Labor-market Volatility in Matching Models with Endogenous Separations*

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Abstract

The business-cycle behavior of a matching model with endogenous separations is studied in this paper. We show that whether aggregate productivity shocks have a larger effect on the vacancy–unemployment ratio than in a model with exogenous separations depends on whether worker productivity stochastically increases with tenure. The difference in the response is quantitatively small, however. We also show that the cleansing effect introduced by allowing for endogenous separations can help in reconciling the model with observed fluctuations in the unemployment rate, but not with those in the vacancy rate.

Keywords: Labor-market search; unemployment and vacancies volatility; productivity shocks; endogenous separations

JEL classification: E24; E32; J41; J63; J64

I. Introduction

Recently, the extent of business-cycle fluctuations in key labor-market variables generated by matching models has received a lot of attention in the literature. This interest has been sparked by the influential work of Shimer (2005), in which he showed that a reasonably calibrated version of the textbook matching model with exogenous separations grossly fails to account for the observed volatility of the unemployment and vacancy rates. In Mortensen and Nagypál (2007), we argued that Shimer’s results are too damning for the matching model and showed that, in our specification of the model, correlated separation rate and productivity shocks can account for two-thirds of the observed volatility of the vacancy–unemployment ratio.1 The incorporation of variation in the separation rate, which in the data accounts for about a third of the fluctuation in the unemployment rate

*We acknowledge the financial support of the National Science Foundation.

1 See also Fujita and Ramey (2007b) and Pries (2007).
at business-cycle frequencies, as in Shimer (2007) and Fujita and Ramey (2007a), can be carried out mechanically. This was done by Shimer (2005) and Mortensen and Nagypál (2007), by allowing the separation rate to follow an exogenous stochastic process. While this is simple to do, it is conceptually more appealing to write down a model where the separation rate varies endogenously through the optimizing response of agents in the model to exogenous shocks, since these models put discipline on the amount of model-generated variation in the separation rate.

The most widely used and popular model of endogenous separations is due to Mortensen and Pissarides (1994). We study a generalized version of this model and find that whether the response of the vacancy–unemployment ratio to aggregate productivity shocks is higher or lower with endogenous separations than with exogenous separations depends on how worker productivity is related to tenure. If worker productivity is independent of tenure, we find a surprising result: the response of the vacancy–unemployment ratio to aggregate productivity shocks in our model is observationally equivalent to the one in the canonical model with exogenous separations. This is because the canonical model can be recast as a special case of our model with constant worker productivity. If worker productivity increases with tenure, aggregate productivity shocks have a larger effect on the vacancy–unemployment ratio. In this case, the weight on the value of forgone leisure is somewhat higher in the wage of low-tenure workers than in the wage of high-tenure ones. This means that the wage of low-tenure workers is less volatile, which, in turn, leads to more volatility in the pay-off from creating a new job in response to aggregate technology shocks. The size of this effect depends on the growth in worker productivity with tenure, and we show that, for plausible parameter values, it is quantitatively small.

This does not mean, however, that incorporating endogenous separations has no effect on the model’s business-cycle performance. The reason for this is that the exact correspondence between aggregate productivity and labor productivity that is present in a matching model with exogenous separations does not hold in a model with endogenous separations. Due to the presence of countercyclical selectivity in the decision whether to continue a match (the well-known “cleansing effect”), labor productivity in the endogenous-separations model responds less than one-to-one to aggregate productivity shocks. This implies that, while the change in the vacancy–unemployment ratio relative to that of aggregate productivity is exactly the same with and without endogenous separations as long as worker productivity does not change with tenure, the change of the vacancy–unemployment ratio relative to that of labor productivity is generally larger in the presence of endogenous separations.
In Section II we describe the generalized Mortensen–Pissarides model under study. Section III presents comparative static results that characterize the response of key labor-market variables to aggregate productivity shocks, while our results are further discussed in Section IV.

II. Matching Model with Endogenous Separations

Consider the following generalization of the Mortensen and Pissarides (1994) model with endogenous separations. The economy is populated by \textit{ex ante} identical risk-neutral workers of measure one and firms of a large measure. All agents discount future income flows at the common rate $r$. Let $px$ represent the output of a worker–firm employment match, where $p$ is aggregate productivity (assumed to be constant) and $x$ is match-specific idiosyncratic productivity. Assume that the initial value of idiosyncratic productivity is drawn from a distribution $G : [x_G, \bar{x}_G] \rightarrow [0, 1]$. Subsequently, let $\{x_t\}$ be a jump process characterized by arrival rate $\delta$ and a distribution of new realizations $F : [x_F, \bar{x}_F] \rightarrow [0, 1]$. This specification encompasses the original Mortensen–Pissarides model as a special case when the distribution $G$ is degenerate at $x_G = x_F$.

The flow opportunity cost of employment to the worker and the flow cost of posting a vacancy to the firm, measured in terms of output, are $b$ and $c$, respectively. The flow of new meetings is assumed to be determined by a matching function with the standard properties, denoted $m(u, v)$, where $u$ and $v$ represent the measure of unemployed workers and of vacancies, respectively. The elasticity of the matching function with respect to vacancies is constant, $\frac{\partial \ln m(u, v)}{\partial \ln v} = \eta$.

We assume that wages at each point in time in a relationship are determined according to Nash bargaining, which implies that the match surplus is shared, i.e.,

$$\beta(J(x) - V) = (1 - \beta)(W(x) - U),$$

where $J(x)$ and $W(x)$ are the value of a match in idiosyncratic productivity state $x$ to the employer and to the worker, respectively, $V$ is the value of a vacancy, $U$ is the value of unemployment, and $\beta < 1$ is the worker’s bargaining share.

Characterization of Steady State

Next, we characterize the steady-state equilibrium of the above model where the distribution of workers across idiosyncratic productivities and unemployment is constant. Note that, since agents are forward looking and all variables that determine agents’ payoff can be adjusted instantaneously, agents’ policies and asset values will be the same whether or not in a steady state.
It is only the distribution of workers across idiosyncratic productivities and unemployment that are backward looking and converge over time to their steady-state values.

