected, his past efforts are valuable also in the second period, which we simply model as a second-period signal with the same accuracy as in the first period. The politician then chooses some policy in $\{x, x'\}$. Finally, citizens observe the state and implemented policy of the second period, and decide, if needed, whether to amend it at some cost $\mu$.

Formal description. The state of nature $\tilde{h}$ at each period is independently drawn from the following distribution:

$$\tilde{h} \sim \begin{cases} h & 1 - \rho \\ h' & \rho \end{cases}$$

$\tilde{h}$ summarizes factors influencing how policies $x$ or $x'$ affect citizens. When $\tilde{h} = h$, it is optimal for voters to have $x$ implemented, while they prefer $x'$ when $\tilde{h} = h'$.

A politician (either incumbent or challenger) is competent ($\theta = C$) with probability $1 - q$ and incompetent ($\theta = N$) with probability $q$. Incumbent and challengers have independently distributed types.

The incumbent has an outside option $O_\theta$, which is increasing in his competence $\theta$, capturing the

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10In the terminal period, the incumbent simply wants to minimize costly effort.
11One could model this “learning by doing” with a less informative signal in the second period, without affecting the results.
12For example, it can represent the cost of providing some public good or the transaction cost associated with a certain redistribution scheme.
labor market premium for competence.\(^{13}\) This outside option eliminates unrealistic equilibria.\(^{14}\) The incumbent decides whether or not to run for reelection, trading off the expected payoff of running for reelection (net of effort cost) and the outside option. If he runs for reelection \((e = 1)\), the incumbent may perform a costly search to determine \(\tilde{h}\). More specifically, a search effort \(s\) costs the incumbent \(T_\theta(s)\) and reveals state \(h'\) with probability \(s\), while revealing nothing otherwise. Thus, the incumbent knows that \(\tilde{h} = h'\) if revelation occurred, and uses Bayesian updating to assess \(\tilde{h}\) conditional on no revelation.\(^{15}\)

If the incumbent is reelected, he receives a rent \(R\) in the second period. Otherwise, a challenger whose type distribution is identical to the incumbent’s is elected.

If, instead, the incumbent chooses the outside option \((e = 0)\), he gets a payoff of \(O_\theta\). The default action \(x\) is chosen, and a challenger is elected for the second period.

The cost function \(T_\theta(s)\) is increasing, continuous, and convex in \(s\), vanishes at 0, and is such that \(T_\theta(0) = 0\) and \(T_\theta'(1) \geq \rho R\) for both types \(\theta\). Moreover, we assume that the competent type has a higher marginal productivity of search effort:

**Assumption 1 (Single Crossing)** \(T_N'(s) > T_C'(s)\) for all \(s > 0\).

The next assumption states that the differential in responsiveness across types increases with the strength of the electoral incentives:

**Assumption 2 (Discerning Power of Electoral Incentives)**

\[
\kappa(z) = \frac{T''_C(T_C^{-1}(z))}{T''_N(T_N^{-1}(z))},
\]

is weakly decreasing in \(z\).

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\(^{13}\)Different versions of Assumption 3 have been employed in the literature on political selection. Mattozzi and Merlo (2008), Messner and Polborn (2004), and Caselli and Morelli (2004), among others, assume that productivity in a political job is positively correlated with productivity in a private sector job.

\(^{14}\)The outside option ensures the uniqueness of a “natural” equilibrium (see Section 4). It rules out the extreme equilibrium in which search effort and political selection reach their theoretical upper bounds and are unaffected by direct democracy. Such equilibrium is empirically implausible, as it predicts the absence of any statistical correlation between the cost of proposing initiatives (e.g., signature requirements) and their observed frequency. As discussed in detail in Section 7 all the existing empirical evidence suggests a positive correlation.

\(^{15}\)This choice of an asymmetric signal structure simplifies the analysis and can be modified without affecting the main results of the paper. It follows, among others, Bénabou and Tirole (2006), who employ a similar idea in the different context of a model of endogenous belief formation about society’s fairness. A similar signal structure is also used in Tirole (2009).
This assumption implies that as the marginal gain from search effort (the electoral incentive) increases, the ratio of search intensities of competent and incompetent types increases. Therefore, by inducing higher search from both types, voters are able to make a better inference on the type of the incumbent. As a consequence, when both types are searching, C is not only more productive, but also more responsive than N to the electoral incentive. Next, we assume that the outside option is higher for the C-type, normalizing to zero the one of the N-type. We also require that, at least in some circumstances (depending on voters behavior), the C-type might actually prefer $O_C$ over running for reelection.

**Assumption 3 (Outside Option)** $O_N = 0$ and

$$O_C > \max_{s \in [0,1]} R(1 - \rho) + R\rho s - T_C(s)$$  \hspace{1cm} (1)

This assumption implies that a competent incumbent prefers his outside option over a regime in which voters reelect him only when he chooses the optimal policy. It therefore imposes an upper bound on the reelection pressure that voters can put on the incumbent.

Finally, we introduce an assumption that acts as a “political linkage” across periods.

**Assumption 4 (Learning by doing)** In period 2, a reelected politician who previously exerted effort $s$ is able to discover the state $h'$ with probability $s$ at no additional cost.

This assumption, which is needed to guarantee the existence of an equilibrium with strictly positive effort in the first period, captures the fact that, if the incumbent exerts effort today, he will accumulate experience that benefits voters through better policies tomorrow. The results derived in the paper are easily shown to hold with a weaker version of this assumption in which only a fraction of the previous search effort is exerted at no cost in period 2, or by the introduction of correlation across periods between the draws of $\tilde{h}$.

As a result of these assumptions, the second-period search level $s$ is equal to zero if a challenger is elected, and to the first-period equilibrium search level, $s^*$, if the incumbent is reelected.

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To see why, intuitively, suppose on the contrary that past effort has no impact on the second period. Then, voters reelect the incumbent if and only if their posterior is higher than the challenger’s expected competence. If the incumbent puts strictly positive effort, any mistake makes voters strictly more pessimistic about the incumbent’s type and prompts them to surely elect a challenger. In such case, a competent incumbent prefers his outside option, and the equilibrium unravels.
3.1 Voters

Without initiatives, voters have the same utility function in each period \( t \in \{1, 2\} \): if, in period \( t \), the state of the world is \( \tilde{h} \) and policy \( \tilde{x} \) is chosen, voters' payoff is

\[
U_t = U(\tilde{x}; \tilde{h}) + I_{\{\theta=C\}} M.
\]

The total payoff is \( U = U_1 + U_2 \).

\( M \) is the “valence” of a competent politician (with zero valence for an incompetent one). In addition to the examples provided in earlier sections, \( M \) could be the expected utility gain that a \( C \)-type provides voters by better handling unforeseen contingencies (for example, a terroristic attack or a financial crisis) with respect to a \( N \)-type. Since voters do not observe competence, they are unable to perceive \( M \) until period 2.

The payoff from default policy \( x \) is normalized to zero for all states of nature, \( U(x; h) = U(x; h') = 0 \); the net benefit of a reform in \( h' \) is captured by \( U(x'; h') = \Delta > 0 \) and the net loss from a reform in \( h \) by \( U(x'; h) = -\Delta' < 0 \). Equivalently, \( \Delta \) is the relative loss of not implementing a reform when it is needed (political inertia), and \( \Delta' \) is the loss associated with an unnecessary reform. We assume that reforms are ex ante undesirable:

\[
\rho \Delta < (1 - \rho) \Delta'.
\]

In particular, it is efficient to choose \( x' \) only if the posterior of the state being \( h' \) is strictly higher than \( \rho \).

If the incumbent chooses to run for office, citizens make a reelection decision after observing the state \( \tilde{h} \) and the implemented policy. Voters’ reelection strategy consists of a reelection probability \( f(\tilde{h}, \tilde{x}) \) for each pair \( (\tilde{h}, \tilde{x}) \) of state and policy.

In the second period, the elected politician chooses a new policy. If the politician is a challenger, he makes no effort \( (s = 0) \) and chooses the default policy \( x \), since he has no reelection incentive. By contrast, a reelected incumbent learns the new state of the world \( h' \) with probability \( s_\theta \) (his effort level in the first period, which carries over to the second period), and chooses \( x' \) if he learns that the state is \( h' \), and \( x \) otherwise.

\[17\]One could introduce a discount factor across periods without affecting the results.

\[18\]Under the attack/crisis interpretation, it is enough that \( M \) is not perceived, with positive probability, before the election.
3.2 Initiatives

Earlier models have compared representative democracy to direct democracy in its purest form (citizens directly choose all policies, without any representation). In reality, however, direct democracy institutions usually coexist with representative democracy institutions, and may complement or substitute each other within a political system. In our model, elected representatives are present, but some of their decisions may be amended by citizens by initiatives.

Citizens can propose an initiative in the second period. An initiative is the process of calling a special election in which voters have the possibility of amending the default policy \( x \) with a reform \( x' \). In order to propose an initiative, voters have to pay a cost \( \mu \), which captures both legal costs (e.g., signature requirements) and technological costs (e.g., the cost of producing and disseminating information) of proposing an initiative. An initiative is proposed (and accepted) if and only if \( \mu \leq \Delta \). In our model, adding citizen initiatives is tantamount to reducing the cost \( \mu \) of initiatives below \( \Delta \).

