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Vote Buying: Legislatures and Lobbying*

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ABSTRACT

We examine the consequences of lobbying and vote buying, assuming this practice were allowed and free of stigma. Two *lobbyists* compete for the votes of legislators by offering up-front payments to the legislators in exchange for their votes. We analyze how the lobbyists' budget constraints and legislators' preferences determine the winner and the payments. When lobbyists are budget constrained then the preferences of all legislators can matter, and a lobbyist's relative strength increases more steeply with a budget increase than with an increase of equal magnitude to the legislators' original preferences for this lobbyist's positio. When lobbyists are not budget constrained then only the preferences of *near* median legislators matter and the preferences of these legislators and the budget enter equally in determining the winner.

Keywords: Vote buying; lobbying; legislatures; political economy.

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INTRODUCTION

Consider a legislature that will vote over two alternatives, where two opposing lobbyists compete by bidding for legislators' votes. We study how the legislative outcome depends on the lobbyists' budgets and preferences and the legislators' preferences. We show that the outcome generally fails to fully reflect legislators' preferences. Moreover, we find that lobbyists' budget constraints can change the outcome in interesting and significant ways from situations where lobbyists' budgets exceed their maximal willingness to pay.

We model the lobbying process via a complete-information game in which lobbyists alternate in increasing their offers to legislators. Legislators care about how they cast their vote, and any payments they receive from lobbyists, rather than about the eventual outcome. The idea is that legislators care about how their voting record is perceived by their constituency, regardless of the actual outcome.[1]

Naturally, the difference in the budgets of the lobbyists plays a critical role in determining which lobbyist is successful when lobbyists are budget constrained, and the difference in their maximal willingness to pay plays an important role when they are not budget constrained. However, legislators' voting preferences enter into the determination of the winner in subtle ways, and are markedly different in how they matter depending on whether or not lobbyists are budget constrained.

The main analytical result (Proposition 2 in Section 3.1) concerns the case where lobbyists are budget constrained. There we show that a lobbyist wins if her budget plus *half* of the total value that legislators attach to voting in her favor exceeds the corresponding magnitude for the other lobbyist. The result that preferences are weighed half as much as budgets in determining the outcome stems from the strategic aspects of the vote-buying game. In making a bid for any given legislator's vote, the lobbyist cares not only about how much he or she must promise to pay, but also about how much this offer will free up for the other lobbyist to use in bidding for other votes.

In contrast, when budgets are unbounded, the role of legislator preferences is very different. What matters then are the lobbyists' valuations and the intensity of preferences of a particular *near-median* group of legislators. The lobbyist with *a priori* minority support wins when its valuation exceeds the other lobbyist's valuation by more than a magnitude that depends on the preferences of that near-median group (Proposition 3).

Thus, the voting preferences of the legislators have quite different effects in the two scenarios. When budget constraints are important, the intensity of the preferences of all legislators *can* matter; when budgets do not constitute the important constraints, only the intensity of preferences of a particular near-median group of legislators matter.

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1 The extent to which legislators care about outcomes would matter only when they are pivotal. However, as discussed in Section 2.1, the probability of being pivotal is often negligible, especially in the context of vote buying where the lobbyist can intentionally make the legislators non-pivotal by buying slightly more than the minimal number of votes they need (see Dal-Bo 2007). This would render the legislators' preferences over outcomes unimportant and they would thus be willing to tender their vote to the highest bidder. In contrast, the extent to which legislator cares about how the vote is cast significantly affects the outcome (both payments and who wins).

The discussion in Section 4 collects a number of additional issues, among them the case of unknown legislators' preferences, welfare implications and related literature. It is noted there that, in general, the outcome of the vote-buying game need not be efficient and might involve higher or lower total surplus than what will arise in its absence. It is also claimed that, when lobbyists' budgets are raised by a certain donation game in which all of the population participates, then the lobbyists' budgets reflect the population preferences and the overall outcome is efficient.

Much of the formal literature on lobbying is concerned with influencing a single decision maker (e.g., a regulator). Our works belongs to a somewhat different strand of the literature that examines the lobbying of a voting body like a legislature. This strand begins with the Colonel Blotto game, which in the context of lobbying has lobbyists making simultaneous offers of payments to legislators. Beyond the notorious difficulties in solving even the very simplest cases, the one-bid simultaneous mixed strategy nature of the game is not appropriate in most applications and that makes it unclear that the Colonel Blotto game provides much robust insight into lobbying. In the fundamental contribution of Groseclose and Snyder (1996), the lobbyists move sequentially and each makes only one final offer. This makes the game much more tractable than the original Colonel Blotto setting. The Groseclose and Snyder analysis focuses on the advantage that this asymmetric procedure confers on the second mover — the first mover can win only by buying a sufficiently significant supermajority.[2] Although the game is tractable and in some scenarios some formal procedure may create such an asymmetry, there are other situations there is no such formal structure and the lobbying process resembles more a repeatedly reactive bidding process like the one we model. Our analysis shows that this changes the strategic interaction and outcomes significantly, and so even though our model is also highly stylized, it is still important to characterize how allowing lobbyists to react to each other's bidding behavior affects lobbying outcomes. We contrast the implications of the our model from that of Groseclose and Snyder (1996) and the rest of the literature in more detail in Section 4.6.

Our paper is also related to Dekel *et al.* (2008), henceforth DJW (2008), that models a general-election scenario rather than a legislative setting. The main difference is that, in the lobbying setting that we examine here, legislators care about how they cast their vote, whereas in DJW (2008) voters care only about the outcome. This change is more natural for the scenario of lobbying in a legislative setting compared to more general elections.[3] A second difference is in the focus here on the effect of budget constraints, which does not appear in DJW (2008).[4] These differences in setting lead to very different conclusions regarding the structure of equilibria. Finally, the vote-buying model itself

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2 See also the subsequent article by Banks (2000) that develops a finite version of that model.

3 Voters in a general election might also care significantly about how they cast their votes, which is, of course, suggested by the fact that people vote despite it being costly and their pivot probability being negligible. To the extent that the voting preferences are more important than preferences over outcomes, the present paper provides a more relevant model for general elections.

4 Budget constraints do not have the same impact in settings where voters care only about outcomes, and so their role is only interesting in this paper.

differs: in DJW (2008) we consider a uniform-offer model where the vote buyers cannot make different offers to different voters.[5]

A MODEL OF VOTE BUYING

Two lobbyists, X and Y, try to influence the voting of a legislature with an odd number, N, of legislators by directly buying votes of legislators prior to the vote. To simplify matters, we assume that vote buying is an ordinary transaction: the lobbyist gets full control of the vote in exchange for an up-front payment to the legislator.[6]

The Lobbying Game

The lobbyists alternate in making offers. Lobbyist k in its turn announces an up-front offer $p_i^k \geq 0$ to each legislator i for her vote. There is a small additional cost, $\gamma > 0$, incurred each round in which a lobbyist makes an offer. A fresh offer (or promise) made to a legislator cannot be lower than those previously made by the same lobbyist to the same legislator. It is not obvious whether this assumption is more or less realistic than the opposite assumption that allows offers to be withdrawn at the end of each round. However, our assumption circumvents technical difficulties familiar from the *Colonel Blotto* literature. The combination of sequential bidding and irreversible promises simplifies the analysis and allows us to incorporate heterogeneity of the voters and the lobbyists that would be very hard or even impossible to analyze if offers could be withdrawn arbitrarily.

The utility that legislator i gets from selling to lobbyist k at the price p_i^k is $p_i^k + V_i^k$, where the parameters V_i^k are the utility legislator i gets from voting for the outcome supported by lobbyist $k = X, Y$. Thus, if the bidding ends with prices p_i^k, it is a dominant strategy for legislator i to sell her vote to lobbyist X if and only if

$$p_i^X + V_i^X > p_i^Y + V_i^Y. \tag{1}$$

To simplify the discussion, we will assume from now on that the legislators play their dominant strategy. Thus, given the outstanding offers at any stage, for each legislator there is a unique lobbyist to whom that legislator would tender her vote if the process were to stop at that stage. Let I_t^k denote the set of legislators who would tender to lobbyist $k = X, Y$ if the process were to stop at the beginning of period t:

$$I_t^k = \{i : p_i^k + V_i^k > p_i^j + V_i^j\}.$$

The bidding ends at the beginning of period t with a win by k if both $|I_t^k| > |I_t^j|$ and $|I_{t-1}^k| > |I_{t-1}^j|$, i.e., if j passed up an opportunity to outbid k.

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[5] In situations where voting preferences do not matter, targeting specific voters is less consequential. In the legislative application, lobbyists have strong incentives to target certain legislators.

[6] In Section 4.5, we also consider the possibility of offering indirect promises to legislators that are only contingent on the outcome of the vote.

Once the bidding process ends, legislators simultaneously tender their votes to the lobbyists. The lobbyist who collects more than half the votes wins.

