Two-part marginal cost pricing in a pure fixed cost economy

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Submitted March 1991; accepted May 1994

Abstract

Two-part marginal cost pricing is the pricing scheme where firms, in addition to the linear price equated to the marginal cost of production, charge non-uniform access fees to customers. Using a general equilibrium model with non-convex technologies, we examine the optimality and existence of this pricing scheme. First, it is shown that two-part marginal cost pricing is an optimal pricing regulation for monopolies when increasing returns to scale arise solely from the presence of fixed costs. Second, a sufficient condition for the existence of two-part marginal cost pricing equilibria is provided under the bounded losses condition. This generalizes the result of Brown et al. (Journal of Economic Theory, 1992, 57, 52–72).

JEL classification: D42; D51; D52; D61

Keywords: Two-part marginal cost pricing; Increasing returns to scale; Fixed cost; Natural monopoly

1. Introduction

What is the optimal pricing regulation of monopolies when increasing returns to scale are present? In a general equilibrium framework, it is known that with production non-convexities marginal cost pricing is a necessary but not a sufficient condition for optimality. In such situations, the generic inefficiency of marginal cost pricing regulation accompanied by lump-sum taxation is well established (Guesnerie, 1975; Brown and Heal, 1980). Beginning with Brown and Heal (1980), there have been several attempts to recover optimality by introducing a two-part tariff pricing scheme into the general equilibrium literature. The additional flexibility offered by the two-part tariff gives it an advantage over marginal cost pricing, as we will see below, and hence there is reason to expect more
positive results. However, to date, the results in this regard have been negative. Both Vohra (1990) and Brown et al. (1992) have shown that the two-part tariff can fail to achieve an efficient outcome.

In light of this previous work, the main purpose of this paper is to focus on a particular form of non-convexity and prove the optimality of a two-part tariff scheme. As such, in Theorem 1 it is shown that if increasing returns to scale is attributed, solely, to the presence of fixed costs, then a two-part tariff, or more precisely, two-part marginal cost pricing, is indeed an optimal pricing policy.

One also wonders about the existence of such equilibria when two-part marginal cost pricing is applied. A sufficient condition for the existence of such equilibria with general non-convexities is provided in Theorem 2. In so doing, the existence result of Brown et al. (1992) is generalized by introducing the bounded losses condition developed by Bonnisseau and Cornet (1988). The existence result for the pure fixed cost case follows as a corollary of our theorem.

As early as 1946, Coase proposed two-part tariffs as an alternative pricing scheme to marginal cost pricing. With a two-part tariff, buyers first have to pay a fixed charge in order to access the market. They then pay a variable charge proportional to the units they purchase. For two-part marginal cost pricing, this latter part is set equal to the marginal cost of production. Any losses remaining after the variable marginal cost charge are made up by the collection of fixed charges to allow the monopoly to break even. In contrast, with marginal cost pricing any such losses are covered by lump-sum taxation. According to Coase (1946), the main distinction between these two pricing rules is as follows. In marginal cost pricing accompanied by lump-sum taxation, an individual is deprived of the right to consume an initial endowment because of the lump-sum tax, but in two-part marginal cost pricing, an individual can avoid all fixed charges by not purchasing any increasing returns commodity. A fixed charge is paid only if an individual is willing to pay, and hence the collected fixed charges reflect the surplus that the commodity brought into a society. In contrast with lump-sum taxation, a fixed charge can be considered as one form of benefit taxation since it taxes away a part of the buyer’s benefit provided by the commodity. For a detailed discussion, see also Clay (1994) and Vohra (1990).

The above argument naturally requires a careful determination of fixed charges. A uniform fixed charge is generally inefficient since each individual has a different appreciation of the commodity. For some consumers fixed charges are too high, thus excluding them from the market. Hence, we shall introduce a non-uniform fixed charge, i.e. a fixed charge that varies among individuals according to their willingness to pay. Consequently, two-part marginal cost pricing is a more flexible scheme compared with marginal cost pricing. Despite this advantage, as noted earlier, Vohra (1990) and Brown et al. (1992) provided examples in which a two-part marginal cost pricing equilibrium fails to be efficient. In other words, our device of a non-uniform fixed charge is not sufficient to carry out the first-best outcome under general production non-convexities.
To understand the motivation of this paper, we can regard two-part marginal cost pricing as a linear approximation of a particular non-linear pricing scheme. In this context, Brown and Heal (1980) showed that any Pareto-optimal allocation supported by a non-linear pricing scheme is implementable by a generalized two-part tariff. Recently, Kamiya (1993) proposed an optimal non-linear pricing rule in a model with increasing returns to scale. As such, it represents a rare positive result in the literature. In his paper, the non-linear pricing contains information about the cost functions of increasing returns to scale firms so that it successfully provides a separating hypersurface between the better-off set and the production possibility frontier. When firms operate under the simplest form of non-convexity, non-linear pricing reduces to its simplest form, namely a two-part tariff. This paper develops this idea to its full extent. Precisely speaking, our result shows that, if no fixed charge excludes a buyer and all monopolies are active, then two-part marginal cost pricing equilibria in a pure fixed cost economy are efficient. 2

There is an alternative another approach that explores the conditions under which marginal cost pricing recovers its efficiency. Dierker (1986) and Quinzii (1988) each developed a sufficient condition for marginal cost pricing to be optimal. In their papers, restrictions are imposed on both technologies and preferences. Their assumptions on technologies are weaker than our assumptions, but those on preferences are stronger. 3 Hence, the results are not directly comparable.

After considering optimality, in a later section we generalize the existence result of Brown et al. (1992) by applying the method developed by Bonnisseau and Cornet (1988) under the bounded losses condition. The existence of two-part marginal cost pricing equilibria in a pure fixed economy is provided as a corollary in the final section. The generalization of the result of Brown et al. (1992) is required since our model incorporates (a) several natural monopolies, (b) the joint production of multiple goods by each monopoly, (c) a fixed charge levied not only on consumers but also on competitive firms, and (d) non-convex technologies which may eventually represent constant returns to scale. 4

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1 For a precise definition, see their paper.
2 Independently, this result was observed by Edlin (1993) for the case of one firm.
3 For both Dierker and Quinzii, preferences are assumed so that no indifference surfaces intersect the commodity axis. Since in a pure fixed cost economy this is strong enough to make all marginal cost pricing equilibria efficient, we avoid this assumption.
4 The existence result of Brown et al. (1992) was contemporaneously extended by Edlin and Epelbaum (1993a) to the case including (a), (b), and (c) above. Following Beato and Mas-Colell (1985), their paper assumes that a production set of a monopoly is given by $Y = K - R^m$, where $K$ is compact. Therefore, a production set which represents constant returns to scale is excluded from their analyses. In a pure fixed cost economy, however, constant returns to scale technologies associated with fixed costs cannot be ignored.
2. Optimality of two-part marginal cost pricing in a pure fixed cost economy

2.1. The model

In this section we consider an Arrow-Debreu model with non-convexity in production due to fixed costs. There are two production sectors in an economy. The first sector, the public sector, consists of a finite number of natural monopolies with pure fixed technologies indexed by \( j \in M \). These firms are regulated as public utilities to follow a certain pricing scheme specified below. The second sector, the private sector, consists of a finite number of consumers, indexed by \( i \in T \), and a finite number of competitive firms with convex technologies, indexed by \( k \in N \). Agents in the private sector are price-takers. There are \( l = l_1 + l_2 \) commodities in the economy, the first \( l_1 \) goods are produced by monopolies and the last \( l_2 \) goods are produced by competitive firms. Let \( L_1 \) be the index set of the monopoly goods and \( L_2 \) be the index set of the competitive goods. Joint production is allowed and each natural monopoly may produce more than one good. We assume private ownership so that each consumer has shareholdings in both natural monopolies and competitive firms. Moreover, the shareholders are assumed to face limited liability, i.e. they receive dividends only if firms' profits are positive.