The value of unemployment solves

\[ rU = b + f(\theta) \left[ \int \max (W(y), U) dG(y) - U \right], \]

where \( f(\theta) \equiv m(u, v)/u = m(1, \theta) \) is the job-contact rate of workers as a function of “market tightness” \( \theta \equiv v/u \). Note that for a general distribution function \( G \), not all meetings result in match formation, since the worker and the firm might decide that the value of initial productivity is too low to make it worthwhile to form a match. The value of employment, in turn, is the solution to

\[ rW(x) = w(x) + \delta \left[ \int \max (W(y), U) dF(y) - W(x) \right]. \]

Similarly, the value of posting a vacancy is

\[ rV = -c + q(\theta) \left[ \int \max (J(y), V) dG(y) - V \right], \]

where \( q(\theta) \equiv m(u, v)/v = f(\theta)/\theta \) is the rate at which vacancies contact workers, and the value of a filled job is

\[ rJ(x) = px - w(x) + \delta \left[ \int \max (J(y), V) dF(y) - J(x) \right]. \]

Given free entry, so that \( V = 0 \), summing up the appropriate Bellman equations implies that the match surplus, \( S(x) = J(x) + W(x) - V - U \), is characterized by

\[ rS(x) = px - b - f(\theta) \beta \int \max (S(y), 0) dG(y) \]

\[ + \delta \left[ \int \max (S(y), 0) dF(y) - S(x) \right], \]

implying that \( S'(x) = p/(r + \delta) > 0 \). Since the surplus function is increasing, the worker and the firm will choose to adopt a reservation policy, i.e., they will choose to form and continue any match that has an idiosyncratic productivity \( x \geq R \). Note that the reservation productivity at the time the match is formed is the same as the one at match dissolution even though the initial distribution of productivities is different. This is because, conditional on the realization of the initial match value, what determines a match’s payoff both initially and after subsequent productivity draws is the match’s current productivity and the option value of drawing a new productivity realization from the distribution \( F \). Also note that an equilibrium with positive employment and unemployment requires that \( G(R) < 1 \) and

$F(R) > 0$, so we assume that the parameters of the model are such that these inequalities hold. Since $S(R) = 0$,

$$
\int_{R}^{\bar{x}_H} S(y)dH(y) = \int_{R}^{\bar{x}_H} S'(y)[1 - H(y)]dy
$$

follows from integration by parts for any function $H$ such that $H(\bar{x}_H) = 1$. The free-entry condition then can be written using equation (4) and the reservation property as

$$
\frac{c}{q(\theta)} = (1 - \beta) \int_{R} S(y)dG(y) = \frac{p (1 - \beta)}{r + \delta} \int_{R} [1 - G(y)]dy. \quad (7)
$$

Evaluating equation (6) at $x = R$, in turn, and using equation (7) gives the optimal separation condition

$$
p \left( R + \frac{\delta}{r + \delta} \int_{R} [1 - F(y)]dy \right) = b + \frac{\beta c \theta}{1 - \beta}. \quad (8)
$$

It is straightforward to show that equations (7) and (8) uniquely determine the equilibrium values of market tightness, $\theta$, and reservation productivity, $R$, as long as $R$ is interior, i.e., $R \in (\bar{x}_G, \bar{x}_G) \cup (\bar{x}_F, \bar{x}_F)$ so that some matches do not get formed or are not dissolved in response to a very low realization of idiosyncratic productivity. If $R$ is not interior (so that $G(R) = 0$ and $F(R) = 1$), then while the equilibrium value of $\theta$ is uniquely defined, there could be multiple possible reservation productivities that result in identical job-finding and separation rates in the economy.

To characterize the steady-state distribution of workers, we derive the measure of workers who are employed, but have not yet experienced a shock to their initial productivity, $M_G$, the measure of workers who are employed and have experienced a shock to their initial productivity, $M_F$, and the measure of unemployed workers, $u$. This is sufficient to characterize the complete distribution of workers across idiosyncratic productivities because the distribution of $x$ among workers in $M_G$ is given by $(G(x) - G(R))/(1 - G(R))$ and that among workers in $M_F$ (assuming there are such workers, i.e., $F(R) < 1$) is given by $(F(x) - F(R))/(1 - F(R))$. The two balance equations that equate the flow into and out of $M_G$ and $M_F$, respectively, are given by

$$
\delta M_G = f(\theta)(1 - G(R)) u \quad (9)
$$

$$
\delta F(R) M_F = \delta (1 - F(R)) M_G. \quad (10)
$$

Together with the adding-up constraint, $u = 1 - M_G - M_F$, these imply that

$$
u = \frac{\delta F(R)}{f(\theta)(1 - G(R)) + \delta F(R)} \quad (11)
$$

In addition to the distribution of workers, for the results below, it is useful to characterize labor productivity, i.e., average output per employed worker in the economy, and the ratio of the opportunity cost of employment to the worker, \( b \), to this average output (the “replacement rate”, for short).

Average output among the employed can be expressed as

\[
MG = \frac{f(\theta)(1 - G(R))F(R)}{f(\theta)(1 - G(R)) + \delta F(R)}
\]

(12)

\[
MF = \frac{f(\theta)(1 - G(R))(1 - F(R))}{f(\theta)(1 - G(R)) + \delta F(R)}.
\]

(13)

when \( F(R) < 1 \) and simply as

\[
P \int_R \frac{dG(y)}{1 - G(R)}
\]

when \( F(R) = 1 \), so that

\[
\bar{p} = p \left[ R + F(R) \frac{\int_R (1 - G(y))dy}{1 - G(R)} + \int_R (1 - F(y))dy \right]
\]

(14)

holds for all possible values of \( F(R) \), where we used that

\[
\int_R \frac{dH(y)}{1 - H(R)} = R + \int_R \frac{(1 - H(y))dy}{1 - H(R)}
\]

for any function \( H \) such that \( H(\bar{x}_H) = 1 \). Using equations (7) and (8) to substitute for \( \int_R (1 - G(y))dy \) and \( \int_R (1 - F(y))dy \) and introducing the notation \( \tilde{\delta} = \delta F(R) \) and \( \tilde{f} = f(\theta)(1 - G(R)) \) for the separation and job-finding rates implied by the model gives that the replacement rate in the model is

\[
\tilde{b} = \frac{b}{\bar{p}} = \frac{(b/c)(1 - \beta)\tilde{f}\delta}{\theta (r + \delta)(\delta + \beta\tilde{f}) + (b/c)(1 - \beta)\tilde{f}\delta - r \frac{pR - b}{c} (1 - \beta) \tilde{f}}.
\]

(15)

### III. Comparative Static Results

**Response of Market Tightness**

Next, we derive comparative statics results that describe how the equilibrium market tightness implied by the model changes with aggregate productivity across steady states.
Proposition 1. The elasticity of market tightness with respect to aggregate productivity across steady states if \( F(R) = 1 \) is

\[
\frac{\partial \ln \theta}{\partial \ln p} = \varepsilon_0 \equiv \frac{r + \delta + \beta \hat{f}}{(1 - \eta)(r + \delta) + \beta \hat{f}} \frac{1}{1 - \tilde{b}}. 
\] (16)

Otherwise,

\[
\frac{\partial \ln \theta}{\partial \ln p} \preceq \varepsilon_0 \] (17)

if and only if

\[
E_G [x - R \mid x \geq R] \preceq E_F [x - R \mid x \geq R]. 
\] (18)

Proof: See the Appendix.