The lobbyists finance their payments out of budgets denoted B^X and B^Y. The total payments that lobbyist k would have to pay at any stage of the game, assuming that the game were to end at that stage, cannot exceed B^k. That is, at the beginning of every period t it has to be that $\sum_{i \in I_t^k} p_i^k + \gamma\tau^k(t) \leq B^k$, where $\tau^k(t)$ is the number of periods in which k has made an offer up to the beginning of t. It is important that at each stage the budget constraint has to hold only with respect to those obligations that are still relevant at that stage. If lobbyist k's up-front offer p_i^k has been outbid by the other lobbyist, so that at that point legislator i would sell her vote to the other lobbyist, then lobbyist k does not have to count this up-front offer against its budget.

Each lobbyist has a value W^k for winning. If the game ends in period $t < \infty$ then lobbyist k's payoff is $W^k - \sum_{i \in I_t^k} p_i^k - \gamma\tau^k(t)$ if k wins, and $-\sum_{i \in I_t^k} p_i^k - \gamma\tau^k(t)$ if k loses. The payoff is $-\infty$ if the game never ends.

Assuming that the legislators always follow their dominant strategy, the game we analyze is between the lobbyists. This is a game of perfect information. The lobbyists' budgets and valuations and the legislators' preferences are commonly known to the lobbyists. When a lobbyist makes offers, he or she observes the past offers and promises received by each legislator.

The lobbyists' strategies are defined in the obvious way — they specify how much more is offered to each legislator over past offers, contingent on the history of offers. The solution concept is subgame perfect equilibrium.

The focus is on the legislators' voting preferences rather than on their preferences over outcomes, since it is natural to assume that for re-election considerations legislators care a great deal about how they vote regardless of what the actual outcome is.[7] It is natural to think of the V_i^k's as being related to the preferences of i's constituency over the actual outcome. We will indeed make this connection later when discussing efficiency in Section 4.2.

Notice that, even if legislators have direct preferences over the outcomes, those would probably be of secondary importance as they would matter only when the legislator's vote is pivotal which might occur only with low probability. Furthermore, in a vote-buying scenario, pivot considerations are even less prominent than in other scenarios, as the vote buyers can effectively eliminate them by offering to buy slightly more than the minimal number of votes they need (see Dal-Bo 2007 for a discussion of this issue).

Further Assumptions and Notation

Let $V_i = V_i^X - V_i^Y$. The analysis that follows depends on the V_i^k's only through V_i and we will therefore represent the preferences in terms of V_i. We order the i's so that V_i is non-increasing and let m be the median legislator ($m = (N+1)/2$). Without loss of generality we assume $V_m > 0$, so that the median prefers to vote for X. Therefore,

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7 The related lobbying literature (Groseclose and Snyder 1996, Banks 2000) also assumes voting preferences.

without any vote-buying X would prevail. Let $n = \arg\max\{i : V_i > 0\}$, i.e., n has the weakest preference for X over Y from among all those who prefer X over Y.

There is a smallest money unit $\varepsilon > 0$. Both the offers and the budgets are whole multiples of ε. To avoid dealing with ties, which add nothing of interest to the analysis, we assume that the V_i's and W^k's are not whole multiples of ε.

Given a number z, let $\lceil z \rceil^{\varepsilon}$ denote the minimal multiple of ε greater than z, and $\lfloor z \rfloor_{\varepsilon}$ the maximal multiple of ε smaller than z. Assuming as above that each legislator votes for X (respectively Y) if and only if V_i plus the amount of money that legislator receives for that vote is strictly positive (respectively negative), then Y must spend at least $\bar{V} = \sum_{i=m}^{n} \lceil V_i \rceil^{\varepsilon}$ to obtain a majority. We assume that both B^Y and W^Y are greater than $\bar{V}$ as otherwise the solution is trivial.

In Figure 1, the solid step function is $\lceil V_i \rceil^{\varepsilon}$, the line crosses the axis at n, the long dashed vertical segment is at m, and the marked area is $\bar{V}$.

VOTE BUYING

The vote-buying game is a sort of a multi-unit auction with a special form of complementarity (only a bundle of more than half the units is valuable). It resembles an all-pay auction in that the loser may end up paying for some votes. But it is not a pure all-pay auction, since at most one lobbyist ends up paying for any given vote. If there were only

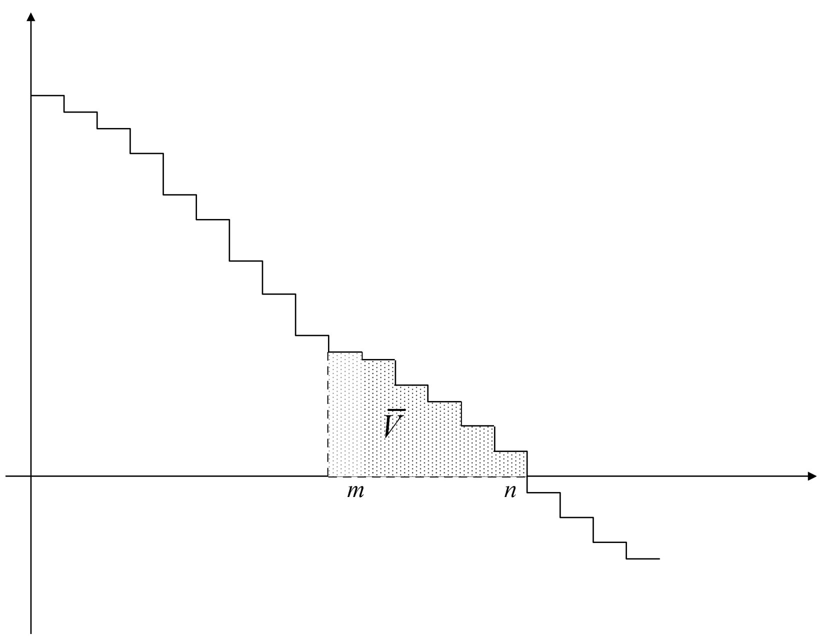

Figure 1. Preferences and parameters: $\lceil V_i \rceil^{\varepsilon}$, $\bar{V}$, m, n.

one legislator, then this would be a complete-information English auction (that allows jump-bidding).

We start with the following observation that applies to both constrained and unconstrained bidders.

Proposition 1 *Assume $\gamma > 0$. The vote-buying game has an equilibrium in pure strategies. In every equilibrium the same lobbyist wins, and the losing lobbyist never makes any offers.*

The existence of a unique winner when budgets are binding follows because this is a finite game of perfect information and ties are ruled out by assumption. In the unconstrained game, since offers are nondecreasing, they eventually reach a point where they must be greater than the value. While it is possible, in principle, that the bidder at that point expects to be outbid by the opponent and hence does not expect to pay that full amount, the fixed cost of making an offer ($\gamma > 0$) implies that such an offer will never be made. Thus the game is equivalent to a finite truncated version, and hence has a unique outcome. That the loser never makes offers also follows from the positive bidding cost γ.

Budget-Constrained Lobbyists

In this part, the budgets are the relevant constraints on the lobbyists. That is, $W^k > B^k$ for $k = X, Y$, and so each lobbyist is willing to spend up to the budget in order to win. The winner is determined by a combination of the relative strengths in terms of the budgets and the intensity of the legislators' voting preferences. Roughly speaking, Y wins if its budget advantage, $(B^Y - B^X)$, exceeds a measure of the preference advantage of X measured by *one half* of the *total* utility advantage of X over Y, i.e., $\sum_i V_i/2$. To understand why the utilities of all legislators matter, but only count half as much as the size of the budgets, it is useful to understand the structure of the winning strategies. The following example helps developing the intuition for this problem by pointing out that the natural least expensive majority, LEM, strategy, which secures the least expensive minimal majority at each stage, may not be optimal.

Example 1 *Optimal vs naive strategies — Why utility has a shadow price of* $1/2$.

There are three legislators with $V_1 = V_2 = 0.5$ and $V_3 = -30.5$. The grid size is $\varepsilon = 1$. Budgets are $B^X = 100$ and $B^Y = 80$.

Note that $B^X - B^Y = 20 < 29.5 = -\sum_i V_i$, so the total utility advantage for Y is greater than the absolute budget advantage of X. Nevertheless, as we show in Proposition 2, X should win, because X's budget exceeds Y's budget plus half of the total utility difference. That is, basically what matters is the budget advantage relative to one half the total preference advantage (setting aside small corrections that are explained in the proof of the result). Let us see how X should play to win.

Suppose that X follows the naive LEM strategy of always spending the least amount necessary to guarantee a majority at any stage. Suppose (just for the purpose of illustration) that at the first stage Y makes offers of 55 to legislator 1 and 25 to legislator 3.

The cheapest legislator for X to buy back is legislator 1 at a cost of 55. Assume Y now offers 55 to legislator 2. At this point X has 45 left in her budget, and cannot afford to buy back either legislator 2 or 3.

What was wrong with this strategy? The problem is that, while X bought the cheapest legislator in response to Y's offer, X also freed up a large amount of Y's budget for Y to spend elsewhere, while X's budget was committed. X needs to worry not only about what she herself is spending at any given stage, but also about how much of Y's budget is freed up. Effectively, freeing up a unit of Y's budget is *just* as bad for X as spending an extra unit of X's budget.