2.1.1. The private sector

Let \( X_i \subset R^l \) be the consumption set and \( \omega_i \in R^l \) be the initial endowment of the \( i \)th consumer. The preference relation is represented by the utility function \( u_i : X_i \rightarrow R \). The \( i \)th consumer's shareholding in the \( k \)th competitive firm is denoted by \( \theta_{ik} \), and in the \( j \)th natural monopoly by \( \tau_{ij} \). All shareholdings are non-negative, \( \sum_{i \in T} \theta_{ik} = 1 \) and \( \sum_{i \in T} \tau_{ij} = 1 \) for all \( k \in N, j \in M \). We make the following assumptions.

Assumption [C1]. For any consumer \( i \in T \),
1. \( X_i = R^l_+ \);
2. (local non-satiation in competitive goods): for every \( x_i \in X_i \) and \( \epsilon > 0 \) there exists \( x'_i \in X_i \cap B_\epsilon(x_i) \) such that \( u_i(x'_i) > u_i(x_i) \), where \( B_\epsilon(x) = \{ y \in R^l \mid \| x - y \| < \epsilon \} \) and \( y_h = x_h \) for all \( h \in L_1 \); and
3. \( \omega_{ih} = 0 \) for all \( h \in L_1 \) and \( \omega_{ih} \geq 0 \) for all \( h \in L_2 \).

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5 The following notations are used. Let \( x \) and \( y \) be vectors in \( R^l \), then \( x \cdot y = \sum_{h \in T} x_h y_h \) denotes the inner product. The notation \( x \geq y \) means that \( x_h \geq y_h \) for all \( h \in L \). We define \( R^l_+ = \{ x \in R^l \mid x \geq 0 \} \) and \( R^l_* = \{ x \in R^l \mid x \leq 0 \} \). Let \( |x| \) be the absolute value and \( \| x \| = (x \cdot x)^{1/2} \) be the Euclidean norm of \( x \). For a set \( A \) in \( R^l \), \( \text{int} A \) and \( \partial A \) denote the interior and the boundary of \( A \), respectively.
The consumption set is non-empty, closed, convex, and bounded from below by Assumption [C1](1). No consumer is endowed with any monopoly good by Assumption [C1](3).

Let $Z_k \subset R^l$ be the production set of the $k$th competitive firm satisfying the following assumption.

**Assumption [F]**. For any competitive firm $k \in N$,

1. $Z_k$ is a closed, convex set and $0 \in Z_k$;
2. (free disposal): $Z_k - R_+^l \subset Z_k$; and
3. for every $z_k \in Z_k$, $z_{kh} \leq 0$ for all $h \in L_1$.

It is assumed that competitive firms cannot produce any monopoly good by Assumption [F](3).

### 2.1.2. The public sector

For a natural monopoly we assume that the index sets of inputs and outputs are given a priori. Let $O_j$ be the non-empty subset of $L_1$ which consists of the goods produced by the $j$th monopoly. Let $\{O_j\}_{j \in M}$ be a partition of $L_1$, i.e. $L_1 = \bigcup_{j \in M} O_j$ and $O_i \cap O_j = \emptyset$ for $i \neq j$. Then the index set of inputs of the $j$th monopoly is given by $I_j = (L_1 \cup L_2) \setminus O_j$. Let $Y_j \subset R^l$ be the production set of the $j$th monopoly. The following assumptions are made.

**Assumption [M]**. For any monopoly $j \in M$,

1. $Y_j$ is closed and $0 \in Y_j$;
2. (free disposal): $Y_j - R_+^l \subset Y_j$;
3. $Y_j \subset R^l_+$, where $R^l_+ = \{ y_j \in R^l \mid y_{jh} \leq 0 \text{ for all } h \in I_j \}$; and
4. (pure fixed cost technology): $Y_j^+ = \{ y_j \in Y_j \mid \text{there exists } h \in O_j \text{ such that } y_{jh} > 0 \}$ is convex.

Assumption [M](4) defines a pure fixed cost technology. That is to say, once a natural monopoly starts producing a strictly positive amount of outputs, then the technology exhibits constant or decreasing returns to scale. In other words, the production frontier is allowed to be non-convex only if there exits a fixed set-up cost. Although this is the simplest form of non-convexity, it is widely observed in the actual economy. It is the most popular non-convex technology in economic models as well (see, for example, Dixit and Stiglitz, 1977; Murphy et al., 1989; Romer, 1990).

### 2.1.3. Two-part marginal cost pricing schemes

All natural monopolies are regulated to follow two-part marginal cost pricing rules when they sell goods to the private sector, namely consumers and competi-
five firms. The first part of the pricing rule consists of a non-negative fixed charge imposed on each agent in the private sector in order for them to access the market and purchase any positive amount of the monopoly good. A fixed charge may vary among agents, a situation which we refer to as a non-uniform fixed charge. The second part of the pricing rule consists of a variable charge which is proportional to the units of purchase of the monopoly good. Hence the payment by an agent who has a fixed charge $F$ imposed and who chooses to buy $x$ units of the good with a constant marginal price $p$, is $px + F$ if $x$ is strictly positive, and zero if $x$ is zero. The constant marginal price is common to all agents and should be equated to the marginal cost of production. Typically, natural monopolies run losses when they price at the marginal cost. Therefore, we consider the break-even constraint that each monopoly should recover the losses resulting from marginal cost pricing through collecting fixed charges from agents. Since two-part marginal cost pricing is the simplest form of non-linear pricing, it is only applicable to those goods which cannot be resold and with which no consumer is endowed.

Let $\varphi_j: \partial Y_j \to \mathbb{R}^1$ be the marginal cost pricing correspondence of the $j$th monopoly defined by the Clarke normal cone. It is worth noting that if $y \in \partial Y_j^+$, then since $Y_j^+$ is convex $\varphi_j(y)$ degenerates to the normal cone, i.e.

$$\varphi_j(y) = \{ q \in \mathbb{R}^1 \mid q \cdot y \geq q \cdot y' \text{ for all } y' \in Y_j^+ \} \text{ for } y \in \partial Y_j^+. $$

For notational simplicity, let $\iota: \mathbb{R} \to \{0, 1\}$ be the indicator function defined by

$$\iota(x) = \begin{cases} 1, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Given the marginal price $p_h$ and fixed charge $F_{ih}$, the outlay of the $i$th consumer who demands $x_{ih}$ of the good $h$ is $p_h x_{ih} + \iota(x_{ih})F_{ih}$. Similarly, given the fixed charge $F_{kh}$, the outlay of the $k$th competitive firm that purchases input $z_{kh} \leq 0$ of the good $h$ is $-p_h z_{kh} + \iota(z_{kh})F_{kh}$. Let $F_j = (F_{ih})_{h \in L_i}$ and $F_k = (F_{kh})_{h \in L_k}$ be the sets of fixed charges for the $i$th consumer and the $k$th competitive firm, respectively. The profit function of the $k$th competitive firm is denoted by

$$\pi_k(p, F_k) = \max \left\{ p \cdot z - \sum_{h \in L_k} \iota(z_{kh})F_{kh} \mid z_k \in Z_k \right\}. $$

A two-part marginal cost pricing equilibrium is defined as follows.