To appreciate the significance of these results, note that the canonical model with exogenous separations is a special case of our model, where the idiosyncratic component of productivity initially has a degenerate distribution, \( G(x) = 0 \) for all \( x < \bar{x}_G \) and \( G(\bar{x}_G) = 1 \), and is permanent until separation takes place, i.e., \( F(R) = 1 \). Hence, equation (16) holds with \( \tilde{f} = f, \tilde{\delta} = \delta \) and \( \tilde{b} = b/p \). The first part of Proposition 1 implies that the elasticity is not affected by the introduction of dispersion into the initial idiosyncratic productivity as long as \( F(R) \) remains 1, i.e., all shocks to the idiosyncratic component lead to separation, which makes separations essentially exogenous. This is due to the assumption of risk-neutral workers and firms.

Let us next consider the case when \( F(R) < 1 \) and equation (18) holds with equality, which means that the expected flow surplus conditional on forming/continuing the match is the same under the two distributions \( G \) and \( F \). One special case of this, of course, is when \( G = F \). Under this assumption, the elasticity of market tightness can be expressed as a function of the discount rate, the separation rate, the job-finding rate, the worker’s bargaining share, the replacement rate, and the elasticity of the matching function with respect to vacancies, independently of the distributions \( F \) and \( G \). Moreover, given equal expected surplus, the expression for the elasticity of market tightness, \( \varepsilon_0 \), is also observationally equivalent to the one in the canonical model with exogenous separations analyzed by Shimer (2005). To see why this is, notice that the equilibrium of our model when equation (18) is satisfied is also the equilibrium of a more general model where two types of shocks arrive, exogenous destruction shocks at rate \( \delta_0 = \delta F(R) \) and shocks to the idiosyncratic component of productivity at rate \( \delta_1 = \delta(1 - F(R)) \) from a distribution \( F_1 : [R, \bar{x}_F] \rightarrow [0, 1] \), where

\[
F_1(x) = \frac{F(x) - F(R)}{1 - F(R)}. 
\]
The proposition then states that as long as the expected surplus following productivity shocks does not change, allowing for additional dispersion in output after initial productivity is realized does not play a role in determining the elasticity of market tightness, again due to risk neutrality.

Proposition 1 further states that if the expected flow surplus is larger under the initial distribution $G$ than under the subsequent distribution $F$, then the expression from the canonical model serves as an upper bound on the elasticity of market tightness. According to Proposition 1, however, there is a version of the model that can produce more response in market tightness to aggregate productivity shocks than predicted by the model with exogenous separations. This is the case when the expected flow surplus conditional on continuing the match exceeds that at formation. Since such an increase in expected flow surplus can be interpreted as growth in firm-specific capital due to training or learning, this is a promising result. As Proposition 2 states, however, there is a limit to how different a response in market tightness the model with endogenous separations can generate.

**Proposition 2.** If $F(R) < 1$ and $E_F[x - R \mid x \geq R] > E_G[x - R \mid x \geq R]$, then the elasticity of market tightness with respect to aggregate productivity satisfies

$$\frac{\partial \ln \theta}{\partial \ln p} < \left(1 + \frac{\tilde{b}r}{r + \tilde{\delta} + \beta \tilde{f}} \left(\frac{E_F[x - R \mid x \geq R]}{E_G[x - R \mid x \geq R]} - 1\right)\right) \varepsilon_0.$$  \hfill (19)

**Proof:** See the Appendix. ■

Proposition 2 places a bound on how much higher response in market tightness one can get from a model with endogenous separations by assuming that the expected flow surplus conditional on forming/continuing a match grows over time. In particular, if the flow surplus is expected to double over time, then the elasticity given a quarterly discount rate of $r = 0.012$, separation rate of $\tilde{\delta} = 0.10$, job-finding rate of $\tilde{f} = 1.35$, worker bargaining power $\beta = 0.5$ and a replacement rate of $\tilde{b} = 0.73$ calibrated by Hall (2006), is less than 1.2% higher than in the standard model with exogenous separation. Even with a tripling of the expected flow surplus, no worker bargaining power, and a replacement rate of $\tilde{b} = 0.9$, the elasticity is less than 20% higher than in the canonical model. Thus the amplification one gets by assuming growth in the surplus over time is quantitatively limited.
Response of the Unemployment and Vacancy Rates

To the extent that one wants to characterize the response of unemployment itself to aggregate productivity shocks, it is not enough to characterize the response of market tightness, since turnover also depends on the response of the other equilibrium object, reservation productivity.

**Lemma 1.** The elasticity of reservation productivity with respect to aggregate productivity can be expressed as

\[
\frac{\partial \ln R}{\partial \ln p} = \frac{E_G \left[ \max(0, x - R) \right]}{R \left[ 1 - (1 - \eta) \frac{\partial \ln \theta}{\partial \ln p} \right]}.
\]

(20)

*Proof*: See the Appendix. ■

While the second term in this expression can be bounded using the results in Propositions 1 and 2, the first term depends on the distribution \(G\), so one cannot derive general results about its value (a result that we quantitatively demonstrate below). It is worth noting that this elasticity need not be negative, since the sign of the second term cannot be generally determined, and, technically, \(R\) can take on a negative value. The elasticity is negative, however, for empirically relevant values of the parameters determining the second term and for a positive value of \(R\).