So, instead of following the naive LEM strategy of buying the cheapest legislators, let X always follow a strategy of measuring the *shadow price* of a legislator as the amount that X must spend *plus the amount of* Y*'s budget that is freed up*. If X had followed that strategy, then in response to Y's first stage offer above, X would have purchased legislator 3 at a price of 56. Then Y would have 25 free, and could only spend it on legislators 1 and 2. Regardless of how Y spends this budget, X can always buy legislator 2 at the next stage at a price of at most 25, against which Y has no winning response.

The example shows that keeping track of the shadow price is a good strategy. In fact, for large budgets, it guarantees a win for the winning candidate characterized in Proposition 2. Let us see how we get from this understanding of *shadow prices* to the expressions underlying Proposition 2.

Under the strategy suggested in the above example, X keeps track of the offer that X has to make to buy a legislator given the current offer of Y, plus the amount of Y's budget that is freed up. The amount that X has to offer to buy a given legislator i when Y has an offer of p_i^Y in place is $p_i^Y - V_i$. The amount of Y's budget that is freed up is p_i^Y. So the *shadow price* of buying legislator i is $2p_i^Y - V_i$. Dividing through by 2 gives us $p_i^Y - V_i/2$. In the proof this translates into the *strength* of Y being Y's budget less the sum of $V_i/2$ over legislators that prefer Y, X's *strength* being X's budget plus the sum of $V_i/2$ over those legislators that prefer X, and the winner being approximately the stronger lobbyist.

This is captured in Proposition 2, which includes some slight modifications to account for the grid size and some other details that are covered in the formal proof. The result requires that budgets be sufficiently large as specified next:

$$B^X > \left|\frac{mV_1}{2}\right| - \frac{\sum_{i=m+1}^{N} V_i}{2} - \frac{V_N}{2} + (2m+1)\,\varepsilon, \tag{2}$$

$$B^Y > \left|\frac{mV_N}{2}\right| + \frac{\sum_{i=1}^{m-1} V_i}{2} + \frac{V_1}{2} + (2m+1)\,\varepsilon. \tag{3}$$

Proposition 2 *If the budgets are large enough so that* (2) *and* (3) *are satisfied*, *then*, *for sufficiently small* γ, X *wins at no cost if*

$$B^X > B^Y - \sum_i \frac{V_i}{2} - \frac{V_N}{2} + (2m+1)\,\varepsilon \tag{4}$$

and Y wins at cost $\bar{V}$ paid to the legislators m (median) through n (almost-indifferent) if

$$B^Y > B^X + \sum_i \frac{V_i}{2} + \frac{V_1}{2} + (2m+1)\,\varepsilon. \tag{5}$$

The interesting feature is that, very roughly, increasing a legislator's preference for a given lobbyist by \$1 is equivalent, in terms of who wins, to increasing the budget of that lobbyist by \$0.5. Thus, money is worth much more to a lobbyist than having its bill being liked, as might be expected due to the use of funds being more flexible. Nevertheless, one of the implications of Proposition 2 is that a lobbyist with strong minority support can win despite having a lower budget than the opposition.

The result says that a lobbyist, say X, benefits from a dollar spent on buying votes twice as much as from effecting a change in a legislators' preferences that increases V_i (the relative preference for X) by \$1. How might this translate into lobbyist behavior? As noted in the introduction, it is natural to view legislator's preferences for voting as determined by the constituency preferences.[8] So the result is informative about the optimal investment by lobbyists (or the interest groups whom they represent) on influencing constituent's preferences. (This can be achieved by investing in local public goods, advertising, etc.). At the optimum the marginal dollar spent on influencing constituents should translate into the equivalent of \$2 spent directly on the legislator.[9]

Another interesting aspect (which contrasts with the unrestricted-budgets case, discussed subsequently) is that the outcome can depend on *all* the legislators' preferences. This dependence is not immediate as Proposition 2 is not a complete characterization. First, there is the measurement unit, ε; but clearly as $\varepsilon \to 0$ this effect disappears. Second, even with $\varepsilon = 0$ it is possible that neither Equation (4) nor Equation (5) holds. If $B^X \in \left[B^Y - \sum_i V_i/2 - V_1/2, B^Y - \sum_i V_i/2 - V_N/2\right]$, which can be nonempty since in general $V_1 > V_N$, then the result does not specify the winner. However, as V_1 and V_N become small relative to $\sum_i V_i$ then clearly the characterization becomes tighter. Moreover, given any preferences for the legislators and any budgets such that one position wins, then — if the budgets are large enough — one can change the preferences of *any* single legislator, *a fortiori* any subset of legislators, by enough to reverse the winner.[10] In this sense, *all* the preferences matter.

Note that if voting preferences are relatively unimportant, i.e., $\sum_i V_i$ is negligible relative to the budgets, then the comparison boils down to a comparison of the budgets. That is, the lobbyist with the highest budget wins. When this is the case, the optimal strategy simplifies to the strategy that seeks to obtain the least expensive majority at

8 It might also be that the legislator does not fully internalize constituents' preferences, in which case V_i is proportional to the constituents' valuations and the discussion below holds with an appropriate rescaling.

9 The model might suggest very little direct investment by interest groups in public opinion. Examining a richer dynamic and multi-agent model in which public opinions are relevant for more than one legislator and more than one vote, while payments to legislators apply to one legislator's vote, could lead to more significant investments in public opinion and in legislators.

10 The budgets must be large enough so that after the change Equations (2) and (3) must continue to be satisfied.

each point (LEM strategy), which is not optimal in general. A scenario with negligible voting preferences would arise, if legislators cared only about outcomes (and not how they vote) and the probability of being pivotal were negligible (as it would be in many plausible cases), since then the preferences over outcomes essentially do not affect the vote tendering considerations of the legislators.

As the proof makes clear, in fact only one large-budget condition is needed in each case. That is, X wins if Equations (2) and (4) hold, and Y wins if Equations (3) and (5) are satisfied. When budgets are not large enough (as given by (2) and (3)) the game becomes quite complex and the formula for determining the winner is involved. As we see little insight and great complication in such an analysis, we do not pursue it. The following example serves to show that an assumption of large enough budgets is necessary.

Example 2 *Large vs small budgets.*

Let $B^Y = 0$, $B^X = 30.2$, $\varepsilon = 0.1$, $N = 3$, $V_1 = -10$, $V_2 = -20$, and $V_3 = -30$. Here X can win by buying legislators 1 and 2 at prices of 10.1 and 20.1.

In this example

$$B^X + \frac{\sum_i V_i}{2} + \frac{V_1}{2} = -4.8 < B^Y - (2m+1)\varepsilon = -0.5,$$

and so if we applied the expressions from Proposition 2, we would mistakenly conclude that Y should win.

If we did not assume small costs of making offers (i.e., if $\gamma = 0$), then the characterization of the winning lobbyist would be unchanged. There could potentially be multiple equilibria which differ from one another with respect to the total payments made by the winner and the identities of their recipients. The loser would still make no payments in equilibrium, but by making bids that will be outbid by the winner, the loser could force the winner to spend more than the minimum sum necessary to obtain a majority in the absence of active opposition.

Unconstrained Lobbyists

We now analyze the case in which the budgets are not binding. The identity of the winner depends on the relative magnitudes of the lobbyists' valuations and the intensity of the voting preferences of the legislators whose index i falls between m (median) and n (weakest supporter of X). Recall that $\bar{V}$ is the sum that Y has to commit to the m through n legislators in order to outbid X in the first step in the least expensive way. Roughly speaking, Y wins at the cost $\bar{V}$ when Y's valuation, W^Y, exceeds W^X by a magnitude related to $\bar{V}$; since X enjoys a preference advantage, it wins at zero cost when $W^X > W^Y$; in the intermediate range in which W^Y exceeds W^X but is not sufficiently larger, the identity of the winner depends on who moves first.[11]

[11] The proof of the following result (in the appendix) is somewhat related to the proof of Proposition 3 in Dekel *et al.* (2007), henceforth DJW (2007), which studies single-object all-pay auctions, though the vote-buying game of the present paper is not a pure all-pay auction. That result in DJW in turn was preceded by a similar result due to Leininger (1991).

Proposition 3 *There exists* $\lambda \in \left[\lfloor W^X \rfloor_\varepsilon, \lfloor W^X \rfloor_\varepsilon + \bar{V}\right]$ *such that, for sufficiently small* $\gamma > 0$, *in any equilibrium*:

1. *If* $\left\lfloor W^Y \right\rfloor_\varepsilon > \lambda$, *then* Y *wins at cost* $\bar{V}$ *paid to the legislators* m (*median*) *through* n (*almost-indifferent*).
2. *If* $\left\lfloor W^Y \right\rfloor_\varepsilon < \left\lfloor W^X \right\rfloor_\varepsilon$ *then* X *wins at no cost.*
3. *If* $\left\lfloor W^Y \right\rfloor_\varepsilon \in \left(\left\lfloor W^X \right\rfloor_\varepsilon, \lambda\right)$ *then* Y *wins at cost* $\bar{V}$ *if it moves first*, *and* X *wins at possibly non-zero cost if it moves first.*

The cutoff level λ has the following meaning. Suppose that X moves first and commits the maximal sum that does not exceed its value, $\left\lfloor W^X \right\rfloor_\varepsilon$, in a manner that makes it as costly as possible for Y to obtain the majority. Then λ is the minimal sum that Y would have to commit to voters in order to obtain a majority. The precise characterization of λ in terms of the parameters of the model is provided in the proof.

