Amplification of Productivity Shocks: Why Don’t Vacancies Like to Hire the Unemployed?¹

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Abstract

In this paper, I study a new amplification mechanism in search models that arises when workers can choose to search on the job and when, for endogenous reasons, employers reap higher benefits from contacting employed searchers. The motivation for on-the-job search in the model is job shopping, where workers look for jobs they find appealing and the appeal of a job to the worker is not observed by the firm. In equilibrium, workers arriving from unemployment are more likely to leave a job for a more appealing one. Knowing this, firms prefer to contact already-employed searchers. Employers’ preference for contacting already-employed searchers introduces a new amplification mechanism into search models. Vacancies in this type of model respond more to aggregate shocks than in standard search models: the probability that a newly encountered searcher is employed rises in a boom, thereby making it more attractive for firms to post vacancies. Using simulations of the proposed model, I explore the extent to which this new amplification mechanism helps in explaining the volatility of unemployment and vacancies over the business cycle. The calibration results show that, for standard parameter values, this mechanism can generate eight times more amplification in response to productivity shocks compared to the baseline model. Moreover, the proposed model, unlike the standard search model, predicts that unemployment and vacancies respond with different signs to shocks in the rate of exogenous separation. This brings back these type of shocks as plausible driving forces behind labor market fluctuations over the business cycle.

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1 Introduction

Close to half of all labor-market transitions are job-to-job transitions — moves of workers from one employer to another without any intervening unemployment (Fallick and Fleischman (2004), Nagypál (2005b)). Despite their magnitude, such job changes have often been ignored by macroeconomists in the formal modelling of aggregate labor market dynamics over the business cycle. In this paper, I consider a model of on-the-job search with the novel feature that firms reap higher benefits from contacting already employed workers, meaning that firms prefer a pool of searchers with more employed workers. While most recruitment professionals are aware of this preference, macroeconomists have not yet considered ways to incorporate such a feature into models of the aggregate labor market. In fact, all of the existing models of on-the-job search implicitly feature a preference by firms for contacting the unemployed. I show that this novel feature gives rise to a new amplification mechanism in search models.

The model I consider builds on the search and matching framework. In the last two decades, search and matching models have gained wide popularity in the analysis of aggregate labor markets. This is largely due to their ability to explain several labor-market phenomena that the standard, neo-classical growth model cannot tackle, such as the existence of equilibrium unemployment. Several authors have asserted that these models can also quantitatively explain the cyclical variation in key labor-market variables. Recently, however, this view has been challenged, and the search and matching approach has been criticized for its lack of amplification (Shimer (2005)). Shimer argues that earlier works were wrong to declare success, since they did not consider either the magnitude of the exogenous driving forces (Blanchard and Diamond (1989), Mortensen and Pissarides (1994), Cole and Rogerson (1999)) or the highly negative correlation between unemployment and vacancies (Andolfatto (1996), Merz (1995), Ramey and Watson (1997), Gomes, Greenwood, and Rebelo (2001)). He shows that, in response to shocks to the productivity of employment relationships, the standard search model results in elasticities of unemployment and of vacancies that are approximately a tenth of the elasticities observed in the data. In response to shocks to the separation rate, the
standard search model results in a positive correlation between unemployment and vacancies, while the data show a strong negative correlation.

A commonly adopted simplification in models of search is that there are no job-to-job transitions (this holds, for example, in all the works mentioned above), even though this has long been recognized as a serious shortcoming (Tobin (1972)). There are a handful of models in the literature that explicitly model on-the-job search and the resulting job-to-job transitions, and these provide many important insights into the ways in which on-the-job search alters labor-market outcomes. These models, though, either do not study cyclical fluctuations (Burdett and Mortensen (1998), Burdett, Imai, and Wright (2004)), or do not consider the extent of amplification generated by the model (Mortensen (1994), Pissarides (1994), Barlevy (2002)). In this paper, I construct a tractable search model with job-to-job transitions that is capable of generating more amplification than the standard search model and that can match both the magnitude and the cyclicality of job-to-job transitions.

Existing frameworks with on-the-job search (Pissarides (1994), Mortensen (1994), Burdett and Mortensen (1998), Barlevy (2002)) do not help in resolving the amplification puzzle because, in all of them, the expected payoff to employers is higher when contacting unemployed searchers than when contacting employed searchers.\(^3\) This exacerbates the lack of amplification, since, during recessions, the extent of on-the-job search declines, the composition of the searching pool shifts towards the unemployed, and the pool of searchers changes in favor of vacancy creation. For example, even though Barlevy (2002) does not report the elasticity of unemployment to productivity shocks in his model, it can be calculated from the numbers that he reports to be hundred times smaller than the elasticity observed in the data.

There are two channels through which preference for contacting unemployed searchers arises in existing models of on-the-job search. First, since unemployed searchers face worse alternatives on average, they are more likely to accept a match of a given quality or productivity.\(^3\) It should be noted, however, that the emphasis of these authors have not been the role of on-the-job search in amplifying productivity shocks. A notable exception is Shimer (2003), who studies amplification with on-the-job search. The mechanism in his model is very different from the one I study in this paper, since he departs in several ways from the standard search model. In his model, a higher expected benefit from contacting unemployed searchers is what helps in resolving the amplification puzzle.
Second, employed searchers have better outside options, which means that they might command higher wages than unemployed searchers, depending on the nature of wage setting. In a standard search model, firms prefer higher acceptance rates and lower wages, meaning that they prefer unemployed searchers.

I argue that models where employers benefit more from contacting employed searchers are more plausible and provide scope for amplification. In this case, there is a complementarity between vacancies and employed searchers in the matching market. Increased search activity from either of them during a boom encourages search activity from the other party, leading to increased amplification. I also argue, based on the work of Eriksson and Lagerström (2004), that there is compelling evidence that employers indeed expect to reap a higher payoff from contacting employed searchers.

What changes in the model could allow firms to reap higher benefits from contacting an employed searcher? While Nagypál (2005a) discusses this issue in a general framework, here I adopt a specific set of assumptions that generates this feature. The key component is that firms do not always prefer to contact searchers with higher acceptance rates, because some matches lead to a negative payoff to the firm. Such a negative payoff at the time of creation of the employment relationship is not included in the standard search model; in that model all costs of creating a vacancy are borne prior to meeting a worker through vacancy creation costs. It is natural to assume, however, that the firm has to expend some additional resources (on training, relocation, and other match-specific investments) at the time the match is formed. Of course, in order for firms to be willing to enter matches that lead to a negative payoff, it is necessary that the firm has less information about the match than the worker. This is naturally the case when the quality of the match is observed only by the worker and directly enters only into the utility function of the worker.

Given this asymmetric-information setup, the basic mechanism of the model is simple. Workers can undertake job shopping at a cost both while unemployed and while employed, where job shopping simply means searching for a match with a higher idiosyncratic value to the worker. Unemployed searchers are “desperate:” they are willing to accept any idiosyncratic
value above some minimum threshold. Employed searchers, on the other hand, are more selective, and accept only matches that have a value above that of their current match. Turnover declines with the idiosyncratic value of the match for two reasons. First, the probability of finding a better match declines. Second, as a consequence, the incentives to search for a better job also decline, leading to lower endogenous search effort. This means that the expected turnover of previously unemployed workers is higher than that of previously employed workers, making them less attractive candidates for firms to hire. Of course, this higher turnover has to be weighed against the higher acceptance rate of unemployed workers. As Nagypál (2005a) shows, the turnover effect can outweigh the acceptance-rate effect when there is a possibility that the firm will make negative profits in a match. Amplification is a direct result of this mechanism. Vacancies that reap higher expected payoffs from contacting employed searchers respond more to aggregate shocks than in the standard search model: the probability of contacting an employed searcher increases in a boom, making it more attractive for firms to post vacancies during these good times.