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6 The break-even constraint is rather a normative condition that we require. For the optimality of two-part marginal cost pricing, it is not necessary to be assumed. Hence, if we want to design incentive regulation it is possible to allow monopolies to earn a profit resulting from fixed charges.

7 If $Y$ is a closed subset of $\mathbb{R}^n$, then the Clarke tangent cone at $y \in Y$ is defined by

$$T(y) = \{ x \in \mathbb{R}^n \mid \exists x^k \to x \text{ such that } y^k + t^k x^k \in Y \text{ for all } (y^k, t^k) \to (y, 0) \text{ with } t^k > 0 \text{ for } k \text{ large enough} \}. $$
Definition. A two-part marginal cost pricing equilibrium is a vector \(((x^*_i), (y^*_j), (z^*_k), (F^*_i), (F^*_k), p^*)\) ∈ \(\prod_{i \in T} X_i \times \prod_{j \in M} Y_j \times \prod_{k \in N} Z_k \times \prod_{i \in T} R^i_+ \times \prod_{k \in N} R^k_+ \times R^+_+/\{0\}\) such that

(i) (utility maximization): for all \(i \in T\),

\[
x^*_i \in \arg \max \left\{ u_i(x_i) \mid x_i \in X_i \quad \text{and} \quad p^* - \sum_{h \in L_i} \eta(h) F^*_i \right\} \leq \sum_{k \in N} \theta_{ik} \pi_k(p^*, F^*_k) + \sum_{j \in M} \tau_{ij} \max\left[0, p^* - y^*_j\right]
\]

(ii) (profit maximization): for all \(k \in N\),

\[
z^*_k \in \arg \max \left\{ p^* - \sum_{h \in L_k} \eta(h) F^*_k \right\} \]

(iii) (marginal cost principle): for all \(j \in M\), \(p^* \in \varphi_j(y^*_j)\);

(iv) (break-even constraint): for all \(j \in M\),

\[
\sum_{h \in O_j} \left( \sum_{i \in T} \eta(x^*_ih) F^*_i + \sum_{k \in N} \eta(z^*_kh) F^*_k \right) = \max\left[0, -p^* \cdot y^*_j\right]
\]

(v) (no excess demand and Walras’ law):

\[
\sum_{i \in T} x^*_i \leq \sum_{j \in M} y^*_j + \sum_{k \in N} z^*_k + \sum_{i \in T} \omega_i,
\]

\[
p^* \cdot \left( \sum_{i \in T} x^*_i - \sum_{j \in M} y^*_j - \sum_{k \in N} z^*_k - \sum_{i \in T} \omega_i \right) = 0.
\]

In the above definition note that an equilibrium fixed charge may exclude some agent from the market. In other words, if a fixed charge were lower, then the agent would have purchased a positive amount of the monopoly good and be better-off. To obtain optimality, we shall avoid the obvious inefficiency associated with an exclusionary fixed charge. Thus we introduce the following definition.

Definition. An equilibrium \(((x^*_i), (y^*_j), (z^*_k), (F^*_i), (F^*_k), p^*)\) is said to be with no exclusion if for all \(i \in T\), \(k \in N\) and \(h \in L_i\), \(F^*_ih > 0 \Rightarrow x^*_ih > 0\) and \(F^*_kh > 0 \Rightarrow -z^*_kh > 0\).

Furthermore, the following definitions are introduced.

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8 Precise description of a non-exclusionary fixed charge will be given in Subsection 3.1.
Definition. An equilibrium \(((x^*_i), (y^*_j), (z^*_k), (F^*_i), (F^*_k), p^*)\) is said to be with active monopolies if for all \(j \in M\) there exists some \(h \in O_j\) such that \(y^*_{jh} > 0\).

Definition. A vector \(((x_i), (y_j), (z_k))\) is said to be a feasible allocation if \(\sum_{i \in T} x_i \leq \sum_{j \in M} y_j + \sum_{k \in N} z_k + \sum_{i \in T} \omega_i\).

Definition. A vector \(((x_i), (y_j), (z_k))\) is said to be Pareto optimal if it is a feasible allocation and there exists no feasible allocation \(((x'_i), (y'_j), (z'_k))\) such that \(u_i(x'_i) > u_i(x_i)\) for all \(i \in T\) and strict inequality holds for some \(i \in T\).

2.2. Optimality theorem and proof

We now present the first theorem.

Theorem 1. Under Assumptions \([C1], [F],\) and \([M]\), if a vector \(((x^*_i), (y^*_j), (z^*_k), (F^*_i), (F^*_k), p^*)\) is a two-part marginal cost pricing equilibrium with active monopolies and no exclusion, then it is Pareto optimal.

Proof. Suppose, on the contrary, that there exists a feasible allocation \(((x'_i), (y'_j), (z'_k))\) such that \(u_i(x'_i) > u_i(x_i)\) for all \(i \in T\) and strict inequality holds for some \(i \in T\). We define the index sets of active monopolies and the monopoly goods produced under the allocation \(((x'_i), (y'_j), (z'_k))\) respectively by

\[ J = \{ j \in M \mid \exists h \in O_j \} \]

and

\[ H = \{ h \in L_1 \mid \exists j \in M \} \]

Furthermore, we define the index sets of the monopoly goods purchased by the \(i\)th consumer as \(H(i) = \{ h \in L_1 \mid x_{ih} > 0 \}\) and by the \(k\)th competitive firm as \(H(k) = \{ h \in L_1 \mid z_{kh} < 0 \}\), respectively. Assumptions \([C1](3), [F](3)\) and feasibility imply that if \(y_{jh} = 0\), then \(x_{ih} = 0\) and \(z_{kh} = 0\) for all \(i \in T\) and \(k \in N\). Therefore, \(H(i) \subseteq H\) and \(H(k) \subseteq H\) for all \(i \in T\) and \(k \in N\). By Definition (i) of the equilibrium and Assumption \([C1](2)\):