DISCUSSION

Budget Constraints

At a first glance one might conjecture that the only difference between the scenarios with and without budget constraints is that in the constrained scenario the budgets play the same role that the valuations play in the unconstrained scenario. In some auction models this is indeed the case. However, it turns out that the outcomes of the two vote-buying scenarios with and without binding budget constraints are markedly different from one another. When the budget constraints are not binding only the preferences of the legislators whose index i falls between m (median) and n (weakest supporter of X) matter for the determination of the winner. These are the legislators whom Y must buy in order to outbid X in the least expensive way. In contrast, when budget constraints are the decisive element, the preferences of all the legislators can (as explained in Section 3.1) affect the outcome. The weight given to the preferences that matter also differ across these two cases. In the case of budget-constrained lobbyists, the preferences of the legislators enter with half the weight given to the budgets of the lobbyists. In the unconstrained case, the preferences of the legislators indexed m to n enter with same weight as the lobbyists' valuations.[12]

The important difference between budget constraints and valuations is that the former constitute hard constraints on the outstanding commitments while the latter can be exceeded despite it being unprofitable. In static scenarios, this distinction might not matter because bids in excess of the valuation are dominated. However, in a dynamic scenario in which past bids become sunk, the distinction between budgets and valuations might

[12] The reader might wonder if the one-half weight on legislator preferences results from the simple majority rule considered above, rather than primarily from the budget constraint as we argue. At the end of the proof of Proposition 2, we observe how the result would change if a supermajority were required, and show that the $V_i/2$ part of the calculation remains, multiplied by a factor that depends on the super-majority required.

become very meaningful for behavior off the equilibrium path. When the budget constraints bind, a central strategic consideration concerns how much budget is being freed up for the opponent. Therefore, the most effective strategy does not necessarily minimize the payments promised to legislators at each stage and the preferences of those who are not the least expensive to acquire also enter the calculations. When the budget constraints do not bind, this consideration is irrelevant, as past offers are essentially sunk costs and the most effective strategy entails acquisition of the least expensive votes at each stage.

Efficiency

In the absence of any mechanism for buying and selling votes, the outcome of voting will in general be inefficient. There is simply nothing to make legislators take into account the effect of their vote on others. A natural hypothesis is that allowing the lobbyists to compete for the votes will help align the outcome with overall societal values for the alternatives, presuming that the lobbyists' budgets represent the utility of some (possibly unmodeled) agents. Our analysis shows that this is not always so.

Under what circumstances will vote buying result in efficiency? If budgets are binding, then equilibrium will be (approximately) efficient if for some reason the budgets are proportional to the true surpluses of the agents in the society, and the legislators' voting preferences are too. That is, let $V^X = \sum_i \lceil V_i \rceil^\varepsilon$ and $V^Y = \sum_i \lceil -V_i \rceil^\varepsilon$, then the equilibria will be efficient if $B^X/V^X = B^Y/V^Y$, and V^X and V^Y represent the preferences of the legislators' constituents. If budgets are raised by a donation game with forward-looking donors who can anticipate the willingness to pay in favor of each alternative, then the game essentially becomes an all-pay auction one of raising donations, where one side begins with an initial advantage. This is a variation on the games studied in DJW (2007). While certain such games could lead to an efficient outcome, it is clear that the set of circumstances in which the outcome would necessarily maximize total societal utility are quite stringent.

The Role of Bidding Costs

The following example clarifies the role of the bidding cost γ in Proposition 3. With $\gamma = 0$ there are equilibria in which the higher value lobbyist, say Y, loses since, if Y tries to win, the other lobbyist, X, can make Y pay out substantial sums without X incurring any cost itself. This is accomplished by offers made by X that are later outbid by Y. Even though this does not directly benefit X in the subgame where X is eventually outbid, it makes it less attractive for Y to reach such a subgame and thus can change bidding behavior earlier in the game, and ultimately can even change who wins the game.

Example 3 *Bidding costs.*

$W^Y = 12.5$, $W^X = 9.5$, $\varepsilon = 1$, $N = 3$, $V_1 = V_2 = V_3 = 0.5$, $\gamma = 0$. Thus, $\bar{V} = 2$ and, if γ were positive, then by Proposition 3 Y would win at the cost 2. To see that with $\gamma = 0$ the situation might be different, suppose that X starts with $p_1^X = 9$ (the full offer is $p_1^X = 9, p_2^X = p_3^X = 0$ but for brevity here and hereafter we will often specify in each stage only the part of the outstanding offer that is being increased). We claim that there is an equilibrium in the ensuing subgame in which Y quits immediately, since it can win in the continuation only by paying more than 12.5. To construct such a continuation, observe that in any equilibrium continuation Y would never commit more than W^Y in one step. This is because the expected incremental sum of payoffs of X and Y from that point on would be negative which is inconsistent with any equilibrium continuation. Thus, Y responds to $p_1^X = 9$ with one of the following profiles of promises: (i) $p_1^Y = 10$, $p_2^Y = p_3^Y = 1$; (ii) $p_1^Y \geq 10, p_2^Y = 1$ or $2, p_3^Y = 0$ (or the same with the roles of 2 and 3 interchanged); and (iii) $p_1^Y = 0, p_2^Y \geq 1, p_3^Y \geq 1$ s.t. $p_2^Y + p_3^Y \leq 12$. The following is a SPE in the subgame following (i). X regains the majority with $p_2^X = 2, p_3^X = 2$, to which Y responds with $p_2^Y = p_3^Y = 3$ and X quits. If Y deviates to a cheaper offer like $p_2^Y = 3$, then on the path of the continuation X responds with $p_2^X = 9$ to which Y responds with $p_2^Y = 10$ and X quits. If, after $p_2^X = 9$, Y continued instead with $p_3^Y \in [2, 9]$, it would not save anything, since X would respond with $p_3^X = 9$ to which Y would respond with $p_2^Y = 10$ or $p_3^Y = 10$. Thus, if Y continues according to (i) and wins, it would end up spending more than W^Y. A SPE continuation after (ii) is essentially the same as in (i). That is, X responds with $p_2^X = 2, p_3^X = 2$ to which Y responds with $p_2^Y = p_3^Y = 3$ and X quits, etc. A SPE continuation following (iii) is as follows. Assuming that $p_2^Y \leq 9$, X responds with $p_2^X = 9$ (otherwise, $p_3^Y \leq 9$ and X would respond with $p_3^X = 9$) to which Y would respond with $p_2^Y = 10$. If at that point $p_2^Y + p_3^Y > 12.5$, then X would quit. If not, X would continue with $p_3^X = 9$, to which Y would respond with $p_3^Y = 10$ and X would quit. If after $p_2^X = 9$, Y continued instead with $p_3^Y \in [2, 9]$, it would not save anything, since X would respond with $p_3^X = 9$ to which Y would respond with $p_2^Y = 10$ or $p_3^Y = 10$. Thus, if Y continues according to (iii) and wins, again it would end up spending more than W^Y.

Notice that, at any point along the continuations described in the example, X behaves optimally, since it expects to be relieved from any commitments that it makes by subsequent promises by Y. This is why this construction requires $\gamma = 0$. With positive γ, X would not want to continue bidding when it is certain to lose, even if its commitments would be later annulled.

Unknown Preferences

Our analysis so far has assumed that legislators' voting preferences are known. This seems reasonable in the lobbying scenario. Nevertheless, it is worthwhile exploring the effect of lobbyists' uncertainty over legislators' voting preferences.

Suppose that, for all i, V_i is an independent draw from a continuous distribution F. We assume that F has a connected support and a continuous and positive density on its support, and is such that $z + F(z)/f(z)$ and $z + (F(z) - 1)/f(z)$ are both increasing on the support of F. There are many prominent distributions satisfying this, such as the uniform distribution. Let $\hat{V} = F^{-1}(0.5)$ be the median of the distribution F.

In this environment our results take the form of conditions under which, for sufficiently large legislatures, the winner is lobbyist k with high probability. The size of the legislature, N, needs to be large so we can apply the law of large numbers. Specifically, the proof is based on an argument that a lobbyist can obtain an expected share that is greater than $1/2$ if some conditions hold, and then by the law of large numbers for large N that almost surely guarantees winning. How large N has to be depends on the distribution and the probability with which one wants the conclusion to hold.

Because of the uncertainty and since we consider different sizes of legislatures we make the following changes in the model. First, we impose the constraint that the *expected* costs of lobbyists' offers must be within their budgets at each point in the game, assuming it ends at that point. Second, we assume the objective of the lobbyists is to maximize the probability of winning subject to the expected budget constraints. Finally, we state the conditions on who wins in terms of "per-legislator" budgets, denoted $b^k \equiv B^k/N$.