In representative democracy \( (\mu \geq \Delta) \), if a politician of type \( \theta \) who chose effort \( s_\theta \) gets reelected, voters’ expected utility in period 2 equals \( U^r(\theta) = s_\theta \Delta \rho + I_{\theta=C}M \). If a challenger is elected, voters’ expected utility in period 2 equals \( U^r_c = (1 - q)M \). With direct democracy \( (\mu < \Delta) \), the expected second-period utility associated with a \( \theta \)-type is

\[
U^d(\theta) = s_\theta \Delta \rho + (1 - s_\theta)(\Delta - \mu) \rho + I_{\theta=C}M.
\]

If a challenger is elected, the expected utility for voters is given by

\[
U^d_c = (1 - q)M + \rho(\Delta - \mu).
\]

We assume that \( \rho \Delta < (1 - q)M \), which implies that

\[
U^r(C) > U^r_c > U^r(N) \quad (3)
\]

\[
U^d(C) > U^d_c > U^d(N) \quad (4)
\]

for all levels of \( s_C \) and \( s_N \). This assumption means that, if voters could observe the incumbent’s type, they would always reelect a competent type and always oust an incompetent type, no matter how much effort they put in the first period.

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20 The model could also allow citizens to amend a bad reform \( x' \), reverting to the status quo \( x \). Bad reforms, however, never occurs in natural equilibria, because politicians choose \( x' \) only if they learn that the state is \( h' \).

21 Our model abstracts from explicitly describing a proposer who would bear the cost of raising the initiative. Such feature would become more relevant in a model with heterogeneous voter preferences.
A lower initiative cost makes voters relatively less concerned about search and relatively more concerned about political selection, keeping the payoff $M$ constant.

We assume without loss of generality that $\mu \leq \Delta$: $\mu = \Delta$ corresponds to pure representative democracy (since there is no gain in amending policy), while $\mu < \Delta$ implies that citizens strictly benefit (ex post) from amending the policy.

## 4 Equilibrium and Political Contract

In this model, the “natural policy” is to implement the reform if and only if the state $h'$ is revealed. Citizens’ ideal policy would be for politicians to put maximal search effort (i.e., $s = 1$), followed by the natural policy. In the present setting, there are three possible kinds of equilibria. First, there always exists a “no search” equilibrium, in which the incumbent puts no effort, hence revealing nothing about his type, and is reelected by citizens, who are indifferent between him and his challenger. This equilibrium is suboptimal whenever an equilibrium with positive search exists.

The second and most natural type of equilibrium, when it exists, is one in which citizens surely reelect the incumbent whenever he chose the optimal policy given the state ($x$ when the state is $h$ and $x'$ when the state is $h'$) and otherwise punish him. Assuming for now that the incumbent follows the natural policy, the only mistake that happens in equilibrium is to choose the default option $x$ when the state was $h'$. Punishment takes the form of a lower reelection probability conditional on a mistake happening. In equilibrium, that reelection probability must induce just the right amount of effort from both incumbent types so that, conditional on the mistake happening, voters are indeed indifferent between reelecting the incumbent or not. Such type of equilibrium is characterized by a single number, the probability $f$ of reelection conditional on observing $(h', x)$.

Finally, there exists a “mirror equilibrium” in which the incumbent does the exact opposite of what his signal tells him to do, i.e., chooses $x'$ if he does not learn the state, and $x$ if $h'$ is revealed. This equilibrium is clearly suboptimal for voters. However, since that behavior does reveal the incumbent’s type, and since citizens highly value competence and lack commitment power, they will reelect the incumbent with high enough prior despite this suboptimal policy.

We focus on natural equilibria, as they maximize welfare within the class of all self-enforcing contracts that citizens, lacking commitment, can credibly propose to politicians.
In the rest of this section, we will show that there exists exactly one positive-search equilibrium of this kind, provided that $\Delta$, the gain from implementing a needed reform, is high enough. We characterize this equilibrium, and verify that it is indeed welfare maximizing within the class of self-enforcing contracts.

### 4.1 Reelection probabilities

Let $\bar{p}$ denote the threshold for the probability that the incumbent is competent, given search efforts $s_C, s_N$ and conditional on observing mistake $(h', x)$, above which voters reelect the incumbent. This threshold depends on search effort levels in the first period as those efforts carry over to the second period (learning-by-doing). The next proposition characterizes the threshold.

**Proposition 1** Upon observing $(\tilde{h}, \tilde{x})$, voters strictly prefer to reelect the incumbent if and only if their posterior $p(\tilde{h}, \tilde{x})$ satisfies

$$p(\tilde{h}, \tilde{x}) > \bar{p} = \frac{(1 - q)M - s_N \mu \rho}{M + \mu \rho(s_C - s_N)},$$

and are indifferent across all reelection probabilities if $p(\tilde{h}, \tilde{x}) = \bar{p}$.

**Proof.** Voters’ expected second-period utility from reelecting the incumbent, given posterior $p$ is

$$p[M + \rho s_C \Delta + \rho (1 - s_C)(\Delta - \mu)] + (1 - p)[\rho s_N \Delta + \rho (1 - s_N)(\Delta - \mu)],$$

while their expected second-period utility with a challenger is $(1 - q)M + \rho(\Delta - \mu)$. Comparing these expressions yields the threshold.

### 4.2 Incumbent’s Choice

The incumbent’s strategy in the first period consists of the following dimensions:

- Whether to run for reelection ($e = 1$), or not ($e = 0$). We allow randomization and let $\lambda = Pr[e = 1]$.
- Search effort $s \in [0, 1]$
- Policy $y$ upon receiving no signal.
• Policy $y'$ upon learning that the state of the world is $h'$.

An incumbent of type $\theta$ chooses his strategy so as to solve

$$\max_{e,s,y,y'} \{ R \left[ (1-\rho)f(y,h) + \rho s f(y',h') + (1-s)\rho f(y,h') \right] - T_{\theta}(s) \} + (1-e)O_{\theta}$$

The incompetent type always runs for reelection since his outside option is zero. In any equilibrium with positive search, the competent type must also run with positive probability $\lambda > 0$.

Convexity of the cost function implies that the optimal search effort is characterized by

$$s_{\theta} = T_{\theta}'^{-1} \left( (f(h', y_{\theta}') - f(h', y_{\theta}))R\rho \right).$$

In particular, $s_{\theta}$ is independent of $f(h, x)$ and $f(h, x')$.

### 4.3 Optimal Self-Enforcing Political Contract

This section establishes that implementing the “natural policy” where the incumbent implements the reform if and only if he learns the state $h'$ is optimal from voters’ viewpoint, and that this policy is implemented by always reelecting the incumbent if he made no mistake, randomizing between reelection and ousting if he chose the default option when the reform was needed, and punishing him with certain ousting if he implemented a bad reform. We will also show that there exists at most one such equilibrium, and exactly one provided that $\mu$ is high enough.

An equilibrium of the kind described above, with $f(h, x) = f(h', x') = 1$, $f(h, x') = 0$, and $f(h', x) \in [0,1)$, clearly dominates any no-search equilibrium. There remains to show that it is also optimal among all possible equilibria inducing positive search.

First, consider voters’ reelection strategy conditional on observing $h$. In that case, the incumbent cannot have received the signal revealing $h'$, irrespective of his type and search level. Type selection cannot occur in that case, and the best voters can hope for, given $h$, is that the incumbent chooses the optimal strategy, which is to take action $x$, which is ex ante optimal, and a fortiori optimal conditional on not receiving any signal. Moreover, voters want to reelect the incumbent, other things equal, for two reasons. First, their posterior about his competence is the same as their belief about the competence of the challengers. Second, the incumbent has

\[\text{22}\text{Otherwise, running for reelection would reveal that the incumbent is incompetent. As a consequence, voters would never reelect the incumbent (see 3) and the running type would optimally choose zero effort.}\]
the strict advantage, compared to his challengers, of having performed positive effort, which
carries over to the second period. Therefore, an optimal strategy is to set $f(h, x) = 1$ and
$f(h, x') = 0$, as it achieves exactly this.23

Now consider the case where the state is $h'$. In this case, voters want the incumbent to choose
the reform $x'$ when he learns the state. Thus, abstracting for now from the adverse selection
problem, voters want the incumbent to choose $x'$ if and only if he received the signal. We now
take into account adverse selection. In any equilibrium with positive search, the competent type
searches more than the incompetent type because it is less costly for him to do so. Therefore, the
natural policy not only maximizes voters’ first-period payoff, but it also increases the probability
of selecting a more competent politician. Thus, setting $f(h', x') = 1$ is optimal. The remaining
probability $f(h', x)$ is pinned down by equilibrium conditions. If $f(h', x) = 1$, the incumbent is
always reelected if he chooses the default action $x$ has no incentive to search.24 This establishes
the following result.