The qualitative and quantitative results of the calibration exercise are promising. They show that on-the-job search and job-to-job transitions vary positively with aggregate productivity shocks when comparing stationary equilibria for different aggregate productivity levels. They also show that the amplification mechanism embedded in the model shows up clearly when considering the response of the unemployment and the vacancy rate to changes in aggregate productivity. In the standard model, as Shimer (2005) has shown, the elasticity of the vacancy-unemployment ratio with respect to labor productivity (which, in the standard model, is equal to market tightness) is below 2 for reasonable parameter values. In my model, the elasticity of the vacancy-unemployment ratio is 12.5 for the same parameter values, due both to the decline in unemployment (elasticity of $-3.9$) and to the increase in vacancy rate (elasticity of 8.6).

Moreover, my results show that the vacancy rate varies negatively and the unemployment rate varies positively with job-destruction shocks. Shimer (2005) argues that destruction shocks induce a positive correlation between vacancies and unemployment rate in the standard model, in stark contrast to the data. In my model, on the other hand, a higher destruc-
tion rate discourages vacancy creation, since it shifts the composition of searchers towards the unemployed. Higher destruction shocks thus lead to higher exogenous turnover through separations into unemployment and to lower endogenous turnover via job-to-job transitions, consistent with the data (Nagypál (2005b)). This brings destruction shocks back into the picture as a plausible source of business-cycle variation in vacancies and unemployment.

2 Environment

The model is set in continuous time with an infinite horizon. There is a unit measure of infinitely-lived workers, who are ex-ante identical. Workers can be either employed or unemployed, and the objective of workers is to maximize

$$\int_{t=0}^{\infty} e^{-rt} u_t dt, \quad (1)$$

where

$$u_t = \begin{cases} \ w_t + \mu_t - c(s_t) & \text{if employed,} \\ \ b - c(s_t) & \text{if unemployed.} \end{cases} \quad (2)$$

Here, $w_t$ is the wage received when employed at time $t$; $\mu_t$ denotes the attractiveness or appeal to employed workers of their current employment match (the utility they derive from having a job they “like”); and $s_t$ denotes the search effort of the worker at time $t$. The appeal, $\mu$, of a job to the worker (which I also refer to below as match quality) is determined upon meeting a potential employer and is drawn from the distribution $F(\cdot)$, where $F : [\mu, \Pi] \to [0, 1]$ is a continuous, twice differentiable, strictly increasing distribution function, $\mu \in \mathbb{R} \cup \{-\infty\}$, and $\Pi \in \mathbb{R} \cup \{\infty\}$. In addition, workers can choose to engage in search at a flow cost of $c(s_t)$, where

$$c(s_t) = \begin{cases} \ c_0 + \tilde{c}(s_t) & \text{if } s_t > 0, \\ \ 0 & \text{if } s_t = 0, \end{cases} \quad (3)$$
where $\hat{c}(\cdot)$ is a strictly increasing, strictly convex, twice continuously differentiable function with $\hat{c}(0) = 0$. This means that there is both a fixed cost and a variable cost of searching, so that, when the incentives become sufficiently low, the worker stops searching altogether. Below, I will assume that the variable cost function takes the particular functional form $\hat{c}(s) = c_1 s^{1+p}$. Finally, $b$ denotes the constant utility flow that a worker receives while unemployed (derived from leisure and from unemployment-insurance benefits).

There is a large measure of ex-ante identical firms. The objective of the firms is to maximize

$$
\int_{t=0}^{\infty} e^{-rt}(\pi_t - K \xi_t)dt,
$$

where

$$\pi_t = \begin{cases} 
0 & \text{if the firm is inactive}, \\
-\kappa & \text{if the firm is active with a vacant job}, \\
p - w_t & \text{if the firm is active with a filled job}, 
\end{cases}
$$

and $\xi_t$ is the Dirac delta function, indicating whether a match was created at time $t$. This means that, as in the standard search model, any firm can enter the market and become active by posting a vacancy at flow cost $\kappa$. If a firm posts a vacancy then it participates in the matching market for creating new matches. When a firm contacts a worker and the worker-firm pair decides to create a match, the firm needs to pay a one-time match-specific start-up cost of $K$, in addition to having paid the vacancy creation costs. Subsequently, the firm receives a flow profit of $(p - w_t)$ until the match dissolves, where $p$ is the output of a match, assumed to be the same for all matches. It is the existence of the fixed cost of vacancy creation that ensures that some matches with sufficiently high turnover will lead to a negative expected payoff to the firm. If the firm could distinguish these matches from matches with longer expected duration, it would choose not to form them.4

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4It is important to note that the optimal allocation in this environment would command that the fixed cost be paid by the worker, the informed party, and not the firm. This is ruled out due to borrowing constraints or adverse selection problems.
Employment matches dissolve for exogenous reasons at rate $\delta$ and for endogenous reasons when the worker decides to form a new employment relationship as a consequence of on-the-job search.

There is a single matching market with a meeting function determining the number of meetings, $m_t$, as a function of the total amount of search effort of workers, $s_t$, and the number of vacancies posted, $v_t$:

$$m_t = m(s_t, v_t),$$  \hspace{1cm} (6)

such that $m_s(s, v) > 0$, $m(0, v) = 0$ for any $v$, and $m_v(s, v) > 0$, $m(s, 0) = 0$ for any $s$. I assume that $m(s, v)$ has constant returns to scale, so that the meeting rate per unit of search effort for workers can be written as

$$\lambda_t = \lambda(\theta_t) = \frac{m(s_t, v_t)}{s_t} = m \left(1, \frac{v_t}{s_t}\right) = m \left(1, \theta_t\right),$$  \hspace{1cm} (7)

where $\theta_t = \frac{v_t}{s_t}$ is the appropriately defined market tightness in the model at time $t$. Similarly, the meeting rate for firms can be written as

$$\eta_t = \eta(\theta_t) = \frac{m(s_t, v_t)}{v_t} = \frac{m(s_t, v_t)/s_t}{v_t/s_t} = \frac{\lambda(\theta_t)}{\theta_t}.$$  \hspace{1cm} (8)

The timing of match formation is as follows. If a worker and a firm meet, the worker observes the appeal of the potential match and can decide whether or not to form the match. The firm does not observe neither the appeal of the match to the worker nor whether the worker was previously unemployed or employed.

As for wages, they are assumed to be set as a fixed fraction $\psi \in (0, 1)$ of output $p$; i.e., the worker and the firm split the output of the match in fixed proportions. This is, of course, a stark assumption regarding wage setting (also adopted for its simplicity in other works with on-the-job search, such as Shimer (2001) and Burdett, Imai, and Wright (2004)). The reason for choosing it is twofold. First, we know from Shimer (2005) that, in the standard model, the elasticity of wages to productivity shocks is very close to unity, while the elasticity of
wages to separation-rate shocks is close to zero. Thus, adopting this simple wage setting gives quantitatively similar results regarding wages to those given by the standard model. Any amplification in the model considered will thus come from the proposed mechanism and not from the wage-setting mechanism. Second, there is little agreement in the literature about how to resolve the problem of wage setting with on-the-job search (for related work, see Postel-Vinay and Robin (2002) and Shimer (2005)), let alone with asymmetric information. The question of how to address wage setting in this environment in a way that has more micro-foundation is an interesting topic for future study.\footnote{In a previous version of the paper, I assumed that wages were determined upon the formation of the match by splitting the expected surplus such that the worker received $\beta$ fraction of the surplus, where the outside option of the worker was assumed to be unemployment, i.e., there was no recall. Wages were assumed to be subsequently renegotiated only if otherwise the participation constraint of the parties would have been violated conditional on the worker not being able to credibly communicate the existence of outside offers. It was possible to show in that environment that wages would never be renegotiated after the match was formed, resulting in a constant wage in steady state. This alternative wage setting mechanism generated very similar results to what is reported below. Since there is no axiomatic foundation for surplus sharing with on-the-job search (see Shimer (2005)), I opted for the simpler wage setting mechanism described in the text.}

