How Monopsonistic is the (Danish) Labor Market?

Dale T. Mortensen*
Northwestern University
d-mortensen@northwestern.edu

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Abstract

An equilibrium model of search effort, recruiting effort, and wage determination is constructed for a labor market characterized by matching friction. In equilibrium, low paid workers invest more in finding a better paying job and more productive employers pay higher wage and engage is more recruiting activity in such an environment. In their study of job separation behavior, Christensen et al. (2001) confirm that workers flow from low to high paying jobs in Denmark as the theory suggests. In this paper, I show that distributional information on wages paid and firm size found in the same data set are consistent with the model's predictions about employer wage and recruiting policy. The results support the view that matching costs are substantial and that employers exercise considerable monopsony power as a consequence.

1 Introduction

Empirical evidence challenges the validity of the 'law of one price' in the labor market. Large industry and employer size differentials provide evidence for wage dispersion in the sense that observably identical workers are paid differently.[1] Two main explanations for wage dispersion are offered in the literature: Either employers pursue

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[1]Papers that provide empirical documentation for this kind of wage dispersion include Kruger and Summers (1988), Katz and Summers (1989), Davis and Haltiwanger (1991), Doms, Dunne and Troske (1997), Abowd, Kramarz and Margolis (1999, 2000a, 2000b) and Oi and Idson (1999).

different wage policies and/or high wage firms attract more able workers.[2] Recent empirical studies by Abowd and Kramerz (2000a, 2000b), based on the analysis of matched employer-worker data for both the U.S. and France, conclude that the two are roughly equally important as explanations for inter-industry differentials and that wage policy differences explain 70% of the size differential in both countries. In this paper, the empirical implications of employer heterogeneity in labor productivity and monopsony power arising as a consequence of search friction are explored. The model and cross firm wage and size distribution data for Denmark are used to answer the question: How monopsonistic is the (Danish) labor market?

Little is known about actual firm wage policies. This fact is surprising since the essential elements of a theory of wage policy have appeared in editions of Samuelson's principles of economics textbook since 1951. In the third edition, Samuelson writes:[3]

> **Wage policy of firms.** The fact that a firm of any size *must* have a wage policy is additional evidence of labor market imperfections....
>
> But just because competition is not 100 per cent perfect does not mean that it must be zero. The world ... is a blend of (1) competition and (2) some degree of monopoly power over the wage to be paid. If you try to set your wage too low will soon learn this. At first nothing much need happen; but eventually it will find its workers quitting a little more rapidly than would otherwise be the case. Recruitment of new people of the same quality will get harder and harder, and you will notice slackening off in the performance and productivity of those who remain on the job.
>
> Availability of labor supply does, therefore, affect the wage you set under realistic conditions of imperfect competition. If you are a very small firm you may even bargain and haggle with prospective workers so as to not pay more than you have to. But if you are of any size at all, you will name a wage for each type of job; then decide how many of the applicants will be taken on ... (1955, p. 541).

Subsequently, Phelps (1970) and Mortensen (1970) pursued Samuelson's insight in the "Phelps volume". They argued that employers face an

[2] Krueger and Summers (1987, 1988) emphasized the former explanation, while Murphy and Topel (1987) argued that unmeasured differences in individual ability are the principal explanation. Although work by Dickens and Katz (1987) and Gibbons and Katz (1991) attempted to resolve the debate, their efforts and those of others were hampered by lack of appropriate matched worker-employer data.

[3] I thank George Neumann for reminding me of this passage.

> inelastic labor supply as consequence of search friction because applicants are willing to accept low wage offers and cannot instantaneously move to higher paying employers. In this environment, a high paying employer profits by attracting and retaining a larger labor force. Still, a low paying employer can survive in spite of high turnover. In other words, every employer has the power to set its own wage even when many competitors populate the market.

The nature of the labor market equilibrium was not fully spelled out by either Mortensen or Phelps, a critique poignantly made by Rothschild (1973). Later, papers by Butters (1976), Burdett and Judd (1983) and Mortensen (1990) resolved the problem by formulating the model as a noncooperative price setting game. In addition, they demonstrated that dispersion in wage policy can be the only equilibrium outcome of imperfect wage competition induced even when all employers are identical. However, persistent cross-firm heterogeneity in productive efficiency is the more likely case and when present induces differences in wage policies that reflect differential labor productivity as Burdett and Mortensen (1998) and Bontemps et al. (2000) point out.

Does the empirical evidence support the hypothesis that monopsony power and cross employer productivity differences are a major source of wage dispersion? Providing some answers to this questions is the one purpose of this paper. The quantitative method used to accomplish this goal build on the analysis of Bontemps et al. (2000) and Christensen et al. (2001). The approach is in the spirit of the founders of econometrics: Given an equilibrium model of worker and job flows and employer wage policy, empirical inferences about the structure of the decision problem of each worker and employer in the model are obtained from quantitative information implicit in the observed cross firm distributions of wages offered, wages paid, and labor force size when these are interpreted as outcomes of the market equilibrium solution to the model. The observed market distributions are regarded as consistent if inferred behavior relationships can be interpreted as solutions to the decision problems that generate the distributions as equilibrium outcome of the model.

The paper extends the model structure used by Bontemps et al. (2000) in their analysis of French data and applies the general approach that they pioneered to similar Danish data. Given the same basic assumption that labor productivity varies across firms, added model features include endogenous search effort choice by workers and recruiting effort choice by employers. The data studied are cross firm observations drawn from the Danish Integrated Database for Labour Market Research (IDA) developed by Christensen et al. (2001) for their study of job separation flows. For all privately owned employing establishments in Denmark, the IDA observations of interest include the number of employees in November of 1994, the number of these who were still employed in the same firm the next November, the number of workers hired during the year by each firm and the prior employment status of each new hire, and the hourly wage paid each employee during the year. From these observation, cross-firm distributions of hourly wage offers, average hourly wages earned and labor force size are constructed.

The labor market model underlying the analysis includes the following components. A continuum of workers populate the supply side of the labor market. All are identical, live forever and act to maximize expected wealth. A continuum of expected wealth maximizing employers who are differentiated by the productivity of their technology represent the demand side of the market. At a point in time, each worker is either employed or not. A worker looks for a job paying an acceptable wage when unemployed and a higher wage when employed at an endogenously chosen search intensity taking as given the wage offer distribution. Similarly, employers with job vacancies seek workers by investing in recruiting effort. The total resources allocated to search and recruiting effort determine the rates at which workers and employers meet. The fact that this matching process is not instantaneous and costless is the source of friction in the model. Finally, each employer chooses her wage and recruiting policy optimally given the choices made by all other employers and workers. An equilibrium is a non-cooperative solution to the market game implicit in this description.

Given dispersion in employer wage offers, both the expected frequency with which a worker locates a higher paying job and the expected gain in future earnings obtained when one is found decrease with the wage currently earned by an employed worker. As a consequence, one expects search effort by employed workers to decrease with the wage earned. Christensen et al. (2001) find supporting evidence for these predictions in cross firm job-separations data found in the Danish IDA. Indeed, they are able to recover estimates of turnover parameters and the search effort function. These estimates imply that workers in lower paying jobs search hard and that search effort diminishes as the wage earned increases. Furthermore, their empirical model of job separations explains the observed market level relationship between the distribution of wage offers that workers face and the steady state distribution of wages earned by employed workers. This fact adds addition support for the hypothesis that workers flow from lower to higher paying jobs at rates that reflect individual rationality.

Because a more productive employer can expect to earn more profit per employee in the modeled economy, she has an incentive to acquire and retain a larger labor force. Offering a higher wage and making a larger recruiting effort are optimal means of doing so. Given the turnover parameter and search function estimates, these propositions can be tested using observable distribution information on wages offered, wages earned and firm size. As noted by Bontemps et al. (2000), one can test whether monopsony is a possible explanation for the wage dispersion observed in the data by solving the first order conditions for the optimal wage policy function. The argument proceeds as follow. If the distributions of wages offered and wages earned represent outcomes that are consistent with the equilibrium solution to the model sketched above, then the first order conditions for optimal wage choices across the support of the distribution of employer productivity implicitly defines a strictly increasing function relating wage and productivity.

The reader is warned at this point that the wage policy function derived in this way is an inference based on the model and the distributional information derived from the data; it is not a relationship actually observed. Never the less, the results are of some interest. First, the model can be rejected in the sense that the low wages observed, those earned by the lowest paid 3.5% of the privately employed labor force, are not profit maximizing. However, wages 100 DKK per hour can be interpreted as optimal for some firm in the sense that the inferred relationship between the wage offered and employer productivity is increasing over the remainder of the support of the distribution of wages offered. Second, over this range, the monopsony rent, defined as the difference between the value of productivity and the wage, is large and increasing. In other words, together the model and data suggest that employers in the Danish labor market possess considerable monopsony power.