**Proposition 2 (Natural Positive-Search Equilibrium)** Any welfare-maximizing equilib-
rium can be implemented by setting $f(h, x) = f(h', x') = 1$ and $f(h, x') = 0$. The remaining
reelection probability $f = f(h', x) \in [0, 1]$ is determined by equilibrium conditions.

For this equilibrium, equilibrium search is given by

$$s_\theta(f) = T_\theta^{s-1}((1 - f)\rho R).$$  \hspace{1cm} (5)

Because the cost function $T$ is convex, $s_\theta(f)$ is decreasing in $f$. Since $T_\theta(s)$ satisfies the single-
crossing property in $(s, \theta)$, the search levels are also ranked: $s_C(f) \geq s_N(f)$ for all $f$.

In our setting, there may exist two kinds of natural equilibria. We call “interior equilibrium” one
in which the competent type surely runs for reelection ($\lambda = 1$), and “constrained equilibrium”
one in which the outside option of the competent incumbent is binding. In a constrained
equilibrium, a competent incumbent runs for reelection with probability $\lambda < 1$ and takes
his outside option with the remaining probability. The incumbent’s outside option precludes
equilibria in which mistakes are punished by certain ousting (i.e., such that $f(h', x) = 0$).

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23Setting $f(h, x')$ small enough would also yield an optimal strategy, provided that the incumbent follows the
“natural policy,” since $(h, x')$ does not arise on the equilibrium path.

24In that equilibrium, voters are indifferent between reelecting the incumbent or the challenger, as both have
the same expected competence and no search carries over to the next period.
Proposition 3 (Reelection and Search Bounds) The competent type runs for reelection only if $f(h', x) \geq \underline{f}$, where $\underline{f}$ solves

$$T_C(s_C(\underline{f})) + O_C = R(1 - \rho) + R\rho s_C(\underline{f}) + R\rho(1 - s_C(\underline{f}))\underline{f}$$

Consequently,

$$s_N \leq \bar{s}_N \text{ and } s_C \leq \bar{s}_C,$$

where $\bar{s}_N = s_N(\underline{f})$ and $\bar{s}_C = s_C(\underline{f})$.

Proof. The expected payoff for a competent incumbent who runs for reelection and chooses effort $s$ is, assuming that $f(h, x) = f(h', x') = 1$,

$$R(1 - \rho) + R\rho s + R\rho(1 - s)f - T_C(s).$$

The objective is increasing in $f$ and submodular in $s$ and $f$, implying that the optimal search level is decreasing in $f$ and that the maximum is increasing in $f$. Running for reelection is optimal if and only if the maximum is higher than $O_C$. Therefore, running for reelection is optimal if and only if $f \geq \underline{f}$. This bound also holds if $f(h, x)$ and/or $f(h', x')$ are strictly less than one, as it reduces the value of effort even more. The rest of the proposition follows from the previous analysis. ■

Corollary 1 In any equilibrium with positive search, $f(h', x) \geq \underline{f}$.

Proof. Suppose that $f(h', x) < \underline{f}$. Then, the competent incumbent does not run for reelection. This implies that an incumbent who runs for reelection immediately reveals himself as incompetent. However, in that case, voters do not reelect him, which is a direct consequence of \[4\], and equilibrium search must be zero. ■

Thus, voters cannot use a “tough” strategy where a mistake $(h', x)$ is punished by a very likely removal from office. The reason is that this would create a lemons problem where only incompetent types would seek office for a second term and, hence, immediately reveal themselves as incompetent.

We now show that there exists at most one (natural) positive-search equilibrium. Let $\phi_1 = \frac{\mu \rho}{M(1 - q)}$, $\phi_2 = \frac{\mu \rho}{s_M q}$, and define

$$\mu_1 = \left[ \frac{\rho}{Mq} + \frac{\rho}{M(1 - q)} \frac{T_C''(0)}{T_N''(0)} \right]^{-1} \left[ 1 - \frac{T_C''(0)}{T_N''(0)} \right],$$

$$\mu_2 = \left[ \frac{\rho}{Mq}(1 - \bar{s}_C) + \frac{\bar{s}_N}{\bar{s}_C} \frac{\rho}{M(1 - q)}(1 - \bar{s}_N) \right]^{-1} \left[ 1 - \frac{\bar{s}_N}{\bar{s}_C} \right].$$
Proposition 4 (Equilibrium) If \( \mu \leq \mu_1 \), there is no equilibrium with positive search. If \( \mu_1 < \mu \leq \mu_2 \), there is a unique equilibrium with positive search. Both types surely run for reelection, and the reelection probability \( f^* \) conditional on \((h', x)\) solves

\[
s_N(f^*)[1 + \phi_1] - s_N^2(f^*)\phi_1 = s_C(f^*)[1 - \phi_2] + s_C^2(f^*)\phi_2.
\]

(9)

If \( \mu > \mu_2 \), there is a unique equilibrium with positive search. The competent type runs for reelection with probability

\[
\lambda = \left[ 1 - s_N[1 + \phi_1] + \phi_1 s_N^2 \right] \left[ 1 - s_C[1 - \phi_2] - \phi_2 s_C^2 \right]^{-1} \tag{10}
\]

and the reelection probability conditional on \((h', x)\) is \( f \).

The next section illustrates how the introduction of direct democracy and, more generally, changes in the cost of direct democracy, affect equilibrium search and selection.

5 Selection and Disincentive Multiplier

How well does the election filter out incompetent politicians? The incumbent risks not being reelected if he made a mistake, and the only kind of mistake that may occur in equilibrium is \((h', x)\): the incumbent chooses the default policy when the reform is needed. Ideally, voters would like such a mistake to reveal, as much as possible, an incompetent type. The separating power of the electoral process is its ability to weed out incompetent politicians. This is measured by the posterior probability \( p(h', x) \) that the incumbent is competent, conditional on observing \((h', x)\): the lower that posterior, the more selective the electoral process. The next proposition shows that this posterior is decreasing with electoral incentives: a lower probability \( f \) of reelection conditional on the mistake \((h', x)\) lowers the probability of ousting a competent type.

Let \( p(f) = \frac{(1-q)(1-s_C(f))}{(1-q)(1-s_C(f)) + q(1-s_N(f))} \) denote the probability that the incumbent is competent, conditional on \((h', x)\).

Proposition 5

i) \( p(f) \) is increasing in \( f \).

ii) \( s_C(f) - s_N(f) \) is decreasing in \( f \).
The condition for $f$ to induce an interior equilibrium is

$$\frac{(1-q)}{(1-q) + q\tau(f)} = \frac{(1-q)M - \mu s_N(f)}{M + \mu(s_C(f) - s_N(f))},$$

(11)

where

$$\tau(f) = \frac{1 - s_N(f)}{1 - s_C(f)} > 1$$

and $s_C, s_N$ are the equilibrium search levels of both types of incumbent given reelection disincentive $f$. Let $f(\mu)$ denote the solution of (11), which is unique by Proposition 4.

The left-hand side of (11) is the posterior probability that the incumbent is competent, conditional on making a mistake. It is decreasing in $\tau$. The right-hand side is the posterior level at which voters are indifferent between reelecting the incumbent and electing a challenger, given i) the equilibrium search levels $s_N(f)$ and $s_C(f)$ chosen by the incumbent, and ii) the cost $\mu$ of correcting mistakes in the second period. The right-hand side is decreasing in $\mu$.

Decreasing the cost $\mu$ of direct democracy weakens electoral incentives in a self-reinforcing pattern, as we now explain. First, it raises the posterior at which voters accept to reelect the incumbent. Indeed, voters now care more about competence per se and less about the value of past effort, since they can amend mistakes at lower cost. Other things equal, thus, $\tau$ must decrease: the equilibrium posterior conditional on a mistake must increase, in order to keep voters indifferent. We have shown in part i) of Proposition 5 that $\tau(f)$ is decreasing in $f$. Therefore, $f$ must increase in equilibrium. The intuition is that, to achieve a higher posterior conditional on a mistake, the incumbent must have searched less, which is possible only if the search disincentive $f$ is higher.

From part ii) of Proposition 5, a higher $f$ implies that $s_N(f)$ and $s_C(f) - s_N(f)$ decrease. Other things equal, this increases the right-hand side of (11), which must again be offset by a lower $\tau$, hence a higher $f$, and so forth. Therefore, the impact of a lower direct democracy cost has a spiraling negative effect on electoral incentives.

Figure 1 illustrates the equilibrium reelection probability for two values of $\mu$ associated with an interior equilibrium. The solid lines are obtained from a value of $\mu$ twice as large as the one generating the dotted lines. As stated earlier, as the cost of direct democracy decreases, the responsibility substitution effect makes voters more willing to reelect an incumbent who mistakenly chose the status quo policy over the reform.

The analysis implies that, in a constrained equilibrium, the probability of reelecting an incumbent who made a mistake is the same with and without initiatives. Instead, if the equilibrium
is interior, adding direct democracy increases that probability. More generally, whether the equilibrium is interior or constrained seems to be crucial in order to assess the effect of direct democracy on effort, selection and overall welfare. In the following section we will show that this is indeed the case. In the subsequent one, we will argue that only the implications generated by the interior equilibrium can be compatible with a rich set of empirical findings from previous studies.