One thing that the current wage-setting mechanism does not allow for is wage differences between previously employed and unemployed workers. In a bargaining setting, such differences could arise through two channels. First, if the outside option of employed searchers were treated as their previous employment match and no subsequent renegotiation were allowed, then employed searchers would command a higher wage. Second, the expected match quality of employed searchers is higher than that of unemployed searchers. This means that in a surplus-sharing framework they would be willing to accept a lower wage, since they would receive a higher fraction of their payoff from a good match quality. These two forces would affect the wages of employed and unemployed searchers in different directions, leaving open the possibility that the firm would still prefer to contact employed searchers.
3 Equilibrium

3.1 Definition of stationary equilibrium

For the sake of simplicity, and to keep the analysis tractable, I consider the stationary equilibrium of the above model. I let the value of unemployment be $U$, the value of a worker employed in a match of quality $\mu$ be $W(\mu)$, the value for the firm of a vacancy be $V$, and the value of an employment match of tenure $\tau$ be $J_\tau$.

Definition 1. A recursive stationary search equilibrium is unemployment rate $u$, vacancy rate $v$, asset values $\{U, V, W(\mu), J_\tau\}$, wage $w$, unemployed searchers’ search effort $s_u$, employed searchers’ search effort function $s(\mu)$, and a distribution of employed workers $G(\mu)$, such that

- $U$ and $W(\cdot)$ are the value of unemployment and of working for workers making optimal searching and matching decisions, given $u, v, w,$ and $G(\cdot)$. $s_u$ and $s(\cdot)$ are the corresponding optimal search effort.

- $V$ and $J_\tau$ are the value of a vacancy and of a filled job of tenure $\tau$ for firms making optimal vacancy creation decisions, given $u, v, w,$ and $G(\cdot)$.

- Agents update their beliefs rationally.

- There is free entry of vacancies.

- Wages are set as a fraction $\psi$ of output.

- The distribution $G(\cdot)$, the unemployment rate $u$, and the vacancy rate $v$ are consistent with the decisions of the agents in the economy.
3.2 Worker side

The Bellman equation characterizing the value of being a worker with quality $\mu$ is

$$rW(\mu) = \max_{s \geq 0} \left\{ \mu + w - c(s) + \lambda(\theta)s \int_{\mu}^{\theta'} \max[W(\mu') - W(\mu), 0]dF(\mu') + \delta(U - W(\mu)) \right\}.$$  

(9)

The flow payoff from working is the utility derived from being in a match of quality $\mu$ and from the wage $w$. An employed worker needs to choose her search effort at cost $c(s)$. If she encounters a new firm, she needs to decide whether to form the new match, given its quality $\mu'$ drawn from the distribution $F$. Moreover, the worker suffers a loss of asset value due to exogenous separation at rate $\delta$.

The Bellman equation characterizing the value of being an unemployed worker is

$$rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda(\theta)s \int_{\mu}^{\theta'} \max[W(\mu') - U, 0]dF(\mu') \right\}.$$  

(10)

An unemployed worker also needs to choose her search effort at cost $c(s)$. If she encounters a firm, she needs to decide whether to form the new match given its quality $\mu'$ drawn from the distribution $F$.

Equation (9) defines a contraction, and therefore the Contraction Mapping Theorem implies that $W(\mu)$ is continuously increasing in $\mu$ and, given the assumptions on $F(\cdot)$ and $c(\cdot)$, differentiable except at the points where the search decision changes discontinuously. This in turn implies that, as intuition would dictate, acceptance decisions have the reservation property, with the quality of the current match being the reservation match quality.

Taking derivatives with respect to $\mu$ on both sides of the worker’s asset equation and rearranging gives the following for the derivative of the function $W$, where it exists:

$$W'(\mu) = \frac{1}{r + \delta + \lambda(\theta)s(\mu)F(\mu)}.$$  

(11)
I next turn to studying the worker’s search decision. Given the structure of the search cost and using the reservation property of acceptance decisions, the worker’s decision problem can be rewritten as follows:

\[
\begin{align*}
    rW(\mu) &= \max \left\{ \mu + w + \delta(U - W(\mu)); \max_{s > 0} \left\{ \mu + w - c(s) + \right. \right. \\
    &\quad \left. \left. + \lambda(\theta)s \int_{\mu}^{\mu'} [W(\mu') - W(\mu)]dF(\mu') + \delta(U - W(\mu)) \right\} \right\}, \\
\end{align*}
\]

where the search decision has been broken down into two steps: a decision of how much to search if searching and a decision whether to search at all. I assume that, if the worker is indifferent between searching and not searching, then she chooses to search. The first-order condition characterizing the first of these maximization problems is given by

\[
c'(s(\mu)) = \lambda(\theta) \int_{\mu}^{\mu'} [W(\mu') - W(\mu)]dF(\mu') = \lambda(\theta) \int_{\mu}^{\mu'} W'(\mu')\overline{F}(\mu')d\mu',
\]

where the second equality follows from integration by parts, and \(\overline{F} = 1 - F\) is the survival function of the distribution \(F\). Clearly, the right hand side of Equation (13) is declining in \(\mu\). Given the strict convexity of \(c\), then search effort is strictly declining in \(\mu\).

Turning to the second maximization problem: since the payoff from search is declining with \(\mu\), the optimal policy regarding whether to search has the reservation property. This means that there exists a \(\mu_s\) above which the worker will choose not to search at all and below which she will choose to search. At \(\mu_s\), the condition of optimality states that

\[
c(s(\mu_s)) = \lambda(\theta)s(\mu_s) \int_{\mu_s}^{\mu'} [W(\mu') - W(\mu)]dF(\mu') = \lambda(\theta)s(\mu_s) \int_{\mu_s}^{\mu'} W'(\mu')\overline{F}(\mu')d\mu',
\]

where again the second equality follows from integration by parts. From these two optimality conditions, given the properties of \(c(\cdot)\), the following proposition follows. (This, together with all other propositions, is proven in the Appendix).

\textbf{Proposition 1.} There exists a \(\mu_s\) such that, for all \(\mu > \mu_s\), \(s(\mu) = 0\). At \(\mu_s\), the search effort, \(s(\mu_s)\), of the worker is a constant \(s > 0\), which is determined by the parameters...
characterizing the cost of search function. The optimal search effort, \( s(\mu) \), of the worker is continuous and strictly declining in \( \mu \) for all \( \mu < \mu_s \). In addition, \( \mu_s \) and \( s(\mu) \) are increasing in \( \lambda(\theta) \) for all \( \mu < \mu_s \).

Notice that the discontinuous jump in the search effort introduces a kink in the function \( W(\mu) \) at \( \mu_s \), since there is a positive difference between its derivative from the left \( (1/(r + \delta + \lambda(\theta)s(\mu_s)F(\mu_s))) \) and its derivative from the right \( (1/(r + \delta)) \).