The nature of the inferred relationship between hiring effort and productivity provides a second test to the model. Information contained in the distribution of wages earned and the firm side distribution can be used to infer a relationship between recruiting effort and employer productivity. This relationship is increasing as theory suggests. Furthermore, the implied marginal cost of hiring increases slowly over a wide initial range of hiring rates but eventually increases sharply as though an upper limit exists on the flow of workers that any one firm can hire. As a consequence, the profit of low productivity firms are relatively small but high productivity firms reap large monopsony profits.

Finally, the unobserved cross firm distribution of labor productivity is identified when both wage offers and recruiting effort increase with productivity. The Danish data and the model suggest that almost all firms have relatively low productivity but a few very high productivity employers are present in the market who employ a significant fraction of the workers. In other words, the cross firm distribution of productivity is highly left skewed with a long right tail. Given the model, this skew is required to accounts for similar shapes of both the observed distribution of firm size and the wage offer distributions. The theory and evidence presented in the paper suggests that high productivity outliers are among the highest paying, largest, and most profitable firms.

2 The Labor Market Model

2.1 Worker Search Behavior

In the modeled labor market, all worker are identical, live forever, and act to maximize expected wealth. Let the unit interval represent the (normalized) set of workers. Suppose that offers arrive at a Poisson frequency proportional to search effort. Specifically, let λs represent the offer arrival rate where s is search effort and λ is a contact parameter. The worker's maximal wealth value of a job match paying wage w solves

$$rW(w) = \max_{s \geq 0} \left\{ \begin{array}{c} w - c_w(s) + \delta \left(U - W(w) \right) \\ +\lambda s \int \max \langle W(x), W(w) \rangle - W(w)] dF(x) \end{array} \right\} \tag{1}$$

where $c_w(s)$ denotes the cost of search effort, δ is the exogenous job destruction rate, and U represents the value of unemployment. The parameter r represent the discount rate and $F(w)$ denotes the fraction of vacancies that offer wage w or less.

Equation (1) is a continuous time Bellman equation that reflects the optimal search effort and separation choices that the worker makes while employed and the fact that a transition to unemployment may occur. Specifically, the worker quits to take an outside employment option if and only if it offers a higher value than the current job, the optimal search effort choice maximizes the difference between the expected net gain in value attributable to search on the jobs, and the worker expected future income values to the value of unemployment U when the worker transits to unemployment as a consequence of the destruction of the match. Similarly, the value of unemployment solves

$$rU = \max_{s \geq 0} \left\{ b + \lambda s \int \left[\max \langle W(w), U \rangle - U \right] dF(w) - c_w(s) \right\} \tag{2}$$

where b an income flow received when unemployed.

The solution to (1) is increasing in w. Indeed, because

$$W'(w) = \frac{1}{r + \delta + \lambda s(w)[1 - F(w)]} > 0$$

by the envelope theorem, an employed worker quits to take an alternative job offer if and only if it pays a higher wage. Hence, the rate at which workers quit an employer paying w to take a job elsewhere is $\lambda s(w)[1-F(w)]$ where optimal search effort is

$$s(w)=\arg\max_{s\geq 0}\left\{\lambda s\int_{w}^{\overline{w}}\left[W(x)-W(w)\right]dF(x)-c_w(s)\right\}.$$

Integration by parts yields the following representation of the first order condition for an optimal choice:

$$\begin{aligned} c_w'(s(w)) &= \lambda\int_{w}^{\overline{w}}\left[W(x)-W(w)\right]dF(x) \\ &= \lambda\int_{w}^{\overline{w}}[W'(x)[1-F(x)]dx \\ &= \lambda\int_{w}^{\overline{w}}\frac{[1-F(x)]dx}{r+\delta+\lambda s(x)[1-F(x)]}. \end{aligned} \tag{3}$$

Because the likelihood of finding a better job declines with the wage earned, search intensity declines with an employed workers current wage, i.e., $s'(w)<0$.

When unemployed, the typical worker accepts an offer only if it is no smaller than the reservation wage, denoted as R. As the reservation wage equates the value of employment to the value of unemployed search, denoted by U, acceptance requires that $w\geq R$ where

$$U=W(R). \tag{4}$$

As a consequence, equations (1) and (2) together imply that the search intensity of an unemployed worker is the same as that of a worker employed at the reservation wage provided that the reservation wage is the unemployment benefit,

$$s_0=s(R), \tag{5}$$

and the reservation wage is equal to the unemployment benefit,

$$R=b. \tag{6}$$

2.2 Steady State Conditions

Given that workers flow from lower to higher paying employers, the cumulative distribution of wages offered, denoted $F(w)$, and the cumulative distribution of wages paid to employed workers, represented as $G(w)$ in the sequel, are closely linked by steady state flow conditions. Because all equilibrium wage offers are acceptable, unemployed workers find employment at rate $\lambda s(R)$. Given this fact, the steady state non-employment rate is

$$u = \frac{\delta}{\delta + \lambda s(R)} \tag{7}$$

where δ represents the exogenous job destruction rate.

Because workers employed at wage w quit at rate $\lambda s(w)[1 - F(w)]$ given endogenous search effort, the balance between flows into and out of the set of workers who are employed at wage w or less, required to define the steady state distribution of wages across employed workers, is

$$\delta G(w) + \lambda[1 - F(w)] \int_{\underline{w}}^{w} s(x) dG(x) = \frac{\lambda s(R) F(w) u}{(1 - u)} = \delta F(w) \tag{8}$$

where the last equality follows from equation (7). The first term on the left side accounts for the fraction of employment who are in the wage category who are laid off per period and the second term is the fraction who quit to take higher paying jobs while the right side is the flow into the category from unemployment all expressed as a fraction of employment. In sum, given the exogenous job destruction rate δ, the contact frequency parameter λ, the search effort function $s(w)$, and the wage offer distribution function $F(w)$, the steady state wage cdf $G(w)$ is the unique solution to the functional equation (8).[4]

The horizontal difference between the distribution of wages earned, $G(w)$, and the distribution of wage offers, $F(w)$, represents an *employment effect*. According to the on-the-job search theory outlined above, the difference is positive because an employed worker is more likely to have found a higher paying job during her current employment spell while a worker hired from non-employment earns a random draw from the wage offer distribution. As the steady state condition relating the wage and offer distribution, equation (8), can be written as

$$\frac{F(w) - G(w)}{1 - F(w)} = \frac{\lambda}{\delta} \int_{\underline{w}}^{w} s(x) dG(x) \tag{9}$$

[4]By differenting both sides of (8) with respect to w and then substituting from the equation to eliminate the integral, one obtains a first order differential equation in $G(w)$. The solution of interest is the particular solution associated with the initial condition $G(\underline{w}) = 0$.

the theory implies that the horizontal differences between the wage offer and wage earned distribution functions depend on the extent of friction in the market as reflected in the ratio of the offer arrival rate to the job separation rate and the average search effort of the workers earning no more than w.

2.3 Wage and Recruiting Policy

Let p represent the value of output per worker employed in a firm of type p and let $\Gamma(p)$ denote the measure of employers with productivity no greater than p. All workers are perfect substitutes, search with intensity $s(w)$ when employed at wage w, and find any wage not less than the reservation $R = b$ acceptable when unemployed as established in the previous section. For the moment, let the triple (p, w, v) characterize an employer where w represents per period wage paid and v denotes recruiting effort per period as reflected in say the number of workers contacted per period.

A specification of the relationship between recruiting effort and a firm's hire flow requires that one account for the endogenous search effort choices of the workers. Here we assumed that the probability that the particular worker is contacted by an employer is proportional to that worker's relative search effort. Because any unemployed worker will accept a wage no less than R but an employed worker accepts only if currently paid less than the wage offered, the unconditional probability that a randomly contacted worker will accept is

$$h(w) = \begin{cases} \frac{us(R)+(1-u)\int_{\underline{w}}^{w} s(z)dG(z)}{us(R)+(1-u)\int_{\underline{w}}^{\overline{w}} s(z)dG(z)} \text{ if } w \geq R \\ = 0 \text{ otherwiese} \end{cases} . \tag{10}$$

The sampling assumption underlying (10) captures the following idea. Employers contact worker by advertising in newspapers, on line, and through employment agencies. A particular worker is contacted at relative frequency that depends on the worker's awareness as reflected in the time and effort that the worker spends searching. All unemployed workers accept offers above their common reservations wage and employed worker accept only if the wage offer exceeds that currently earned.