6 Welfare, Direct Democracy, and Political Agency

This section studies the impact of direct democracy on citizens’ welfare. Our benchmark corresponds to the case where the cost $\mu$ of direct democracy is equal to its policy benefit $\Delta$. Starting from $\mu = \Delta$, we study how citizens’ expected welfare changes as $\mu$ decreases. As observed in earlier sections, there are two cases to consider: when representative democracy is in an interior equilibrium ($\mu_1 < \Delta \leq \mu_2$), and when representative democracy is in a constrained equilibrium ($\Delta > \mu_2$).\footnote{When $\Delta < \mu_1$, the incumbent makes no effort. Therefore, direct democracy only has a direct, beneficial impact.}

Figure 1: Equilibrium Reelection Probabilities for $\mu = 50$ (solid lines) and $\mu = 25$ (dashed lines). Other parameter values: $M = 100$, $\rho = 0.8$, $q = 0.56$. 
We denote by $EU(\mu)$ the expected utilities under direct democracy when the cost is $\mu$. If $EU(\Delta) \geq EU(\mu)$, direct democracy at cost $\mu$ reduces citizens’ ex ante welfare with respect to pure representative democracy.

For our next result, we denote by $\lambda(\Delta)$ the equilibrium probability of participation of a competent incumbent, as characterized by (10), and by $s^*_\theta$ the equilibrium level of search for each competence type $\theta \in \{N, C\}$ under pure representative democracy. Finally, “costless amendment” will refer to the case in which $\mu = 0$.

**Proposition 6** The introduction of costless amendment reduces citizens welfare in each of the following cases:

1. $qs^*_N + (1-q)s^*_C > 1/2$.

2. $qs^*_N + (1-q)\lambda(\Delta)s^*_C > (1 + q(1 - \lambda(\Delta)))(2\phi_1)^{-1}$.

**Responsibility Substitution** Provided that equilibrium search is high enough, Proposition 6 implies that introducing costless direct democracy reduces expected welfare. The result is driven by a responsibility substitution mechanism. Decreasing the cost of direct democracy has two effects on welfare: a direct reduction, from $\Delta$ to $\mu$, of the welfare cost stemming from not implementing a needed reform; and an increase of the probability that the incumbent will not be implementing a needed reform, which is caused by responsibility substitution. Relative to a pure representative democracy with high equilibrium search, the direct effect is weaker than the responsibility substitution effect.

This suggests that adding direct democracy is likely to improve citizens’ welfare only when the initial quality of the political process, measured by the initial level of equilibrium search, is low. When politicians’ ability to identify needed reforms is high, or when voters can elicit more search through tougher punishment strategies, one should expect high equilibrium search under pure representative democracy, and introducing direct democracy may bring little benefit or even be harmful to citizens’ welfare, as responsibility substitution becomes more salient in such environments.

**Selection Complementarity** The impact of $\mu$ on the constrained equilibrium stands in sharp contrast with the responsibility substitution arising in the interior equilibrium. A lower $\mu$ results in a higher competence threshold above which citizens reelect the incumbent, as they care more about competence per se (due to the second policy dimension) and less about search. In equilibrium, this requires a higher probability $\lambda$ that the competent type seeks reelection,
This implies a complementarity between direct democracy and selection: a lower cost of direct democracy increases the proportion of competent politicians who seek reelection and, as a consequence, the average search level. Ironically, this virtuous selection complementarity implies that a lower cost of direct democracy makes it less likely that citizens will actually need to use direct democracy. The reason is that, in expectation, the politician is more competent and makes fewer mistakes.

Proposition 6 compares two extremes: no (or prohibitively expensive) direct democracy institution vs. costless direct democracy institution. The next proposition shows a stronger result. It provides conditions under which citizens welfare is increasing in $\mu$ over the entire domain $[\mu_1, \mu_2]$. We recall the definition of $\kappa$:

$$\kappa(z) = \frac{T_C''(T_C^{-1}(z))}{T_N''(T_N^{-1}(z))}$$

**Proposition 7** When $\kappa$ is constant and $\Delta \leq \mu_2$, $EU(\mu)$ is strictly increasing on $(\mu_1, \Delta]$.

$\kappa$ is constant when search costs take the form $T_\theta(s) = \alpha_\theta s^\beta$, with $0 < \alpha_C < \alpha_N$, and where $\beta > 1$ guarantees convexity of the cost function. This family includes, for example, quadratic cost functions, scaled by a competence factor. This additional assumption acts as a regularity condition on the derivative of the welfare function with respect to the cost, and allows to establish a uniform monotonicity result that is otherwise not guaranteed when $\kappa$ is highly responsive to the electoral incentives.

Figures 2 and 3 plot $EU(\mu)$ in the two types of initial equilibria for the simple quadratic cost function case.

In our model, then, initiatives improve welfare only in the following circumstances: when search in representative democracy was already low; when the separating power of elections is exceptionally high; and when elections have become completely unsuccessful in both selecting politicians and providing them with incentives to search.

The situation in which $\mu < \mu_1$ is empirically implausible, for two reasons. First, the real-world analog of $\mu$ can be lowered up to a certain level: together with legal costs (e.g. signature requirements for petition, geographical representation requirements), usually mandated by the state constitution, there are technological costs associated with proposing an initiative (e.g. legislative knowledge required to write a bill, disseminating information among voters, organizing a

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26 In a constrained equilibrium, reelection probability and search levels are independent of $\mu$.

27 All figures are obtained using a quadratic cost function for each type, with $T_C = .6T_N$. 

21
special election) that cannot be arbitrarily lowered. Second, and more generally, \( \mu < \mu_1 \) implies that voters can exert direct policymaking at a lower cost than using representative democracy because the informational effort exerted by politicians has a negative social value. Therefore, representative democracy itself is unnecessary in this case. Additionally, a rapidly decreasing \( \kappa(\mu) \) seems unrealistic since many stages of the policymaking process have become delegated to bureaucracies, thereby making the signal extraction problem relatively more sophisticated.

In summary, when there is a complementarity between voters’ payoffs and improved selection (constrained equilibrium) initiatives can be beneficial if the cost of proposing them preserves these complementarities. When there is a substitutability between voters’ payoffs and effort (constrained equilibrium) initiatives reduce equilibrium search. This effect is likely to be welfare decreasing and is stronger when the initial equilibrium search is higher. This result is somewhat surprising as, in our setting, initiatives are ex post welfare improving and—by construction—immune from their standard drawbacks (tyranny of the majority, interest groups).
7 Empirical Implications

This section discusses the empirical predictions of the model. We find, in particular, that only the interior equilibrium generates a relationship between the cost of direct democracy and the frequency of initiatives that is consistent with the data.

7.1 Frequency and Cost of Initiatives

The equilibrium probability that an initiative occurs in the model can be interpreted, for empirical purposes, as the frequency at which initiatives occur in the data.

Interior and constrained equilibria yield different predictions about the frequency of initiatives. When the equilibrium is constrained, a lower cost $\mu$ of initiatives reduces the probability of initiatives, due to the selection complementarity analyzed in Section 6. When the equilibrium is interior, lowering $\mu$ worsens both search effort and political selection, which translates into an increased need for initiatives. This observation provides a way to empirically distinguish between interior and constrained equilibrium.

**Proposition 8** The expected frequency of initiatives may decrease in $\mu$ only if the equilibrium is interior.
In the Appendix, the expected frequency of initiatives is shown to be constant in \([0, \mu_1]\) and increasing above \(\mu_2\) (that is, when the equilibrium is constrained). A condition on the expected search level \(qs_N(f) + (1 - q)s_C(f)\) is also provided, under which the expected frequency is decreasing in \(\mu\) on the entire range \((\mu_1, \mu_2)\).

Intuitively, the expected frequency of initiatives decreases if expected search is sensitive enough to the reelection probability \(f\), and if equilibrium search is high enough. These conditions are closely related to the condition under which initiatives have a negative welfare effect that was derived in Section 6, and hold in a simple example with quadratic cost functions that is provided in the Appendix.

In summary, a negative relationship between cost and frequency can only arise when the responsibility substitution effect is in place and initiatives are likely to be welfare reducing.

The available empirical evidence suggests that the relationship between both the legal (Matsusaka and McCarty, 2004; Bowler and Donovan, 2004) and technological (Matsusaka, 2005c) components of the cost of proposing initiatives and their frequency is indeed negative.\(^{28}\) As a consequence, the effect of the unprecedented increase in the frequency of initiatives that has occurred in the United States in the last three decades can be interpreted in light of the findings of this paper, which provides an alternative channel for the increased criticism that direct democracy institutions have received in recent years.\(^{29}\)

### 7.2 Political Selection

As was shown in Section 5, the model predicts that direct democracy institutions lead to a more permissive reelection probability *conditional* on a policy mistake. The prediction is more subtle with respect to the *ex ante* reelection probability, \(1 - \rho(1 - s_0)f\), owing to the disincentive multiplier: although voters are more lenient with policy mistakes, those mistakes also become

\(^{28}\)Matsusaka and McCarty (2004) show, using data from 22 states between 1953 to 1993, that the frequency of initiatives is negatively related to two key variables related to \(\mu\): the signature requirement for the petition and a dummy variable for the requirement of some geographic dispersion in signatures. Bowler and Donovan (2004) obtain similar results using a more sophisticated measure capturing the cost of proposing an initiative. For the technological component of \(\mu\), Matsusaka (2005c) argues that in the last three decades there has been a sharp decrease in the costs of gathering and disseminating information, coupled with a steady increase in the number of initiatives, especially in western states (California, Oregon).