We can characterize the optimal search decision further by using the particular functional form for the variable search cost function introduced above. With this functional form, Equation (13), evaluated at \( \mu = \mu_s \), and Equation (14) imply that

\[
\begin{align*}
    s(\mu_s) &= \bar{s} = \left[ \frac{c_0}{c_1 \rho} \right]^{\frac{1}{1+\rho}}.
\end{align*}
\]  

(15)

This, in turn, together with Equation (13), evaluated at \( \mu = \mu_s \), implies that

\[
\begin{align*}
    c_1(1+\rho) \left[ \frac{c_0}{c_1 \rho} \right]^{\frac{1}{1+\rho}} &= \lambda(\theta) \int_{\mu_s}^{\bar{\mu}} W'(\mu')F(\mu')d\mu' = \frac{\lambda(\theta)}{r + \delta} \int_{\mu_s}^{\bar{\mu}} F(\mu')d\mu',
\end{align*}
\]  

(16)

where the derivative of \( W \) has been substituted from Equation (11). This gives an equilibrium condition that determines \( \mu_s \) as a function of \( \lambda(\theta) \) and of exogenous parameters.

Substituting Equation (11) into the optimality condition for search for \( \mu < \mu_s \), and using the functional form for the search function gives

\[
\begin{align*}
    (1+\rho)c_1 s(\mu)^\rho &= \lambda(\theta) \int_{\mu}^{\mu_s} \frac{\bar{F}(\mu')}{r + \delta + \lambda(\theta)s(\mu')F(\mu')}d\mu' + \frac{\lambda(\theta)}{r + \delta} \int_{\mu_s}^{\bar{\mu}} \bar{F}(\mu')d\mu'.
\end{align*}
\]  

(17)

Taking derivatives on both sides with respect to \( \mu \) gives

\[
\begin{align*}
    s'(\mu) &= -\frac{\lambda(\theta)\bar{F}(\mu)s(\mu)^{1-\rho}}{(1+\rho)c_1 \left( r + \delta + \lambda(\theta)s(\mu)\bar{F}(\mu) \right)}.
\end{align*}
\]  

(18)

This differential equation together with the boundary condition in (15) fully characterizes the search decision of workers as a function of the quality of their match, and can be solved
numerically for a given value of $\lambda(\theta)$.

Finally, given that $W(\mu)$ is increasing as argued above, an unemployed worker will clearly adopt a reservation match quality policy when searching for a job. Hence,

$$rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda(\theta)s \int_{\mu_m}^{\bar{T}} [W(\mu') - U]dF(\mu') \right\},$$

where $\mu_m$ is an unemployed worker’s reservation match quality implicitly defined by

$$W(\mu_m) = U.$$  \hfill (20)

Comparing the asset equation of a worker at match quality $\mu_m$ and that of an unemployed worker, it follows that

$$s_u = s(\mu_m),$$  \hfill (21)

and

$$\mu_m = b - w = b - \psi p.$$  \hfill (22)

These results imply that an unemployed searcher takes any job where the total payoff, $(w + \mu)$, compensates for the foregone flow payoff, $b$, from unemployment. Moreover, a worker in a marginal match searches with the same intensity as an unemployed searcher. These results follow from the fact that unemployed and employed searchers have access to the same search technology, so that there is no option value of search lost or gained when switching employment status (unlike in Burdett (1978)).
3.3 Firm side

The value of being a firm with a match of tenure $\tau$ is

$$J_\tau = \int_\mu^{\bar{\mu}} J(\mu) dH_\tau(\mu)$$  \hspace{1cm} (23)

where $H_\tau(\mu)$ is the distribution of match quality across matches of tenure $\tau$ and $J(\mu)$ is the value of having a match with a worker of quality $\mu$. Recall that $\mu$ is not observable to the firm, which is why expectations need to be taken with respect to the distribution of match quality. Also, given that the firm does not take any actions after the match is formed, it is sufficient to focus on $J_0$.

The Bellman equation characterizing $J(\mu)$ is

$$rJ(\mu) = p - w + \lambda(\theta)s(\mu)F(\mu)(V - J(\mu)) + \delta(V - J(\mu)),$$  \hspace{1cm} (24)

The flow payoff of a match to the firm is $(p - w)$. In addition, the firm needs to take into account that the match might end for exogenous reasons at rate $\delta$ and for endogenous reasons if the worker decides to move to another job. The latter happens at rate $\lambda(\theta)s(\mu)F(\mu)$. Since endogenous turnover is decreasing with $\mu$ (and becomes zero once $\mu > \mu_s$), the value of a match to the firm increases in $\mu$.

Given that free entry drives the value of a vacancy to zero, (i.e., $V = 0$), and given the search policy of the worker, we can write the above as

$$J(\mu) = \begin{cases} \frac{p - w}{r + \delta + \lambda(\theta)s(\mu)F(\mu)} & \text{if} \quad \mu \leq \mu_s \\ \frac{p - w}{r + \delta + \lambda(\theta)s(\mu)F(\mu)} & \text{if} \quad \mu > \mu_s \end{cases}.$$  \hspace{1cm} (25)
The Bellman equation characterizing the value of a vacancy can be expressed as

\[
    rV = -\kappa + \eta(\theta) \left[ \gamma \int_{\mu}^{\bar{\mu}} (J(\mu) - K - V) A_e(\mu) dF(\mu) + (1 - \gamma) \int_{\mu}^{\bar{\mu}} (J(\mu) - K - V) A_u(\mu) dF(\mu) \right],
\]

(26)

where \( \gamma \) is the probability that the contacted worker is employed, \( A_e(\mu) \) is the probability that an employed searcher accepts a match of type \( \mu \), and \( A_u(\mu) \) is the probability that an unemployed searcher accepts a match of type \( \mu \). Given free entry and the fact that unemployed workers accept all matches above the threshold \( \mu_m \), this can be rewritten as

\[
    \frac{\kappa}{\eta(\theta)} = \gamma \int_{\mu}^{\bar{\mu}} (J(\mu) - K) A_e(\mu) dF(\mu) + (1 - \gamma) \int_{\mu_m}^{\bar{\mu}} (J(\mu) - K) dF(\mu).
\]

(27)

### 3.4 Equilibrium distribution of workers

In order to determine \( A_e(\mu) \) and \( \gamma \), I turn to the derivation of \( G(\mu) \), which is the stationary measure of employed workers below match quality \( \mu \), and \( u \), which is the stationary unemployment rate. Clearly, given the acceptance policy of the searchers the support of \( G \) is \([\mu_m, \bar{\mu}]\), with \( G(\bar{\mu}) = 1 - u \).

The stationary measure of unemployment can be derived from equating the flow into and out of unemployment:

\[
    u \lambda(\theta) s(\mu_m) F(\mu_m) = \delta (1 - u).
\]

(28)

The flow out of unemployment is the term on the left, which takes into account the search intensity decision, \( s_u = s(\mu_m) \), of the unemployed and the fact that unemployed workers accept all matches above \( \mu_m \). The flow into unemployment is the term on the right, which is simply the result of exogenous separations at rate \( \delta \). Therefore,

\[
    u = \frac{\delta}{\delta + \lambda(\theta) s(\mu_m) F(\mu_m)}.
\]

(29)
To determine the distribution $G(\mu)$, one can equate the flow into and out of $G(\mu)$ (just as in the Burdett and Mortensen (1998) model). The flow into the pool of employed workers with match quality $\mu$ or lower is

$$u\lambda(\theta)s(\mu_m)(F(\mu_m) - F(\mu)), \quad (30)$$

while flow out of the pool of employed workers with match quality $\mu$ or lower is

$$\delta G(\mu) + \lambda(\theta)F(\mu) \int_{\mu_m}^{\min(\mu, \mu_s)} s(\mu')dG(\mu'). \quad (31)$$

The inflow clearly consists of those unemployed workers (searching at intensity $s_u = s(\mu_m)$) who find a match above $\mu_m$ but below $\mu$. The outflow consists of workers that separate exogenously and workers that find a match that is better than $\mu$, where one has to take into account that only workers below match quality $\mu_s$ are searching and workers with different match quality search with different intensity.