An employer's expected profit flow per worker contacted is the product of the hire probability per worker contacted and the value of filling a job vacancy, i.e.,

$$\pi_p(w) = h(w)J_p(w)$$

where $J_p(w)$ denotes the expected present value of the future profit flow generated by a job-worker match given the employer's productivity p and wage w. Because an employed worker quits when aware of a higher outside offer, the employer's value of a continuing match solves the Bellman equation

$$rJ_p(w) = p - w - \delta J_p(w) - \lambda s(w)[1 - F(w)]J_p(w).$$

Equivalently,

$$J_p(w) = \frac{p - w}{r + d(w)}$$

where

$$d(w) = \delta + \lambda s(w)[1 - F(w)] \tag{11}$$

represents the job separation rate for an employer paying wage w. Hence, expected profit per worker contacted can be written as

$$\pi_p(w) = \begin{cases} \frac{h(w)(p-w)}{r+d(w)} \text{ if } w \geq R \\ 0 \text{ otherwise} \end{cases}. \tag{12}$$

The optimal wage and recruiting policy maximizes the employer's total expected profit flow, i.e.,

$$(w(p), v(p)) = \arg \max_{(w,v) \geq 0} \{\pi_p(w)v - c_f(v)\} \tag{13}$$

where v represents the number of workers contacted and $c_f(v)$ is the cost of recruiting. As

$$w(p) = \arg \max_w \pi_p(w) \tag{14}$$

follows as a corollary and the derivative $\pi'_p(w)$ is increasing in p and decreasing in w at an optimal choice of the wage, the optimal wage paid $w(p)$ is increasing in p. The optimal number of workers contacted, $v(p)$, equate the marginal cost and expected profit per contact, i.e.,

$$c'_f(v(p)) = \max_w \pi_p(w). \tag{15}$$

Hence, recruiting effort as reflected in $v(p)$ also increases with p by the second order condition, $c''_f(v) < 0$, and the envelope theorem. In short, more productive employers pay more and contact more workers.

Because more productive employers offer a higher wage and invest more in recruiting effort more productive employers both attract workers more quickly and retain workers more easily. A increasing relationship between size and productivity is a consequence. Formally, an employer's average labor force size over then long run, that which equates worker flows into and out of the firm, is

$$n(p) = \frac{h(w(p))v(p)}{d(w(p))}. \tag{16}$$

2.4 Labor Market Equilibrium

As already noted, the search effort behavior of the workers determine the steady state relationship between the distribution of wages offered and wages earned represented by equation (8). The employer wage and recruiting policy functions determine the wage offer distribution and the contact or offer frequency per worker. Specifically, the total number of employer contacts made per worker is

$$\lambda = \int_{\underline{p}}^{\overline{p}} v(x) d\Gamma(x) \tag{17}$$

and the fraction of these that offer a wage no greater than $w(p)$ can be expressed as

$$F(w(p)) = \frac{\int_{\underline{p}}^{p} v(x) d\Gamma(x)}{\int_{\underline{p}}^{\overline{p}} v(x) d\Gamma(x)} = \frac{\int_{\underline{p}}^{p} v(x) d\Gamma(x)}{\lambda}. \tag{18}$$

In other words, the steady state and these aggregation conditions imply that the search strategy $s : [\underline{w}, \overline{w}] \rightarrow \Re_+$, wage policy $w : [\underline{p}, \overline{p}] \rightarrow \Re_+$, and recruiting strategy $v : [\underline{p}, \overline{p}] \rightarrow \Re_+$ functions determine the contact frequency λ, the cumulative distribution of wages offered $F : [\underline{w}, \overline{w}] \rightarrow \Re_+$ and the cumulative distribution of wages earned $G : [\underline{w}, \overline{w}] \rightarrow \Re_+$ In turn, equations (3), (14), and (15) imply the optimal strategies of the market participants depend only on the agents' expectations regarding these market outcomes. A *rational expectations labor market equilibrium*, then, is a fixed point of the map implicitly defined by these relationships from the space of search, wage and recruiting strategy functions to itself. An existence proof is presented in the Appendix.

3 Empirical Inference

In this section, information about the distributions of wages offered, wages earned, and labor force size across employers drawn from the Danish Integrated Database for

Labor Market Research (IDA) is used to test the plausibility of the model's implications. Following the method suggested and applied by Bontemps et al. (2000), I ask whether these distributions could have been generated as equilibrium market outcomes. Formally, the test is whether the necessary second conditions for the optimal choice of search effort, wage and recruiting effort are satisfied.

3.1 The Data

The distributional information used here is from data drawn from the IDA by Christensen et al. (2001) for their study of on-the-job search behavior as an explanation for observed differences in worker separation rates across employers. For the 113,525 privately owned firms, the observations of interest include an hourly wage, w, defined as the average hourly wage paid to all employees during the year beginning in November 1994 expressed in Danish Crowns (DKK), the number of workers hired by the firm who were not previously employed during the year, and the size of the firm's labor force, n, in that month. The observed wage cdf, $G(w)$, is the fraction of workers employed by a firm paying an average hourly wage of w or less. An unbiased estimate of the observed wage offer cdf, $F(w)$, is the fraction of workers hired from non-employment by all firms paying average wage w or less.

The constructed wage and offer cumulative distribution functions for the complete sample are illustrated in Figure 1. As search on-the-job theory implies, the wage distribution, denoted as G in the figure, stochastically dominates the offer distribution, represented by F. In other words, the positive employment effect characterized by equation (9) uniformly exists on the common support of the wage and offer distributions. The wage density function, $g(w) = G'(w)$, is illustrated in Figure 2. A log normal density function with the same mean and variance is plotted along with the observed wage density in the figure. The close fit provides evidence that cross-employer wage dispersion as reflected in the distribution of the average wage paid is well approximated by the log normal.

Finally, the distribution of firm size as characterized by its relative frequency distribution is illustrated in Figure 3. Note that almost all firms are quite small. Indeed, 26.5% employ a single wage earner and 62% have 4 or fewer. Only, 9% of the firms are composed of more than 20 employees and only 3% have more than 50 workers. Still, there are a few very large firms. The obvious skew in the distribution is reflected in the fact that the median size lies between 2 and 3 employees while the average is 13.2.