Proposition 4 *For any $\delta > 0$, there is $N(\delta)$ and $\bar{\varepsilon}$ such that for all $N > N(\delta)$ and all grids with $\varepsilon \in (0, \bar{\varepsilon})$ the following hold*:

- *If $b^Y > b^X + \hat{V}/2 + \delta$, then Y wins with probability of at least $1 - \delta$.*
- *If $b^X > b^Y - \hat{V}/2 + \delta$, then X wins with probability of at least $1 - \delta$.*

When δ is sufficiently small, the lobbyist who is likely to lose will not enter the bidding and the winning lobbyist will bid the minimum necessary to secure majority with sufficiently high probability. The reason for the minimum payment in equilibrium is clear. As in all other cases, the loser would like to avoid payment.[13,14]

Although the result is derived here only for the case of bounded budgets, the proof provides a key step for analyzing the case of unbounded budgets. Namely, it shows that the lobbyists should still use *flat* strategies that make the same offer to all legislators. Then the game resembles a simple dynamic all-pay auction. The analysis of such an auction would require significant work and go beyond the scope of this paper, but with the preliminary result we obtain it seems clear how to proceed to solve the unbounded budgets case.

Indirect Promises

Due to legal or ethical reasons or plainly because the voting is confidential, it might be the case that lobbyists cannot acquire legislators' votes directly or make payments contingent on how the legislator actually votes. Instead, a lobbyist can influence the voting only by making promises that will be fulfilled if and only if this lobbyist wins.

To model this, suppose that, in its turn to propose, lobbyist k promises legislator i a payment c_i^k (instead of the bribes p_i^k) that will be paid out if k wins, independently of

[13] In this case, the loser would avoid bidding even if $\gamma = 0$, since, owing to the uncertainty, it cannot be sure that all of its offers will be outbid.

[14] As explained in the remark following the proof of this proposition, the $1/2$ here results from the simple majority rule (and not from the shadow price of the budget constraint that led to a factor of $1/2$ on legislators' preferences in Proposition 2.

how i voted. Again the process ends if two rounds go by without a change in who would be the winner.

Since the winner must pay *all* the promises it made, at any point along the process, it has to be that $\sum_{i=1}^{N} c_i^k \leq B^k$. This is in contrast with the up-front buying scenario where the payment offered to i counts against k's budget only if i prefers to tender to k. The payoff to lobbyist k is $W^k - \sum_{i=1}^{N} c_i^k$ if k wins; 0 if k loses (and $-\infty$ if the game never ends). Thus, the winner honors its promises to all legislators regardless of how they cast their votes, while the loser is not making any payments.

Since they are not directly paid for their votes, they are assumed to vote according to their voting preferences V_i^k. Thus, legislator i votes for lobbyist X's proposal if and only if $V_i^X > V_i^Y$.

In the most compelling interpretation of this scenario, the lobbyist makes the promises to the constituency of legislator i. If, for example, the lobbyist can influence the structure of the bill being voted upon, the c_i^k's could represent *pork* to a given legislator's district. The V_i^k's are derived from the preferences of i's constituency over the actual outcomes including the promises (be it because the legislator cares about the constituency's benefit or because of reelection considerations). To formalize the connection between the promises and legislators' voting preferences, let U_i^k measure the benefit to i's constituency of lobbyist k's win. The simplest way to think about it is that all the voters in i's district share the same preferences over the outcomes. We assume that V_i^k is an increasing function of $c_i^k + U_i^k$. Thus, legislator i will support lobbyist X if and only if

$$c_i^X + U_i^X > c_i^Y + U_i^Y. \tag{6}$$

The above is, of course, just an interpretation. Alternatively, one may simply think of U_i^k as legislator i's personal utility of k's win and of the c_i^k's as promises that benefit i directly.

Other than the above, the game remains essentially as before. It is important to emphasize that the main difference is that here the legislator maintains control of the vote and payments are contingent only on the outcome, whereas in the up-front buying scenario considered before payments were contingent on the individual's vote but not on the outcome of the vote.

Let $U_i = U_i^X - U_i^Y$ and relabel legislators so that U_i is non-increasing in i. Under this labeling, let $m = (N+1)/2$, suppose (w.l.o.g) that $U_m > 0$ and let $n = |\{i : U_i > 0\}|$. Also assume that for all i and k, the values U_i and W^k are not multiples of ε. Recall that, given a number z, $\lceil z \rceil^{\varepsilon}$ is the smallest multiple of ε greater than z, and let $\overline{U} = \sum_{i=m}^{n} \lceil U_i \rceil^{\varepsilon} > 0$ be the minimal sum that Y has to promise to legislators in order to secure the support of a minimal majority, in case X does not promise anything.

The analysis is now the same as in the case where voters (legislators) care only about outcomes and not how they cast their vote. This is similar to the model in DJW (2008) where the case of non-binding budgets is considered. Further details of the game and proofs of the subsequent results are not provided here as they follow immediately from DJW (2008).

Result *There exists an equilibrium in the indirect-promises game. In any equilibrium*, Y *wins if and only if* $\min\left\{B^Y, \lfloor W^Y \rfloor_\varepsilon\right\} \geq \min\left\{B^X, \lfloor W^X \rfloor_\varepsilon\right\} + \overline{U}$.

The idea behind this result is easily explained. Lobbyist Y must spend at least $\overline{U}$ in order to secure a majority. After that, X could try to obtain some of these votes back (or others, if Y has overspent on these marginal votes), with the competition back and forth leading to the winner being the lobbyist with the largest budget (or willingness to pay) once an expense of $\overline{U}$ has been incurred by Y.

This game has many equilibria because the loser's behavior is not pinned down, as it is certain to lose and will not have to honor the promises it makes. The introduction of uncertainty over the other lobbyist's budget (or value) and of an equilibrium refinement singles out equilibria where the lobbyists in turn purchase a winning majority in the least expensive way possible, provided that their total commitment does not exceed their budget (or value). The identity of the winner would still be the same as above, but the total payment of the winner would be the smaller of the loser's budget and its value adjusted by the magnitude $\overline{U}$. That is, if Y wins then Y promises exactly $\min\left\{B^X, \lfloor W^X \rfloor_\varepsilon\right\} + \overline{U}$ and if X wins then X promises exactly $\max\left\{\min\left\{B^Y, \lfloor W^Y \rfloor_\varepsilon\right\} - \overline{U} + \varepsilon, 0\right\}$. Moreover, only *near-median* voters, between $\hat{m} = \{\min i : \lceil U_i \rceil^\varepsilon = \lceil U_m \rceil^\varepsilon\}$ and $\hat{n} = \{\min i : U_i > -\varepsilon\}$, receive positive payments.

To sum up, the lobbying competition with indirect promises has the flavor of an English auction. Focusing on the refined equilibria of the perturbed game, the winner ends up paying the second highest budget or value (adjusted by a measure of the preference advantage that one has over the other among the legislators). Only the intensity of the preferences of a group of near median legislators affect the outcome and only members of this group get promises in equilibrium. Notice that the significant sum that ends up being paid is in contrast to the minimal sums paid out by the winner in the upfront purchase scenarios.

Related Literature

The most closely related work is Groseclose and Snyder (1996) that models the lobbying of a legislature by a targeted offers game where each vote-buyer gets to move only once, and in sequence. A subsequent paper by Banks (2000) modifies their continuum voters model to one with a finite number of voters. The conclusions of Propositions 2 and 3 are quite different from theirs. Their model provides a significant second-mover advantage, which contrasts sharply with the open-ended sequential nature of our game. Specifically, in their game, in order to win, the first mover needs to be able to bid in such a way that it would be unprofitable for the second mover to buy any majority. In a game without an exogenously determined last mover, as the one we analyze, if one lobbyist is (temporarily) outbid for some legislator, it can remobilize those resources, which places lobbyists on a more equal footing. Also, owing to the single move that each lobbyist has in the Groseclose and Snyder model, the distinction between budgets and values has no importance in their model, while in our model budgets and values have rather different effects on the outcome. It is conceivable that in some scenarios a formal procedure indeed creates asymmetry of the sort on which the work of Groseclose and Snyder focuses. However, in many other situations, there is no such formal structure and the lobbying process resembles more a continuing bidding process like the one we model. Our analysis shows that this changes significantly the strategic interaction and the results.

It is worth emphasizing that the contrast of our results with those of Groseclose and Snyder (1996) provides an illustration of some of the empirical implications of our model and how they are distinct from others in the literature. The Groseclose–Snyder model leads to equilibria that either have a structure where the first lobbyist makes very substantial bids (more than the second lobbyist can afford), or where the first lobbyist drops out and the second wins with the minimal necessary bids. Such a model predicts quite substantial payments to be made in a non-trivial fraction of cases. In contrast, our model predicts that payments will generally be minimal, as the lobbyists need not pre-emptively outbid one another, given their ability to react to each other's bids and also to forecast the outcome of the equilibrium. At least in broad terms, our model's predictions are in line with the seemingly low amounts spent in various aspects of vote buying as pointed out by a number of authors, including Tullock (1972), Lamont and Thaler (2003), and Ansolabehere *et al.* (2003). Our model also differs in terms of the other testable hypothesis noted by Groseclose and Snyder (1996, p. 308): in contrast to their results, in our model, a lobbyist would not make an offer to a legislator who favors the lobbyist's position, and a lobbyist whose position has a definite majority without purchases would not make any offers. And of course, in our model, lobbyists would not obtain supermajorities. (The existence of supermajorities can be explained in other ways, as noted by Groseclose and Snyder (1996, p. 303) and one purpose of our model is to highlight that supermajorities due to competitive bidding would only result in cases where one lobbyist has a last-mover advantage, a period after which no one else can make offers.) Note also that despite the ability of lobbyists to bid multiple times, they will not do so in equilibrium. So the different nature of our model would not be directly evident in terms of bidders exercising their ability to bid multiple times, as that should not occur in equilibrium, even though that option has important consequences in terms of the predicted equilibrium structure.