Equating these two flows when $\mu \leq \mu_s$ gives

$$\frac{u\lambda(\theta)s(\mu_m)(F(\mu_m) - F(\mu))}{F(\mu)} = \frac{\delta G(\mu)}{F(\mu)} + \lambda(\theta) \int_{\mu_m}^{\mu} s(\mu')dG(\mu'). \quad (32)$$

Differentiating both sides with respect to $\mu$ and rearranging gives

$$G'(\mu) = \frac{u\lambda(\theta)s(\mu_m)F(\mu_m) - \delta G(\mu)}{\delta + \lambda(\theta)s(\mu)F(\mu)} \frac{f(\mu)}{F(\mu)}. \quad (33)$$

For $\mu > \mu_s$, the same steps give

$$G'(\mu) = \frac{u\lambda(\theta)s(\mu_m)F(\mu_m) - \delta G(\mu)}{\delta} \frac{f(\mu)}{F(\mu)}. \quad (34)$$

These differential equations together with the boundary condition $G(\mu_m) = 0$ fully characterize the distribution of workers for a given value of $\lambda(\theta)$ and $s(\mu)$.

Given the distribution $G$, the firm's initial belief that a match of quality $\mu$ is accepted by an
employed searcher can be expressed as

\[ A_e(\mu) = \begin{cases} 
1 & \text{if } \mu > \mu_s, \\
\frac{\int_{\mu_m}^{\mu_s} s(\mu')dG(\mu')}{\int_{\mu_m}^{\mu_s} s(\mu')dG(\mu')} & \text{if } \mu_m \leq \mu \leq \mu_s, \\
0 & \text{if } \mu < \mu_m. 
\end{cases} \]  

(35)

Also, the fraction of employed searchers in the searching population is

\[ \gamma = \frac{\int_{\mu_m}^{\mu_s} s(\mu')dG(\mu')}{us(\mu_m) + \int_{\mu_m}^{\mu_s} s(\mu')dG(\mu')} . \]  

(36)

Using these results and substituting in the value of \( J(\mu) \) from Equation (25) and using the wage setting condition, \( w = \psi p \), the free entry condition can be rewritten as

\[ \frac{\kappa}{\eta(\theta)} = \int_{\mu_m}^{\mu_s} \left( \frac{p(1 - \psi)}{r + \delta + \lambda(\theta)s(\mu)F(\mu)} - K \right) A(\mu)dF(\mu) + F(\mu_s) \left( \frac{p(1 - \psi)}{r + \delta} - K \right), \]  

(37)

where

\[ A(\mu) = \frac{us(\mu_m) + \int_{\mu_m}^{\min(\mu,\mu_s)} s(\mu')dG(\mu')}{us(\mu_m) + \int_{\mu_m}^{\mu_s} s(\mu')dG(\mu')} . \]  

(38)

3.5 Characterization of equilibrium

To understand better the mechanism of the model, it is useful to show two more results. One simply follows from the positive selection of employed workers into different match qualities and formally shows that the expected turnover of unemployed searchers is higher than that of employed searchers. This is one of the crucial elements necessary to generate higher profits to the firm from contacting an employed searcher.

**Proposition 2.** The probability that a previously unemployed worker with tenure \( \tau \) leaves a job is always higher in a stationary equilibrium than the same probability for a previously employed worker with the same tenure.
This proposition means that increasing the fraction of employed workers has two effects: it decreases turnover conditional on match formation, but it decreases the probability that a match is formed, since employed searchers are less likely to form a newly contacted potential employment match. The first has a positive effect on vacancy creation, while the second has a negative effect. The crucial question is which of these two effects dominates, that is under what conditions will the firm reap higher benefits from contacting an employed searcher than from contacting an unemployed searcher.

**Proposition 3.** Let $\Pi_u$ be the expected profit of a firm from contacting an unemployed searcher and $\Pi_e$ be the expected profit of a firm from contacting an employed searcher. There exists a $\hat{K}$ such that for $K \leq \hat{K}$, $\Pi_u \geq \Pi_e$, while for $K > \hat{K}$, $\Pi_u < \Pi_e$.

This proposition shows that whether the firm prefers to contact an employed or an unemployed worker depends crucially on the size of the fixed cost $K$ that the firm has to expend in order to create the employment relationship. The larger this cost is, the worse turnover will be for the firm, and the less it will benefit from the high acceptance rate of unemployed searchers.

## 4 Calibration

In this section, I calibrate the model and look at what happens when I compare stationary equilibria with different values of the aggregate productivity parameter, $p$, and the exogenous job destruction rate, $\delta$.

In the calibration exercise, I follow the literature and use a Cobb-Douglas matching function

$$m(s_t, v_t) = m_0 s_t^\alpha v_t^{1-\alpha}. \quad (39)$$

Then $\lambda(\theta) = m_0 \theta^{1-\alpha}$ and $\eta(\theta) = m_0 \theta^{-\alpha}$, so that the expected time for a firm to make a contact with a worker is $\frac{1}{\eta(\theta)} = \frac{1}{m_0} \theta^\alpha = \frac{1}{m_0} \left( \frac{\lambda(\theta)}{m_0} \right)^{\frac{1}{1-\alpha}} = m_0^{\frac{1}{1-\alpha}} \lambda(\theta)^{\frac{\alpha}{1-\alpha}}$.

For the numerical implementation of the model, it is useful to note that two parameters, $c_1$
and \( m_0 \), can be eliminated from the equilibrium conditions. In other words, two normalizations are possible. This can be achieved by letting \( \hat{s}(\mu) = s(\mu)c_1^{1+\rho} \), \( \hat{\lambda}(\theta) = \lambda(\theta)c_1^{-1+\rho} \), and \( \hat{\kappa} = \kappa m_0^{\alpha-1}c_1^{1+\rho}1^{-\alpha} \). These normalizations are possible because the arrival rate of contacts for workers is determined by both \( \lambda(\theta) \) and \( s(\mu) \). An economy with high \( \lambda(\theta) \) and low \( s(\mu) \) is not distinguishable from one with low \( \lambda(\theta) \) and high \( s(\mu) \) as long as \( \kappa \) is adjusted appropriately, since that is the parameter that determines the relative costliness of posting a vacancy compared to the parameter, \( c_1 \), determining the marginal cost of search for the worker. Also, the model cannot identify whether few matches are created because the cost of vacancies is high or because the matching function has a low scale parameter, so a second normalization of the cost of vacancy posting is possible, which gives the expression for \( \hat{\kappa} \).

For the choice of the calibrated parameters, I follow Shimer (2005) as closely as possible, to facilitate direct comparison of my results with his. In particular, the following parameters are set to be the same as his. The aggregate productivity can be normalized without loss of generality to be \( p = 1 \). The model is set to generate monthly series so \( r \) is chosen to be 0.4\%, giving an annual discount rate of 4.8\%, while the exogenous job destruction rate is set to 3.33\%. The value of leisure or of unemployment insurance, \( b \), is set to 0.4 at 40\% of the match output, while the elasticity of the matching function with respect to search effort \( \alpha \) is set to 0.72. Unlike in Shimer (2005), the cost of posting a vacancy is set at 5\% of monthly output. This gives a vacancy-unemployment ratio of around 0.5, reported as the empirical value in Faberman (2005). Somewhat surprisingly, the model generated data are not very sensitive to this cost of vacancy posting, unlike in the standard search model. In my model, most of the costs of creating an employment relationship are the cost borne once the match is formed \( (K) \). Vacancy-creation costs constitute a small part of creating an employment match, and hence matter little for the simulation results.