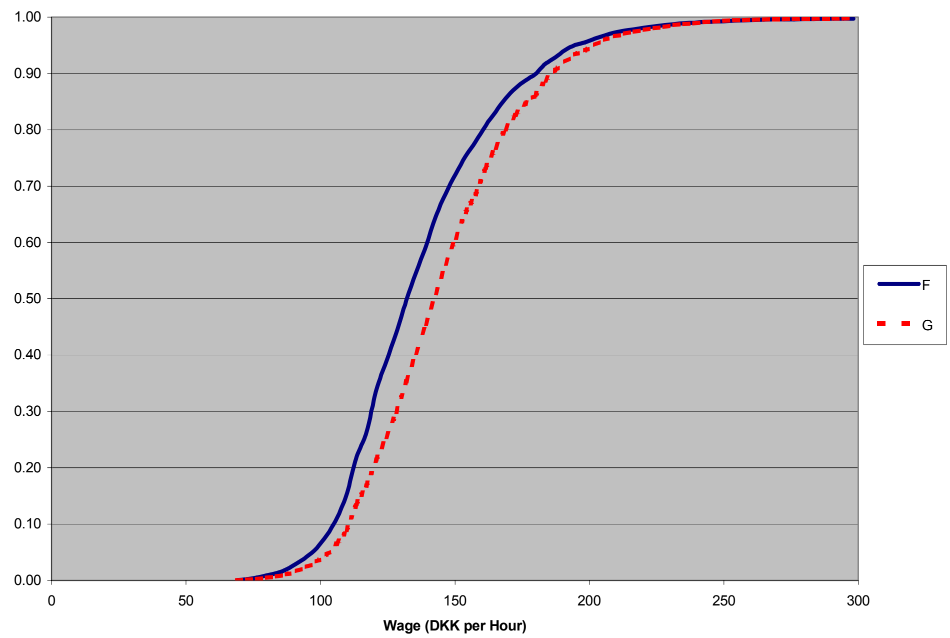

Figure 1: Offer (F) and Wage (G) Cumulative Distributions

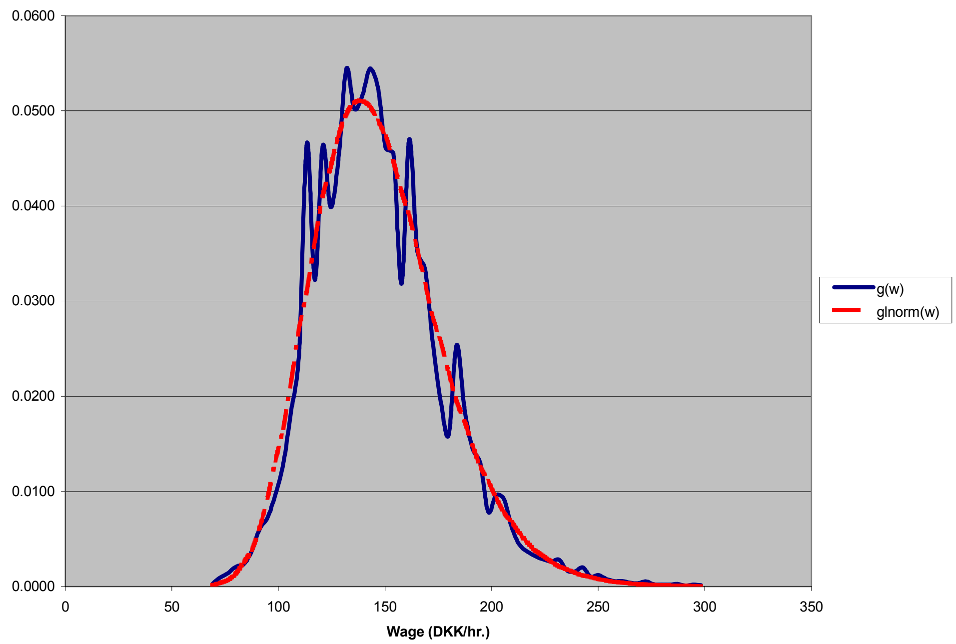

Figure 2: Actual $g(w)$ and Log Normal $g \ln orm(w)$ Wage Densities

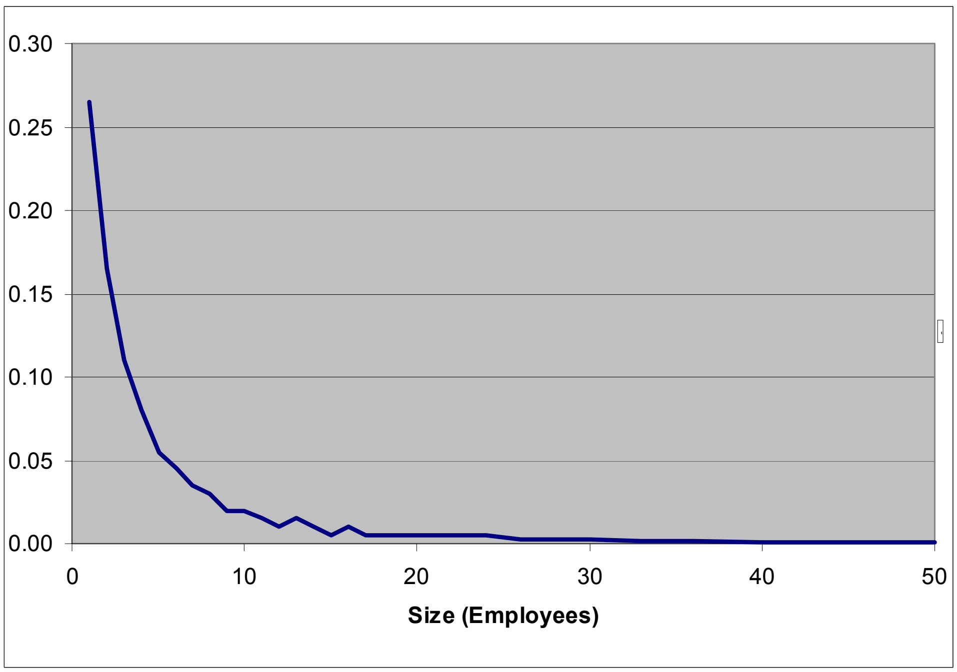

Figure 3: Firm Size Density

3.2 The Search Effort Strategy

Christensen et al. (2001) use observations on worker separations at the firm level to estimate the arrival rate and job destruction parameters, λ and δ respectively, and the search effort function $s(w)$ implied by a power function specification of the cost of search. Specifically, they assume power a cost of search effort function of the form

$$c_w(s) = \frac{c_0 s^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}. \tag{19}$$

In this case, the first order condition, equation (3), can be rewritten as

$$\lambda s(w) = \left(\frac{\lambda^2}{c_0} \int_w^{\overline{w}} \frac{[1-F(x)]dx}{r+\delta+\lambda s(x)[1-F(x)]} \right)^{\gamma}. \tag{20}$$

In other words, for any vector of parameters $(c_0, \delta, \lambda, \gamma)$, the product of the arrival rate and the optimal search effort function, $\lambda s(w)$, is the unique solution to this functional equation.

Estimates of the parameters are obtained by finding those values that maximize the likelihood of the observed number of separations experienced by each firm during the year beginning in November 1994. Since the duration of an job spell in a firm paying wage w is exponential with parameter equal to the separation rate $d(w) = \delta + \lambda s(w)[1 - F(w)]$, the number of workers who stay with the firm is binomially distributed with "sample size" equal to firm size n and "probability of success" equal to $e^{-d(w)}$. Under the assumption that the parameters are identical across firms, the maximum likelihood estimates conditional on the interest rate r and offer distribution F are

$$(c_0, \delta, \lambda, \gamma) = \arg\max \sum_i \left[\ln \begin{pmatrix} n_i \\ x_i \end{pmatrix} - d(w_i)x_i + (n_i - x_i)\ln(1 - e^{-d(w_i)}) \right] \tag{21}$$

where $d(w)$ is the function specified in equation (11), w_i represents the wage, n_i the size, and x_i the number of stayers for firm i. Since c_0 and λ^2 are not separately identified, search effort at the lowest wage $s(\underline{w})$ is normalized to equal unity. Given the offer cdf $F(w)$ observed in the data and an annual interest rate r equal to 4.9% per annum, the estimates of the remaining parameters are $\delta = 0.2872, \lambda = 0.5933$ and $\gamma = 1.1054$.[5] (Because the sample size is very large, the precision of the estimates is virtually certain to the third significant digit.)

[5]These estimates are slightly different from those reported in Christiansen et al. (2001) because they were obtained using a different bin width for the construction of the wage and offer cdfs, F and G. Otherwise, the estimates where obtained using the same procedure.

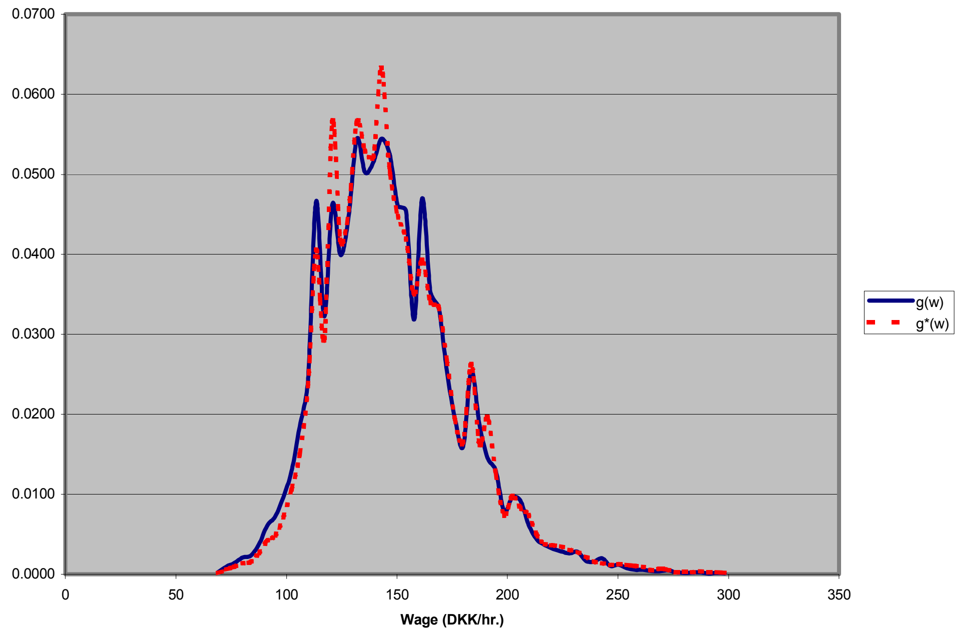

Figure 4: Actual $g(w)$ and Steady State $g^*(w)$ Wage Densities

The arrival rate and job destruction rate parameter estimates reflect relatively high turnover in the Danish labor market. The implied average duration of a job spell in the highest paying job, equal to $1/\delta$, is about 3.5 years while expected duration in the lowest paying job is only $1/(\delta+\lambda) = 1.15$ years. Given the cost function specification in equation (19), the fact that the estimate of the elasticity of search effort with respect to the return to search, γ, is close to unity suggests that the cost of search function is reasonably approximated by a quadratic.

Since the actual wage distribution observed in the data is not used in the estimation, its availability provides an out of sample test of how well the theory explains the employment effect represented in equation (9). In Figure 4, the actual wage density, $g(w)$, and that predicted by the estimates and the steady state condition, denoted $g^*(w)$, are plotted together. As one can see, the differences are generally small. Although the heights are not perfectly replicated at the spikes, irregularities in the actual offer density seem to be echoed in the inferred wage density. In sum, the observed market offer and wage distributions are consistent with the on-the-job search model estimated from firm level turnover data.