Baron (2006) analyzes a game in which two competing lobbyists can make offers to legislators in repeated rounds. Given the difference in game structure and focus, his work and ours are complementary.

There are also related papers on lobbying that have roots in the common agency literature, such as Bernheim and Whinston (1986), Grossman and Helpman (1994), Dixit (1996), Le Breton and Salanie (2003), and Martimort and Semenov (2008), among others.[15] As such models generally look at a single voter (the politician or agent),

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[15] There are also studies of how the structure of the political system interacts with lobbying to determine policy choices, such as Helpman and Persson (2001) and Bennedsen and Feldmann (2002); but those works do not have head-to-head competition between lobbyists and they focus on variation in the political system, and so there is little relationship between those papers and the insights from our work. Similarly, there are papers by Lizzeri (1999) and Lizzeri and Persico (2001)

the complete-information solutions result in efficient outcomes (e.g., see Bernheim and Whinston 1986, Le Breton and Salanie 2003).[16] In particular, the politician as well as each lobbyist ends up being pivotal; because some lobbyist is making a payment that is not pivotal in swaying the politician, then they could lower their payment and not affect the outcome. This reinforces the idea that the inefficiencies that we uncovered are due to the fact that in many contexts at least some players end up not being pivotal in a vote-buying game when the vote is not by unanimity.

Buchanan and Tullock (1962) discuss the rationale for the prohibition of vote buying. They observe that under a unanimity voting rule, free trade in votes would lead to efficiency. They suggest, however, that this might not be the case when a simple majority rule is in force. They do not model the market for votes formally, but argue intuitively that a perfect market for votes would lead to efficiency, but that imperfections are likely to arise and might preclude efficiency. Our analysis can be seen as providing a particular formal interpretation to these ideas. Neeman (1999) points out that, with some uncertainty over legislators' behavior, pivot considerations are of marginal importance and hence vote buying (by a single buyer) need not result in efficiency.[17] Our own analysis of efficiency focuses on the next step — it inquires about the efficiency consequences of competition between vote buyers.

APPENDIX

Proposition 1 *The vote-buying game with up-front payments has an equilibrium in pure strategies. In every equilibrium, the same lobbyist wins, and the losing lobbyist never makes any offers.*

Proof of Proposition 1

The facts that the budget-constrained vote-buying game has an equilibrium in pure strategies follows from the fact that this is a finite game of perfect information, and hence we can find such an equilibrium via backwards induction.

The fact that in every equilibrium the same lobbyist wins, also follows from a backward induction argument. Each terminal node has a unique winner (as the V_i's are not a multiple of ε and so legislators are never indifferent), and lobbyists prefer to win regardless of the payments necessary. Thus, in any subgame, working by induction back from nodes whose successors are only terminal nodes, there is a unique winner. It then follows directly that the losing lobbyist never makes any offers, as they could otherwise deviate to offer nothing and guarantee no payment.

who study games where candidates make policy choices in addition to redistribution; these too are far from issues we study.

16 As such, the focus of many of these models has been on various distributional issues such as taxation and redistribution, or the politics of protectionism and international trade.

17 This and the point made by Buchanan and Tullock regarding efficiency of vote trading under unanimity are just alternative statements of the observation we made above that trading results in efficiency when every legislator is pivotal.

In the unconstrained game, each period the offer to each legislator has to weakly increase, and it must strictly increase for at least one i. Therefore, after lN periods the minimal offer made to some legislators is $(l+1)\,\varepsilon$, and eventually is greater than $\max_k W^k$. An offer greater than W^k is made only if k is certain that $j \neq k$ will outbid k, but in equilibrium it cannot be that both X and Y are certain they will be outbid by the other. So, in equilibrium both players quit in every period after some finite period, and hence the equilibrium is the same as if the game were truncated at any such period. Having reduced the game to a finite game we can complete argument as in the constrained case above. ■

Proposition 2 *If the budgets are large enough so that* $(A.4)$ *and* $(A.6)$ *are satisfied*, *then*, *if* γ *is small enough*, X *wins if*

$$B^X > B^Y - \sum_i \frac{V_i}{2} - \frac{V_N}{2} + (2m+1)\,\varepsilon \tag{A.1}$$

and Y *wins if*

$$B^Y > B^X + \sum_i \frac{V_i}{2} + \frac{V_1}{2} + (2m+1)\,\varepsilon. \tag{A.2}$$

Proof of Proposition 2

We prove the following result assuming $\gamma = 0$.

Lobbyist X has a strategy that guarantees winning at cost bounded by B^X if

$$B^X - B^Y \geq -\sum_i \frac{V_i}{2} - \frac{V_N}{2} + (2m+1)\,\varepsilon \tag{A.3}$$

and

$$B^X \geq \left|\frac{mV_1}{2}\right| - \frac{\sum_{i=m+1}^{N} V_i}{2} - \frac{V_N}{2} + (2m+1)\,\varepsilon \tag{A.4}$$

and lobbyist Y has a strategy that guarantees winning at cost bounded by B^Y if

$$B^X - B^Y \leq -\sum_i \frac{V_i}{2} - \frac{V_1}{2} - (2m+1)\,\varepsilon \tag{A.5}$$

and

$$B^Y \geq \left|\frac{mV_N}{2}\right| + \frac{\sum_{i=1}^{m-1} V_i}{2} + \frac{V_1}{2} + (2m+1)\,\varepsilon. \tag{A.6}$$

This immediately implies Proposition 2 because then for small enough γ when the inequalities are strictly satisfied the same strategies guarantee a win within the budget constraint.

Lobbyist X can guarantee a win using the strategy we describe next. Have X allocate offers as follows. Let t be the period. X will identify a set of legislators S_t to *buy* that has cardinality exactly m. X will make the minimal necessary offers to buy these votes.

To complete the proof we need only describe how X should select S_t, and then show that if X has followed this strategy in past periods, then X will have enough budget to cover the required payments regardless of the strategy of Y.

Let p_i^Y be the current offer that Y has to legislator i. Set this to 0 in the case where Y has never made a viable offer to the legislator, or in a case where X already has the best standing offer to the legislator. Similarly define p_i^X.

X selects to whom to make offers by looking for a set that minimizes the sum of what X has to offer in order to beat Y, plus the offers of Y that X thereby frees up. In particular, let S_t be the set of m legislators that minimizes $\sum_{i \in S_t} 2p_i^Y - V_i$. This is equivalent to choosing the m legislators that have the smallest values of

$$p_i^Y - \frac{V_i}{2}.$$

In the case where there are some i's that are tied under the above criterion, let X lexicographically favor legislators with lower indices. To complete the proof, we simply need to show that this strategy is within X's budget in every possible situation, presuming that X has followed this strategy up to time t.[18]

Notice that the cost of a legislator $i \in S_t$ to X is at most

$$\max\left\{\left\lceil p_i^Y - V_i \right\rceil^{\varepsilon}, 0\right\} + \varepsilon. \tag{A.7}$$

The expression $\max\{\lceil p_i^Y - V_i \rceil^{\varepsilon}, 0\}$ captures the fact that it could be that $p_i^Y < V_i$ in which case no offer is necessary.

The amount that must be offered to a legislator can only rise or stay constant over time, and so if some legislators were *purchased* by X in the past and have not been subsequently purchased by Y, then these legislators are still among the cheapest m available in the current period time and would still be selected under X's strategy (including the lexicographic tie-breaking).

Let i^* denote the most *expensive* $i \in S_t$ in terms of the *adjusted price* $p_i^Y - \frac{V_i}{2}$. If there are several such legislators, pick the one with the lowest index. So, $i^* \in \arg\max_{i \in S_t} \left\{p_i^Y - V_i/2\right\}$, and let $\overline{S}_t$ be the union of $\{i^*\}$ with the complement of S_t.

Given the algorithm followed by X, we know that

$$p_i^Y - \frac{V_i}{2} \le p_{i^*}^Y - \frac{V_{i^*}}{2}$$

for every $i \in S_t$. This can be rewritten as

$$p_i^Y \le p_{i^*}^Y - \frac{V_{i^*}}{2} + \frac{V_i}{2} \tag{A.8}$$

for each $i \in S_t$.

[18] This implies the proposition, as it means that either Y will not respond and the game will end with X the winner, or else X will get to move again and can again follow the same strategy. As the game must end in a finite number of periods, this implies that X must win.