\(^{29}\)See for example Broder (2000), Gerber (1999), or Cronin (1989), which argues in favor of higher signature requirements for state level initiatives and is against the introduction of direct democracy at the federal level.
more likely. When equilibrium search is relatively insensitive to reelection incentives, the ex ante reelection probability is therefore higher when direct democracy is present, but the difference is smaller than for the reelection probability conditional on policy mistakes. These predictions are consistent with the empirical evidence provided by Hugh-Jones (2010) and Bali and Davis (2007): in states that allow initiatives, incumbents generally get reelected more often than in other states, while they are reelected much more often than in non-initiative states when economic conditions are bad.

### 7.3 Likelihood of Reform Optimality

How does the probability $\rho$ that a reform is needed affect the political equilibrium? Our model suggests that a higher value of $\rho$ increases the range of costs $\mu$ for which the equilibrium is constrained, and increases equilibrium search when the equilibrium is interior.

The insights of Sections 5 and 6 can help understand these results. A higher value of $\rho$ increases citizens’ value for the incumbent’s search activity, which reduces the reelection posterior threshold. To match this lower threshold, equilibrium search must be higher, because the posterior conditional on a mistake is lower if more search occurred. Higher search can only be sustained by higher search incentives, i.e., by a lower probability of reelection conditional on a mistake. This explains why equilibrium search is increasing in $\rho$. It also explains why the constrained-equilibrium region gets larger: the value of seeking reelection is now lower for a competent type, because it requires higher effort, and it comes with a lower probability of reelection.

**Proposition 9**

i) In any interior equilibrium, search is increasing in $\rho$.

ii) Both $\mu_1$ and $\mu_2$ are decreasing in $\rho$.

How direct democracy affects welfare therefore depends on $\rho$: a higher $\rho$ can move the equilibrium from the no-search region to the interior-equilibrium region, and from the interior-equilibrium region to the constrained-equilibrium one, reversing the welfare impact of direct democracy.
We now study how the equilibrium depends on the probability $q$ that politicians (both incumbent and challengers) are incompetent. The equations (9) and (10) defining $f^*$ and $\lambda$ suggest that $q$ has an ambiguous effect on both of these variables. This point is illustrated by Figure 4, which displays the relationship between $q$ and $\mu_1, \mu_2$ (the threshold values for $\mu$ that define the three equilibrium regions of the model). $\mu_1$ is the smallest cost of initiative for which search matters to voters and $\mu_2$ is the largest cost of initiative for which responsibility substitution arises.

Search is maximized when $\mu = \mu_2$ (that is when $f = f^*$ and $\lambda = 1$) and is equal to zero for $\mu \leq \mu_1$. Therefore, fixing a value for $\mu$ (say, low enough to be below $\mu_1$ for some $q$) and letting $q$ vary, we see that there is a non-monotonic relationship between $q$ and search. For $q$ low enough, the equilibrium is constrained, and increasing $q$ does not affect the equilibrium search performed by each type, but changes the probability of entry of the competent type due to the selection complementarity, until the $\mu_2$ boundary is reached from below. After crossing $\mu_2$, the figure also relies on the assumption that $(1-\bar{s}_N)\bar{s}_C < \kappa(0)$, which is guaranteed when $\bar{s}_N > .5$ or when $\kappa$ is a constant, and implies that the peak of $\mu_1$ is located to the right of the peak of $\mu_2$. 

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**Figure 4: Equilibrium regions for $q$**

### 7.4 Competence

We now study how the equilibrium depends on the probability $q$ that politicians (both incumbent and challengers) are incompetent. The equations (9) and (10) defining $f^*$ and $\lambda$ suggest that $q$ has an ambiguous effect on both of these variables. This point is illustrated by Figure 4, which displays the relationship between $q$ and $\mu_1, \mu_2$ (the threshold values for $\mu$ that define the three equilibrium regions of the model). $\mu_1$ is the smallest cost of initiative for which search matters to voters and $\mu_2$ is the largest cost of initiative for which responsibility substitution arises.

Search is maximized when $\mu = \mu_2$ (that is when $f = f^*$ and $\lambda = 1$) and is equal to zero for $\mu \leq \mu_1$. Therefore, fixing a value for $\mu$ (say, low enough to be below $\mu_1$ for some $q$) and letting $q$ vary, we see that there is a non-monotonic relationship between $q$ and search. For $q$ low enough, the equilibrium is constrained, and increasing $q$ does not affect the equilibrium search performed by each type, but changes the probability of entry of the competent type due to the selection complementarity, until the $\mu_2$ boundary is reached from below. After crossing $\mu_2$, the figure also relies on the assumption that $(1-\bar{s}_N)\bar{s}_C < \kappa(0)$, which is guaranteed when $\bar{s}_N > .5$ or when $\kappa$ is a constant, and implies that the peak of $\mu_1$ is located to the right of the peak of $\mu_2$.
responsibility substitution starts driving each type’s equilibrium search down, until the latter reaches zero when $\mu$ approaches the $\mu_1$ boundary. Eventually, while $q$ keeps increasing, $\mu_1$ is crossed again and search starts increasing as the responsibility substitution effect becomes less severe. When $\mu_2$ is crossed again each type’s search, after reaching a peak, remains constant and the entry probability of the competent type decreases.

We can now describe how the effect of direct democracy (and, more generally, of a lower $\mu$) depends on $q$. When $q$ is small (most politicians are competent) or large (most politicians are incompetent), the range of $\mu$’s for which the equilibrium is constrained is large, and initiatives are beneficial except for low values of $\mu$. For moderate values of $q$, instead, the range of $\mu$’s associated with interior and no-search equilibria get larger, thereby making initiatives more likely to generate the responsibility substitution effect.

The following proposition formalizes the intuition contained in Figure 4.

**Proposition 10** $\mu_1$ and $\mu_2$ are strictly quasiconcave in $q$ with an interior maximizer.

**Proof.** The thresholds can be expressed as

$$\mu_1 = \frac{M}{\rho} \left[1 - \kappa(0)\right] \left[\frac{1}{q} + \frac{\kappa(0)}{1 - q}\right]^{-1}$$

$$\mu_2 = \frac{M}{\rho} \left[1 - \frac{\bar{s}_N}{\bar{s}_C}\right] \left[\frac{1 - \bar{s}_C}{q} + \frac{\bar{s}_N}{\bar{s}_C} \frac{1 - \bar{s}_N}{1 - q}\right]^{-1}$$

their denominators are strictly quasiconvex in the relevant domain: therefore, they are both decreasing between 0 and their unique interior minimizer, and then increasing from there onwards. Moreover, these denominators are also strictly positive in the whole domain of $q$. Since the reciprocal of a strictly quasiconvex function that is positive on a subset of $\mathbb{R}^n$ is strictly quasiconcave on that subdomain, then $\mu_1$ and $\mu_2$ are both strictly quasiconcave functions, and their maximizer must be the minimizer of their denominators. 

Proposition 10 implies that, as expected competence increases from low to moderate levels, initiatives become more likely to be welfare decreasing. This result further supports the idea that direct democracy is more likely to generate responsibility substitution when the political environment is generally more favorable to citizens (that is, when on average politicians are more competent).

Our analysis thus supports the criticism that direct democracy has generated in the recent years. When, in the United States, initiatives were introduced at the state-level (mostly in

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31Several articles have linked recent failures of California’s political system to direct democracy institutions.
western states), politicians were often perceived as weak and controlled by large corporations. Several aspects captured in our model by the politician’s “competence” have improved over the last century. This implies that, while initiatives were highly beneficial a century ago, responsibility substitution effects have become more serious in recent years.

8 Related literature

Our model shares several characteristics with the public finance model of Besley and Smart (2007). In that model, voters use elections to discipline the incumbent and select “honest” politicians for the second period. Compared to that model, we consider politicians with heterogeneous ability to acquire information and to provide good policies. Moreover, information is endogenously acquired, rather than determined at the outset.

More broadly, our results contribute to the contracting literature on information acquisition. Crémér and Kahlil (1992), Crémér at al (1998a) and Crémér and al. (1998b) study different versions of a principal-agent model in which the agent’s information is endogenous and depends on the contract proposed by the principal. Our paper focuses on the particular form of contracts likely to arise in the context of political delegation, and features limited commitment and amendment ability. Our contribution to this literature also includes a new condition (Assumption 2), different from the standard single-crossing condition, under which an adverse shock on incentives (in our context, a lower cost of initiatives) is magnified by equilibrium effects.