3.3 The Wage Policy Function

In the language of Bontemps et al. (2000), observed wage and offer distributions are said to be *admissible* only if the first order condition for an optimal wage choice, equation (14), implicitly defines an increasing relationship between wage and productivity. In other words, they are admissible only if they can be rationalized as equilibrium outcomes of the monopsony wage determination model. In this section, I show that the distributions are consistent with the theory in this sense.

Equations (13) and (14) imply that the first order condition for an interior optimal wage choice can be written as

$$\frac{\pi_p'(w)}{\pi_p(w)} = \frac{h'(w)}{h(w)} - \frac{d'(w)}{r + d(w)} - \frac{1}{p - w} = 0.$$

The necessary second order condition requires that the solution to the equation, the wage policy function $w(p)$, increases with p over the support of the productivity distribution. Since the inverse of the wage policy function, can be written as

$$p(w) = w\left(1 + \frac{1}{\frac{wh'(w)}{h(w)} - \frac{wd'(w)}{r+d(w)}}\right) \tag{22}$$

one can simply check whether it is monotone increasing over the support of the wage distribution.

The results reported below are based on the following computations. The separation function $d(w)$ is computed using equations (11) where δ, λ and $s(w)$ are the Christensen et al. estimates, the wage distribution $G(w)$ is the log normal approximation to the wage cdf. Substituting the log normal for the actual wage distribution has no effect on the results except to smooth some of the inferred relationships. The curve of the estimated separation rate function is illustrated in Figure 5. The fact that the elasticity is quite large in the upper half of the wage support but then tends to zero as the wage tends to the upper support are the most interesting features of the curve. These characteristics reflect the fact that search effort while employs falls and approaches zero well before the upper support is reached with the wage earned.

The acceptance probability function $h(w)$ can be computed using equation (10) and the offer cdf $F(w)$ derived from the steady state condition (8) and the log normal approximation to $G(w)$ after assigning a value to the unemployment rate u. Under a strict interpretation of the theory, the steady state condition for unemployment, equation (7), and the separation parameters determine the unemployment rate. However, the estimates $\delta = 0.2872$ and $\lambda = 0.5933$, and the normalization $s(\underline{w}) = 1$

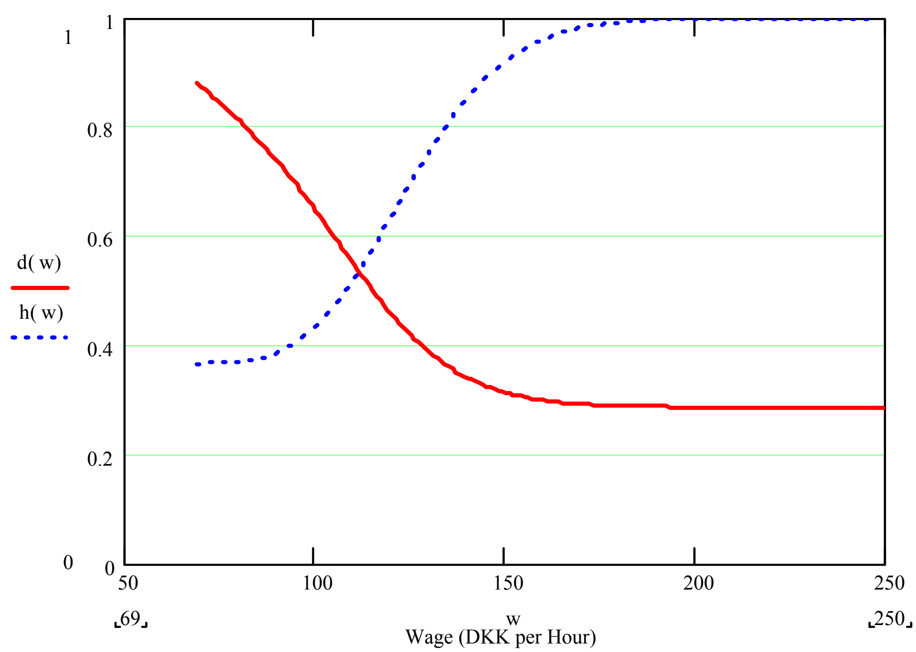

Figure 5: Separation $d(w)$ and Acceptance $h(w)$ Rates

imply a steady state unemployment rate equal to 32.6%, far larger than the 12.2% unemployment rate recorded for Denmark in the relevant year, 1995.

The Christensen et al. estimate of δ seems to be excessive as an estimate of the annual transition rate from employment to unemployment. For example, Rosholm and Svarer (2000) derive an estimate equal to 0.099 per annum using panel data on Danish worker labor market event histories for the 1980s. One possible explanation for the difference is that the Christensen et al. estimate should be interpreted as the intercept of the separation function rather than as the employment to unemployment transition rate, especially since the worker's destination state is not used in the estimation procedure. Indeed, this reasoning suggests that $\delta = \delta_0 + \delta_1$ where $\delta_0 = 0.099$ represents the transition rate to unemployment and δ_1 is that part of the job-to-job transition rate which is not related to wage differences.

Interestingly, the steady state condition (8) continues to hold given the alternative interpretation of δ. To prove the assertion, let $\delta = \delta_0 + \delta_1$ where δ_0 is regarded as the rate of transition from employment to non-employment and δ_1 is the intercept of the job-to-job transition rate function. Under the assumption that workers who move between jobs for non-wage reasons earn a random wage offer and all wage offers are acceptable, the flow of workers to jobs that pay w or less is

$$s(R)\lambda F(w)u + \delta_1 F(w)(1-u),$$

where the first term is the inflow from non-employment and the second term is the inflow from employment. Equating the inflow to the outflow yields an equation equivalent to (12)

$$\begin{aligned} & \delta G(w) + \lambda[1 - F(w)] \int_{\underline{w}}^{w} s(x)dG(x) \\ = & \frac{s(R)\lambda F(w)u + \delta_1 F(w)(1-u)}{1-u} \\ = & (\delta_0 + \delta_1)F(w) = \delta F(w), \end{aligned}$$

because the steady state unemployment rate now solves

$$u = \frac{\delta_0}{\delta_0 + \lambda s(R)}. \tag{23}$$

Under the alternative interpretation, the implied steady state unemployed fraction is $.099/(.099 + .5933) = 0.143$ given $s(R) = s(\underline{w})$, a number which is near the 12.2% unemployment rate actually experienced in the relevant year. A possible reason for the remaining difference between the two is that the lowest wage paid $\underline{w}$ actually exceed the reservation wage R in which case equation (3) implies that the search intensity of an employed worker $s(R)$ exceeds that of the same worker when employed at the lowest wage, $s(\underline{w})$. Since the inferences drawn in the sequel are essentially the same for either unemployed fraction, I choose to use the actual unemployed fraction in the relevant year, $u = 0.122$, in equation (10). The associated acceptance probability function $h(w)$ is illustrated in Figure 5. Note that the probability is also highly elastic at low wages but tends to unity in the upper ranges of the support of the wage distribution.

Finally, the curve of the wage function, $w(p)$, implied by equation (22) and the computed separation and acceptance rate functions is represented in Figure 6. The curve itself is obtained by computing $p(w)$ for a series of wage values in the support of the wage distribution and then by plotting each wage rate against the associated computed value of $p(w)$. As clearly illustrated in the figure, the wage does increase with productivity for all rates about 100 DKK per hour. The fact that the curve for wage rates below that critical value has a negative slope implies that the any wage on the segment minimizes rather than maximizes expected profit per worker contacted. Contrary to the model, none of the observed wage rates in this region of the support can be profit maximizing. As only about 3.5% of all employed workers earn a wage in this region, it is possible that this inconsistency can be attributed to sampling and measurement error. For the sake of argument, we continue the analysis as though the lower support of the offer and wage distributions were at 100 DKK rather than 70 DKK per hour.

Over the range for which the inferred wage policy function is admissible, the wage increases rapidly with productivity initially but then falls dramatically. Monopsony rents, as measured by profit per hour divided by the hourly wage, $(p-w)/w$, are large even at relatively low wage rates For example, at the lowest admissible wage, $w = 100$ DKK per hour, this measure of monopsony rent is 50%. At the median wage earned, equal to 144 DKK per hour, the ratio equal roughly 80% of the wage. Finally, at the 90th percentile, $w = 186$ DKK, $p = 2,720$ DKK per hour. In this case, monopsony rent is more than 13 times the wage! Although this last inference is hardly plausible, the results generally suggest that firms have considerable monopsony power.