The elasticity of the job-finding rate with respect to aggregate productivity can be expressed as

\[
\frac{\partial \ln \tilde{f}}{\partial \ln p} = \frac{\partial \ln[f(1 - G(R))]}{\partial \ln p} = \eta \frac{\partial \ln \theta}{\partial \ln p} + \frac{\partial \ln(1 - G(R))}{\partial \ln p} = \eta \frac{\partial \ln \theta}{\partial \ln p} - \frac{G'(R)R}{1 - G(R)} \frac{\partial \ln R}{\partial \ln p}.
\]

(21)

If the model generates a large negative response in the reservation productivity to aggregate productivity shocks and if the elasticity of the survival function, \(1 - G\), is large enough, then it is possible to generate a significantly larger response in the job-finding rate than would be implied simply by the change in market tightness.

Similarly, the separation rate responds to aggregate productivity shocks due to the impact of such changes on the reservation productivity. In particular,

\[
\frac{\partial \ln \delta}{\partial \ln p} = \frac{F'(R)R}{F(R)} \frac{\partial \ln R}{\partial \ln p}.
\]

(22)

so that if the reservation productivity declines in response to a positive productivity shock, it also decreases the separation rate. To the extent that the separation rate (i.e., the rate at which workers transit from employment
to unemployment) does vary countercyclically, the extension of the model to include endogenous separations can explain this pattern. In fact, this was one of the main motivations for the development of the original Mortensen–Pissarides model.

Putting together the response of the job-finding and separation rates, we get that the response of the unemployment rate, \( u = \frac{\delta}{\delta + \tilde{f}} \), is affected by both through

\[
\frac{\partial \ln u}{\partial \ln p} = \frac{\tilde{f}}{\delta + \tilde{f}} \left( \frac{\partial \ln \delta}{\partial \ln p} - \frac{\partial \ln \tilde{f}}{\partial \ln p} \right)
\]

\[
= \frac{\tilde{f}}{\delta + \tilde{f}} \left( \left( \frac{F'(R) R}{F(R)} + \frac{G'(R) R}{1 - G(R)} \right) \frac{\partial \ln R}{\partial \ln p} - \eta \frac{\partial \ln \theta}{\partial \ln p} \right). \tag{23}
\]

Clearly, the negative response of the reservation productivity can amplify the response of the unemployment rate to aggregate productivity shocks and can do so substantially, as we show in Section IV. Given the small response of market tightness, however, a large negative response of the unemployment rate to an increase in aggregate productivity is accompanied by a countercyclical response in the vacancy rate, since

\[
\frac{\partial \ln v}{\partial \ln p} = \frac{\partial \ln \theta}{\partial \ln p} + \frac{\partial \ln u}{\partial \ln p}. \tag{24}
\]

Thus, while the model extended to include endogenous separations has the potential to explain the volatility of the unemployment rate over the business cycle in response to aggregate productivity shocks, it cannot do much better than the canonical model in explaining jointly the behavior of unemployment and vacancies.

**Response of Labor Productivity**

In choosing how to confront the model with the data, there is a crucial difference between models with exogenous and endogenous separations. In particular, in models with exogenous separations, aggregate productivity (the exogenous process driving labor-market fluctuations) is equivalent to labor productivity, so it can be directly measured in the data. In endogenous separations models, however, there is no one-to-one correspondence between aggregate productivity and labor productivity, since the distribution of idiosyncratic productivity changes in response to aggregate productivity shocks. To see this, note that labor productivity (i.e., average output per employed worker) is given by the expression in equation (14), which is clearly determined jointly by the value of aggregate productivity, \( p \), and the reservation productivity, \( R \).

The divergence between aggregate productivity and labor productivity is relevant, since, when aggregate productivity shocks are considered, the appropriate measure of the change in market tightness relative to the change in labor productivity is given by

\[ \frac{\Delta_p \ln \theta}{\Delta_p \ln \bar{p}} \equiv \frac{\partial \ln \theta / \partial \ln p}{\partial \ln \bar{p} / \partial \ln p}. \] (25)

Thus, when aggregate productivity shocks are the driving force, to the extent that \( \partial \ln \bar{p} / \partial \ln p < 1 \) when \( \partial \ln R / \partial \ln p < 0 \) due to the endogenous response of the distribution of idiosyncratic productivities, \( \partial \ln \theta / \partial \ln p \) understates the model-implied change in market tightness relative to the change in labor productivity. To gauge the importance of this effect, it is useful to consider the following lemma.

**Lemma 2.** If

\[ E_G [x - R | x \geq R] = E_F [x - R | x \geq R], \] (26)

then the elasticity of labor productivity with respect to aggregate productivity is

\[
\frac{\partial \ln \bar{p}}{\partial \ln p} = 1 + (1 - \delta) \left( \frac{\hat{\delta}}{r + \hat{\delta} + \beta \hat{f}} \frac{\partial \ln \hat{f}}{\partial \ln \bar{p}} \right) \left( \frac{\partial \ln \hat{f}}{\partial \ln p} - \eta \frac{\partial \ln \theta}{\partial \ln p} \right) + \frac{r + \hat{\delta}}{r + \hat{\delta} + \beta \hat{f}} \] (27)

**Proof:** See the Appendix. □

This lemma states that, if the expected flow surplus conditional on forming/continuing the match is the same under the two distributions \( G \) and \( F \), an elasticity of labor productivity with respect to aggregate productivity significantly below 1 is possible only if there is a large response in the job-finding and separation rates to an aggregate productivity shock that comes through changes in the reservation productivity. In addition, a low worker bargaining power helps generate a smaller elasticity of labor productivity with respect to aggregate productivity for the same responses in the job-finding and separation rates.

While this lemma implies that the model with endogenous separations has more potential to generate a large enough change in market tightness relative to that in labor productivity than that with exogenous separations, the proposition below highlights a shortcoming of the model when it comes to jointly matching the change in unemployment and vacancies. © The editors of the *Scandinavian Journal of Economics* 2008.
Proposition 3. If

\[ E_G [x - R \mid x \geq R] = E_F [x - R \mid x \geq R], \]  

then the change in market tightness relative to the change in labor productivity is

\[ \frac{\Delta_p \ln \theta}{\Delta_p \ln \bar{p}} = \frac{1}{1 - b} + \frac{r + \delta}{r + \delta + \beta \bar{f}} \frac{\Delta_p \ln \bar{f}}{\Delta_p \ln \bar{p}} - \frac{\delta}{r + \delta + \beta \bar{f}} \frac{\Delta_p \ln \bar{\delta}}{\Delta_p \ln \bar{p}}. \]  

Moreover, if, in addition,

\[ \frac{\Delta_p \ln \bar{f}}{\Delta_p \ln \bar{p}} > 0, \quad \frac{\Delta_p \ln \bar{\delta}}{\Delta_p \ln \bar{p}} < 0 \quad \text{and} \quad \beta \geq \frac{r + \delta}{\bar{f} - \bar{\delta}}, \]

then\(^2\)

\[ \frac{\Delta_p \ln \bar{v}}{\Delta_p \ln \bar{p}} \leq \frac{1}{1 - b}. \]  

Proof: See the Appendix. □

Equation (29) of this proposition gives an empirically testable relationship between the changes in market tightness, the job-finding and separation rates relative to the change in labor productivity in the version of the model where the expected flow surplus conditional on forming/continuing the match is the same under the two distributions \(G\) and \(F\).