Our welfare result is also related to Gilligan and Krehbiel (1987). Comparing the allocation of power between a legislative body and a committee, they study the choice between the “Restrictive Amendment” procedure and the “Unrestrictive” procedure. Their model includes an information-acquisition task, which is better performed when the legislature commits itself not to amend the proposal of the committee, thereby raising the value of information. In contrast to that paper, we introduce uncertainty about politicians’ ability, and study how this adverse

See, for example, Andreas Kluth’s recent column entitled “The people’s will” in The Economist (April, 20th 2011).

32 The improvement in average competence can be related to both better accountability mechanisms and to the improvements in the resources that politicians can access to deliver good policies to voters, which applies especially to complex, multidimensional issues where voters are likely not to be able to use initiatives (captured by the payoff $M$).

33 An honest politician sets a high tax rate only when the cost of providing the public good is high, which is its private information, and never diverts public funds when the cost is low.
selection component interacts with the moral hazard component already present in Gilligan and Krehbiel.

In comparing direct democracy and representative democracy, Persson and Tabellini (1994) show how electing a representative agent can help solve the credibility problem associated with dynamic capital taxation. In Redoano and Scharf (2004) and Loeper (2007), the two procedures are compared in a setting with multiple districts and inter-jurisdictional externalities. In Redoano and Scharf (2004), the representative agent serves as a commitment device for voters in different districts to credibly commit to a compromise policy. As a result, representative democracy is more likely to deliver policy centralization than referenda. In Loeper (2007), direct democracy is immune from strategic delegation problems, which makes it Pareto superior to representative democracy. Maskin and Tirole (2004) find that when voters are uncertain enough about the optimal policy, direct democracy is dominated by representative democracy. By contrast, our results do not rely on any informational disadvantage of direct democracy compared to representative democracy. Besley and Coate (2001) investigate the role of initiatives as “unbundling” institutions. Initiatives allow citizens to reduce the number of issues that are bundled in the electoral competition through candidates, thereby making the outcome likely to be closer to their preferences. This result holds even in absence of any type of agency problem between voters and politicians. Boehmke and Patty (2007) look at how proposed initiatives can play a role of informational cues: by the fact that a certain measure was not enacted by the legislature and became an initiative, voters can infer that the size and the relevance of the set of losers of that measure is significant, and then use this information to update their beliefs.

Hugh-Jones (2010) analyzes a reduced-form model in which direct democracy and representative democracy are complements. That model predicts that politicians get reelected more often in states with direct democracy than in states without it. Hugh-Jones also provides empirical evidence confirming this prediction. By contrast, our less stylized model, where direct democracy and representative democracy can be complements or substitutes, suggests that the relationship between direct democracy and reelection probability found in the data is consistent only with responsibility substitution (interior equilibrium), and not with selection complementarity (constrained equilibrium).

There is a large body of literature in political science on direct democracy institutions in United States. Matsusaka (1992) and Matsusaka (2005b) develop a theory, which Boehmke and Patty (2007) build upon, of the optimal allocation of policies between legislature and direct democracy. His main finding, supported by empirical evidence, is that when a policy creates a substantial set
of losers, direct democracy is more likely to (and more effective at) tackling these issues, while an elected representative will prefer to adopt measures that are likely to benefit most of the population. For this reason, direct democracy institutions are welfare enhancing. Matsusaka and McCarty (2001) is closer to our paper. In that model, direct democracy and representative democracy may coexist. Direct democracy introduces competition in the legislative process by giving amendment power to an interest group over the policy chosen by an elected representative. Initiatives are usually beneficial, unless several conditions are simultaneously met: the incumbent is aligned with voters and highly uncertain about voters’ preferences, and an extreme interest group poses a significant threat to the elected politician. In contrast, our results provides a rationale for having positive costs, e.g., participation quorums, associated with the proposal of an initiative or a referendum.

Rydberg (2009) is closely related to our work. That paper focuses on the dichotomy between policy positions and a candidate’s personal characteristics (such as physical appearance). Building on Gerber (1996), Rydberg shows that, in initiative states, voters care less about policy congruence with candidates, and more about their individual characteristics. Our paper considers a different distinction (policy dimensions that can be amended by initiatives vs. competence, which captures both non-amendable aspects of policy, honesty, and leadership), and studies how initiatives affect the strength of political delegation and voters’ welfare.

9 Robustness

This section briefly discusses which assumptions of the model can be relaxed without altering the results. Although we do not provide formal proofs, most of the extensions are straightforward.

Discount factor. If voters discount second-period utility by a factor $\beta < 1$, their posterior threshold, which is only based on second-period utility, is not affected. If politicians discount their second-period payoff, this is equivalent to reducing their rent from being reelected, which does not affect qualitatively the analysis. Overall, discounting second-period payoffs reduces equilibrium search but does not qualitatively affect the impact on welfare of direct democracy.

Initiatives in both periods. Allowing initiatives in the first period has no impact on the equilibrium: it does not affect voters’ threshold or politicians’ reelection incentives. However,

34 The last assumption, in particular, seems hard to defend. In particular, Gerber (1999) shows how most interest groups are not able to use initiatives to have a direct influence over policy.
it does mitigate the impact of second-period direct democracy on politicians’ first-period effort.

**Symmetric direct democracy.** If citizens can cancel a reform (i.e., move from $x'$ to $x$) this does not affect the analysis, given the signal structure, since in any natural equilibrium politicians only implement the reform if it is optimal to do so (i.e., $(h, x')$ does not arise in equilibrium).

**Symmetric signal structure.** We considered the case where the incumbent’s signal about the state has a symmetric structure: given effort $s$, $\Pr(\text{Learn } h' \mid \tilde{h} = h', s) = \Pr(\text{Learn } h \mid \tilde{h} = h, s) = \frac{1+s}{2}$. This model makes the analysis less tractable but does not affect the qualitative nature of the results.

**Less extreme forms of learning-by-doing.** Assuming that a reelected incumbent inherits only a (possibly type–dependent) fraction $\delta_\theta$ of his first period search effort does not affect the results.

### 10 Conclusion

The current expansion of direct democracy institutions, in the United States and elsewhere, requires a careful analysis of their effects. This paper is one of the few theoretical contributions analyzing how initiatives affect not only implemented policies, but also the behavior and the selection of elected representatives. Using a simple model of political delegation that abstracts from previous criticism of direct democracy, we identify a previously unexplored channel through which initiatives affect policymaking and political selection: responsibility substitution.

The paper identifies three natural conditions that, taken together, generate this effect. First, voters cannot commit on a reelection strategy; second, direct democracy cannot affect all potential channels through which politicians competence affects voters’ utility; third, electoral incentives and citizens’ ability to filter out competent politicians must be monotonically related.\(^{35}\) stronger electoral incentives reduce the probability that citizens oust a competent politician. This last condition has further implications. First, it guarantees uniqueness of a “natural equilibrium,” i.e., one in which conditional on his information, an office-motivated politician chooses the welfare maximizing policy. Second, it generates a vicious cycle: as explained in Section, a lower cost of direct democracy raises voters’ posterior threshold to reelect...

\(^{35}\)This assumption translates into a condition on the cost of information acquisition across competence levels that is related–but different–from the standard single–crossing property,
the incumbent. This higher threshold can be sustained in equilibrium only if electoral incentives become weaker and search decreases, which further raises voters’ posterior threshold, and so forth.

The paper shows that responsibility substitution can more than offset the direct welfare gain from initiatives: even when direct democracy is ex post welfare improving, it can be ex ante welfare decreasing. The comparative statics generated by the model are consistent with several established empirical findings: the negative relationship between cost and frequency of initiatives, the higher probability of reelection conditional on poor economic performance when initiatives are present, and a weaker relationship when focusing on overall reelection probabilities. Our results on the importance of politician’s information acquisition \((\rho)\) and on the average quality of the pool of politicians \((q)\) both suggest that the responsibility substitution effect has become more important over time, which helps explaining the declining popularity that these institutions have suffered in the last decades in United States.

A natural extension of our analysis is to study the impact of the responsibility substitution effect on interest groups organization and influence. In the literature, it has been argued that direct democracy institutions may allow interest groups to bring more competition into the policymaking process, which may improve the policy outcome. Our model offers a useful framework to study whether this result is preserved when information is endogenously acquired.

\section*{Appendix}

\textbf{Proof of Proposition 4.} The proof is based on the following lemma.

\textbf{Lemma 1} For fixed parameter values, there exists at most one equilibrium with \(\lambda = 1\) (competent type enters surely) and at most one equilibrium with \(\lambda \in (0, 1)\). Furthermore, these equilibria cannot coexist.