I next turn to the calibration of the new parameters in this model. For the distribution of match qualities, I use a normal distribution with mean zero and standard deviation \( \sigma \). There is no good empirical counterpart that could guide this choice. In choosing \( \sigma \), I aim to keep the relative payoff that a worker gets in equilibrium from the appeal of a job to be low compared to the payoff from the wage \( w \). This is important, since it would be unappealing
to study a model where most payoffs to workers come from some unobserved amenity of jobs. My choice of $\sigma = 0.2$ implies that, in terms of the total payoff from production, the probability of drawing a match quality that is half as large as output is below 1%. Of course, due to job-to-job transitions, the endogenous distribution of match qualities first-order stochastically dominates the distribution of the initial draw of match qualities, so the question of how important match quality is compared to wages is determined endogenously.

The parameters of the cost of search function are set to generate job-to-job transitions close to what is observed in the data (2.73% on a monthly basis, as reported by Nagypál (2005b)). To achieve this target the parameter $\rho$ is set equal to 0.5 and the fixed cost of search for a worker is set close to zero at 0.1% of output, though even such a small fixed cost leads to no more search once the eighty-sixth percentile of the quality distribution is reached. (The extent of job-to-job transitions is much more sensitive to $\rho$ than to $c_0$, as long as $c_0$ is small enough.)

A crucial element in the calibration is the fraction of output, $\psi$, that is paid out in wages and the fixed cost, $K$, of creating a match. Based on measures of markup, I set $\psi = 0.9$. This is significantly lower than the wage that would result in a standard search model, but this model includes additional costs that the firm has to bear after the match is formed. Given $\psi$, I choose $K$ to match an unemployment rate of 6%. The resulting $K$ is 1.505, roughly one and a half months’ output of a match. There is no clear empirical counterpart to this number, although the calibrated value does seem reasonable. It is clear from Proposition 4 that a relatively high value is necessary for this model to generate amplification. As the model is very sensitive to these two parameters, I will report sensitivity analysis to these parameters below.

For the calibrated parameter values, the equilibrium values of interest are reported in Table 1. As can be seen, the unemployment rate is exactly at its calibrated value, while the vacancy rate and the job-to-job transition rate are very close. The average match quality is 16.6% of output, which I find a plausible value. Even though there are many more employed searchers than unemployed searchers, due to the higher search effort of the unemployed, a firm has
Table 1: Variables of interest generated by the model for the baseline calibration.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>unemployment rate</td>
<td>6.00%</td>
</tr>
<tr>
<td>vacancy rate</td>
<td>2.96%</td>
</tr>
<tr>
<td>job-finding rate of the unemployed</td>
<td>51.7%</td>
</tr>
<tr>
<td>job-to-job transition rate</td>
<td>2.68%</td>
</tr>
<tr>
<td>total separation rate</td>
<td>5.98%</td>
</tr>
<tr>
<td>fraction of employed workers searching</td>
<td>65.8%</td>
</tr>
<tr>
<td>prob. of contacting employed searcher</td>
<td>60.8%</td>
</tr>
<tr>
<td>lowest accepted match quality</td>
<td>-0.5 ($F = 99.38%$)</td>
</tr>
<tr>
<td>search threshold match quality</td>
<td>0.222 ($F = 13.35%$)</td>
</tr>
<tr>
<td>average match quality</td>
<td>0.166</td>
</tr>
</tbody>
</table>

nearly a 40% chance of contacting an unemployed searcher. It would be interesting to know how this number relates to relevant empirical estimates. Also, we can see that unemployed workers in the model are “desperate:” they accept the vast majority of jobs offered to them. They then keep searching on the job until a match quality in the upper thirteenth percentile of the distribution is found.

In Figure 1, I plot the search effort chosen by workers with different match qualities, the distribution of initial match quality for unemployed searchers, and the endogenous distribution of employed workers across match qualities in equilibrium. Due to job-to-job transitions, the second distribution first-order stochastically dominates the first, just as in Burdett and Mortensen (1998). With endogenous search intensity, for a given level of job-to-job transitions, there is an even larger shift in the distribution than in the Burdett and Mortensen (1998) model. This is because it is exactly workers in low-quality matches who choose a high search effort, and hence are most likely to make a job-to-job transition.

4.1 Comparative statics — aggregate productivity

Next, I allow aggregate productivity to vary. As stated earlier, I rely on comparisons of stationary equilibria to assess the response of the model to aggregate shocks. In the standard search model, such a comparative static exercise invariably gives results that are very close to the dynamic response of the full stochastic model. In that model transition dynamics
Table 2: Response of the variables of interest across stationary equilibria to a 1% increase in productivity, their implied elasticity, and the corresponding elasticities in the standard model and the data as calculated from Shimer’s results as a ratio of standard deviations of the log of the variables taking into account their reported correlations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>1% increase in productivity</th>
<th>implied elasticity</th>
<th>standard elast. Shimer</th>
<th>data elast. Shimer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>baseline equil.</td>
<td>new equil.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment rate</td>
<td>6.00%</td>
<td>5.77%</td>
<td>-3.89</td>
<td>-0.45</td>
</tr>
<tr>
<td>vacancy rate</td>
<td>2.96%</td>
<td>3.21%</td>
<td>8.65</td>
<td>1.35</td>
</tr>
<tr>
<td>job-finding rate of unemployed</td>
<td>51.7%</td>
<td>53.9%</td>
<td>4.3</td>
<td>0.5</td>
</tr>
<tr>
<td>job-to-job transition rate</td>
<td>2.68%</td>
<td>2.73%</td>
<td>1.98</td>
<td>—</td>
</tr>
<tr>
<td>total separation rate</td>
<td>5.98%</td>
<td>6.03%</td>
<td>0.89</td>
<td>—</td>
</tr>
<tr>
<td>frac. of empl. workers searching</td>
<td>65.8%</td>
<td>65.9%</td>
<td>0.15</td>
<td>—</td>
</tr>
<tr>
<td>prob. contacting empl. searcher</td>
<td>60.8%</td>
<td>61.5%</td>
<td>1.15</td>
<td>—</td>
</tr>
<tr>
<td>lowest accepted match quality</td>
<td>-0.5</td>
<td>-0.51</td>
<td>-1.8</td>
<td>—</td>
</tr>
<tr>
<td>search threshold match quality</td>
<td>0.222</td>
<td>0.225</td>
<td>1.11</td>
<td>—</td>
</tr>
<tr>
<td>average match quality</td>
<td>0.166</td>
<td>0.168</td>
<td>1.4</td>
<td>—</td>
</tr>
</tbody>
</table>

are very fast, due to the forward-looking and instantaneous adjustment of market tightness and the calibrated high job-finding rate. Due to the presence of on-the-job search, the full stochastic version of my model is much more complex. In particular, the state space includes the complete distribution of match qualities across employed workers and the stochastic dynamic equilibrium is thus more difficult to characterize. Intuition would suggest, though, that the dynamics of the model become more gradual. With on-the-job search, the market tightness does not adjust instantaneously to its new long-run value. While the job-finding rate of unemployed workers is as high as in the standard model, the job-to-job transition rate of employed workers is much lower, and the adjustment towards the new steady state could therefore be much more prolonged.

In Table 2, I report how the variables of interest change across stationary equilibria in response to a 1% increase in aggregate productivity. I also report the implied elasticities and compare these to the elasticities that Shimer (2005) finds in the standard model and in the data.