\[
\sum_{i \in T} \left( p^* \cdot \hat{x}_i + \sum_{h \in L_1} \nu \cdot (\hat{x}_{ih} F^*_{ih}) \right) > \sum_{i \in T} \left( p^* \cdot \omega_i + \sum_{k \in N} \theta_{ik} \pi_k (p^*, F^*_{k}) + \sum_{j \in M} \tau_{ij} \max\{0, p^* \cdot y^*_j\} \right).
\]
Using the fact that
\[ \pi_k(p^*, F^*_k) \geq p^* \cdot \bar{z}_k - \sum_{h \in L_i} \epsilon(\bar{z}_{kh}) F^*_k = p^* \cdot \bar{z}_k - \sum_{h \in H(k)} F^*_k, \]
we have
\[
\sum_{i \in T} \left\{ p^* \cdot \bar{z}_i + \sum_{h \in H(i)} F^*_{ih} \right\} > \sum_{i \in T} p^* \cdot \omega_i + \sum_{i \in T} \left\{ p^* \cdot \bar{z}_k - \sum_{h \in H(k)} F^*_{kh} \right\} + \sum_{j \in M} \max[0, p^* \cdot y^*_j].
\]
By \( F^*_{ih} \geq 0 \), \( F^*_{kh} \geq 0 \), \( H(i) \subseteq H \), \( H(k) \subseteq H \) for all \( i \in T \), \( k \in N \), and \( J \subseteq M \),
\[
\sum_{i \in T} p^* \cdot \bar{z}_i - \sum_{i \in T} p^* \cdot \omega_i - \sum_{k \in N} p^* \cdot \bar{z}_k > \sum_{j \in J} \max[0, p^* \cdot y^*_j] - \left\{ \sum_{i \in T} \sum_{h \in H} F^*_{ih} + \sum_{k \in N} \sum_{h \in H} F^*_{kh} \right\}.
\]
By Definition (iv) of the equilibrium and the no-exclusion condition, for all \( j \in M \),
\[
\sum_{h \in O_j} \left\{ \sum_{i \in T} F^*_{ih} + \sum_{k \in N} F^*_{kh} \right\} = \max[0, -p^* \cdot y^*_j].
\]
Since \( H \subseteq \bigcup_{j \in J} O_j \), we obtain
\[
\sum_{h \in H} \left\{ \sum_{i \in T} F^*_{ih} + \sum_{k \in N} F^*_{kh} \right\} \leq \sum_{j \in J} \max[0, -p^* \cdot y^*_j].
\]
Therefore,
\[
p^* \cdot \left\{ \sum_{i \in T} \bar{z}_i - \sum_{k \in N} \bar{z}_k - \sum_{i \in T} \omega_i \right\}
> \sum_{j \in J} \max[0, p^* \cdot y^*_j] - \sum_{j \in J} \max[0, -p^* \cdot y^*_j] = \sum_{j \in J} p^* \cdot y^*_j.
\]
From Assumption \([M](3)\) it follows that \( \sum_{j \in J} \hat{y}_j \geq \sum_{j \in M} \hat{y}_j \) and thus
\[
p^* \cdot \left\{ \sum_{i \in T} \bar{z}_i - \sum_{k \in N} \bar{z}_k - \sum_{i \in T} \omega_i \right\} \leq p^* \cdot \sum_{j \in M} \hat{y}_j \leq p^* \cdot \sum_{j \in J} \hat{y}_j.
\]
Consequently, we have
\[
p^* \cdot \sum_{j \in J} \hat{y}_j > p^* \cdot \sum_{j \in J} y^*_j.
\]
If $J = \emptyset$, then this is a contradiction. Suppose $J \neq \emptyset$. Since all natural monopolies are active at an equilibrium, $y_j^* \in \partial Y_j^+$ for all $j \in J$. By the definition of $J$, $\hat{y}_j \in \partial Y_j^+$ for all $j \in J$. Note that $p^* \in \Phi_j(y_j^*)$ implies $p^*$ is a normal vector at $y_j^* \in \partial Y_j^+$, i.e.

$$p^* \cdot (y_j^* - \hat{y}_j) > 0,$$

for all $j \in J$.

This contradicts the above inequality. □

**Remark.** To see that the active monopolies condition is necessary for optimality, we provide the following counterexample. Consider a two-good economy, with a numeraire good $x$ and a monopoly good $y$, which consists of one consumer and one natural monopoly. The consumer is endowed with $\omega$ units of good $x$. In Fig. 1, the shaded curve represents the production possibility frontier of the monopoly with a pure fixed cost technology. Indifference curves are shown by $UU$ and $U'U'$. Under the budget constraint $AB$ with zero fixed charge and marginal price $p$, the consumer chooses a point $B$ consuming only good $x$, which is a two-part marginal cost pricing equilibrium with no exclusion and an inactive monopoly. However, under the budget constraint $CD$ with zero fixed charge and marginal price $p$, the consumer can afford a point $E$ since now the monopoly makes positive profit $BD$ which goes into the consumer's income. The allocation $E$ is a two-part marginal cost pricing equilibrium with no exclusion and an active monopoly, which Pareto dominates the allocation $B$.

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9 Vohra's (1990) example 4.1 is indeed the case of a pure exists fixed cost economy. He showed that no efficient two-part marginal cost pricing equilibrium exists. According to our Theorem 1, this corresponds to the case where there is one equilibrium with an inactive monopoly which is inefficient.
3. Existence of a two-part marginal cost pricing equilibrium

In this section we first consider an economy with a general type of non-convexity and then develop a sufficient condition for the existence of two-part marginal cost pricing equilibria with active monopolies and no exclusion. We apply the method introduced by Bonnisseau and Comet (1988) assuming the bounded losses condition. This is a generalization of the existence result of Brown et al. (1992). Second, the existence of the equilibria in a pure fixed cost economy will be provided as a corollary. Note that the corollary does not follow from the original result of Brown et al. (1992) because, for example, their assumptions exclude a pure fixed cost technology with constant returns to scale.

3.1. Existence result under the bounded losses condition

3.1.1. Assumptions and theorem

In addition to Assumptions [C1] and [F] in Section 2, the following assumptions are introduced. Let \( S = \{ q \in R^l_+ | \sum_{h \in L_1 \cup L_2} q_h = 1 \} \) be the unit simplex. Let

\[
\Omega \equiv \left\{ \left( (x_i), (y_j), (z_k) \right) \in \prod_{i \in T} X_i \times \prod_{j \in M} Y_j \times \prod_{k \in N} Z_k \left| \sum_{i \in T} x_i \leq \sum_{j \in M} y_j \right. \right. \\
+ \left. \sum_{k \in N} z_k + \sum_{i \in T} \omega_i \right\}
\]

be a feasible set of the economy.

Assumption \( [M^*] \). For all \( j \in M \), \( Y_j \) is a non-empty closed set and satisfies \( Y_j - R^l_+ \subset Y_j \subset R^l_+ \).

Assumption \( [BL] \) (bounded losses condition). For all \( j \in M \), there exists a real number \( \delta_j \) such that for all \( y_j \in \partial Y_j \) and \( q \in \varphi_j(y_j) \cap S \), \( q \cdot y_j \geq \delta_j \) holds.

Assumption \( [C2] \). For all \( i \in T \),

1. \( u_i : X_i \to R \) is a continuous, quasi-concave function, and
2. \( \tau_{ij} > 0 \) for all \( j \in M \), and \( \omega_{ih} > 0 \) for all \( h \in L_2 \).

Assumption \( [B] \). For all \( \omega' \geq \sum_{i \in T} \omega_i \), the set defined by

\[
\Omega(\omega') \equiv \left\{ \left( (x_i), (y_j), (z_k) \right) \in \prod_{i \in T} X_i \times \prod_{j \in M} Y_j \times \prod_{k \in N} Z_k \left| \sum_{i \in T} x_i \leq \sum_{j \in M} y_j \right. \right. \\
+ \left. \sum_{k \in N} z_k + \omega' \right\}
\]

is bounded.
Under Assumption \([M^*]\) we allow not only pure fixed cost technologies but a general kind of non-convex technologies. However, the bounded losses assumption, Assumption \([BL]\), imposes a restriction on technologies, i.e. that the losses of a natural monopoly resulting from marginal cost pricing should be bounded. For a detailed discussion see Bonnisseau and Comet (1988).

The next step is to formalize a non-uniform fixed charge that causes no exclusion of agents. Brown et al. (1992) introduced the concept of willingness to pay in a general equilibrium framework which enables us to define a non-exclusionary fixed charge. We extend their definition to an economy with multiple monopoly goods as follows.\(^\text{10}\)

**Definition.** The set of production equilibria is defined by

\[
PE = \left\{ (p, y) \in S \times \prod_{j \in M} \partial Y_j \mid p \in \bigcap_{j \in M} \varphi_j(y_j) \cap S \right\}.
\]

For all \((p, y) \in PE\) the \(i\)th consumer's revenue function is given by

\[
R_i(p, y) = p \cdot \omega_i + \sum_{k \in N} \theta_{ik} \pi_k(p, F_k) + \sum_{j \in M} \tau_{ij} \max[0, p \cdot y_j].
\]

Let \(p \in \text{int } S\) and \((p, y) \in PE\). The \(i\)th consumer's reservation utility for the monopoly good \(h\) is

\[
\bar{u}_{ih}(p, y) = \max \left\{ u_i(x_i) \mid p \cdot x_i + \sum_{s \in L_i} \varphi_i(x_is) F_{is} \leq R_i(p, y) \text{ and } x_i \in X_i \right\},
\]

with \(x_{ih} = 0\).