\textit{Proof.} If \(\lambda = 1\), equilibrium search solves

\[
\frac{(1 - q)(1 - s_C)}{(1 - q)(1 - s_C) + q(1 - s_N)} = \bar{p} = \frac{(1 - q)M - \mu \rho s_N}{M + \mu \rho (s_C - s_N)},
\]

which can be reexpressed as

\[a(f) = b(f), \quad (12)\]
where
\[ a(f) = s_N(f)[1 + \phi_1] - s_N^2(f)\phi_1, \]  
\[ b(f) = s_C(f)[1 - \phi_2] + s_C^2(f)\phi_2. \]  
(13)  
(14)

If \( \lambda \in (0, 1) \), equilibrium search solves
\[ \frac{(1 - q)(1 - s_C)\lambda}{(1 - q)\lambda(1 - s_C) + q(1 - s_N)} = \bar{p}, \]
which becomes
\[ 1 - \bar{s}_N(1 + \phi_1) + \bar{s}_N^2\phi_1 = \lambda \left\{ 1 - \bar{s}_C(1 - \phi_2) - \bar{s}_C^2\phi_2 \right\}. \]  
(15)

Since (15) entirely pins down \( \lambda \), at most one constrained equilibrium can exist. Since each side of (15) is positive this implies that
\[ a(f) > b(f) \]  
(16)

It is easy to show that \( a \) is decreasing in \( f \), and that \( b \) is either everywhere decreasing in \( f \) or first decreasing and then increasing. An equilibrium \( f \) is a root of the function \( a - b \). We show that the function \( a - b \) is strictly quasiconcave (i.e., strictly increasing, then strictly decreasing). Let \( f_b \) denote the (possibly, equal to 1) value of \( f \) such that \( b \) is decreasing on \([0, f_b]\) and increasing on \([f_b, 1]\). Since \( a \) is decreasing, the function \( a - b \) is decreasing on \([f_b, 1]\).

To conclude the proof of quasiconcavity, it suffices to show that if \( a - b \) is decreasing around any value \( f < f_b \), then it is decreasing on \([f, 1]\). For \( f < f_b \), we have \( a'(f), b'(f) \) negative, and
\[ \frac{a'(f)}{b'(f)} = \kappa((1 - f)\rho R)\frac{(1 + \phi_1) - 2s_N(f)\phi_1}{1 - \phi_2 + 2s_C(f)\phi_2}, \]
which is increasing in \( f \), since \( \kappa \) is decreasing by assumption and \( 1 - \phi_2 + 2s_C(f)\phi_2 \) is positive (since \( b' \) is negative). Now suppose that \( (a - b)'(\hat{f}) \leq 0 \) for any \( \hat{f} < f_b \). Then, \( a'/b'(\hat{f}) > 1 \) and hence \( a'/b'(f) > 1 \) and \( (a - b)'(f) < 0 \) for all \( f \in (\hat{f}, f_b) \). In particular, given that \( a(1) - b(1) = 0 \), the function \( a - b \) can only cross the \( x \)-axis once, and it does so from below. This shows that there exists at most one level of \( f \) for which (12) holds and, therefore, at most

\[ \text{Indeed, the left-hand side may be rewritten as } (1 - \bar{s}_N)(1 - \phi_1\bar{s}_N), \text{ which is positive, with a similar factorization for the right-hand side.} \]

\[ \text{Seen as a function of } s_N, \text{ a is a quadratic concave polynomial that reaches its peak at } (1 + \phi_1)/(2\phi_1) > 1. \text{ Therefore, a is increasing in } s_N \text{ on } [0, 1] \text{ and, therefore, decreasing in } f. \text{ Similarly, } b \text{ is a quadratic convex polynomial in } s_C, \text{ whose minimizer } s_b \text{ is always strictly less than 1, and is negative if } \phi_2 < 1. \text{ Therefore there exists } f_b \text{ such that } s_C(f) \text{ is in the increasing range of } b \text{ if and only if } f > f_b. \]
one interior equilibrium. Now suppose that \( a(f) > b(f) \). If \( a(f) = b(\hat{f}) \) for some \( \hat{f} \in (f, 1) \), strict quasi-concavity implies that \( a(f) < b(f) \) for all \( f > \hat{f} \), which contradicts the fact that \( a(1) = b(1) \). This shows that (12) and (16) cannot hold together.

Lemma 1 shows that exactly three cases are possible: i) no equilibrium with positive search, ii) an equilibrium with \( \lambda = 1 \), and iii) an equilibrium with \( \lambda < 1 \). There remains to identify which of these cases holds. Existence of an interior equilibrium obtains if and only if (12) is satisfied for some \( f \in (f, 1) \). Since \( a(1) = b(1) = 0 \) and \( a'(f)/b'(f) \) is increasing in \( f \), a necessary condition for the existence of \( f < 1 \) such that \( a(f) = b(f) \) is that \( a'(1)/b'(1) \geq 1 \). Otherwise, \( b'(f) < a'(f) < 0 \) for all \( f \) (recalling that \( a, b \) are decreasing) and, therefore, \( a(f) > b(f) \) for all \( f < 1 \). Using that \( a'(1)/b'(1) = \kappa(0)(1 + \phi_1)/(1 - \phi_2) \), we get the condition \( \mu \geq \mu_1 \).

An equilibrium with \( \lambda < 1 \) exists if and only if (16) holds. Using the definitions of \( \phi_1 \) and \( \phi_2 \), this is equivalent to \( \mu > \mu_2 \). If \( \mu > \mu_2 \), there exists a constrained equilibrium. If \( \mu < \mu_2 \), then (15) implies (using again positivity of both sides) that \( a(f) < b(f) \). If \( \mu > \mu_1 \), we have \( a'(1)/b'(1) > 1 \) and, more precisely \( a'(1) < b'(1) < 0 \). Since \( a(1) = b(1) \), this implies that, in a left-neighborhood of 1, \( a(f) > b(f) \). Combined with the fact that \( a(f) < b(f) \), we conclude by continuity of the functions \( a, b \) that there exists some \( f \in (f, 1) \) such that \( a(f) = b(f) \), which shows the existence of an equilibrium with \( \lambda = 1 \), whenever \( \mu > \mu_1 \) and \( \mu < \mu_2 \). To conclude, we show that \( \mu_1 < \mu_2 \). This is immediate: when \( \mu = \mu_2 \), as we saw, \( a(f) = b(f) \). This implies that \( \mu > \mu_1 \), since we also saw that in the opposite case \( a(f) > b(f) \) for all \( f < 1 \).

**Proof of Proposition 5.** i) Equivalently, we need to show that the ratio \( \frac{1-s_N(f)}{1-s_C(f)} \) is decreasing in \( f \). Since \( T''_C(s) < T''_N(s) \) for all \( s \) and \( T'_C(0) = T'_N(0) = 0 \), we necessarily have \( T''_C(0) < T''_N(0) \), or \( \kappa(0) < 1 \).\(^{38}\) Since \( \kappa \) is decreasing by assumption, this implies that \( \kappa \) is everywhere less than 1. Since \( s_\theta(f) = T^{-1}((1 - f)R\rho) \), we have, by differentiation, \( \kappa((1 - f)R\rho) = \frac{s'_N(f)}{s'_C(f)} \). Since \( \kappa \) is less than 1 and \( s'_\theta < 0 \), we conclude that for all \( f \), \( s'_C(f) < s'_N(f) \). Intuitively, reducing electoral incentives (i.e., increasing \( f \)) has a worse impact on the competent type than on the incompetent one. The derivative of the ratio \( \frac{1-s_N(f)}{1-s_C(f)} \) has the same sign as

\[
-s'_N(f)(1-s_C(f)) + (1-s_N(f))s'_C(f).
\]

As observed earlier, \( 1-s_N(f) > 1-s_C(f) \). Combined with \( s'_C(f) < s'_N(f) < 0 \), this implies that this sign is negative.

\(^{38}\)We assume away the knife-edge case \( \kappa(0) = 1 \).
ii) Since \( s_\theta(1) = 0 \) for \( \theta = \{N, C\} \), \( s_\theta(f) = \int_f^1 (-s_\theta(\tilde{f})) d\tilde{f} \). Therefore,

\[
s_C(f) - s_N(f) = \int_f^1 (1 - \kappa((1 - \tilde{f}) R\rho)) (-s_C'(|\tilde{f}|)) d\tilde{f},
\]

(17)

using that \( \kappa((1 - f) R\rho) = \frac{s_C'(f)}{s_C(f)} \) for all \( f \). Since \( \kappa \) is everywhere less than 1 and \( s''_C \) is negative, the integrand in (17) is positive, which shows that \( s_C(f) - s_N(f) \) increases with the interval \([f, 1]\) of integration and, therefore, decreases in \( f \).

**Proof of Proposition 6.**

i) The difference between \( EU(\Delta) \) and \( EU(\mu) \) in an interior equilibrium is given by

\[
D^I(\mu) = \begin{cases} 
D^I(\mu) = 2\Delta \rho [qs_N(\Delta) + (1 - q)s_C(\Delta)] + 2M(1 - q) + 
\end{cases}
\]

\[
-(\Delta - \mu) \rho - (\mu + \Delta)\rho [qs_N(\mu) + (1 - q)s_C(\mu)] - 2M(1 - q).
\]

and the condition follows from computing that difference for \( \mu = 0 \).

ii) The difference between \( EU(\Delta) \) and \( EU(\mu) \) in a constrained equilibrium is given by

\[
EU(\Delta) - EU(\mu) = \begin{cases} 
2[q\tilde{s}_N + (1 - q)\lambda \tilde{s}_C] \Delta \rho + q(1 - q) M(\lambda - \mu) + 
\end{cases}
\]

\[
-(\Delta - \mu) \rho - [q\tilde{s}_N + (1 - q)\lambda \tilde{s}_C] \rho (\mu + \Delta) + 
\]

\[
2[[q\tilde{s}_N + (1 - q)\lambda \tilde{s}_C] \Delta \rho - q(1 - q) M(1 - \lambda) + 
\]

\[
-(\Delta - \mu) \rho - (\Delta + \mu)[\rho q s_N(\mu) + (1 - q)s_C(\mu)]
\]

(18)

and the condition follows from computing that difference for \( \mu = 0 \).