The amplification mechanism of the model is clearly evident in these results. In the standard
model, the vacancy-unemployment ratio (which is, in that model, equal to market tightness) has an elasticity with respect to labor productivity below 2, for reasonable parameter values. In my model, the elasticity of the vacancy-unemployment ratio is 12.54, which is due both to the decline in unemployment (elasticity of −3.89) and to the increase in the vacancy rate (elasticity of 8.65). These elasticities are short of those observed in the data reported in Table 2, but are much closer than the ones generated by the standard model.

To demonstrate more clearly why my model is generating more amplification, I plot in Figure 2 the payoff to a firm from creating matches of different quality, and the probability that these different matches are accepted by unemployed and employed searchers. Clearly, there is a strong complementarity between vacancies and employed searchers, since it is exactly the matches that generate negative payoffs to the firm that employed searchers are likely to reject, while unemployed searchers accept jobs indiscriminately. This gives firms a preference for contacting an employed worker. Since the probability of contacting an employed worker rises with a drop in unemployment, firms are willing to wait much longer to fill a vacancy in these markets. This allows the contact rate for workers to rise 8 times more than in the standard model.6

With regards to the response of the other variables, I find that the job-to-job transition rate is strongly procyclical, although only about half as much as the job-finding rate of the unemployed. This translates into a procyclical total separation rate. As for the average match quality, it is also strongly procyclical due to the higher level of job-to-job transitions.

Of the above elasticities, the most important one is that of the unemployment rate. To study its robustness with respect to the choice of the variables $\psi$ and $K$, I calculate its sensitivity to $\psi$, while correspondingly varying $K$ to keep the unemployment rate in the baseline case equal to 6%. The results are plotted in Figure 3. As the variable $\psi$ increases, the corresponding value of $K$ declines significantly. The elasticity of unemployment (in absolute value) also decreases; however, even for $\psi = 0.99$, the elasticity is well above 2. (It is worth noting that, at $\psi = 0.999$, there is no value of $K$ that generates a 6% unemployment rate, and, for

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6The elasticity of $\lambda(\theta)$ is 2.49 in this model, while, in the standard model, it can be shown that, for the wage-setting mechanism assumed in this paper, it would be exactly $\frac{1-\alpha}{\alpha} = 0.39$. 

K = 0, the elasticity is very low.) This means that the reported results are fairly robust to the choice of ψ and K.

### 4.2 Comparative statics — destruction rate

Next, I allow the destruction rate to vary. In Table 3, I report how the variables of interest change across stationary equilibria in response to a 1% increase in the destruction rate. I also report the implied elasticities and compare these to the elasticities that Shimer (2005) finds in the standard model and in the data. Here, the principle difficulty in the standard model is not with regards to magnitudes; in fact, the magnitude of the elasticities generated by the standard model is closer to the data with respect to destruction-rate shocks than with respect to productivity shocks. A more significant problem is the sign of the elasticities. The standard model predicts that both unemployment and vacancies vary positively with destruction rate shocks, thereby introducing a strong positive correlation between them, greatly at odds with the data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>1% increase in destruction rate</th>
<th>implied elasticity</th>
<th>standard elast. Shimer</th>
<th>data elast. Shimer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>baseline equil. value</td>
<td>new equil. value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unemployment rate</td>
<td>6.00%</td>
<td>6.18%</td>
<td>2.94</td>
<td>0.87</td>
</tr>
<tr>
<td>vacancy rate</td>
<td>2.96%</td>
<td>2.82%</td>
<td>-4.71</td>
<td>0.79</td>
</tr>
<tr>
<td>job-finding rate of unemployed</td>
<td>51.7%</td>
<td>50.6%</td>
<td>-2.07</td>
<td>0.5</td>
</tr>
<tr>
<td>job-to-job transition rate</td>
<td>2.68%</td>
<td>2.65%</td>
<td>-0.9</td>
<td>—</td>
</tr>
<tr>
<td>total separation rate</td>
<td>5.98%</td>
<td>5.99%</td>
<td>0.15</td>
<td>—</td>
</tr>
<tr>
<td>frac. of empl. workers searching</td>
<td>65.8%</td>
<td>65.7%</td>
<td>-0.13</td>
<td>—</td>
</tr>
<tr>
<td>prob. contacting empl. searcher</td>
<td>60.8%</td>
<td>60.1%</td>
<td>-1.12</td>
<td>—</td>
</tr>
<tr>
<td>lowest accepted match quality</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>search threshold match quality</td>
<td>0.222</td>
<td>0.219</td>
<td>-1.09</td>
<td>—</td>
</tr>
<tr>
<td>average match quality</td>
<td>0.166</td>
<td>0.163</td>
<td>-1.85</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3: Response of the variables of interest across stationary equilibria to a 1% increase in the destruction rate, their implied elasticity, and the corresponding elasticities in the standard model and the data as calculated from Shimer’s results as a ratio of standard deviations of the log of the variables taking into account their reported correlations.
This model, unlike the standard model, predicts that the vacancy rate decreases in response to an increase in the destruction rate. A high destruction rate discourages vacancy creation, since it shifts the composition of searchers towards the unemployed. This brings destruction shocks back into the picture as a plausible source of business-cycle variation in vacancies and unemployment. If anything, the model overestimates the response of unemployment and of vacancies to a change in the separation rate, since the implied elasticities are larger than those empirically found by Shimer.

An interesting feature of the model is that, in response to changes in the destruction rate, the total separation rate is nearly acyclical. A higher destruction rate discourages job-to-job transitions, since it reduces the payoff from finding a high-quality match. Thus, in response to the higher destruction rate, the rate of separations into unemployment increases, but the rate of separations that lead to job-to-job transitions declines, in qualitative agreement with the data (Nagypál (2005b)). The lower rate of job-to-job transitions also implies that workers have a lower chance of securing high-quality matches, so that the average match quality also declines.

5 Conclusion

Since the work of Shimer (2005), it has become well-known that the textbook search model does not generate nearly enough volatility of the correct sign in response to shocks to labor productivity or to the rate of separations. A vibrant recent body of work explores how this failure can be remedied through different generalizations. Most of these papers focus on the role of wage setting in generating volatility (Hall (2005), Menzio (2005), Kennan (2005)). While these explanations certainly provide important insights, this paper offers a complementary explanation of how the search model can generate more amplification. This explanation is based on the idea that, if there is complementarity between vacancies and employed searchers in the creation of employment matches, then in a model with on-the-job search a recession is a bad time to create a vacancy. This is because the extent of on-the-job search and of job-to-job transitions is low during these bad times.
In order to introduce such a complementarity between vacancies and employed searchers, I introduced two new elements. One is that firms cannot observe how much a worker likes a potential employment match. Firms anticipate, however, that unemployed searchers are “desperate” and are willing to take any job, while employed searchers have better existing options and are thus more selective. This element by itself is not enough, though, to overcome the fact that unemployed searchers are much more likely to accept a job meaning that it is easier thus more profitable for a firm to hire them. In addition, it is necessary that the firm make negative profits from very short matches. To achieve this, I add the second element, a cost that the firm has to pay when an employment relationship is formed, such as a cost of training. This natural addition means that firms prefer lower turnover individuals even if they are initially more difficult to attract.

Given this complementarity between vacancies and employed searchers, calibration of the proposed model provides important results. First, this model generates a much larger response than the standard model in unemployment and vacancies to changes in labor productivity. The extent of amplification is still not enough to generate the variability observed in the data, but the results are much closer than those from the standard model. Second, this model generates responses in vacancies to an increase in the separation rate of the correct sign, unlike the standard model. In my model a high destruction rate discourages vacancy creation, since it shifts the composition of searchers towards the unemployed. In other words, changes in the destruction rate are a plausible source of business-cycle fluctuations.