That is to say, the maximum utility level one could attain if the good \(h\) were not available. Note that \(\bar{u}_{ih}(p, y)\) is well defined since for \(p \in \text{int } S\) the budget set is compact.\(^\text{11}\) The expenditure function evaluated at the reservation utility is

\[
E_{ih}(p, y) = \min \left\{ p \cdot x_i + \sum_{s \in L_i \setminus \{h\}} \varphi_i(x_is) F_{is} \mid u_i(x_i) \geq \bar{u}_{ih}(p, y) \right\},
\]

and \(x_i \in X_i\),

which represents the income level necessary to attain \(\bar{u}_{ih}\) when good \(h\) is available. The willingness to pay is defined as follows.

\(^{10}\) See also Edlin and Epelbaum (1993a) for more general treatment with detailed discussions.

\(^{11}\) Note that when the competitive goods are free, \(\bar{u}_{ih}\) may be unbounded and \(S_{ih}\) will not be well defined.
Definition. The willingness to pay of the $i$th consumer for the monopoly good $h$ is defined by

$$S_{ih}(p, y) = R_i(p, y) - E_{ih}(p, y),$$

for $(p, y) \in PE$ and $p \in \text{int } S$,

$$S_{ih}(p, y) = \lim_{(p^k, y^k) \to (p, y)} S_{ih}(p^k, y^k),$$

for $(p, y) \in PE$ and $p \in \partial S$,

where $(p^k, y^k) \in PE$ and $p^k \in \text{int } S$.\(^{12}\)

By definition, it is always true that $0 \leq S_{ih}(p, y) \leq R_i(p, y)$. Similarly, the willingness to pay of the competitive firms is defined as follows.

Definition. The willingness to pay of the $k$th competitive firm for the monopoly good $h$ is defined by

$$S_{kh}(p, y) = \pi_{kh}(p, y) - \pi_{kh}(p, y),$$

for $(p, y) \in PE$,

where

$$\pi_{kh}(p, y) = \max_{s \in L \setminus \{h\}} (p \cdot z_k - \sum_{s \in L \setminus \{h\}} \nu(z_{ks}) F_{ks} | z_k \in Z_k),$$

$$\pi_{kh}(p, y) = \max_{s \in L_1} (p \cdot z_k - \sum_{s \in L_1} \nu(z_{ks}) F_{ks} | z_k \in Z_k \text{ with } z_{kh} = 0).$$

If we consider the break-even constraint, then we can represent a fixed charge as a share of the losses of a monopoly incurred from marginal cost pricing. Let $\alpha_{ih} : PE \to [0, 1]$ be the share of the $i$th consumer associated with the good $h$. Let $\beta_{kh} : PE \to [0, 1]$ be the share of the $k$th competitive firm associated with the good $h$. In other words, a fixed charge is now denoted by

$$F_{ih} = \alpha_{ih}(p, y) | p \cdot y_{j(h)}|,$$

$$F_{kh} = \beta_{ih}(p, y) | p \cdot y_{j(h)}|,$$

when $p \cdot y_{j(h)} < 0$, where $j(h) \in M$ is an index of the monopoly that produces the good $h$. The following assumptions are made.

Assumption [NE]\(^{13}\) For all $j \in M$,

(1) (break-even constraint): for all $(p, y) \in PE$, if $p \cdot y_j < 0$, then

$$\sum_{h \in 0} \{\sum_{i \in T} \alpha_{ih}(p, y) + \sum_{k \in N} \beta_{kh}(p, y)\} = 1;$$

\(^{12}\) Note that since utility functions are concave, for $p \in \text{int } S$, $S_{ih}(p, y)$ is continuous in $(p, y)$.

\(^{13}\) Assumption [NE] is imposed without specifying how to set $\alpha(\cdot)$ and $\beta(\cdot)$. In an economy with $m = l_i = 1$, Brown et al. (1992) proposed the proportional fixed charge, which is a fraction of the consumer’s willingness to pay, i.e. $\alpha(p, y) = S(p, y) / \sum_{i \in T} S(p, y) \beta(p, y) = 0$, which clearly satisfies [NE]. The most general case with $n$ monopolies is studied by Edlin and Epelbaum (1993a).
(2) (non-negativity): for all \((p, y) \in PE\), if \(p \cdot y_j \geq 0\), then \(\alpha_{ih}(p, y) = \beta_{kh}(p, y) = 0\) for all \(h \in O_j, i \in T\) and \(k \in N\);

(3) (non-exclusionary condition): for all \((p, y) \in PE, h \in O_j\) and \(i \in T\), \(S_{ih}(p, y) = 0 \Rightarrow \alpha_{ih}(p, y) = 0\) and \(S_{ih}(p, y) > 0 \Rightarrow \alpha_{ih}(p, y) > 0\) and \(S_{kh}(p, y) = 0 \Rightarrow \beta_{kh}(p, y) = 0\) and \(S_{kh}(p, y) > 0 \Rightarrow \beta_{kh}(p, y) > 0\); and for all \((p, y) \in PE, h \in O_j\) and \(i \in T, S_{ih}(p, y) = 0 \Rightarrow \beta_{ih}(p, y) = 0\) and \(S_{ih}(p, y) > 0 \Rightarrow \beta_{ih}(p, y) > 0\);

(4) (continuity): for all \(h \in O_j, i \in T\) and \(k \in N, \alpha_{ih}: PE \rightarrow [0, 1]\) and \(\beta_{ih}: PE \rightarrow [0, 1]\) are continuous functions;

(5) (homogeneity of degree zero): for all \((p, y) \in PE, h \in O_j, i \in T, k \in N, \lambda \in R^+\), \(\alpha_{ih}(\lambda p, y) = \lambda \alpha_{ih}(p, y)\) and \(\beta_{kh}(\lambda p, y) = \lambda \beta_{kh}(p, y)\).

Assumption [S] (surplus condition). For all \(j \in M\) and all \((p, y) \in PE\), \(\sum_{h \in O_j} \sum_{i \in T} S_{ih}(p, y) + \sum_{k \in N} S_{kh}(p, y) > |p \cdot y_j|\).

Assumption [NE](1) ensures that the sum of all fixed charges set by each monopoly exactly makes up the losses incurred from marginal cost pricing. Assumption [NE](2) says that if there is non-negative profit, then no fixed charge should be imposed. Assumption [NE](3) describes the condition for a non-exclusionary fixed charge and (4) is required to derive continuous demand functions. It is worth noting that Assumption [NE] alone implies that the total willingness to pay among the private sector for all the monopoly goods produced by one monopoly is more than or equal to the losses incurred by the monopoly. We assume, furthermore, in the surplus condition [S] that it should hold with strict inequality, which is a sufficient condition for every natural monopoly to be viable.

Clearly, the above assumptions are restrictive. Since the willingness to pay for a certain monopoly good depends on a fixed charge for other monopoly goods as well, if there are close substitutes in an economy, then Assumptions [NE] and [S] may not be satisfied. However, if the monopoly goods are complements or essential goods by themselves, then those assumptions are likely to be satisfied. Moreover, if there are sufficiently many agents in the private sector relative to the public sector and the losses from marginal cost pricing are bounded under Assumption [BL], then Assumptions [NE] and [S] are less restrictive.

We now present the second theorem.

Theorem 2. Under Assumptions [C1] [C2], [F], [M*], [BL], [B], [NE], and [S] there exists a two-part marginal cost pricing equilibrium with active monopolies and no exclusion.