The reason that monopsony power is large and increases with the wage offered is reflected in equation (22) and the shapes of $h(w)$ and $d(w)$ illustrated in Figure 5. The first order condition for an optimal choice, equation (22), implies that the monopsony rent measure $(p - w)/w$ is approximately equal to the sum of the elasticities of the separation rate function and the acceptance probability function. Both $h(w)$ and $d(w)$ are relatively elastic at low wages but converge to constants as the wage tends

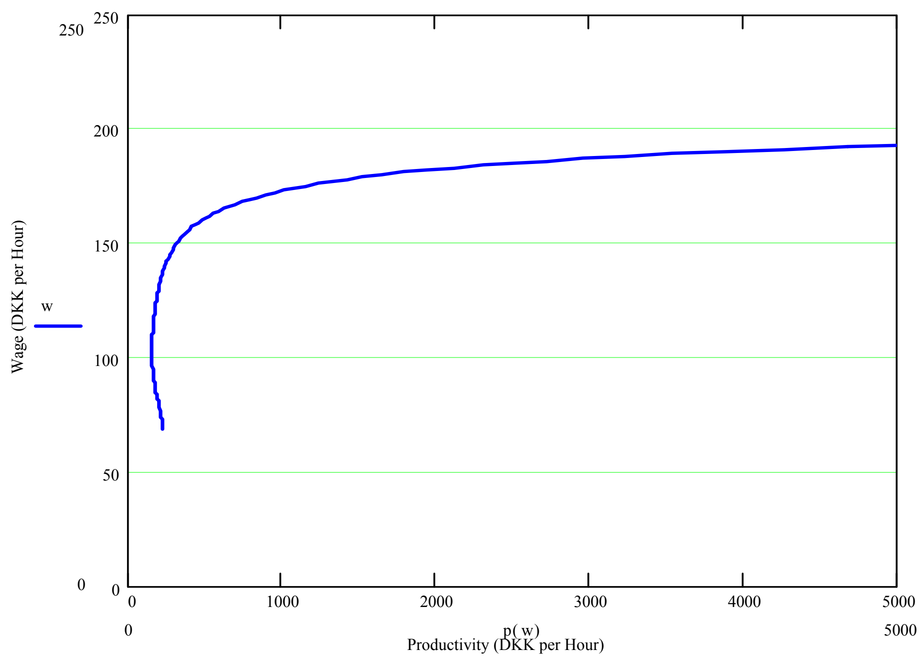

Figure 6: Inferred Wage Policy Function $w(p)$

to the upper support. In other words, a small wage differential in the lower reaches of the support has a substantial impact on an employer's ability to both attract and retain workers but the same differential in the upper reaches has little effect on either. Clearly, the reason for the differences in the response of the separation rate function $d(w)$ at different wage levels is that well paid workers have less incentive to invest in search than do low paid workers. For the same reason, an employer is more likely to contact a lower than a higher paid worker given the assumption that contact rates reflect search intensity. This fact plus the concentration of employment in relatively low paying jobs combine to explain the observed variation in the elasticity of the acceptance probability $h(w)$.

3.4 The Recruiting Policy Function

To make inferences about recruiting effort, one can use information contained in the firm size distribution illustrated in Figure 3. Given that firm size, $n(p)$, is monotone increasing in firm productivity from equation (16), it follows that the fraction of workers employed at wage no greater than $w = w(p)$ is the fraction of workers employed by firms with productivity no greater than p. Formally,

$$G(w(p)) = \frac{\int_{\underline{p}}^{p} n(x)d\Gamma(x)}{\int_{\underline{p}}^{\overline{p}} n(x)d\Gamma(x)}. \tag{24}$$

For the same reason, the measure of firms of size no greater than $n(p)$ is equal to the measure with productivity no greater than p, i.e.,

$$Q(n(p)) = \Gamma(p) = \int_{\underline{p}}^{p} d\Gamma(x). \tag{25}$$

By first differentiating these two equations with respect to p and then using the result to eliminate $\Gamma'(p)$, one obtains the ordinary following differential equation in $n(p)$

$$G'(w(p))w'(p) = \frac{n(p)\Gamma'(p)}{En} = \frac{n(p)Q'(n(p))n'(p)}{En} \tag{26}$$

where

$$En = \int_{\underline{p}}^{\overline{p}} n(x)\Gamma'(x)dx = \int_{n(\underline{p})}^{n(\overline{p})} nQ'(n)dn. \tag{27}$$

is the mean of the size distribution. Equivalently,

$$N'(w) = \frac{EnG'(w)}{N(w)Q'(N(w))} \tag{28}$$

where by definition

$$N(w) \equiv n(p(w)) \tag{29}$$

is firm size expressed as a function of the wage paid rather than productivity.

As all the terms on the right side of (28) are observable on the support of G, the function $N(w)$ can be computed as the particular solution to the ODE associated with the initial condition $N(\underline{w}) = 1$, the lower support of the observed firm size distribution. The result of the calculation, which is necessarily increasing, is plotted against inferred productivity $p(w)$ over the relevant range of wage rates in Figure 7. Initially, firm size $n(p)$ increases at an increasing rate with productivity but then reverses its curvature and flattens out at high levels. It is of interest to note that this relationship together with the observed size and wage distributions used to compute it imply that about half of the employed work for the largest 1% of the firms, those with 118 or more employees.

The solution to equation (16) for the number of workers contacted by an employer of productivity p yields

$$v(p) = \frac{n(p)d(w(p))}{h(w(p))}. \tag{30}$$

Given $v(p)$ is an optimal choice in the sense defined by (13), the second order necessary condition implies that $v(p)$ must be increasing as noted in the theory section of the paper. This implication can be tested by computing the associated function $v(p(w)) = N(w)d(w)/h(w)$ using the components of the right side already derived above. Because $d(w)$ is decreasing and $h(w)$ in increasing in w, the necessary condition need not hold in the data even though $N(w)$ is increasing.

In fact, the wage and size distributions are consistent with recruiting effort choice theory in the sense that $v(p(w))$ is increasing throughout the wage support. However, it is even more informative to plot the expected profit per employed worker against the associated annual rate at which workers are hired. The steady state hire flow is