More importantly, the change in the vacancy rate relative to the change in labor productivity can be bounded above by \(1/(1 - b)\). This implies that, in order to match the volatility of the vacancy rate observed in the data, one needs a replacement rate close to 1. This finding is reminiscent of that of Hagedorn and Manovskii (2007) for the simple version of the model with exogenous separations.

IV. Discussion

How Good an Approximation is Comparative Statics?

A natural question that arises when using comparative static exercises around the non-stochastic steady state to gauge the cyclical response of

\(^2\)Empirically, the sign restrictions on the change in the job-finding and separation rates are unrestrictive. Moreover, the lower bound on \(\beta\) is also not very restrictive empirically, since, given a quarterly discount rate of \(r = 0.012\), separation rate of \(\bar{\delta} = 0.10\) and job-finding rate of \(\bar{f} = 1.355\), the value of the lower bound is 0.0061. Alternatively, if one only uses that \(\beta \geq 0\), then an upper bound on \(\Delta_p \ln \bar{v}/\Delta_p \ln \bar{p}\) is given by

\[ \frac{1}{1 - b} - \frac{u}{1 - u} \frac{\Delta_p \ln u}{\Delta_p \ln \bar{p}}. \]
a model is how good such an approximation is. It is by now well known that due to the high job-finding rate that determines the speed of adjustment of the unemployment rate and the high persistence of productivity, in the standard model with exogenous separations, comparative static results are essentially equivalent to the response of the full dynamic system; see the discussion in Mortensen and Nagypál (2007).

It is also the case that, in the model with endogenous separations, both market tightness and the reservation productivity are purely forward-looking jump variables that respond immediately to aggregate productivity shocks. Thus the dynamic response of these two endogenous variables is very similar to the comparative static results as long as the productivity process is highly persistent. Since the job-finding and separation rates are determined uniquely by market tightness and the reservation productivity, they are also jump variables whose response is well approximated by the comparative static exercises if the productivity process is persistent enough. The only endogenous variable in the model that does not instantaneously adjust is the distribution of workers across different employment and productivity states, which determines the unemployment rate and labor productivity in the economy. Just like in the model with exogenous separations, the speed of adjustment of the unemployment rate, equal to the sum of the separation and job-finding rate, is large enough in practice that it very quickly converges to its new state-contingent value. This means that the only relevant variable that is potentially slow to adjust to its new state-contingent value is labor productivity. Studying the issue of dynamic adjustment with an emphasis on the adjustment of labor productivity in a fully specified dynamic stochastic model is beyond the scope of this paper.

Some Quantitative Results

Next, we present some quantitative results. In our baseline calculations, we use parameter values used by Shimer (2005), where possible, to facilitate direct comparison of our results. Hence, aggregate productivity is normalized to $p = 1$, the quarterly discount rate is $r = 0.012$ and $\delta$, $b$ and $c$ are set so that the implied quarterly separation, job-finding and replacement rates are $\tilde{\delta} = 0.10$, $\tilde{f} = 1.355$ and $\tilde{b} = 0.40$, respectively. We set the elasticity of the matching function to $\eta = 0.28$ and the bargaining power of workers to $\beta = 1 - \eta = 0.72$. There is no obvious empirical counterpart to which the

---

3 Of course, for lower levels of persistence, aggregate productivity shocks do not result in as large a response in a dynamic system as in the comparative static exercises. Thus, in this case, the comparative static results give an upper bound on the cyclical response of these forward-looking choice variables.

4 Unlike in the model with exogenous separations, $\eta$ is no longer equivalent to the elasticity of the job-finding rate with respect to the vacancy–unemployment ratio since changes in the
Table 1. Model-implied changes in relevant labor-market variables in response to aggregate productivity shocks and relative to changes in labor productivity for the baseline case and for four variations

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Variation 1 ($b = 0.55$)</th>
<th>Variation 2 ($\beta = 0.05$)</th>
<th>Variation 3 ($E_F(lnx) = 1$)</th>
<th>Variation 4 ($\sigma_F(lnx) = 2.5$)</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\theta,p}$</td>
<td>1.72</td>
<td>2.29</td>
<td>2.02</td>
<td>1.73</td>
<td>1.76</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{f,p}$</td>
<td>0.70</td>
<td>1.18</td>
<td>1.21</td>
<td>0.82</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{s,p}$</td>
<td>-3.25</td>
<td>-11.10</td>
<td>-0.43</td>
<td>-0.68</td>
<td>-0.35</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{u,p}$</td>
<td>-3.68</td>
<td>-11.40</td>
<td>-1.52</td>
<td>-1.40</td>
<td>-1.11</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{v,p}$</td>
<td>-1.96</td>
<td>-9.15</td>
<td>0.50</td>
<td>0.32</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\theta,\overline{p}}$</td>
<td>0.81</td>
<td>0.52</td>
<td>0.62</td>
<td>0.94</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{f,\overline{p}}$</td>
<td>2.13</td>
<td>4.44</td>
<td>3.27</td>
<td>1.83</td>
<td>1.83</td>
<td>7.56</td>
</tr>
<tr>
<td>$\varepsilon_{s,\overline{p}}$</td>
<td>0.87</td>
<td>2.30</td>
<td>1.96</td>
<td>0.87</td>
<td>0.87</td>
<td>2.34</td>
</tr>
<tr>
<td>$\varepsilon_{u,\overline{p}}$</td>
<td>-4.03</td>
<td>-21.60</td>
<td>-0.69</td>
<td>-0.72</td>
<td>-0.37</td>
<td>-1.97</td>
</tr>
<tr>
<td>$\varepsilon_{v,\overline{p}}$</td>
<td>-4.56</td>
<td>-22.20</td>
<td>-2.46</td>
<td>-1.49</td>
<td>-1.15</td>
<td>-3.88</td>
</tr>
<tr>
<td>$\varepsilon_{\theta,\overline{p}}$</td>
<td>-2.43</td>
<td>-17.80</td>
<td>0.81</td>
<td>0.34</td>
<td>0.68</td>
<td>3.68</td>
</tr>
</tbody>
</table>

The distributions of idiosyncratic productivity should be matched. Given the evidence that wages are log-normally distributed, we set $G = F = \text{Log} N(0, 1)$ in our baseline calculations.