\(^{14}\) When monopoly goods are substitutable or complementary between each other, the measurement of the social surplus requires more delicate argument. This question is fully analyzed by Edlin and Epelbaum (1993b).
3.1.2. Proof of Theorem 2  
Our approach is the following. First, we show that if a fixed charge is non-exclusionary, then we can treat it as a lump-sum tax. As a result, two-part marginal cost pricing equilibria reduce to quasi marginal cost pricing equilibria. Next, we apply the result of Bonnisseau and Cornet (1988) to show the existence of a quasi marginal cost pricing equilibrium.

We provide the following Lemmas 1 and 2.

Lemma 1. Under assumption [NE], for all \( k \in N \) and for all \( (p, y) \in \mathcal{P}E \),

\[
\arg\max \left\{ p \cdot z_k + \sum_{h \in L_1} t(z_{kh}) \beta_{kh}(p, y) p \cdot y_{(h)} \right\} = \arg\max \left\{ p \cdot z_k \right\}.
\]

That is to say, the competitive firms maximize profit as though they faced linear pricing.

Proof. See the appendix.

Intuitively, since a non-exclusionary fixed charge is defined so that a firm is willing to pay it anyway, we can regard it as if it were a lump-sum tax. In other words, the fixed charge terms are irrelevant to profit maximization. By Lemma 1 and convexity of the production set, the supply correspondence of the \( k \)th competitive firm is given simply by the normal cone correspondence. Let \( \phi_k : \partial Z_k \rightarrow \mathbb{R}^l \) be the normal cone correspondence. The profit function of the \( k \)th competitive firm can then be written as

\[
\pi_k(p, y) = p \cdot z_k^* + \sum_{h \in L_1} \beta_{kh}(p, y) p \cdot y_{(h)}, \quad \text{where} \ p \in \phi_k(z_k^*).
\]

A similar result holds for consumers. Recall that the \( i \)th consumer's revenue function is

\[
R_i(p, y) = p \cdot \omega_i + \sum_{k \in N} \theta_{ik} \pi_k(p, y) + \sum_{j \in M} \tau_{ij} \max\{0, p \cdot y_j\}.
\]

We define the quasi-revenue function as

\[
\bar{R}_i(p, y) = p \cdot \omega_i + \sum_{k \in N} \theta_{ik} \pi_k(p, y) + \sum_{j \in M} \tau_{ij} \max\{0, p \cdot y_j\} + \sum_{h \in L_1} \alpha_{ih}(p, y) p \cdot y_{(h)}.
\]

Namely, in the quasi-revenue function the fixed charges are subtracted from revenue as though they were lump-sum taxes.
Lemma 2. Under assumption [NE], for all \((p, y) \in PE\) and \(i \in T\),
\[
\arg \max \left\{ u_i(x_i) \mid p \cdot x_i - \sum_{h \in L_1} v(x_{ih}) \alpha_{ih}(p, y) p \cdot y_{kh} \leq R_i(p, y) \text{ and } x_i \in X_i \right\}
\]
\[= \arg \max \{ u_i(x_i) \mid p \cdot x_i \leq \tilde{R}_i(p, y) \text{ and } x_i \in X_i \}.\]

Proof. See the appendix.

Next, we define aggregate production equilibria which explicitly include a competitive firm's production vector.

Definition. The set of aggregate production equilibria is defined by
\[
APE = \left\{ (p, y, z) \in S \times \prod_{j \in M} \partial Y_j \times \prod_{k \in N} \partial Z_k \mid p \in \left( \bigcap_{j \in M} \varphi_j(y_j) \right) \right\}
\]
\[\cap \left( \bigcap_{k \in N} \Phi_k(z_k) \right) \cap S \right\}.
\]

For notational convenience, using Lemma 1 we can rewrite the quasi-revenue function as being defined on \(APE\):
\[
\tilde{R}_i(p, y, z) = p \cdot \omega_i + \sum_{k \in N} \theta_{ik} \left\{ p \cdot z_k + \sum_{h \in L_1} \beta_{ih}(p, y) p \cdot y_{kh} \right\}
\]
\[+ \sum_{j \in M} \tau_{ij} \max \left\{ 0, p \cdot y_j \right\} + \sum_{h \in L_1} \alpha_{ih}(p, y) p \cdot y_{kh}.\]

To apply the theorem in Bonnisseau and Cornet (1988) we prove that the following assumptions are satisfied.

Assumption C. For all \(i \in T\),
(i) \(X_i\) is a non-empty, closed, convex, bounded-below subset of Euclidean space;
(ii) a preference relation is continuous, convex and locally non-satiated; and
(iii) quasi-revenue \(\tilde{R}_i(p, y, z): APE \to R\) is continuous, homogeneous of degree one in \(p\), and for all \((p, y, z) \in APE\),
\[
\sum_{i \in T} \tilde{R}_i(p, y, z) = p \cdot \left\{ \sum_{j \in M} y_j + \sum_{k \in N} z_k + \sum_{i \in T} \omega_i \right\}.
\]

Assumption P. For all \(j \in M\) and \(k \in N\), \(Y_j\) and \(Z_k\) are non-empty, closed, and satisfy free disposal.
Assumption B. For all \( \omega' \geq \sum_{i \in T} \omega_i \), the set defined by

\[
\Omega(\omega') = \left\{ (x, y, z) \in \prod_{i \in T} X_i \times \prod_{j \in M} Y_j \times \prod_{k \in N} Z_k \mid \sum_{i \in T} x_i \leq \sum_{j \in M} y_j + \sum_{k \in N} z_k + \omega' \right\}
\]

is bounded.

Assumption PR. For all \( j \in M \) and \( k \in N \), \( \varphi_j \) and \( \phi_k \) are upper hemi-continuous and non-empty, convex, compact values.

Assumption BL. For all \( j \in M \), there exists a real number \( \delta_j \) such that for all \( y_j \in \partial Y_j \) and \( q \in \varphi_j(y_j) \cap S \), \( q \cdot y_j > \delta_j \) holds, and for all \( k \in N \) there exists a real number \( \mu_k \) such that for all \( z_k \in \partial Z_k \) and \( q \in \phi_k(z_k) \cap S \), \( q \cdot z_k \geq \mu_k \) holds.

Assumption SA (survival assumption). \((p, y, z) \in APE\) implies \( p \cdot \{\sum_{j \in M} y_j + \sum_{k \in N} z_k + \sum_{i \in T} \omega_i\} > \inf p \cdot \sum_{i \in T} X_i \).

Assumption R. \((p, y, z) \in APE\) and \( p \cdot \{\sum_{j \in M} y_j + \sum_{k \in N} z_k + \sum_{i \in T} \omega_i\} > \inf p \cdot \sum_{i \in T} X_i \) imply \( \bar{R}(p, y, z) > \inf p \cdot X_i \) for all \( i \in T \).

Lemma 3. Assumptions \([C1], [C2], [F], [M^*], [BL], [B], [NE], \) and \([S]\) imply Assumptions C, P, B, PR, BL, SA, and R.

Proof. See the appendix.

Lemma 3 allows us to invoke the following theorem.