$$H(w) = v(p(w))h(w) = d(w)N(w) \tag{31}$$

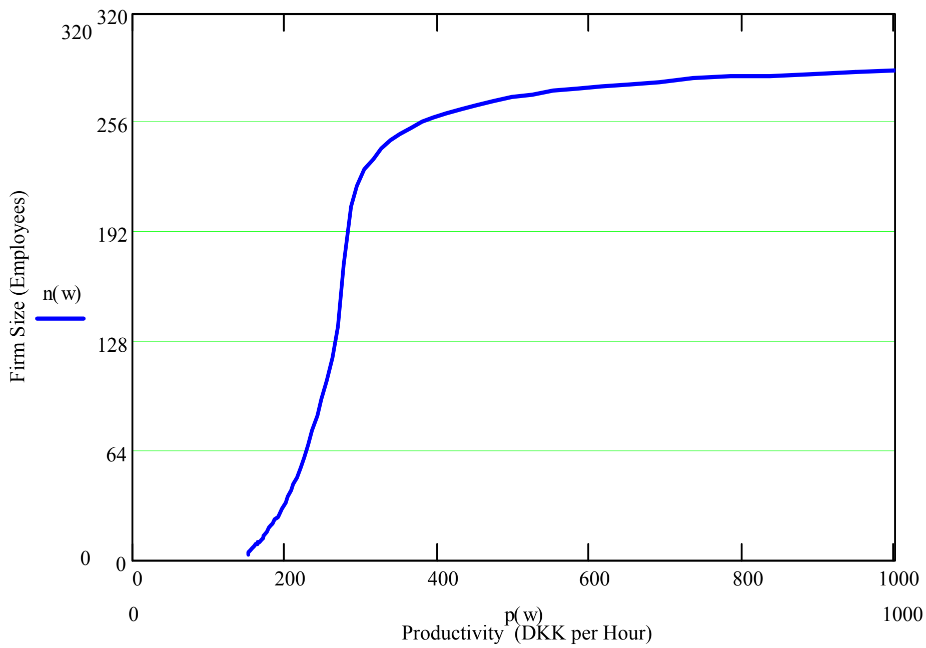

Figure 7: Firm Size vs Productivity $n(p)$

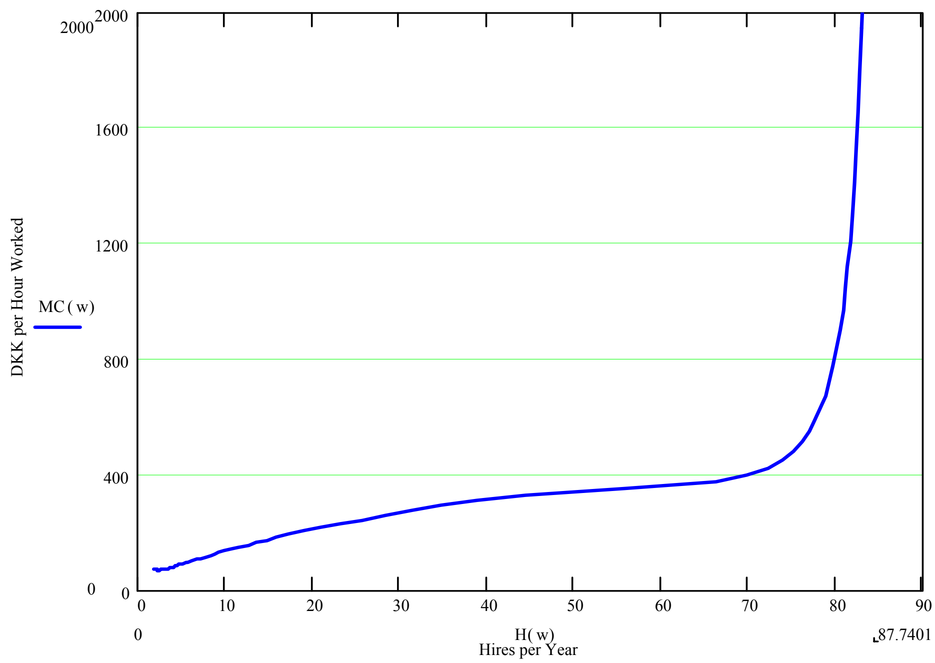

Figure 8: Marginal Hiring Cost

while the cost of hiring the marginal worker,

$$MC(w) = \frac{c_f'(v(p(w))}{h(w)} = \frac{p(w) - w}{r + d(w)}, \tag{32}$$

is equal to the expected profit from the first order condition equation (15).

To compute the right side of the marginal cost equation on an annual basis, one must know the number of hours worked per annum. However, as this parameter is simply a scale factor which does not effect the shape of the inferred marginal cost curve, a plot of the marginal cost per hour worked per year against the vacancy rate is equally informative. Such is the plot illustrated in Figure 8.

Over the lower half of the wage support, the hire rate increases slowly from 1.8 workers per year at $w = 100$ DKK per hour, to 39 workers per year at the median wage $w = 144$ DKK per hour. Over this range, the marginal cost of hiring a worker increases from 72 DKK per annual hour worked to 310 DKK. In other word, the cost of hiring the marginal worker increases from 70% of the worker's annual earning at a hire rate of 1.8 workers per year to 215% of annual earnings at a hire rate of 39 worker per year. Beyond a hire rate of about 80 workers per year, which is the inferred optimal rate for a firm paying at the 70th percentile of the wage distribution $w = 160$ DKK per hour, marginal cost jumps dramatically. Indeed, the inferred marginal cost curve suggests that an effective upper bound exist on the flow of worker that any firm can hire in a year equal to about 85 workers per year.

The marginal hiring costs inferred by the model and the IDA data are large. Consequently, a substantial fraction of monopsony rent is actually dissipated by recruiting investment, particularly for those operating on the flat segment of the marginal cost of hiring curve. By equation (13), net profit earned by any firm is represented in Figure 8 as the area of the rectangle defined by any particular hire rate H, the origin, and the corresponding point on the marginal cost of hiring curve in the figure minus the area under the marginal cost curve. Hence, net profit is less than half the product of expected profit and the contact rate πv up to a hire rate of about 75 workers per year. However, for firms hiring more than 75 workers per year, monopsony profit increases rapidly with the hire rate as a consequence of the rapid rise in the marginal hiring costs. These firms account for about 42% of employment.

3.5 The Distribution of Employer Productivity

Finally, equation (28) also provides the means to compute the distribution of productivity across employers implied by the model and the observed wage and firm size distributions. The implied density function evaluated at any wage w,

$$\Gamma'(p(w)) = \gamma(p(w)) = \frac{q(N(w))N'(w)}{p'(w)},$$

where $q(n) = Q'(n)$ is the size density, is plotted in Figure 9. The most obvious characteristic of the fact that its mode is equal to its lower support and it has a long right tail. Computation imply that the productivity of 99% of the firms ranges between 150 DKK per hour and 260 DKK per hour. However, only half of the worker force is employed by these firms. Among the remaining 1%, the inference is that there are a very few highly productive, large and well paying firms.

4 Conclusions

As a consequence of friction, search theory implies that unemployed workers accept any offer above some reservation wage and then seek a higher paying job once employed at an intensity that reflects expected potential gain in future income. Given

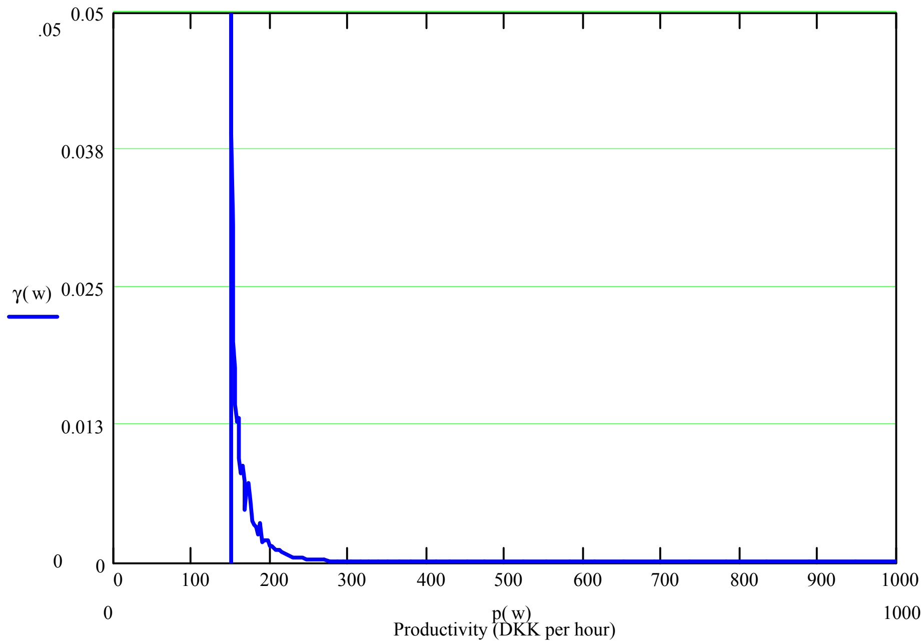

Figure 9: Productivity Density $\gamma(p)$

this strategy, employers with different labor productivity coexist in the market although the more productive employers find it optimal to acquire and retain larger worker forces by offering a higher wage and by investing more in recruiting effort. I find that Danish evidence on the labor turnover at the firm level and the observed distributions of wages offered, wages earned and firm size reported in the Integrated Database for Labor Market Research (IDA) is broadly consistent with this explanation of wage dispersion in the sense that the optimal choices of search effort, wage policy and recruiting effort are for the most part well defined when the observed distributions are regarded as the model's equilibrium outcomes.

Christensen et al. (2001) demonstrate that firm separation behavior observed in the IDA supports the proposition that employed workers search at intensities that reflect the expected gain in future income attributable to search effort. Furthermore, the estimated separation function and the observed wage offer distribution imply a steady state distribution of wages earned which is virtually identical to that observed in the data. In this paper, I show that the estimated separation function and the acceptance probability implied by the parameters of the separation function and the wage distribution observed in the data are consistent with the hypothesis that employers choose wage offers optimally in response to worker search behavior for all but the lowest wages paid in the Danish labor market. I also show that the wage distribution and the size distribution are consistent with the requirement that recruiting effort increases with employer productivity.

The Danish wage and size distribution data imply a strong monotone relationship between the labor force size and the wage paid which is consistent with the well known size-wage premium. Furthermore, these data and the model imply that the marginal cost of hiring is relatively constant over a wide range of hiring rates but eventually increases dramatically as the hiring flow rises above 75 workers per year. Indeed, the results suggest the existence of an upper bound on any firm's ability to hire workers. As a consequence of the apparent limit, the most productive firms, who pay well, also reap substantial monopsony profit. Finally, the model and data imply that the labor productivity of almost all employers is relatively low but that a few employers are highly productive and large. Indeed, the most productive 1% of the firms employ 50% of the work force.

Overall the results suggest that employers have substantial monopsony power. For example, the inferred difference between labor productivity and the wage is 80% of the wage paid at the median of the distribution. However, a large part of the rent represents return on investments made in recruiting effort. Indeed, the model implies the firm paying the median wage faces a marginal cost of hiring a new worker equal to about two years of earnings. Still these numbers support the view that Danish employer have substantial monopsony power, particular the large and productive few who employ a large fraction of the total labor force.

Of course, there are numerous caveats, but these suggest fruitful avenues for future research. The inferred differences between productivity and wage at high rates are simply to large to be plausible. To verify this conjecture, one needs actual observations on employer productivity. To the extent that firm level productivity measures exist for the Danish data, they are available only for the largest 9% of all private firms, those with twenty of more employees. Still, my co-authors from the separations project reported in Christensen et al. (2001) and I are currently in the process of determining the extent to which observed cross-firm differences in these measures explain observed differences in hire rates and wage offers.