Table 1 reports the model-implied changes in relevant labor-market variables in response to aggregate productivity shocks (measured by the appropriate elasticity) and relative to changes in labor productivity (measured by the appropriate ratio of elasticities) for the baseline case and for four variations. It also reports the relevant responses to changes in labor productivity based on Table 1 of Shimer (2005). As argued in Mortensen and Nagypál (2007), the empirical equivalent to the change in $x$ relative to the changes in $y$ (denoted here by $\varepsilon_{x,y}$) in the model with exogenous separations—where adjustment of all endogenous variables takes place instantaneously or very fast—is the ordinary least squares (OLS) coefficient $\rho_{xy} \sigma_x / \sigma_y$, where $\sigma_x$ and $\rho_{xy}$ represent the standard deviation of $\ln x$ and the correlation between $\ln x$ and $\ln y$, respectively. We report the same OLS coefficients here, though in the model with endogenous separations, due to the slow adjustment of labor productivity, the OLS coefficient is no longer the exact counterpart to the calculated relative changes.

Since the distributions $G$ and $F$ are the same in the baseline case, the result of Proposition 1 applies with equality, so that the elasticity of market tightness with respect to aggregate productivity is the same as in the model.

reservation productivity also affect the job-finding rate. When considering aggregate productivity shocks, the elasticity of the job-finding rate with respect to the vacancy–unemployment ratio is larger than $\eta$ if the elasticity of the reservation productivity with respect to aggregate productivity has the expected negative sign. In particular, the elasticity of the job-finding rate with respect to the vacancy–unemployment ratio is 0.408 in the baseline calculation.
with exogenous separations studied by Shimer (2005), 1.72. Of course, the change in market tightness relative to changes in labor productivity is somewhat larger, 2.13, since labor productivity in the model does not increase with aggregate productivity one-to-one. While the model underpredicts the change in the job-finding rate relative to changes in labor productivity, it has no trouble in generating a large enough (negative) change in the separation rate relative to changes in labor productivity; in fact, in the baseline case it generates too much of a relative change in the separation rate. By underpredicting the change in the job-finding rate and overpredicting the change in the separation rate, the model generates sufficient volatility in the unemployment rate, though clearly the model misses on the share of variation in the unemployment rate due to its two margins.

A serious shortcoming of the model using the baseline parameters is that it predicts a countercyclical vacancy rate. This turns out to be a general shortcoming of the model with endogenous separations since, by increasing the volatility of unemployment for a given response of market tightness, the volatility of the vacancy rate is dampened. Given the results in Propositions 1 and 2, the only way to significantly increase the change in market tightness relative to changes in labor productivity is to decrease the response of labor productivity to aggregate productivity shocks. Variation 1 in Table 1 demonstrates one way of doing this: by increasing the replacement rate. In variation 1, we set $\tilde{b} = 0.55$. This reduces the elasticity of labor productivity with respect to aggregate productivity significantly, so that the implied change in market tightness relative to changes in labor productivity is more than twice as large as in the baseline case. Improvement along this dimension comes at a cost, though, since now the model grossly overpredicts the response of the separation rate and predicts a very strongly countercyclical vacancy rate, very much at odds with the data. Variation 2 shows that a lowering of the worker’s bargaining power to 0.05 also leads to a decrease in the elasticity of labor productivity with respect to aggregate productivity.\footnote{For lower values of $\beta$, there do not exist values of $\delta$, $b$ and $c$ that allow the model to match the targeted separation, job-finding and replacement rate.} This variation is more promising in the sense that it also explains unemployment variability more through variation in the job-finding rate than through variation in the separation rate, in line with the data. While the model predicts a procyclical vacancy rate for these parameter values, it still cannot explain the magnitude of variation in the vacancy rate, which is not surprising given Proposition 3.

So far we have maintained that the two distributions, $G$ and $F$, are the same. We know from Proposition 1 that if the expected surplus under $F$ is larger than under $G$, then the elasticity of market tightness can be increased above its baseline value, though it is clear from Proposition 2
that the size of this effect is limited. In variation 3, we assume that $G$ is kept at $\text{Log} N(0, 1)$, but the mean of $F$ is increased so that the elasticity of vacancies turns slightly positive, corresponding to $F$ being $\text{Log} N(1, 1)$. For this configuration of parameters, in equilibrium the expected surplus under $F$ is 2.38 times as large as that under $G$. While this increases $\varepsilon_{\theta, p}$ slightly, $\varepsilon_{\theta, p}$ actually declines compared to the baseline due to the increased responsiveness of labor productivity to aggregate productivity shocks. In terms of the other responses, their relative magnitude and sign is correct, but the absolute magnitude of the responses, especially that of vacancies, is still significantly smaller than their counterpart in the data. Finally, in variation 4, we assume that $G$ is kept at $\text{Log} N(0, 1)$, but the variance of $F$ is increased so that the elasticity of vacancies turns slightly positive, corresponding to $F$ being $\text{Log} N(0, 2.5)$. For this configuration of parameters, in equilibrium the expected surplus under $F$ is 10.34 times as large as that under $G$. While this increases $\varepsilon_{\theta, p}$ slightly, again $\varepsilon_{\theta, p}$ actually declines compared to the baseline. In terms of the other responses, their relative magnitude and sign is similar to that in variation 3, except that the response of the separation rate is smaller.

Our analytical results do not allow us to rule out that there exists an empirically plausible configuration of parameters and choice of the distribution functions $G$ and $F$ that allows the model to replicate the variation in the relevant variables observed in the data (although, given Proposition 3, this clearly requires $G \neq F$). It seems to us, however, that achieving this goal requires a very particular choice of the distribution functions $G$ and $F$ and that with these choices, other aspects of the model may fail to fit the data (e.g. the observed distribution of wages or the return to tenure on the job). Our analytical results on the response of market tightness to aggregate productivity shocks indicate that, in order to match the data, the model needs to predict a small response of labor productivity to aggregate productivity shocks. This in turn requires a large response in the mass of matches that are worthwhile to operate during good times but become unproductive to operate during bad times. In other words, the model needs to predict a large “cleansing effect”. Whether there is such a large cleansing effect present in the data is a question one can potentially address using microdata, but which is beyond the scope of this paper.