Theorem 2.1 (Bonnisseau and Cornet, 1988, p. 123). Under Assumptions C, P, B, PR, BL, SA, and R there exists an equilibrium \((x^*_i, y^*_j, z^*_k, p^*)\) which satisfies

(a) for all \( i \in T \), \( x^*_i = \arg\max \{u_i(x_i) \mid x_i \in X_i \} \) and \( p^* \cdot x_i \leq \bar{R}(p^*, y^*, z^*) \);
(b) for all \( j \in M \), \( p^* \in \varphi_j(y^*_j) \);
(c) for all \( k \in N \), \( p^* \in \phi_k(z^*_k) \); and
(d) \( \sum_{i \in T} x^*_i \leq \sum_{j \in M} y^*_j + \sum_{k \in N} z^*_k + \sum_{i \in T} \omega_i, p^* \geq 0, \) and \( p^* \cdot \{\sum_{i \in T} x^*_i - \sum_{j \in M} y^*_j - \sum_{k \in N} z^*_k - \sum_{i \in T} \omega_i\} = 0 \).

By Lemmas 1 and 2, conditions (a) and (c) imply that \( (x^*_i) \) and \( (z^*_k) \) satisfy Definitions (i) and (ii) of two-part marginal cost pricing equilibrium. Condition (b) implies Definition (iii) and condition (d) implies Definition (v), respectively. Under Assumption [NE] no exclusion occurs and monopolies break even. Finally, we show that all natural monopolies are active. If a monopoly makes positive
profit at the equilibrium, then clearly it is active. If a monopoly runs at a loss, then Assumption [S] ensures sufficient willingness to pay for the goods produced by the monopoly and under Assumption [NE] there is positive demand. Since none of the monopoly goods is endowed in the economy, this implies that every natural monopoly produces positive outputs and is active. Consequently, we have proved Theorem 2.

3.2. Existence result for a pure fixed cost economy

It should be noted that for the pure fixed cost economy described in Section 2 we need the slight modification of Theorem 2. The reason is the following. When natural monopolies have pure fixed cost technologies and marginal cost correspondences are defined by the Clarke normal cone, the survival assumption, Assumption SA, may not hold. An example in a two-good economy with one monopoly is illustrated in Fig. 2(a). The shaded curve represents the production frontier of the monopoly that produces the monopoly good $y_2$. The numeraire good $y_1$ is endowed to consumers. At the point $\mathbf{y}$, the price vector $\mathbf{\beta}$ is an element of the Clarke normal cone shown by the shaded cone, and hence $(\mathbf{\beta}, \mathbf{y}) \in PE$. Since $\beta_2$ is zero, the profit of competitive firms is non-positive and the endowment values are zero. At $\mathbf{y}$ the monopoly's output level $\hat{y}_2$ is zero. Therefore,

$$\mathbf{\beta} \cdot \left( \sum_{j \in M} \delta_j + \sum_{i \in T} \omega_i \right) = 0,$$

which violates the survival assumption.\textsuperscript{15} Theorem 2 is not generally applicable to a pure fixed cost economy.

Although it is quite common to define the marginal cost correspondence by the Clarke normal cone in the general equilibrium literature, the Clarke normal cone

\textsuperscript{15} The survival assumption is violated only at those points where all monopolies produce no goods even though they invested some of the fixed costs.
does not necessarily match marginal cost pricing when the production frontier exhibits an inward kink. We define the marginal cost correspondence slightly differently from the Clarke normal cone as follows. Let \( \bar{Y}_i^+ \) be the closure of \( Y_i^+ \) to define a truncated production set

\[
\bar{Y}_j = \bar{Y}_j^+ - R_+. 
\]

By Assumption [M], \( \bar{Y}_j \) is convex. We define the (modified) marginal cost correspondence \( \bar{\varphi}_j : \partial \bar{Y}_j \to R^1 \) by the normal cone. On the set \( \partial \bar{Y}_j \cap \partial Y_j \) two correspondences, \( \bar{\varphi}_j \) and \( \varphi_j \), coincide. However, at the point of inward kinks these two differ (see Fig. 2(b)). The pricing rule \( \bar{\varphi}_j \) tells us that a firm equates price with marginal cost if output is strictly positive and may lower the price if a firm does not produce at all. This form of marginal cost pricing is found, for example, in Dierker et al. (1985) and Bonnisseau and Cornet (1990). Since our focus is on the region where all monopolies are active, the modified rule \( \bar{\varphi}_j \) does not affect the optimality result given in Theorem 1. In terms of the marginal cost correspondence \( \bar{\varphi}_j \) and the truncated production set \( \bar{Y}_j \), we can appropriately redefine all the assumptions in Section 2. In particular, the survival assumption holds. Furthermore, for pure fixed cost technologies Assumption [B] is trivially satisfied. Hence, a little modification of the proof of Theorem 2 provides the following corollary.

Corollary. Under assumptions [C1], [C2], [F], [M], [B], [NE], and [S] there exists a two-part marginal cost pricing equilibrium with active monopolies and no exclusion.

4. Conclusion

We have shown that in a pure fixed cost economy with regulated natural monopolies, two-part marginal cost pricing leads to a Pareto-optimal outcome. In our two-part pricing scheme, however, a fixed charge has to be assigned to each agent so that it does not exceed anyone's willingness to pay. Needless to say, if we wish to measure the willingness to pay of every agent in an economy, then the informational requirement is tremendous. Although perfect information is assumed throughout this paper, the implementation of two-part pricing in the case of imperfect information is an important issue. Furthermore, for the monopolists there is little incentive to follow the regulation specified by the government. In future research we intend to explore an incentive regulation and, even further, deregulation of monopolies, which implement the first-best outcome, taking the result of this paper as a starting point.

Although the non-convexity considered here is very special, namely that of pure fixed cost technologies, we believe that there is a wide range of applications in the existing literature including macroeconomics, development and international trade.
Acknowledgements

This paper is based on Moriguchi (1991). The author is grateful to Donald Brown and Kazuya Kamiya for their encouragement and insightful discussions. The author also thanks an anonymous referee, Toshihiko Hayashi, Hiroaki Nagatani, Hiroo Sasaki, and the participants of the SITE workshop at Stanford University in 1991 for their helpful comments. Colin McKenzie and Joshua Gans are gratefully acknowledged for their editorial help.

Appendix

We prove Lemmas 1, 2, and 3 below.

Lemma 1. Under Assumption [NE], for all \( k \in N \) and for all \((p, y) \in PE\),

\[
\arg \max \left\{ \sum_{h \in L_i} \iota(z_{kh}) \beta_{kh}(p, y)p \cdot y_{j(h)} \right\}
\]

\[
= \arg \max \left\{ \sum_{h \in L_i} \beta_{kh}(p, y)p \cdot y_{j(h)} \right\}
\]

\[
= \arg \max \left\{ \sum_{h \in L_i} \beta_{kh}(p, y)p \cdot y_{j(h)} \right\}
\]

Proof. First, observe that

\[
\max \left\{ \sum_{h \in L_i} \iota(z_{kh}) \beta_{kh}(p, y)p \cdot y_{j(h)} \right\}
\]

\[
= \max \left\{ \sum_{h \in L_i} \beta_{kh}(p, y)p \cdot y_{j(h)} \right\}
\]

Let \( \bar{\xi}_k \in \arg \max \{ \sum_{h \in L_i} \iota(z_{kh}) \beta_{kh}(p, y)p \cdot y_{j(h)} \} \). Then to conclude the first equality in Lemma 1 it suffices to show that

\[
p \cdot \bar{\xi}_k + \sum_{h \in L_i} \beta_{kh}(p, y)p \cdot y_{j(h)} = p \cdot \bar{\xi}_k + \sum_{h \in L_i} \iota(z_{kh}) \beta_{kh}(p, y)p \cdot y_{j(h)}.
\]

In other words, it must be the case that for all \( s \in L_i \) such that \( \beta_{ks}(p, y)p \cdot y_{j(s)} \) \( > 0, \bar{\xi}_{ks} < 0 \) follows. Suppose, on the contrary, that \( \bar{\xi}_{ks} = 0 \). Under Assumption [NE](3) \( \beta_{ks}(p, y)p \cdot y_{j(s)} > 0 \) implies \( S_{ks}(p, y) > 0 \).