Equations (A.7) and (A.8) imply that the amount required by X to follow this strategy at this stage is at most

$$\sum_{i \in S_t} \left\lceil p_{i^*}^Y - \frac{V_{i^*}}{2} - \frac{V_i}{2} \right\rceil^{\varepsilon} + m\varepsilon. \tag{A.9}$$

If we can get an upper bound on the expression $p_{i^*}^Y - V_{i^*}/2$, then we have an upper bound on how much X has to pay. So we want to maximize $p_{i^*}^Y - V_{i^*}/2$ subject to the following constraints:

(a) $p_i^Y - \frac{V_i}{2} \geq p_{i^*}^Y - \frac{V_{i^*}}{2}$ for every $i \notin S_t$,
(b) $p_i^Y \geq V_i + p_i^X$, and
(c) $\sum_{i \in \overline{S}_t} p_i^Y \leq B^Y$.

To get an upper bound, we ignore (b), and relax (c) by replacing B^Y with $\bar{B}^Y = \max\left\{B^Y, |mV_1/2| + \sum_{i=1}^m V_i/2\right\}$. The solution then involves spending all of $\bar{B}^Y$ in a manner that equalizes $p_i^Y - V_i/2$ with $p_{i^*}^Y - V_{i^*}/2$ for each $i \notin S_t$. (This is feasible due to the lower bound imposed on $\bar{B}^Y$; it is not necessarily feasible for B^Y, but still gives a bound.)

Thus, we end up with

$$p_i^Y = x^Y\left(\overline{S}_t\right) + \frac{V_i}{2},$$

for each $i \in \overline{S}_t$, where

$$x^Y(\overline{S}_t) = \frac{\bar{B}^Y - \sum_{i \in \overline{S}_t} V_i/2}{m} \tag{A.10}$$

From (A.9), for X's strategy to be feasible it is sufficient that

$$B^X \geq \sum_{i \in S_t} \left\lceil x^Y\left(\overline{S}_t\right) - \frac{V_i}{2} \right\rceil^{\varepsilon} + m\varepsilon.$$

Substituting for x^Y from (A.10), this becomes

$$B^X \geq \bar{B}^Y - \sum_i \frac{V_i}{2} - \frac{V_{i^*}}{2} + (2m+1)\,\varepsilon,$$

which has an upper bound when $i^* = N$, and which then yields the claimed expressions by substituting the definition of $\bar{B}^Y$. ■

Remark 1 *It is easy to see the effect in the proof above of requiring a supermajority of* $k > m$ *votes. Specifically the denominator in* $(A.10)$ *would be* $N - k + 1$, *and substituting this would give*

$$B^X \geq \frac{k}{N-k+1}\bar{B}^Y - \frac{k}{N-k+1}\sum_{i \in \bar{S}_t} \frac{V_i}{2} - \sum_{i \in S_t} \frac{V_i}{2} + (2m+1)\,\varepsilon$$

$$\geq \frac{k}{N-k+1}\bar{B}^Y - \frac{k}{N-k+1}\sum_{i\in k}^{N}\frac{V_i}{2} - \sum_{i=1}^{k-1}\frac{V_i}{2} - \frac{V_N}{2} + (2m+1)\,\varepsilon.$$

We see that now some of the V_i terms are multiplied by a factor greater than 1, *that depends on the majority needed*, *but the* 1/2 *factor remains as well.*

Proposition 3 *There exists value* $\lambda \in [\lfloor W^X \rfloor_\varepsilon, \lfloor W^X \rfloor_\varepsilon + \bar{V}]$ *such that*, *for sufficiently small* γ, *in any equilibrium*

1. *If* $\lfloor W^Y \rfloor_\varepsilon > \lambda$, *then* Y *wins at cost* $\bar{V}$ *paid to the legislators* m (*median*) *through* n (*almost-indifferent*).
2. *If* $\lfloor W^Y \rfloor_\varepsilon < \lfloor W^X \rfloor_\varepsilon$ *then* X *wins at no cost.*
3. *If* $\lfloor W^Y \rfloor_\varepsilon \in \left(\lfloor W^X \rfloor_\varepsilon, \lambda\right)$ *then* Y *wins at cost* $\bar{V}$ *if it moves first*, *and* X *wins at possibly nonzero cost if it moves first.*

Proof of Proposition 3

Define $\tilde{\imath}$ and $\tilde{z}$ as the solutions to $\sum_{i=\tilde{\imath}}^{n}\left(\tilde{z} - \lceil V_i \rceil^{\varepsilon}\right) = \lfloor W^X \rfloor_\varepsilon$ where $\tilde{z} \in \left[V_{\tilde{\imath}}, V_{\tilde{\imath}-1}\right)$ and where $V_0 = \infty$. Now, $d = \lfloor W^X \rfloor_\varepsilon - \sum_{i=\tilde{\imath}}^{n}\left(\lfloor \tilde{z} \rfloor_\varepsilon - \lceil V_i \rceil^{\varepsilon}\right)$, and let $\kappa = d/\varepsilon$, where by construction $0 \leq \kappa \leq n - \tilde{\imath}$. Let $\lambda \equiv \min\{n-m, n-\tilde{\imath}-\kappa\} \times \lfloor z \rfloor_\varepsilon + \max\{0, \tilde{\imath}+\kappa-m\}\lceil z \rceil^{\varepsilon}$. To understand this notation observe that if X initially offers $\tilde{z} - \lceil V_i \rceil^{\varepsilon}$ to all legislators in $[\tilde{\imath}, n]$ then X would exhaust the value of winning. Moreover, subject to not offering more than the value, these offers maximize $\tilde{z} \times m$, the amount that Y would need to obtain a majority. However, $\tilde{z} - \lceil V_i \rceil^{\varepsilon}$ is not a feasible offer as it is not a multiple of ε. If X offers only $\lfloor \tilde{z} \rfloor_\varepsilon - \lceil V_i \rceil^{\varepsilon}$ to those legislators then X would have left over an amount d. Therefore to d/ε of these legislators X could offer ε more, i.e., $\lceil z \rceil^{\varepsilon} - \lceil V_i \rceil^{\varepsilon}$, without exceeding his value of winning. Then the minimal cost to Y to obtain a majority would be exactly λ.

Consider any node at which k must offer an additional amount that is more than W^k to obtain a majority. At such a node k will make such an offer only if both lobbyists are certain $j \neq k$ will overbid, which j will do only if both are certain j will win, in which case k loses $\gamma > 0$ by making the offer instead of quitting. So at any node where k must offer at least W^k to obtain a majority, k will quit.

Now assume w.l.o.g. that $\lfloor W^k \rfloor_\varepsilon < \lfloor W^j \rfloor_\varepsilon$. We argue by induction that, at any node where k must spend a strictly positive amount to obtain a majority, k will quit. Assume the inductive hypothesis that k will quit at any node where the minimal offer needed to obtain a majority is $W^k - l\varepsilon$. Consider a node α at which k must spend $W^k - (l+1)\,\varepsilon$. If k makes such an offer, leading to node β, consider a response of j of mirroring k's last bid and adding ε to m of the offers, leading to node α' at which the minimal required for k to obtain a majority becomes $W^k - l\varepsilon$ and hence k will quit at α'. Thus at β the continuation equilibrium must be one at which j wins, and hence k's offer at α leads to an additional loss to k of at least γ. Hence k would prefer to quit at α.

Thus we have the following:

1. If $\lfloor W^X \rfloor_\varepsilon > \lfloor W^Y \rfloor_\varepsilon$ then Y will not make an initial move and X wins without making any offer.
2. If $\lfloor W^Y \rfloor_\varepsilon > \lfloor W^X \rfloor_\varepsilon$ and Y is first to move and Y makes an offer of $\bar{V}$ to obtain a majority then X quits and Y wins.
3. If $\lfloor W^Y \rfloor_\varepsilon > \lfloor W^X \rfloor_\varepsilon + \bar{V}$ and X is the first to move, and X makes any offer less than W^X then Y can reply (at cost below W^Y) by mirroring X's offer and adding $\bar{V}$. At that point X will quit since a positive amount is required for a majority. Hence X's opening offer was not optimal, and the only outcome is for X not to make an initial offer or to make an initial offer greater than W^X, which as already argued cannot be part of an equilibrium. Thus Y wins.
4. If $\lfloor W^X \rfloor_\varepsilon < \lfloor W^Y \rfloor_\varepsilon < \lfloor W^X \rfloor_\varepsilon + \bar{V}$ and X is the first to move, and can force Y to subsequently pay more than $\lfloor W^Y \rfloor_\varepsilon$ for a majority, and if X can do so at a cost less than W^X, then X will do so and win. When can this be done by X? Exactly when $\lfloor W^Y \rfloor_\varepsilon < \lambda$. Thus, if λ is greater than W^Y then X wins since, as argued above, Y must spend more than $\lfloor W^Y \rfloor_\varepsilon$ to obtain a majority after such an initial move by X and would prefer to quit. (The amount that X must spend to win will typically be less than W^X; we do not specify the exact amount as it is even more notationally cumbersome and not of great interest.) If Y moves first then after making an offer of $\bar{V}$ and thereby obtaining a majority we are in case 2 above. On the other hand if λ is less than W^Y then, whatever X does in the first move (so long as it is at a cost under W^X), Y can subsequently obtain a majority at a cost under W^Y whereupon X will need to spend a positive amount to obtain a majority while $W^Y > W^X$. Hence X will quit at this point, so that at any equilibrium X will not make any initial offer when $\lfloor W^Y \rfloor_\varepsilon > \lambda$.