V. Conclusion

In this paper, we have studied via comparative static exercises the business-cycle properties of a generalization of the most widely used and popular matching model of endogenous separations: that due to Mortensen and Pissarides (1994). Allowing the separation rate to vary endogenously is empirically relevant, since variation in the separation rate accounts for about

a third of the fluctuations in the unemployment rate at business-cycle frequencies. We demonstrate a surprising result for the Mortensen–Pissarides model: that the expression derived in the model with exogenous separations for the elasticity of the vacancy–unemployment ratio with respect to aggregate productivity is exactly the same as the corresponding expression in the Mortensen–Pissarides model under the assumption that the distribution of idiosyncratic productivities does not change with tenure.

This does not mean, however, that incorporating endogenous separations cannot improve the model’s fit with the data. Most importantly, unlike in the model with exogenous separations, in the presence of endogenous separations, labor productivity generally responds less than one-to-one to aggregate productivity shocks. This increases the model-implied change in the market tightness relative to changes in labor productivity. Despite this improvement, our results indicate that, even when extended to allow for endogenous separations, the matching model has trouble reproducing all the business-cycle variation in the data. This is especially the case when it comes to explaining the variation in the vacancy rate, which, for many plausible specifications, ends up being countercyclical in the model, in stark contrast with the data.

Several authors have argued that the matching model with exogenous separations can match the empirically observed volatility of labor-market variables if the wage-setting mechanism generates a less procyclical wage than implied by the standard model; see Hall (2005), Shimer (2005) or Gertler and Trigari (2006). It is an interesting open question whether incorporating alternative wage-setting mechanisms into the version of the model with endogenous separations gives the same result. Of course, introducing plausible alternative wage-setting mechanisms into an endogenous separations model is more challenging theoretically than doing so in the presence of exogenous separations. The only paper that attempts to address this issue is by Braun (2005), who finds a limited role for wage rigidity in an estimated matching model with endogenous separations.

Appendix

Proof of Proposition 1

Totally differentiating the two equilibrium conditions (7) and (8) with respect to ln p gives

\[
\frac{\partial \ln \theta}{\partial \ln p} (1 - \eta) = \left( 1 - \frac{(1 - G(R)) R}{\int_R [1 - G(y)] dy} \frac{\partial \ln R}{\partial \ln p} \right)
\]

and

\[
R (r + \delta) + \delta \int_R [1 - F(y)] dy + (r + \delta F(R)) R \frac{\partial \ln R}{\partial \ln p} = \frac{\beta c \theta (r + \delta)}{p (1 - \beta)} \frac{\partial \ln \theta}{\partial \ln p}.
\]

(A2)

Solving these two equations for \(\partial \ln \theta / \partial \ln p\), using equations (7) and (8) to substitute for \(\int_R (1 - G(y)) dy\) and \(\int_R (1 - F(y)) dy\) and using the notation \(\bar{\delta} = \delta F(R)\) and \(\bar{\delta}' = f(\theta)(1 - G(R))\), we get

\[
\frac{\partial \ln \theta}{\partial \ln p} = \frac{r + \bar{\delta} + \beta \bar{\delta}'}{\bar{\delta} + \beta \bar{\delta}'} = \frac{r + \bar{\delta} + \beta \bar{\delta}'}{\bar{\delta} + \beta \bar{\delta}'}.
\]

(A3)

Next, equation (15) implies that

\[
(1 - \beta) \bar{b} c \theta = \bar{b} (r + \delta)(\bar{\delta} + \beta \bar{\delta}') - \bar{b} r A,
\]

(A4)

where

\[
A = \frac{pR - b}{c \theta} (1 - \beta) \bar{\delta}'.
\]

Substituting this into equation (A3) and rearranging gives

\[
\frac{\partial \ln \theta}{\partial \ln p} = \frac{r (1 - \bar{\delta}) + (\bar{\delta} + \beta \bar{\delta}') (\bar{\delta} + \beta \bar{\delta}') - \bar{b} r A}{(1 - \eta)(r + \bar{\delta}) + \beta \bar{\delta}'}.
\]

(A5)

When \(F(R) = 1\), then equilibrium condition (8) implies that \(A = \beta \bar{\delta}'\), which, after simple substitution, implies the first part of the proposition. When \(F(R) < 1\), assume that

\[
\frac{\int_R (1 - G(y)) dy}{1 - G(R)} = \frac{\int_R (1 - F(y)) dy}{1 - F(R)},
\]

(A6)

which is equivalent to assuming that equation (18) holds with equality. Using equations (7) and (8) to substitute for \(\int_R (1 - G(y)) dy\) and \(\int_R (1 - F(y)) dy\) and rearranging shows that this is equivalent to

\[
A = \frac{pR - b}{c \theta} (1 - \beta) \bar{\delta}' = \beta \bar{\delta}' - \bar{\delta} + \bar{\delta}.
\]

(A7)

Substituting for \(A\) in equation (A5) and simplifying gives

\[
\frac{\partial \ln \theta}{\partial \ln p} = \frac{r + \delta + \beta \bar{\delta}'}{(1 - \eta)(r + \bar{\delta}) + \beta \bar{\delta}'} \frac{1}{1 - \bar{\delta}}.
\]

(A8)

Following the same steps and changing the equality in equation (A6) to an inequality in either direction gives the inequality results in Proposition 1. □
Proof of Proposition 2

Let

\[
B = \frac{\int_R (1 - F(y)) dy}{1 - F(R)} > 1.
\]  

(A9)

Using equations (7) and (8) to substitute for \(\int_R (1 - G(y)) dy\) and \(\int_R (1 - F(y)) dy\) and rearranging shows that this is equivalent to

\[
A = \frac{p_R - b}{c \theta} \tilde{f} (1 - \beta) = \beta \tilde{f} - B \delta + B \tilde{\delta}.
\]