Using the definition of willingness to pay,

\[
p \cdot \bar{\xi}_k + \sum_{h \in L_i} \iota(z_{kh}) \beta_{kh}(p, y)p \cdot y_{j(h)} \leq \bar{\pi}_{ks}(p, y)
\]

\[
= \bar{\pi}_{ks}(p, y) - S_{ks}(p, y)
\]

\[
< \bar{\pi}_{ks}(p, y) + \beta_{ks}(p, y)p \cdot y_{j(s)}.
\]
Let $z' \in Z_k$ be the vector that attains the profit $\pi_{k_1}(p, y)$, i.e.

$$\pi_{k_1}(p, y) = p \cdot z' + \sum_{h \in L_1 \setminus \{s\}} \imath(z'_{kh}) \beta_{kh}(p, y) p \cdot y_{kh}.$$ 

Then

$$p \cdot z' + \sum_{h \in L_1} \imath(z'_{h}) \beta_{kh}(p, y) p \cdot y_{kh} < p \cdot z' + \sum_{h \in L_1} \imath(z'_{h}) \beta_{kh}(p, y) p \cdot y_{kh}.$$ 

This contradicts the definition of $\hat{z}_k$. Hence, the first equality holds. The second equality is obvious. □

**Lemma 2.** Under Assumption [NE], for all $(p, y) \in PE$ and $i \in T$,

$$\arg \max \left\{ u_i(x_i) \mid p \cdot x_i - \sum_{h \in L_1} \imath(x_{ih}) \alpha_{ih}(p, y) p \cdot y_{kh} \leq R_i(p, y) \right\}$$

and $x_i \in X_i$.

**Proof.** We define the corresponding budget sets as

$$B_i(p, y) = \left\{ x_i \in X_i \mid p \cdot x_i \leq \bar{R}_i(p, y) \right\}$$

and $\bar{B}_i(p, y) = \left\{ x_i \in X_i \mid p \cdot x_i \leq \bar{R}_i(p, y) \right\}$.

Clearly, $B_i(p, y) \subseteq \bar{B}_i(p, y)$ holds. Let $\hat{x}_i \in \arg\max\{u_i(x_i) \mid x_i \in B_i(p, y)\}$. Then to prove Lemma 2 it suffices to show that $\hat{x}_i \in \bar{B}_i(p, y)$. That is, for any $s \in L_1$ such that $\alpha_{is}(p, y) p \cdot y_{ks} > 0$, $\hat{x}_is > 0$ follows. Assume, on the contrary, that $\hat{x}_is = 0$. Then

$$u_i(\hat{x}_i) \leq \bar{u}_i \equiv \max\{u_i(x_i) \mid x_i \in B_i(p, y) \text{ and } x_i = 0\}.$$ 

Using Assumptions [C1](2) and [C2](1), this is equivalent to

$$p \cdot \hat{x}_i - \sum_{h \in L_1} \imath(\hat{x}_{ih}) \alpha_{ih}(p, y) p \cdot y_{kh} \leq E_{is}(p, y).$$ 

However, by Assumption [NE](3) $\alpha_{is}(p, y) p \cdot y_{ks} > 0$ implies $S_{is}(p, y) > 0$,

$$p \cdot \hat{x}_i - \sum_{h \in L_1} \imath(\hat{x}_{ih}) \alpha_{ih}(p, y) p \cdot y_{kh} = R_i(p, y)$$

$$= E_{is}(p, y) + S_{is}(p, y) > E_{is}(p, y).$$

This is a contradiction. □

**Lemma 3.** Assumptions [C1], [C2], [F], [M⁺], [BL], [B], [NE], and [S] imply Assumptions C, P, B, PR, BL, SA, and R.
Proof. Clearly, Assumptions C(i) and (ii) are implied by [C1] and [C2]. Assumption C(iii) follows since by [FE](1) and (2)

\[ \sum_{i \in T} \tilde{K}_i(p, y, z) = \sum_{i \in T} \left\{ p \cdot \omega_i + \sum_{k \in N} \theta_{ik} \left( p \cdot z_k + \sum_{h \in L_1} \beta_{kh}(p, y)p \cdot y(h) \right) \right\} \\
+ \sum_{j \in M} \tau_{ij} \max[0, p \cdot y_j] + \sum_{h \in L_1} \alpha_{ih}(p, y)p \cdot y(h) \right\} \\
= p \cdot \sum_{i \in T} \omega_i + p \cdot \sum_{k \in N} z_k + \sum_{j \in M} \max[0, p \cdot y_j] \\
+ \sum_{j \in M_{h \in O_j}} \sum_{i \in T} \left\{ \sum_{j \in M} \alpha_{ih}(p, y) + \sum_{k \in N} \beta_{kh}(p, y) \right\} p \cdot y_j \\
= p \cdot \left\{ \sum_{i \in T} \omega_i + \sum_{k \in N} z_k + \sum_{j \in M} y_j \right\}, \]

and [NE](5) ensures homogeneity of degree one in price. Assumption P is implied by Assumptions [F] and [M*]. By the properties of the Clarke normal cone and normal cone correspondences, Assumption PR is satisfied. Since \( Z_k \) is a convex set, Assumptions B and BL follow from Assumptions [B] and [BL].

Next we show Assumption R. Recall that: \( \inf X_i = 0 \) and

\[ R_i(p, y) = p \cdot \omega_i + \sum_{k \in N} \theta_{ik} \pi_k(p, y) + \sum_{j \in M} \tau_{ij} \max[0, p \cdot y_j], \]

Assumption [NE](3) ensures that \( R_i(p, y) > 0 \) implies \( \tilde{R}_i(p, y) > 0 \) for all \( (p, y) \in PE \). Therefore, all we need is to show \( R_i(p, y) > 0 \) for all \( i \in T \). If \( p_h > 0 \) for some \( h \in L_2 \), then by Assumption [C2](2), \( p \cdot \omega_i > 0 \). Since \( \pi_k(p, y) \geq 0 \) for all \( (p, y) \in PE, k \in N \), this implies \( R_i(p, y) > 0 \) for all \( i \in T \). If \( p_h = 0 \) for all \( h \in L_2 \), then \( p \cdot \omega_i = 0 \). Since \( \pi_k(p, y) = 0 \), and \( S_{kh}(p, y) = 0 \) for all \( h \in L_1 \) and \( k \in N \). Suppose \( p \cdot y_j \leq 0 \) for all \( j \in M \). Then \( R_i(p, y) = 0 \) and thus \( S_{ih}(p, y) = 0 \) for all \( h \in L_1, i \in T \). Consequently, for all \( j \in M \)

\[ 0 = \sum_{h \in O_j} \left\{ \sum_{i \in T} S_{ih}(p, y) + \sum_{k \in N} S_{kh}(p, y) \right\} \leq |p \cdot y_j|, \]

which contradicts the surplus condition [S]. Hence, there exists \( j \in M \) such that \( p \cdot y_j > 0 \). By Assumption [C2](1), this implies \( R_i(p, y) > 0 \) for all \( i \in T \).

Finally, Assumption SA follows since \( \tilde{R}_i(p, y, z) > 0 \) for all \( i \in T \) and

\[ \sum_{i \in T} \tilde{R}_i(p, y, z) = p \cdot \left\{ \sum_{i \in T} \omega_i + \sum_{k \in N} z_k + \sum_{j \in M} y_j \right\}. \]
References


