Voting and Experimentation with Correlated Types

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Abstract

This note extends Strulovici (2010) to the case where voter types are correlated and/or ex ante heterogeneous. There are two voters and the risky action is taken according to the unanimity rule. Increasing correlation between the types of the two voters reduces the negative payoff externality that voters have on each other, thereby increases experimentation and efficiency. Moreover, it allows voters to learn from each other and, therefore, increases the speed of experimentation.

1 Introduction

In a dynamic voting model where voters gradually learn about their preferences between a "risky" and a "safe" action, Strulovici (2010) identifies two effects, "winner frustration" and "loser trap," which reduce voters' incentive to experiment with the risky action, thereby causing experimentation to be inefficiently brief. In most of that paper,[1] preferences are independently distributed across voters.

This note shows, for the case of two voters and where the risky action requires unanimity, that increased correlation across voter types alleviates winner frustration and improves efficiency. Thus, it reduces the negative "payoff externality" that voters have on each other. Moreover, increased correlation creates a positive informational externality, as voters can learn about their type by observing each other's payoffs. Ex ante heterogeneity of types also results in more experimentation: the more pessimistic voter prefers, other things equal, the other voter to have a higher probability of being a winner, as this reduces the risk of winner frustration. In the limit, if a voter is a sure winner (i.e., the risky action is surely optimal for him), the other voter behaves as a single decision maker.[2] The intuition for this result is more general: with more ex ante type heterogeneity, the voter who is pivotal today is more likely to be pivotal in the future as well (because it is less likely that voter preferences will "cross" one another) and, therefore, is less subject to winner frustration and loser trap effects. Therefore, provided that heterogeneity is persistent, more heterogeneity today should result in more experimentation.

2 Correlation and Heterogeneity

The setting is the same as the benchmark setting of Strulovici (2010), with the following modifications. There are two voters, 1 and 2, who share a common belief about the initial joint distribution of their types, although this distribution may be asymmetric and exhibit type correlation across voters. Let θ^i denote Voter i's type, and let

$$p^{\vartheta_1\vartheta_2} = Pr[(\theta^1, \theta^2) = (\vartheta^1, \vartheta^2),$$

[1]Section 5 allows for correlation and arbitrary news arrival processes, but does not specifically study the impact of increased correlation on equilibrium and efficiency.

[2]This case implies zero correlation since one voter has a known type.

where $\vartheta^i \in \{g, b\}$ describes the possible types (good or bad) of each voter. Also let

$$p^i = Prob[\theta^i = g]$$

for $i \in \{1, 2\}$, and

$$\alpha = \frac{p^{gg}}{p^1 p^2}.$$

α is a measure of the correlation between voter types. The standard correlation measure and α have a one-to-one relationship for any given p^1 and p^2. If $\alpha = 1$, types are uncorrelated. In general, α takes values in $\mathbb{R}_+$, although not all values of $\mathbb{R}_+$ are achievable for given p^1, p^2. For example, $p^1 = 1$ implies that $\alpha = 1$, since in that case Voter 1's type is deterministic hence uncorrelated with Voter 2's type. Let Δ denote the set of pairs (p^2, α) that are achievable as elementary probabilities vary over the four-dimensional simplex, i.e.,

$$\Delta = \left\{ \left(p^{gg} + p^{bg}, \frac{p^{gg}}{(p^{gg} + p^{gb})(p^{gg} + p^{bg})} \right) : \sum_{\vartheta_1, \vartheta_2} p^{\vartheta_1 \vartheta_2} = 1, p^{\vartheta_1 \vartheta_2} \geq 0 \right\}.$$

The following proposition is a simple exercise of Bayesian updating, whose proof is easy and omitted.

Proposition 1 (State Dynamics) *Beliefs are governed by the following dynamics equations. When no lump-sum is observed,*

- $\frac{dp^{gg}}{dt} = -\lambda p^{gg}(2 - p^1 - p^2)$
- $\frac{dp^{bb}}{dt} = \lambda p^{bb}(p^1 + p^2)$
- $\frac{dp^{gb}}{dt} = -\lambda p^{gb}(1 - p^1 - p^2)$, $\frac{dp^{bg}}{dt} = -\lambda p^{bg}(1 - p^1 - p^2)$
- $\frac{d\alpha}{dt} = -\lambda \alpha (1 - \alpha)(p^1 + p^2)$

When Voter 1 receives a lump-sum,

- $p^{bb}_+ = 0$, $p^{gb}_+ = \frac{p^{gb}}{p^1}$, $p^{bg}_+ = 0$, $p^{bb}_+ = \frac{p^{bb}}{p^1}$
- $\alpha_+ = 1$, $p^1_+ = 1$, $p^2_+ = \alpha p^2$

where the subscript '+' denotes values immediately after the lump-sum is observed, and its absence denotes values immediately before the lump-sum. Symmetric formulas obtain if instead Voter 2 receives a lump sum.

Suppose that R requires unanimity. We assume that the voter who is the less likely of being a winner is in control: if that voter wants to play the risky action, so should the player with a higher expected type. This assumption is consistent with the elimination of weakly dominated strategies. A unanimity equilibrium (UE) is defined as follows: at any time t, if $p^i \leq p^j$, then j votes for R whenever i does.