Then

\[
\frac{\partial \ln \theta}{\partial \ln p} = \frac{r (1 - \tilde{b}) \delta + (\delta + \beta \tilde{f})(\delta + \tilde{b} r) - \tilde{b} r (\beta \tilde{f} - B \delta + B \tilde{\delta})}{(1 - \eta)(r + \tilde{\delta}) + \beta \tilde{f}} \frac{1}{(1 - \tilde{b}) \delta}
\]

(A10)

where the last inequality follows from the fact that \(F(R) > 0\). ■

Proof of Lemma 1

Rearranging equation (A2) and using equation (8) to substitute for \(\int_R (1 - F(y)) dy\) implies that

\[
\frac{\partial \ln R}{\partial \ln p} = \frac{c \theta (r + \delta)}{p R (r + \tilde{\delta})} \left[ \frac{\beta}{1 - \beta} \left( \frac{\partial \ln \theta}{\partial \ln p} - 1 \right) - \frac{b}{c \theta} \right].
\]  

(A11)

Next, note that equation (A4) and the expression for \(\partial \ln \theta / \partial \ln p\) in equation (A5) can be combined to imply

\[
\frac{b}{c \theta} = \frac{\tilde{b} (\delta + \beta \tilde{f})(r + \tilde{\delta}) - \tilde{b} r A}{(1 - \tilde{b}) \delta (1 - \beta) \tilde{f}} = \frac{\partial \ln \theta}{\partial \ln p} \frac{(1 - \eta)(r + \tilde{\delta}) + \beta \tilde{f}}{(1 - \beta) \tilde{f}} - \frac{r + \tilde{\delta} + \beta \tilde{f}}{(1 - \beta) \tilde{f}}.
\]  

(A12)

Substituting into equation (A11) and simplifying gives the result stated in the lemma. ■

Proof of Lemma 2

Using the same steps that we used to arrive at equation (15), we can derive that average output in the economy can be expressed as

\[
\bar{p} = c \theta \frac{(r + \delta)(\delta + \beta \tilde{f}) + \delta \tilde{f} \frac{b}{c \theta} (1 - \beta) - r \tilde{f} (1 - \beta) \frac{pR - b}{c \theta}}{(1 - \beta) \tilde{f} \tilde{\delta}}.
\] (A13)

Then using equation (A4), the definition of \( A \) and the fact that, under the stated condition, equation (A7) holds, after simplifications we get that

\[
\bar{p} = c \theta \frac{r + \delta + \beta \tilde{f}}{(1 - \beta) \tilde{f} (1 - \tilde{\delta})}.
\] (A14)

Therefore

\[
\frac{\partial \ln \bar{p}}{\partial \ln p} = \frac{\partial \ln \theta}{\partial \ln p} + \frac{\partial \ln \delta}{\partial \ln p} + \frac{\beta \tilde{f}}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \tilde{f}}{\partial \ln p} - \frac{\partial \ln (1 - \tilde{\delta})}{\partial \ln p} = \frac{\partial \ln \theta}{\partial \ln p} + \frac{\delta}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \delta}{\partial \ln p}
\]

\[
- \frac{\partial \ln \tilde{f}}{\partial \ln p} - \frac{\partial \ln (1 - \tilde{\delta})}{\partial \ln p} = \frac{\partial \ln \theta}{\partial \ln p} + \frac{\delta}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \delta}{\partial \ln p}
\]

\[
- \frac{\partial \ln \tilde{f}}{\partial \ln p} - \frac{\partial \ln (1 - \tilde{\delta})}{\partial \ln p} = \frac{\partial \ln \theta}{\partial \ln p} + \frac{\delta}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \delta}{\partial \ln p}
\]

so that

\[
\frac{\partial \ln \bar{p}}{\partial \ln p} = (1 - \tilde{\delta}) \left( \frac{\partial \ln \theta}{\partial \ln p} + \frac{\delta}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \tilde{f}}{\partial \ln p} - \frac{r + \delta}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \tilde{f}}{\partial \ln p} \right)
\]

\[
= 1 + (1 - \tilde{\delta}) \left( \frac{\delta}{r + \delta + \beta \tilde{f}} \frac{\partial \ln \tilde{f}}{\partial \ln p} - \frac{r + \delta}{r + \delta + \beta \tilde{f}} \left( \frac{\partial \ln \theta}{\partial \ln p} - \eta \frac{\partial \ln \theta}{\partial \ln p} \right) \right),
\] (A16)

where the last equality follows by using the expression for \( \partial \ln \theta / \partial \ln p \) from equation (A8) that holds under the stated condition. □

Proof of Proposition 3

Equation (A16) implies that

\[
\frac{1}{\partial \ln \bar{p}} = 1 + (1 - \tilde{\delta}) \left( \frac{r + \delta}{r + \delta + \beta \tilde{f}} \left( \frac{\Delta_p \ln \tilde{f}}{\Delta_p \ln \bar{p}} - \eta \frac{\Delta_p \ln \theta}{\Delta_p \ln \bar{p}} \right) \right)
\]

\[
- \frac{\delta}{r + \delta + \beta \tilde{f}} \frac{\Delta_p \ln \delta}{\Delta_p \ln \bar{p}},
\] (A17)

so that, given that equation (A8) holds under the stated condition, substituting into
\[
\frac{\Delta_p \ln \theta}{\Delta_p \ln \bar{p}} = \frac{\partial \ln \theta}{\partial \ln p} = \frac{\partial \ln \bar{p}}{\partial \ln p}
\]
(A18)
and rearranging yields the first result stated in the proposition.

If
\[
\beta \geq \frac{r + \delta \delta}{\bar{f} f}
\]
then
\[
\frac{\delta}{r + \delta + \beta \bar{f}} \leq \frac{r + \delta}{r + \delta + \beta \bar{f}} \leq \frac{\bar{f}}{\delta + \bar{f}},
\]
(A19)
so that, given the stated sign restrictions on the change in the job-finding and separation rates,
\[
\frac{\Delta_p \ln \theta}{\Delta_p \ln \bar{p}} \leq \frac{1}{1 - \bar{b}} + \frac{\bar{f}}{\delta + \bar{f}} \left( \frac{\Delta_p \ln \bar{f}}{\Delta_p \ln \bar{p}} - \frac{\Delta_p \ln \delta}{\Delta_p \ln \bar{p}} \right) = \frac{1}{1 - \bar{b}} - \frac{\Delta_p \ln u}{\Delta_p \ln \bar{p}},
\]
(A20)
which implies the second result stated in the proposition. ■

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