As Nagypál (2005a) shows, the proposed mechanism is not the only one capable of generating complementarity between vacancies and employed searchers. Another large class of models can give the same result using the idea that workers are of different, unobserved qualities, and higher quality workers are more likely to be employed. In such models, unemployed workers are more likely to be “lemons,” and unemployment therefore carries a stigma (as in the model of Vishwanath (1989)). These two alternatives give similar predictions regarding the amplification of labor productivity and destruction shocks. More work will be required to distinguish the two.
References


6 Appendix

6.1 Proof of Proposition 1

As stated in the text, the reservation property of search decisions simply follows from the fact that the payoff from a unit of search \( \left( \lambda(\theta) \int_{\mu}^{\bar{\mu}} W'(\mu')F(\mu')d\mu' \right) \) is decreasing in \( \mu \).

Given the specification of the search cost function in Equation (3), it follows from Equations (13) and (14), evaluated at \( \mu_s \), that \( s(\mu_s) = \bar{s} \) is a solution to the equation

\[
\hat{c}'(s)s = c_0 + \hat{c}(s). \tag{40}
\]

Given the assumptions on \( \hat{c} \), the right-hand side exceeds the left-hand side at \( s = 0 \) and the left-hand side is increasing faster than the right-hand side. This implies that the above equation has a unique solution \( \bar{s} > 0 \), which is independent of the other model parameters.

From Equation (13), the optimal search effort for \( \mu \leq \mu_s \) can be expressed as

\[
s(\mu) = c'^{-1} \left( \lambda(\theta) \int_{\mu}^{\bar{\mu}} W'(\mu')F(\mu')d\mu' \right). \tag{41}
\]

Given the assumptions on \( c \), \( c'^{-1} \) is a continuous and strictly increasing function. Given that its argument is continuously decreasing in \( \mu \), this means that \( s(\mu) \) is continuous and strictly declining in \( \mu \) for all \( \mu < \mu_s \).

The result that \( \mu_s \) is increasing in \( \lambda(\theta) \) simply follows from the fact that, given the constancy of \( s(\mu_s) \), the left-hand side of Equation (13) evaluated at \( \mu_s \) is a constant. The right-hand side is increasing in \( \lambda(\theta) \) and decreasing in \( \mu_s \), hence to maintain equilibrium a higher \( \lambda(\theta) \) must result in a higher level of \( \mu_s \). Finally, the result that \( s(\mu) \) is increasing in \( \lambda(\theta) \) follows from totally differentiating the optimality condition in Equation (13).
6.2 Proof of Proposition 2

Let the initial distribution of previously unemployed workers across match qualities at tenure 0 be \( H_0^u(\mu) \) and let the initial distribution of previously employed workers across match qualities at tenure 0 be \( H_0^e(\mu) \). Given the search and acceptance decisions of the different type of workers, these distributions can be expressed as

\[
H_0^u(\mu) = \frac{F(\mu) - F(\mu_m)}{F(\mu_m)}, \quad (42) \\
H_0^e(\mu) = \frac{\int_{\mu_m}^{\min(\mu, \mu_s)} s(\mu') (F(\mu) - F(\mu')) dG(\mu')}{\int_{\mu_m}^{\mu_s} s(\mu') F(\mu') dG(\mu')} \quad (43)
\]

For \( \mu, \mu' \in [\mu, \bar{\mu}] \), define \( d(\mu, \mu') = \frac{F(\mu) - F(\mu')}{F(\mu')} \). Clearly, given the assumptions on \( F \), \( d(\mu, \mu') \) is strictly decreasing in \( \mu' \) since

\[
\frac{\partial d(\mu, \mu')}{\partial \mu'} = -f(\mu') \frac{F(\mu') + f(\mu') (F(\mu) - F(\mu'))}{F(\mu')^2} = -\frac{f(\mu') F(\mu)}{F(\mu')^2} < 0. \quad (44)
\]

Then

\[
H_0^e(\mu) = \frac{\int_{\mu_m}^{\min(\mu, \mu_s)} s(\mu') (F(\mu) - F(\mu')) dG(\mu')}{\int_{\mu_m}^{\mu_s} s(\mu') F(\mu') dG(\mu')} < \frac{\int_{\mu_m}^{\min(\mu, \mu_s)} s(\mu') F(\mu') dG(\mu')}{\int_{\mu_m}^{\mu_s} s(\mu') F(\mu') dG(\mu')} \leq \frac{\int_{\mu_m}^{\min(\mu, \mu_s)} s(\mu') F(\mu') dG(\mu')}{\int_{\mu_m}^{\mu_s} s(\mu') F(\mu') dG(\mu')} = d(\mu, \mu_m) = H_0^u(\mu), \quad (45)
\]

hence the initial distribution of the employed first-order stochastically dominates the distribution of the unemployed.

As for the distributions \( H_\tau^u(\mu) \) and \( H_\tau^e(\mu) \) for \( \tau > 0 \), these can be derived from \( H_0^u(\mu) \) and \( H_0^e(\mu) \) by applying the same out-flow rates to them as a function of \( \mu \), which is \( \lambda(\theta) s(\mu) F(\mu) \). Since these rates do not depend on whether the worker was previously employed or unemployed, the resulting \( H_\tau^e(\mu) \) first-order stochastically dominates \( H_\tau^u(\mu) \).
6.3 Proof of Proposition 3

Define $\Pi_\Delta = \Pi_u - \Pi_e$. Notice that given the workers’ search and acceptance decisions

$$
\Pi_u = \int_{\mu_m}^{\mu_s} (J(\mu) - K) A_u(\mu) dF(\mu) = \\
= \int_{\mu_m}^{\mu_s} \left( \frac{p(1-\psi)}{r + \delta + \lambda(\theta)s(\mu)F(\mu)} - K \right) dF(\mu) + F(\mu_s) \left( \frac{p(1-\psi)}{r + \delta} - K \right),
$$

(46)

and

$$
\Pi_e = \int_{\mu}^{\mu} (J(\mu) - K) A_e(\mu) dF(\mu) = \\
= \int_{\mu_m}^{\mu_s} \left( \frac{p(1-\psi)}{r + \delta + \lambda(\theta)s(\mu)F(\mu)} - K \right) A_e(\mu) dF(\mu) + F(\mu_s) \left( \frac{p(1-\psi)}{r + \delta} - K \right),
$$

(47)

hence $\Pi_\Delta$ can be expressed as

$$
\Pi_\Delta = \Pi_u - \Pi_e = \int_{\mu_m}^{\mu_s} \left( \frac{p(1-\psi)}{r + \delta + \lambda(\theta)s(\mu)F(\mu)} - K \right) (1 - A_e(\mu)) dF(\mu).
$$

(48)

Given that $1 - A_e(\mu)$ is always positive and is strictly positive for some $\mu \in [\mu_m, \mu_s]$, $\Pi_\Delta$ is strictly decreasing in $K$. Moreover, $\Pi_\Delta$ is positive for $K = 0$ since $\psi < 1$, and is negative for $K = \frac{p(1-\psi)}{r + \delta + \lambda(\theta)s(\mu_s)F(\mu_s)}$. Therefore, there exists a $\hat{K}$ such that, for $K \leq \hat{K}$, $\Pi_\Delta$ is positive, and, for $K > \hat{K}$, $\Pi_\Delta$ is strictly negative.
Figure 1: Search effort as a function of match quality, the distribution of initial match quality for unemployed searchers, and the endogenous distribution of employed workers across match qualities in equilibrium in the baseline calibration.
Figure 2: Firm payoff from forming matches of different quality and the probability that different quality matches are accepted by unemployed and employed searchers.
Figure 3: The sensitivity of the calibrated value of the one-time match-creation cost and of the elasticity of unemployment with respect to productivity changes to the choice of the wage/sharing rule.