This completes the proof of the proposition. ■

Proposition 4 *For any $\delta > 0$, there is $N(\delta)$ and $\bar{\varepsilon}$ such that for all $N > N(\delta)$ and all grids with $\varepsilon \in (0, \bar{\varepsilon})$ the following hold*:

- *If $b^Y > b^X + \hat{V}/2 + \delta$, then Y wins with probability of at least $1 - \delta$.*
- *If $b^X > b^Y - \hat{V}/2 + \delta$, then X wins with probability of at least $1 - \delta$.*

Proof of Proposition 4

Lemma 1 *Suppose that lobbyist Y offers a constant price x to all voters, such that $1 > F(x) > 0$. The least expensive way for lobbyist X to assure itself an expected share $\sigma \in [0, 1]$ of the vote would be offering a constant price to all voters. The same is also true if the roles are reversed.*

Note that we do not assume here that the constant price offered by X is a multiple of ε. If that constraint were added, then the cost to X of obtaining a share σ would be at least as high (and might involve a different strategy).

Proof of Lemma 1

The problem of finding bids p_i^X that lobbyist X can make to assure expected share σ at minimum cost is

$$\min_{\{p_i^X\}} \sum_i p_i^X[1 - F(x - p_i^X)] \quad \text{s.t.} \ \sum_i 1 - F(x - p_i^X) \geq N\sigma, \quad p_i^X \geq 0. \tag{A.11}$$

The first-order conditions to (A.11) can be written as

$$p_i^X f(x - p_i^X) + 1 - F(x - p_i^X) - \frac{\lambda}{N} f(x - p_i^X) - \mu_i = 0, \tag{A.12}$$

where λ and μ_i are nonnegative multipliers.

Given that the support of F is connected and f is positive on F's support, we have three possible ranges for solutions to (A.12): one where $f(x - p_i^X) = 0$ and $F(x - p_i^X) = 0$, one where $f(x - p_i^X) > 0$ and $0 < F(x - p_i^X) < 1$, and one where $f(x - p_i^X) = 0$ and $F(x - p_i^X) = 1$. The first-order conditions cannot be satisfied in the first case, unless $\mu_i = 1$ in which case the non-negativity constraint is binding and $p_i^X = 0$. However, by hypothesis, $0 < F(x - 0)$, which is a contradiction of the presumption of the case that $F(x - p_i^X) = 0$. In the third case, for $f(x - p_i^X) = 0$ and $F(x - p_i^X) = 1$ to hold, since $1 > F(x)$ it must be that $p_i^X < 0$. However, this cannot be a solution given the nonnegativity constraint. Thus, all possible solutions must fall in the second case. In the second case, in order to satisfy the first-order conditions, it must be that $p_i^X \leq \lambda/N$. [If $\mu_i = 0$ then this is clear since $(1 - F) > 0$. If $\mu_i > 0$, then the constraint that $p_i^X \geq 0$ must be binding, in which case $p_i^X = 0$ and again $p_i^X \leq \lambda/N$.] For this case, since $f(x - p_i^X) > 0$, we rewrite (A.12) as

$$x - p_i^X - \frac{1 - F(x - p_i^X)}{f(x - p_i^X)} - (x - \frac{\lambda}{N}) + \frac{\mu_i}{f(x - p_i^X)} = 0. \tag{A.13}$$

Suppose that there are two solutions, p_i^X and p_j^X to (A.13) in this range. Without loss of generality, letting $z^i = x - p_i^X > z^j = x - p_j^X$, we have

$$z^i - \frac{1 - F(z^i)}{f(z^i)} - \left(x - \frac{\lambda}{N}\right) + \frac{\mu_i}{f(z^i)} = 0 = z^j - \frac{1 - F(z^j)}{f(z^j)} - \left(x - \frac{\lambda}{N}\right) + \frac{\mu_j}{f(z^j)}.$$

Since $z - (1 - F(z))/f(z) = z + (F(z)) - 1)/f(z)$ is increasing (in this range where $f(z) > 0$), it follows that $0 = \mu_i < \mu_j$. (Note that μ_i takes on only two values.) But this implies $p_j^X = 0 < p_i^X$, which contradicts the fact that $z^i > z^j$.

Thus, we have shown that any solution to (A.11) necessarily has identical prices offered to all agents.

The proof for Lemma 1 with the roles reversed for the lobbyists has (A.11) replaced by

$$\min_{\{p_i^Y\}} \sum_i p_i^Y [F(p_i^Y - x)] \text{ s.t. } \sum_i F(p_i^Y - x) \geq N\sigma, p_i^X \geq 0,$$

with corresponding first-order conditions

$$p_i^Y f(p_i^Y - x) + F(p_i^Y - x) - \frac{\lambda}{N} f(p_i^Y - x) - \mu_i = 0.$$

Working through similar cases as those above, and this time using the fact that $z + F(z)/f(z)$ is increasing on the support of F, yields the same conclusion. ■

Lemma 2 *If* $(0.5 + \eta)N[B^X/(0.5 - \eta)N + F^{-1}(0.5 - \eta)] < B^Y$, *then* Y *can obtain expected share* $(0.5+\eta)$ *of the vote at each stage. Similarly, if* $(0.5+\eta)N[B^Y/(0.5 - \eta)N - F^{-1}(0.5 + \eta)] < B^X$, *then* X *can obtain a share of* $(0.5 + \eta)$ *at each stage.*

Proof of Lemma 2

We show the first claim, as the second is analogous. We show that there is a strategy that Y can follow through the whole game that guarantees at each stage an expected share of at least $(0.5 + \eta)$ of the vote. Suppose that it is Y's turn and throughout the game so far Y has always offered the identical prices across voters.

Assume that X's current offer is a constant price p_X and X's expected share is σ_X. If $\sigma_X \leq 0.5 - \eta$ then Y does not need to make any offer to obtain $\sigma_Y > 0.5 + \eta$. If $\sigma_X > 0.5 - \eta$ then $p_X \leq B^X / [(0.5 - \eta)\, N]$. This is because if more than share $0.5 - \eta$ in expectation sell to X at price $B^X / [(0.5 - \eta)\, N]$ then X's budget is exhausted, so no higher price for X is possible.

Now, if Y offers all voters the price $p_Y = B^X / [(0.5 - \eta)\, N] + F^{-1}(0.5 + \eta) \geq p_X + F^{-1}(0.5 + \eta)$ then Y will get an expected share of at least $(0.5 + \eta)$ of the vote. We must therefore verify whether such an offer is feasible for Y. For small enough η, $B^X(0.5 + \eta)/(0.5 - \eta) + (0.5+\eta)NF^{-1}(0.5+\eta) < B^Y$. Therefore the price p_Y is feasible for Y when Y's expected share is only slightly above $0.5+\eta$. If at p_Y more than expected share $0.5+\eta$ want to sell to Y then Y has an alternative offer p'_Y under which only $0.5+\eta$ want to seel to Y, and clearly p'_Y is feasible since $(0.5 + \eta)\, Np' < (0.5 + \eta)\, Np < B^Y$. If p'_Y is not a multiple of ε then for any ε small enough there is a p''_Y that is slightly larger than p'_Y that also gives Y an expected majority of $(0.5 + \eta)\, N$ which establishes the claim.

Now consider the case that X's current offer is not a constant price, say it is p^i_X for legislator i, and X's expected share is $\sigma_X > 0.5 - \eta$, and that Y offers all voters the price p''_Y. We know that against this offer of Y the best that X can do is p_X, and that then Y obtains an expected share of $(0.5 + \eta)$, hence p_Y against $\left(p^i_X\right)_{i=1}^N$ achieves at least this expected share. ■

We now show (1) and (2) of the proposition. We concentrate on (1), as the other case is analogous, given the lemmas above. For $\delta > 0$, there exists sufficiently small $\eta > 0$ such that $(0.5+\eta)N[B^X/[(0.5-\eta)N] + F^{-1}(0.5-\eta)] < B^X + N\hat{V}/2 + N\delta$. Therefore, if η is sufficiently small, $b^Y > b^X + \hat{V}/2 + \delta$ together with Lemma 2 imply that Y can obtain an expected share of $(0.5 + \eta)$. When N is made sufficiently large, an expected share of $(0.5 + \eta)$ means an arbitrarily large probability of winning. Therefore, there exists $N(\delta)$

such that, for $N > N(\delta)$, Y's winning probability is above $1 - \delta$. This complete the proof of Proposition 4. ■

Remark 2 *In this result changing the threshold from simple majority to a supermajority of proportion ρ for X would not change Lemma* 1, *and in Lemma* 2 *would require changing* $0.5 - \eta$ *to* $\rho - \eta$ *and changing* $0.5 + \eta$ *to* $1 - \rho + \eta$. *The overall statement of the first part of Proposition* 4 *would change to the following*: *If* $b^X > b^Y - F^{-1}(\rho)\rho + \delta$, *then X wins with probability of at least* $1 - \delta$. *Thus the factor of* $1/2$ *here is driven by the majority rule* (*in contrast to Proposition* 2).

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