Allowing for worker heterogeneity, both in the model and in the empirical analysis, is another needed extension. One should note, however, that doing so is potentially important only to the extent that worker characteristics, say individual ability, and employer productivity are correlated since differences in the average wage paid across employers reflect only differences in employer wage policy if employer and worker characteristics are orthogonal. Of course, matching theory suggests that a positive association should exist given worker and employer complementary in the production process. Although search theories of matching heterogenous partnerships are in their infancy, their empirical estimation with matched worker-employer data should soon be feasible.

5 Appendix

The purpose of the appendix is to establish that the model has a unique market equilibrium solution under reasonable conditions. First, we need a formal definition:

Definition A *labor market equilibrium* is an aggregate contact rate per worker λ, a wage offer distribution $F : [\underline{w}, \overline{w}] \to [0,1]$, and a distribution of wages earned $G : [\underline{w}, \overline{w}] \to [0,1]$ that are the solutions to equations (17), (18) and (8) respectively given that the typical worker's search effort strategy $s : [\underline{w}, \overline{w}] \to \Re_+$ and every employer's recruiting effort $v : \left[\underline{p}, \overline{p}\right] \to \Re_+$ and wage policy $w : \left[\underline{p}, \overline{p}\right] \to \Re_+$ choices are optimal under rational expectation. i.e., they satisfy equations (3), (14) and (15) respectively.

Theorem If the measure of employers over productivity as represented by the density $\gamma : \left[\underline{p}, \overline{p}\right] \to \Re_+$ is strictly positive and $\underline{p} \leq b < \overline{p}$, the search cost function $c_w(s)$ is strictly convex and satisfies $c_w(0) = c_w'(0) = 0$, and the recruiting cost function $c_f(s)$ is strictly convex and satisfies $c_f(0) = c_f'(0) = 0$, then an unique labor market equilibrium exists.

Burdett and Mortensen (1998) and Bontemps et al. (2000) prove existence in the special case in which search and recruiting effort are assumed to be exogenous positive constants. They show that an equilibrium is equivalent to the particular solution to an ordinary differential equal defining the wage policy function $w(p)$. An extension of that approach is taken here.

For any wage policy function $w(p)$, let

$$\sigma(p) = s(w(p)) \tag{33}$$

represent the search effort of an employed worker expressed as a function of the employer's labor productivity and let

$$q(p) = \lambda[1 - F(w(p))] \tag{34}$$

denote the rate at which a worker employed at wage $w(p)$ finds a higher paying job per unit of search effort. The definition (34) and equation (18) implies

$$q'(p) = -v(p)\gamma(p). \tag{35}$$

while equation (3) and (33) and (34) yield

$$c_w''(\sigma(p))\sigma'(p) = \frac{-w'(p)\lambda[1 - F(w(p)]}{r + \delta + \lambda s(w(p))[1 - F(w(p))]} = \frac{-w'(p)q(p)}{[r + \delta + \sigma(p)q(p)]}. \tag{36}$$

Given equation (10), (11) and (12), the first order condition for an optimal wage choice, $\pi_p'(w) = 0$ from (14), can be written as

$$\frac{1}{p - w} = \frac{\lambda s(w)F'(w)}{\delta + \lambda s(w)[1 - F(w)]} - \frac{\lambda s'(w)[1 - F(w)] - \lambda s(w)F'(w)}{r + \delta + \lambda s(w)[1 - F(w)]} \tag{37}$$

since the steady state condition (8) implies

$$\frac{\lambda s(w)G'(w)}{\delta + \lambda \int_b^w s(z)dG(z)} = \frac{\lambda s(w)F'(w)}{\delta + \lambda s(w)[1 - F(w)]}. \tag{38}$$

By multiplying both sides of (37) by $w'(p)$ and then substituting appropriately from equations(34) and (35), one finds that

$$w'(p) = \left(v(p)\gamma(p)\left(\frac{r + 2[\delta + \sigma(p)q(p)]}{\delta + \sigma(p)q(p)}\right) - \sigma'(p)q(p)\right)\left(\frac{p - w(p)}{r + \delta + \sigma(p)q(p)}\right). \tag{39}$$

The first order condition for optimal recruiting effort, equation (15), implies

$$c_f''(v(p))v'(p) = \rho(p) \tag{40}$$

where

$$\rho(p) = \left(\frac{\delta + \lambda \int_b^{w(p)} s(z)dG(z)}{\delta + \lambda \int_b^{w(\overline{p})} s(z)dG(z)} \right) \left(\frac{1}{r + \delta + \lambda s(w(p))[1 - F(w(p))]} \right) \tag{41}$$

since the envelope theorem and equation (12) yield

$$\frac{d \max_w \pi_p(w)}{dp} = \frac{\partial \pi_p(w(p))}{\partial p} = \rho(p).$$

Hence,

$$\frac{\rho'(p)}{\rho(p)} = \frac{w'(p)}{p - w(p)} \tag{42}$$

from equation (37).

To summarize, the five equation, (35), (36), (39), (40) and (42), form a system of ordinary differential equations in the unknown functions $q(p), \sigma(p), w(p), \rho(p)$ and $v(p)$. In matrix form, the system can be written as

$$\begin{bmatrix} q' \\ \sigma' \\ w' \\ \rho' \\ v' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{q}{(r+\delta+\sigma q)c_w''(\sigma)} & 0 & 0 \\ \frac{(p-w)q}{r+\delta+\sigma q} & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\rho}{p-w} & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -v\gamma(p) \\ 0 \\ \frac{[r+2(\delta+\sigma q)](p-w)v\gamma(p)}{(\delta+\sigma q)(r+\delta+\sigma q)} \\ 0 \\ \frac{\rho}{c_f''(v)} \end{bmatrix} \tag{43}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{q^2(p-w)}{(r+\delta+\sigma q)^2 c_w''(\sigma)} & 1 & -\frac{q}{(r+\delta+\sigma q)c_w''(\sigma)} & 0 & 0 \\ -\frac{(p-w)q}{r+\delta+\sigma q} & 0 & 1 & 0 & 0 \\ -\frac{q}{r+\delta+\sigma q}\rho & 0 & \frac{\rho}{p-w} & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -v\gamma(p) \\ 0 \\ \frac{[r+2(\delta+\sigma q)](p-w)v\gamma(p)}{(\delta+\sigma q)(r+\delta+\sigma q)} \\ 0 \\ \frac{\rho}{c_f''(v)} \end{bmatrix}$$

$$= \begin{bmatrix} -v\gamma(p) \\ \frac{-q^2(p-w)v\gamma(p)}{(r+\delta+\sigma q)^2 c_w''(\sigma)} \left(1 + \frac{r+2(\delta+\sigma q)}{(\delta+\sigma q)q}\right) \\ \frac{(p-w)qv\gamma(p)}{r+\delta+\sigma q} \left(1 + \frac{r+2(\delta+\sigma q)}{(\delta+\sigma q)q}\right) \\ \frac{q\rho v\gamma(p)}{r+\delta+\sigma q} \left(1 + \frac{r+2(\delta+\sigma q)}{(\delta+\sigma q)q}\right) \\ \frac{\rho}{c_f''(v)} \end{bmatrix}.$$

To identify the particular solution of interest, one needs five boundary conditions, one for each variable. The definitions (33), (34) and (41) together with equations (3) and (18) imply

$$\sigma(\overline{p}) = 0,\ \ q(\overline{p}) = 0 \text{ and } \rho(\overline{p}) = \frac{1}{r+\delta}. \tag{44}$$

The other two required conditions are obtained by solving for the optimal wage and recruiting effort of the least productive employers. Since no firm with productivity less than $R = b$ can attract a labor force and made a profit, an analysis of (12) and (13) implies that the optimal recruiting effort is the corner solution

$$v(p) = 0 \text{ for all } p \in [\underline{p}, b) \tag{45}$$

and the optimal wage choice is any w less than the reservation wage. Conversely, for any $p \geq b$, the optimal wage satisfies $b \leq w < p$ and the optimal recruiting choice solves the interior first order condition (15). Hence, the set of participating firms is bounded by the reservation wage, i.e.,

$$w(b) = b \text{ and } v(b) = 0. \tag{46}$$

Hence, we are looking for a solution to the ODE system (43) on the interval $[b, \overline{p}]$ consistent with the five boundary conditions specified in (44) and (46). Since the right side of (43) is continuous in the vector of variable and p, a unique particular solution exists. By construction, that solution is a labor market equilibrium.

Two observations regarding the restrictions imposed on the exogenous measure of employer types are in order. First, we know from Mortensen (1990) that there is no pure strategy solution to the wage choice problem when a mass of employers have the same productivity. One can prove existence in this case but the argument is more complicated. Second, the assumption that the lower support is less than the reservation wage is equivalent to a specification in which potential low productivity employers exist. Given this interpretation, the boundary conditions (46) are implied by free entry.

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