Theorem 1 *There exists a unique UE. This equilibrium determined by a cut-off function $\delta : \Delta \to [0,1]$ such that $C(p^1, p^2, \alpha) = R$ if and only if $p^1 > \delta(p^2, \alpha)$ whenever $p^1 \leq p^2$, with the reverse relation if $p^1 > p^2$.*

Proof. First suppose that $p^2 = 1$. Then, Voter 1 has full control over the collective decision. He therefore imposes his optimal policy, which is that of a single decision maker. This defines $\delta(1,1) = p^{SD}$. This also fully determines the value functions of both voters in that case. Let $p \mapsto w(p)$ denote the value function of voter 2, where p is Voter 1's probability of being a winner is p, and v^{SD} is the value function of a single decision maker, which is also voter 1's value function in this case. More generally suppose that at time 0, $p_0^1 \leq p_0^2$. It follows from Proposition 1 that $p_t^1 \leq p_t^2$ for all t preceding the first arrival of a lump-sum. In particular, this implies that 1 has full control of the collective decision (under unanimity) over that period. Therefore, he chooses a policy θ that solves

$$u_t = \sup_\theta E\left[\int_t^\sigma e^{-r(\tau-t)} d\pi^1_{\theta_\tau}(\tau) + e^{-r(\sigma-t)}\left(qw(p^2_{\sigma_+}) + (1-q)v(p^1_{\sigma_+})\right)\right],$$

where, letting σ_i denote the (possibly infinite) time at which i receives his first lump sum, $\sigma = \min\{\sigma_1, \sigma_2\}$ and $q = Prob[\sigma_1 < \sigma_2]$. This is a standard control problem, whose solution is known to be Markov. Voter 1 is indifferent between R and S at probability level p, if p solves the equation

$$pg + \lambda p[w(\alpha p^2) - s/r] + \lambda p^2[v^{SD}(\alpha p) - s/r] = s. \tag{1}$$

The left-hand side is increasing in p, equal to 0 for $p = 0$ and greater than $g > s$ if $p = 1$. Therefore, it has a unique root $\delta(p_2, \alpha)$. This shows that $C(p^1, p^2, \alpha) = R$ if and only if $p^1 > \delta(p^2, \alpha)$. The case $p^1 > p^2$ obtains by symmetry.

Theorem 2 states that a voter's incentive to experiment increases both with the other voter's probability of being a winner and with voters' type correlation. Intuitively, if types are more positively correlated, the risk of winner frustration decreases. The risk of winner frustration is also lower for a given voter if the other voter is more likely to be a winner. In the extreme case in which, say, Voter 2 is a sure winner (i.e., $p^2 = 1$), Voter 1 has full control over collective decisions, and can behave in effect as a single-decision maker. In addition, positive correlation increases

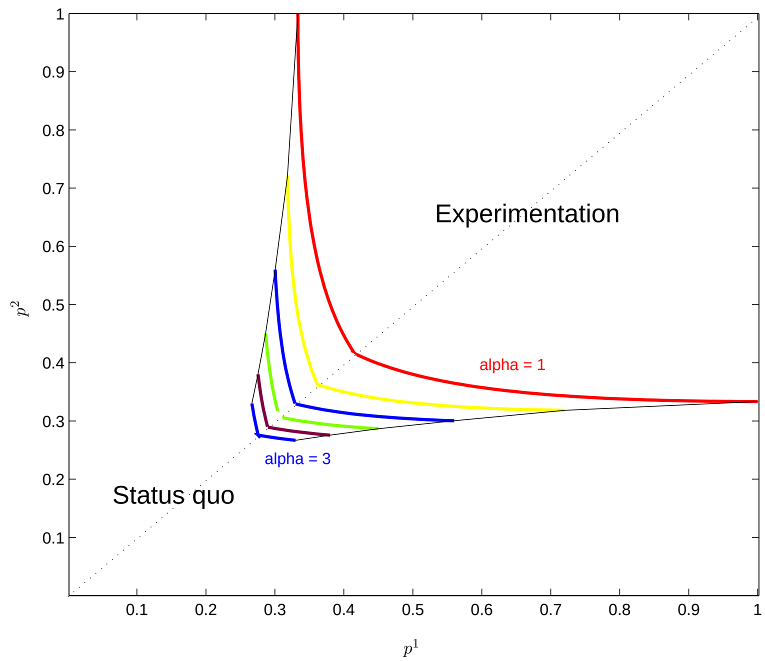

Figure 1: Experimentation Boundary δ as a function of α. $r = 1$, $\lambda = 1$, $s = 1$, $g = 2$.

the speed of learning, which reduces the time cost of experimentation. In the extreme case of perfect type correlation, the setting is equivalent to one with a single decision maker with twice the initial learning intensity. Figure 1 shows numerical computations of the experimentation boundary for several values of the correlation measure α.

Theorem 2 *δ is decreasing in both components.*

Proof. The left-hand side of (1), is increasing in p, p^2 and α. Therefore, keeping α fixed, the root $\delta(p^2, \alpha)$ must be decreasing in p^2, and similarly keeping p^2 fixed, $\delta(p^2, \alpha)$ must be decreasing in α. ∎

References

STRULOVICI, B. (2010) "Learning While Voting: Determinants of Collective Experimentation," forthcoming in *Econometrica*.