**Proof of Proposition 7.**

As long as equilibrium search is positive, the derivative of the expected utility with respect to \( \mu \) is given by

\[
\frac{d}{d\mu} EU(\mu) = -[q(1 - s_N) + (1 - q)(1 - s_C)] + (\mu + \Delta) \frac{d f^*}{d\mu} [qs'_N + (1 - q)s'_C]
\]

(18)

where \( s'_\theta = \frac{\partial s_\theta(\mu)}{\partial f} < 0 \) and \( \frac{d f^*}{d\mu} < 0 \).

This derivative has two components: the first, \(-[q(1 - s_N) + (1 - q)(1 - s_C)]\) is negative while the second, \((\mu + \Delta) \frac{d f^*}{d\mu} [qs'_N + (1 - q)s'_C]\), is positive.

**Step 1** We have

\[
(\mu + \Delta) \frac{d f^*}{d\mu} [qs'_N + (1 - q)s'_C] > 2\mu \frac{d f^*}{d\mu} [qs'_N + (1 - q)s'_C].
\]

Therefore, (18) will be established if we show that

\[
2\mu \frac{d f^*}{d\mu} [qs'_N + (1 - q)s'_C] - [q(1 - s_N) + (1 - q)(1 - s_C)] > 0.
\]

(19)
Step 2 By applying the implicit function theorem to the equation defining \( f^* \) we obtain

\[
\frac{df^*}{d\mu} = \frac{(1 - s_N) s_N \phi_1 + (1 - s_C) s_C \phi_2}{s_N'[2s_N \phi_1 - \phi_1 - 1] + s_C'[2s_C \phi_2 - \phi_2 + 1]}
\]

Therefore, (19) is equivalent to

\[
\frac{qs'_N + (1 - q)s'_C}{s_N'[2s_N \phi_1 - \phi_1 - 1] + s_C'[2s_C \phi_2 - \phi_2 + 1]} > \frac{[q(1 - s_N) + (1 - q)(1 - s_C)]}{2(1 - s_N)s_N \phi_1 + 2(1 - s_C)s_C \phi_2}.
\] (20)

To simplify the task, let’s use the following notation:

\[
(1 - s_N)(1 - s_C) = \tau(\mu), \quad s'_N s'_C = \kappa((1 - f(\mu))\rho R) = \hat{\kappa}(\mu), \quad \nu(\mu) = \frac{s_N}{s_C}.
\]

Rearranging yields

\[
\frac{1}{2s_C} \frac{1 - q}{\nu \tau + \frac{1 - q}{\nu}} - \frac{1 - q}{\nu} \frac{(1 - \nu)(\tau - \hat{\kappa})}{\tau + \frac{1 - q}{\nu}} \geq 0.
\]

The condition is equivalent to

\[
\frac{1}{2s_C} \left[ \frac{1 - q}{\nu} + \hat{\kappa} - \frac{1 - \hat{\kappa}}{\phi_1} \right] - \frac{1 - q}{\nu} \frac{(1 - \nu)(\tau - \hat{\kappa})}{\tau + \frac{1 - q}{\nu}} \geq 0
\]

That is

\[
[\phi_2 + \phi_1 \hat{\kappa} - 1 + \hat{\kappa}] - 2s_C \phi_2 \frac{(1 - \nu)(\tau - \hat{\kappa})}{\tau + \frac{1 - q}{\nu}} \geq 0
\] (21)

Step 3 If \( \kappa \) is a constant, then \( \kappa = \nu \) for all \( \mu \). Rearranging the inequality \( \mu \geq \mu_1 \), we obtain that \( \kappa \geq 1/2 \). Therefore, (20) simplifies to

\[
\frac{q\kappa + 1 - q}{q\kappa^2 + 1 - q} \geq \frac{q\tau(\mu) + 1 - q}{2q\kappa \tau(\mu) + 2(1 - q)}
\]

the right hand side of the former is then smaller than

\[
\frac{q\tau(\mu) + 1 - q}{2q\kappa^2 \tau(\mu) + \kappa(1 - q)} < 1.
\]

As a result, the condition holds, since \( RHS > 1 > LHS \).

Proof of Proposition 8 When \( \mu \leq \mu_1 \), there is no search and the probability of a reform is constant, equal to \( \rho \). In a constrained equilibrium, the only variable that depends on \( \mu \) is \( \lambda(\mu) \). We also know that \( \lambda(\mu) \) is decreasing in \( \mu \). Therefore, it suffices to consider the case for which the incumbent is competent, since the behavior of the other type is independent of \( \mu \).

Given that the type is \( C \), we have:

\[
F^C(\mu) = [1 - \lambda(\mu)]\rho + \lambda(\mu)(1 - \bar{s}_C)\rho(1 - f) + \lambda(\mu)[1 - (1 - \bar{s}_C)(1 - f)]\rho(1 - \bar{s}_C)\rho
\]

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which is decreasing in $\lambda(\mu)$. As a consequence, $F^C(\mu)$ in increasing.

In an interior equilibrium, instead, the ex ante probability of observing an initiative is given by

$$F^I(\mu) = (1 - s^E)\rho^2 (1 - f) + (1 - (1 - s^E)\rho(1 - f))\rho(1 - s^E)$$

where $s^E = q s_N + (1 - q) s_C$ is the expected search. Therefore,

$$\frac{d}{d\mu} F^I(\mu) = -\frac{ds^E}{d\mu} \rho (1 + 2\rho (1 - f) (1 - s^E)) - \frac{df}{d\mu} \rho^2 (1 - s^E)^2$$

since $\frac{ds^E}{d\mu} = \frac{df}{d\mu} (qs'_N + (1 - q)s'_C) > 0$ and $\frac{df}{d\mu} F^I(\mu) < 0$ is equivalent to

$$|qs'_N + (1 - q)s'_C| > \frac{\rho (1 - s^E)^2}{1 + 2\rho (1 - f) (1 - s^E)}$$

which, depending on the shape of the cost functions and the values of $q$ and $\rho$, can hold or not.

**Example for which (22) holds.** Suppose that $\Delta = M$ and that the cost functions are given by $T_C(s) = R\rho s^2$ and $T_N(s) = R\rho s^2$. Then, $|qs'_N + (1 - q)s'_C| = 1 - q/2$, and (22) becomes

$$1 - q/2 > \frac{\rho (1 - s^E)^2}{1 + 2\rho (1 - f) (1 - s^E)}.$$ 

This condition holds whenever $1 - q/2 > \rho$. When $M = \Delta$, this condition is implied by our maintained assumption that $M(1 - q) > \Delta \rho$.

**Proof of Proposition 9.** i) From (5) and convexity of $T_\theta$, $s_\theta(f; \rho)$ is increasing in $\rho$.

The equilibrium reelection probability $f^*$ is the unique solution of the equation

$$F(f, \rho) = s_N(f)[1 + \phi_1(\rho)] - s^2_N(f)\phi_1(\rho) - s_C(f)[1 - \phi_2(\rho)] - s^2_C(f)\phi_2(\rho) = 0$$

The proof of Lemma 1 implies that $F(\cdot, \rho)$ has the single-crossing property in $f$, being negative below $f^*$ and positive above. Moreover,

$$\frac{\partial F}{\partial \rho}(f^*, \rho) = s_N(f^*)(1 - s^2_N(f^*)) \frac{\Delta}{M(1 - q)} + s_C(f^*)(1 - s_C(f^*)) \frac{\Delta}{Mq} > 0$$

which shows that $F$ is increasing in $\rho$. Therefore, $f^*(\rho)$ is decreasing in $\rho$.

ii) (7) implies that $\mu_1$ is decreasing in $\rho$. For $\mu_2$, we apply the implicit function theorem to (6), substitute the first-order condition defining $s_C$, and obtain

$$\frac{\partial f}{\partial \rho} = \frac{(1 - \bar{s}_C - (1 - \bar{s}_C)f)}{\rho((1 - \bar{s}_C)f)} > 0.$$
This implies that $\bar{s}_N$ and $\bar{s}_C$ are decreasing in $\rho$, while $\frac{\bar{s}_N}{\bar{s}_C}$ is increasing in $\rho$. Therefore, the numerator entering the definition of $\mu_2$, $1 - \frac{\bar{s}_N}{\bar{s}_C}$, decreases in $\rho$, while the denominator
\[
\left[ \frac{\rho}{Mq} (1 - \bar{s}_C) + \frac{\bar{s}_N}{\bar{s}_C} \frac{\rho}{M(1 - q)} (1 - \bar{s}_N) \right]
\]
increases in $\rho$, concluding the proof.

References


